CHAPTER 2

ANALYSIS OF PROJECTED ORTHOGONAL MATCHED FILTER RECEIVER SYSTEM

2.1 INTRODUCTION

A generic problem which has been studied extensively is that of detecting one of a set of known signals is received over an additive noise channel. When the additive noise is white and Gaussian, it is well known that the receiver maximizes the probability of correct detection. The receiver consists of an MF demodulator comprised of a bank of correlators with correlating signals equal to the transmitted set, followed by a detector which chooses as the detected signal the one for which the output of the correlator is maximum.

If the noise is not Gaussian, then the MF receiver does not necessarily maximize the probability of correct detection. However, it is still used as the receiver of choice in many applications since the optimal detector for non-Gaussian noise is typically nonlinear and depends on the noise distribution which may not be known. One justification often given for its use is that if a signal is corrupted by Gaussian or non-Gaussian additive white noise, then the filter matched to that signal maximizes the output SNR.
In this chapter, the modifications of the MF receiver have been developed by imposing inner product constraints on the measurement vectors describing the MF receiver. The vectors are constrained with a specified inner product structure and equal to the transmitted signals. The resulting receiver again consists of a bank of correlators followed by the same detector used in the MF receiver. In this situation, the LS inner product shaping, inspired by the quantum detection problem, is a versatile method with applications spanning many different areas. A solution to a previously unsolved problem in this field, for a very important special case that often arises in practice has been derived. In the ensuing chapters the focus on more subtle applications of LS inner product shaping to problems with no inherent inner product constraints.

The concept of optimal QSP measurements to derive new processing methods can be followed by considering a generic detection problem where one of a set of signals is transmitted over a noisy channel. By describing the conventional MF detector as a QSP measurement, and imposing inner product constraints on the MF measurement vectors, a new class of detectors has been derived. The demonstration shows that, when the additive noise is non-Gaussian these detectors can significantly increase the probability of correct detection over the MF receiver, with only a minor impact in performance when the noise is Gaussian.

In the development process, the focus primarily on the case in which the correlating signals are chosen to be orthonormal and orthogonal or to form a normalized tight frame, so that the outputs of the receiver are uncorrelated on a space formed by the transmitted signals. However, the ideas developed in this chapter can be readily applied to other forms of inner product constraints on the correlating signals.
2.2 LITERATURE REVIEW

The MF detector has been described with QSP parameters and imposing an inner product constraint on the measurement vectors. The proposed POMF receiver system consists of a bank of correlators with correlating signals that are matched to a set of signals with a specified inner product structure $R$ and are closest in an LS sense to the transmitted signals.

These receivers depend only on the transmitted signals, so that they do not require knowledge of the noise distribution or the channel Signal to Noise Ratio (SNR). When the transmitted signals are linearly independent this receiver is referred to as an Orthogonal Matched Filter (OMF) receiver and the transmitted signals are linearly dependent are referred to as a Projected Orthogonal Matched Filter (POMF) receiver.

The problem of detecting a transmitted signal using orthogonal matched filter detector, is matched to a set of orthogonal signals that are closest in a least square sense was discussed by Eldar and Oppenheim (2001). The problem of detecting a set of known signals in presence of additive noise, referred to as an orthogonal matched filter receiver, and when the transmitted signals are linearly dependent it is referred to as a Projected Orthogonal Matched Filter receiver was described by Eldar et al (2004).

To minimize the probability of a detection error for the least squares measurement, when distinguishing between collections of mixed quantum states. Using this condition the optimal measurement for state sets with a broad class of symmetries was discussed by Eldar et al (2004). To minimize the probability of detection error when distinguishing collections
of possibly non orthogonal mixed quantum states and this quantum states ensemble consists of linearly independent density operators (Eldar 2005). The problem of constructing measurements optimized to distinguish between collections of possibly nonorthogonal quantum states was considered. A collection of pure states seeks a positive operator-valued measure consisting of rank one operator with measurement vectors closest in squared norm to the given states dealt with Eldar and Forney (2001).

Data selection for detection of a known signal in coloured Gaussian noise was performed. The performance of the matched filter detector for a specific subset is parameterized in terms of quadratic form (Sestok 2004). The problem of designing an optimal quantum detector to minimize the probability of a detection error when distinguishing among a collection of quantum states can be formulated as a SDP by Eldar et al (2003). The problem is distinguishing among a finite collection of quantum states, when the states are not entirely known. In each state it was a collection of a known state and an unknown state dealt with Elron and Eldar (2005).

The problem of designing an optimal quantum detector distinguishes unambiguously between collections of mixed quantum states. The optimal measurement minimizes the probability using arguments of duality in vector space optimization by Eldar et al (2004). The least square tight frame was found in which the scaling of the frame is specified to least square error by Eldar and Froney (2002). A general framework for consistent linear reconstruction in infinite dimensional Hilbert spaces was introduced. The linear reconstruction scheme coincides with oblique projection, which refers to as an ordinary orthogonal projection when adapting inner product constraint by Eldar and Tobias werther (2005).
Eldar (2006) discussed with the problem of constructing an optimal set of orthogonal vectors referred to as the least squares orthogonal vectors from a given set of vectors in a real Hilbert space can be formulated as a SDP problem. Eldar and Ole Christensen (2006) proposed pseudo inverse of the frame operator can be computationally more efficient for a general frame on an arbitrary Hilbert space, shift-invariant frames with multiple generators. An analytical comparison between the matched filter detector and the orthogonal subspace projection detector for the sub pixel target detection in hyper spectral images (Bajorski Peter 2007). An alternate receiver such as Kalman Filter (KF) was analyzed to achieve an unbiased signal estimate (Hoang Nguyen 2005).

The proposed receiver system is applied with speech signal and the performance of the system can be analyzed with the probability of the signal detection and probability of error correction. The analysis shows that POMF receiver can perform better than Matched Filter, Kalman Filter and the adaptive filters like Recursive Least Square (RLS) and normalized Least Mean Square (n-LMS) filters. The probability of the signal detection and probability of error correction is approximately one and zero respectively for the POMF receiver. Since the primary applications of the POMF detector are greatly involved in the context of communication, the Bit Error Rate (BER) levels are calculated over a wide range of Signal to Noise Ratio (SNR).

2.3 POMF PARAMETERS

The QSP framework is primarily concerned with Orthogonal and Orthonormal Functions, Hilbert space and Singular Value Decomposition.
2.3.1 Orthogonal and Orthonormal Functions

Two vectors \( x \) and \( y \) in an inner product space \( V \) are orthogonal if their inner product \( \langle x, y \rangle \) is zero and denoted as \( x \perp y \). Two vector subspaces \( A \) and \( B \) of vector space \( V \) are called orthogonal subspaces if each vector in \( A \) is orthogonal to each vector in \( B \). A linear transformation \( T : V \to V \) is called an orthogonal linear transformation if it preserves the inner product. That is, for all pairs of vectors \( x \) and \( y \) in the inner product space \( V \) represented as
\[
\langle Tx, Ty \rangle = \langle x, y \rangle. \tag{2.1}
\]

This means that \( T \) preserves the angle between \( x \) and \( y \), and that the lengths of \( T_x \) and \( x \) are equal. In particular, orthonormal refers to a collection of vectors that are both orthogonal and normal (of unit length). Several vectors are called pairwise orthogonal if any two of them are orthogonal, and a set of such vectors is called an orthogonal set. An orthogonal set is an orthonormal set if all its vectors are unit vectors. The nonzero pairwise orthogonal vectors are always linearly independent.

It is common to use the inner product for two functions \( f \) and \( g \) is given by
\[
\langle f, g \rangle_\omega = \int_a^b f(x)g(x)\omega(x)dx. \tag{2.2}
\]
a nonnegative weight function \( \omega(x) \) is the definition of this inner product and those functions are orthogonal if their inner product is zero mentioned as
\[
\int_a^b f(x)g(x)\omega(x)dx = 0. \tag{2.3}
\]
the norms are written with respect to this inner product and the weight function as
\[
\|f\|_\omega = \sqrt{\langle f, f \rangle_\omega} \tag{2.4}
\]
2.3.2 Singular Value Decomposition

It is an important factorization of a rectangular real or complex matrix, with several applications in signal processing and statistics. The spectral theorem says that normal matrices can be unitarily diagonalized using a basis of eigenvectors. Suppose $M$ is an $m$-by-$n$ matrix whose entries come from the field $K$, which is either the field of real numbers or the field of complex numbers. Then there exists a factorization of the form

$$M = U \Sigma V^*$$  \hspace{1cm} (2.5)

where $U$ is an $m$-by-$m$ unitary matrix over $K$, the matrix $\Sigma$ is $m$-by-$n$ with nonnegative numbers on the diagonal and zeros off the diagonal, and $V^*$ denotes the conjugate transpose of $V$ is an $n$-by-$n$ unitary matrix over $K$. Such a factorization is called a singular value decomposition of $M$.

2.3.3 Hilbert Spaces

Underlying the development of QSP, in the signal space view point toward signal processing in which signals are regarded as vectors in an abstract Hilbert space referred to as the signal space. A complex vector space $V$ over the complex numbers $C$ is a set of elements called vectors, together with vector addition and scalar multiplication by elements of $C$ such that $V$ is closed under both operations. The inner product on the vector space $V_i$ as denoted by $\langle x, y \rangle$, is a mapping from $V$ to $C$ that satisfies

1. $\langle x, y \rangle = \langle y, x \rangle^*$. \hspace{1cm} (2.6a)
2. $\langle x, ay + bz \rangle = a \langle x, y \rangle + b \langle x, z \rangle$. \hspace{1cm} (2.6b)
3. $\langle x, x \rangle \geq 0$ and $\langle x, x \rangle = 0$ if and only if $x = 0$. \hspace{1cm} (2.6c)

where $(.)^*$ denotes the conjugate. The norm of a vector $x$ is defined by $\|x\| = \sqrt{\langle x, x \rangle}$ and the distance between $x$ and $y$ is defined by $\|x - y\|$. Any mapping satisfying the above properties is said to be valid inner product.
2.4 ADAPTIVE FILTERS

An adaptive filter is a filter that self-adjusts its transfer function according to an optimising algorithm. Most adaptive filters are digital filters that perform digital signal processing and adapt their performance based on the input signal. Generally, the adaptive process involves the use of cost function, which is a criterion for optimum performance of the filter to feed an algorithm which determines how to modify the filter coefficients to minimize the cost on the next criterion.

2.4.1 Least Mean Square Filter

Least Mean Squares (LMS) algorithms are used in adaptive filters to find the filter coefficients to produce the least mean squares of the error signal.

Figure 2.1 Problem formulation for Adaptive Filter

Figure 2.1 shows that the unknown system $h(n)$ is to be identified and the adaptive filter attempts to adapt the filter $\hat{h}(n)$ to make it as close as possible to $h(n)$, while using only observable signals $x(n), d(n)$ and $e(n)$ but $y(n), v(n)$ and $h(n)$ are not directly observable. The idea behind
LMS filters is to use the method of steepest descent to find a coefficient vector $h(n)$ which minimizes a cost function. The cost function is defined as

$$C(n) = \left\{ \left| e(n) \right|^2 \right\}$$  \hspace{1cm} (2.7)

where $e(n)$ denotes error signal and $E\{ . \}$ denotes the expected value.

### 2.4.2 RECURSIVE LEAST SQUARE FILTER

Recursive Least Squares (RLS) algorithm is used in adaptive filters to find the filter coefficients that relate to recursively producing the least squares of the error signal. The idea behind RLS filters is to minimize a cost function $C$ by appropriately selecting the filter coefficients $w_n$, updating the filter as new data arrives. The error signal $e(n)$ and desired signal $d(n)$ are defined in the Figure 2.2.

The error implicitly depends on the filter coefficients through the estimate $\hat{d}(n)$

$$e(n) = d(n) - \hat{d}(n)$$  \hspace{1cm} (2.8)
2.5 KALMAN FILTER (KF)

The Kalman filter is an efficient recursive filter that estimates the state of a dynamic system from a series of incomplete and noisy measurements. The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the squared error.

The Kalman filter model assumes the true state at time $k$ is evolved from the state at $(k - 1)$ according to

$$X_k = F_k x_{k-1} + B_k u_k + w_k$$

(2.9)

where $F_k$ is the state transition model which is applied to the previous state $x_{k-1}$, $B_k$ is the control input model which is applied to the control vector $u_k$ and $w_k$ is the process noise which is assumed to be drawn from a zero mean multivariate normal distribution with covariance $Q_k$.

2.6 RECEIVER DESIGN

Suppose that one of $m$ signals $\{s_i(t), 1 \leq i \leq m\}$ is received over an additive noise channel with equal probability, the signal lies in the real Hilbert space $H$ with inner product and span an $n$-dimensional subspace. The basic assumptions are signals which are to be normalized. The received signal $r(t)$ is also assumed to be in $H$ and is modeled as $r(t) = s_i(t) + n(t)$ where $n(t)$ is a stationary white noise process with zero mean and spectral density $\sigma$.

The receiver shown in Figure 2.3 consists of the correlation demodulator that cross-correlates the received signal $r(t)$ with $m$ signals.
\{q_i(t) \in U, 1 \leq i \leq m\}, \text{ where the signals lie in the Hilbert space } H \text{ and span a subspace } U \subseteq H. \text{ The received signals } \{q_i(t)\} \text{ are to be determined so that } a_i = \langle q_i(t), r(t) \rangle. \text{ The declared detected signal is } s_i(t) \text{ where } i = \arg\max a_i. \text{ The difference between the modified receiver and the MF receiver lies in the choice of signals } q_i(t).

For a correlation demodulator, choose the signals } q_i(t) \text{ so that the noise is non-Gaussian. The resulting detector leads to improved performance over MF detection. Drawing from the quantum detection problem, the inner product constraint on the signals is equivalent to imposing a constraint on the correlation between the demodulator outputs.

![Figure 2.3 Correlation Demodulator](image)

Building upon the results regarding optimal QSP measurements, a new class of correlation receivers was developed. The MF receiver consider as correlation receiver depend only on the transmitted signals. It leads to improved performance of the MF for some classes of non-Gaussian noise, with essentially negligible loss of performance for Gaussian noise.
The correlation between the outputs $a_i$ of the correlation demodulator is proportional to the inner products between the signals $q_i(t)$.

$$\text{cov}(a_i, a_k) = E(\langle q_i(t), n(t) \rangle \langle n(t), q_k(t) \rangle) = \sigma^2 \langle q_i(t), q_k(t) \rangle.$$ (2.10)

The modification of the MF demodulator leads to shaping the correlation of the outputs prior to detection. Thus, the signals $q_i(t)$ have a specified inner product structure, so that the outputs $a_i$ have the desired correlation. Here the signals $q_i(t)$ are preferred so that the outputs $a_i$ are uncorrelated and considered the signals $q_i(t)$ are chosen to have an arbitrary inner product structure. The subsequent development considered separately in which the transmitted signals are linearly independent or linearly dependent. If the signals $s_i(t)$ are linearly independent, decorrelate the outputs $a_i$ by choosing the signals $q_i(t)$ to be orthonormal.

The resulting demodulator is referred to as Orthogonal Matched Filter (OMF) demodulator, and the overall detector is referred to as the OMF detector. If the signals $s_i(t)$ are linearly dependent, so that they span an n-dimensional subspace $U$, then there are at most n orthonormal signals in $U$, and it cannot choose the correlating signals to whitened the outputs $a_i$ in the conventional space.

Instead of this correlating signals, projections of a set of orthonormal signals in a larger space containing $U$ and the correlating signals are chosen to form a normalized tight frame for $U$ and the outputs $a_i$ are then uncorrelated on a space formed by the transmitted signals. The resulting demodulator is the Projected Orthogonal Matched Filter demodulator or POMF detector.
2.7 OMF DEMODULATOR

The signals are considered being linearly independent \( \{s_i(t), 1 \leq i \leq m\} \), if the correlating signals are not orthogonal, then the outputs of the demodulator are correlated. To improve the performance of the detector, eliminate this common information prior to detection, so that the detection is based on uncorrelated outputs. Therefore, the correlating signals must be orthonormal and denoted by \( q_i(t) = g_i(t) \). Then the outputs of the correlation demodulator are uncorrelated and have equal variance.

\[
SNR = \frac{\langle q_i(t), s_i(t) \rangle^2}{E(\langle q_i(t), n(t) \rangle^2)} = \frac{1}{\sigma^2} \langle q_i(t), s_i(t) \rangle^2 \leq \frac{1}{\sigma^2} \langle q_i(t), q_i(t) \rangle^2 \langle S_i(t), S_i(t) \rangle^2 = \frac{1}{\sigma^2} (2.11)
\]

There are many ways of choosing a set of orthonormal signals \( g_i(t) \). In the modification of MF receiver like to choose these signals so that noise is non-Gaussian and the resulting detector leads to improved performance over MF detection. In addition, design criterion depends only on the transmitted signals so that the modified receiver does not depend on the noise distribution or the channel SNR.

The MF demodulator has the well known property if the transmitted signal is \( s_i(t) \), then choosing \( q_i(t) = s_i(t) \) maximizes the SNR of \( a_i \). The choice \( q_i(t) = s_i(t) \) of course maximizes the total SNR, since the individual terms are maximized by this choice. To derive a modification of the MF receiver, choosing a set of orthonormal signals \( g_i(t) \) to maximize the output SNR. If the signals \( g_i(t) \) to be orthonormal, then in general it cannot maximize the SNR of \( a_i \) individually subject to this constraint. This set of orthonormal signals \( g_i(t) \) have to maximize the total output SNR,

\[
SNR_T = \frac{1}{\sigma^2} \sum_{i=1}^{m} |\langle g_i(t), S_i(t) \rangle|^2
\]

(2.12)
Here a set of orthonormal quantum measurement vectors are chosen to maximize the probability of correct detection in a quantum detection problem which is equivalent to a set of orthonormal correlating signals to maximize SNR$_T$. The above equation is a quantum detection problem, and then applies results derived in that context. In particular for arbitrary signals $s_i(t)$, there is no closed form analytical expression for the orthonormal signals $g_i(t)$ maximizing SNR$_T$. Thus in the OMF demodulator, the signals $g_i(t)$ are chosen to be orthonormal, and to minimize the LS error

$$\varepsilon_{LS} = \sum_{i=1}^{m} \langle S_i(t) - g_i(t), S_i(t) - g_i(t) \rangle$$

(2.13)

This is equivalent to LS orthonormalisation problem so that the minimizing signals $\hat{g}_i(t)$, refer to as the OMF signals. The symbol $S$ and $\hat{G}$ denote the set transformation corresponding to the signals $s_i(t)$ and $\hat{g}_i(t)$ respectively. Thus, the OMF demodulator consists of a correlation demodulator with orthonormal signals $\hat{g}_i(t)$, that are closest in LS sense to the signals $s_i(t)$.  

$$\hat{G} = S(S^*S)^{-1/2}$$  

(2.14)

### 2.8 PROJECTED ORTHOGONAL MATCHED FILTER RECEIVER

Suppose the transmitted signals \{s_i, 1 \leq i \leq m\} are linearly dependent, and span an n-dimensional subspace, where n < m. As in the case of linearly independent signals, choose the signals $q_i(t)=g_i(t)$ to be orthonormal and to minimize the LS error as shown in Figure 2.4. Since $\langle g_i(t), g_i(t) \rangle = 1$ for any signals $g_i(t)$, minimizing the LS error is equivalent to maximizing
\[ \sum_{i=1}^{m} \langle g_i(t), S_i(t) \rangle = \sum_{i=1}^{m} \left\{ g_i^n(t), S_i(t) \right\} \]  

(2.15)

where the signals form a normalized tight frame for U.

![Figure 2.4 Projected Orthogonal Matched Filter Receiver](image)

For any normalized frame for U maximizing is equivalent to minimizing

\[ \sum_{i=1}^{m} \left\langle g_i^n(t) - S_i(t), g_i^n(t) - s_i(t) \right\rangle \]  

(2.16)

Thus when the signals \( s_i(t) \) are linearly dependent, choosing a set of orthogonal signals to maximize SNR is equivalent to choosing a normalized tight frame for U to minimize the LS error. Furthermore if the transmitted signal is \( s_i(t) \), then the \( i^{th} \) output of the correlation demodulator with signals \( g_i(t) \) is

\[ a_i = \langle g_i(t), r(t) \rangle = \langle g_i^n(t), S_i(t) + n(t) \rangle + \langle g_i^n(t), n(t) \rangle = r_i + n_i \]  

(2.17)

Since \( r_i \) and \( n_i \) are uncorrelated \( n_i \) does not contain any linear information that is relevant to the detection of \( s_i(t) \). Therefore in the case of linearly dependent signals \( s_i(t) \), the signals \( q_i(t) \) to be a normalized tight
frame for U denoted by \( q_i(t) = f_i(t) \), that minimizes the LS error. Thus the signals \( \{f_i(t), 1 \leq i \leq m\} \) corresponding to F is to minimize

\[
\varepsilon_{LS} = \sum_{i=1}^{m} \langle s_i(t) - f_i(t), s_i(t) - f_i(t) \rangle
\]  

(2.18)

Subject to the constraint \( FF^* = Pu \)

This problem is equivalent to the LS tight frame problem, so that the minimizing signals \( f_i(t) \) referred to as the POMF signals, follow immediately from theorem with F denoting the set transformation corresponding to the signals \( f_i(t) \).

\[
\hat{F} = S((S^* S)^{1/2})^t
\]  

(2.19)

Thus the POMF demodulator consists of a correlation demodulator with signals \( f_i(t) \) defined that form a tight frame for U, and are closest in the LS sense to the signals \( s_i(t) \). To implement the POMF demodulator with the OMF demodulator, it is more convenient to reformulate the POMF signals in terms of their coefficients is a basis expansion for the n-dimensional space U, which has been viewed as vectors in \( \mathbb{C} \).

In many practical receivers, the MF demodulator serves as a front end whose objective is to obtain a vector representation of the received signal. Thus many applications do not have control over the correlating signals of the correlation demodulator, but rather they are given by the MF outputs. In this section the OMF and POMF demodulator can be implemented by processing the MF outputs. Specifically, an equivalent implementation of the OMF and POMF demodulators was derived that consists of an MF demodulator followed by an optimal whitening
transformation on a space formed by the transmitted signals, that optimally decorrelates the outputs of the MF prior to the detection.

2.9 RESULTS AND DISCUSSION

The design of POMF receiver will be a compromise between Matched Filter (MF) receiver, adaptive filter receivers and Kalman Filter (KF) receiver. The analog input is obtained through the channel 1 at a sample rate of 8000 and duration of 1.25 seconds and number of samples obtained from the speech signal is about 10000. These samples are converted into feature vectors and Hilbert transform is performed on the matrix followed by FFT so that the spectral values of the signal can be obtained. As the concept of QSP, the spectral matrix is being converted into orthogonal matrix using Gram-Schmidt orthogonalization procedure.

Figure 2.5 Block diagram for Feature vector Generation

Figure 2.6 Input Speech Signal
In the orthogonal matrix, white noise with zero mean and unit standard deviation is added to the signal. Figure 2.5 shows the block diagram representation of feature vector generation. Analog input is a continuous speech signal given through microphone and the signal is plotted with its amplitude with respect to time shown in Figure 2.6.

In Figure 2.7, the speech signal is added with white Gaussian noise. The output of the receiver is the addition of original speech signal and white Gaussian noise shown in Figure 2.8.

The exact and approximate expressions for the probability of error with the knowledge of SNR are also obtained. The normalized LMS adaptive filter receiver can be designed for estimating the instantaneous state of linear system perturbed by white noise. The analysis suggests that over a wide range of channel parameters the POMF receiver outperform both the Matched Filter and the adaptive filters. The probability of the signal detection and probability of error correction is exactly one and zero respectively for POMF receiver.
In this section, the theoretical performance of the POMF receiver is discussed. The analysis can be extended for large system performance of the receiver assuring random Gaussian signature vectors. The analysis indicates that in many cases the POMF receiver can lead a substantial improvement in performance over the MF, Kalman and adaptive filter receivers, which motivates the use of this receiver.

Figure 2.9(a) and Figure 2.9(b) evaluates in the case with exact probability of input signal detection \( P(S_d) = 1 \) and the probability of input signal error prediction \( P(S_p) = 0 \). The corresponding curve for the MF, n-LMS, RLS and KF receivers are plotted for comparison. The POMF receiver performs similarly to the MF at various levels of SNR and better than RLS, n-LMS and KF receivers.

The comparison part is carried with n-LMS adaptive filter, where the same input is taken and the noise is appended with speech signal which is shown in Figure 2.10(a) and Figure 2.10(b).
Further Recursive Least Square adaptive receiver (RLS), Kalman Filter receiver and Matched Filter receiver can be designed for estimating the instantaneous state of linear system perturbed by white noise. The results show in Figure 2.11(a) and Figure 2.11(b) that the probability of signal detection and probability of error prediction is less than one and greater than zero respectively.
The results shown in Figure 2.12(a) and Figure 2.12(b) for Matched Filter receiver gives the probability of signal detection and probability of error Prediction is less than one and greater than zero respectively. The analysis of Kalman Filter receiver shows that probability of signal detection and probability of error Prediction is less than one and greater than zero respectively as shown in Figure 2.13(a) and Figure 2.13(b). The results are summarized in Table 2.1.
Table 2.1 Analysis of various Receivers with Probability of Error

<table>
<thead>
<tr>
<th>Probability of Error Detection</th>
<th>POMF</th>
<th>n-LMS adaptive</th>
<th>RLS adaptive</th>
<th>Matched Filter</th>
<th>Kalman Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of Error Prediction</td>
<td>1.0</td>
<td>0.65</td>
<td>0.92</td>
<td>0.94</td>
<td>0.93</td>
</tr>
<tr>
<td>Probability of Error Prediction</td>
<td>0.0</td>
<td>0.485</td>
<td>0.12</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The simulation results proved that the POMF detector outperforms than others and suggest that the POMF detector may hold considerable promise in a variety of applications. The behaviors of the detectors were simulated in Gaussian noise using random signal generations.

The Bit Error Rate (BER) levels have been obtained in the simulations is generally acceptable in a communication context. The primary applications of the POMF detector are greatly involved in the context of communication. So in these contexts, Bit Error Rate (BER) of POMF receiver is better than other receivers. The probability of bit error
rate of the POMF receiver is plotted as a function of Signal to Noise Ratio (SNR).

Figure 2.14 to Figure 2.17 evaluates the theoretical probability of Bit Error Rate for Matched Filter receiver, Kalman Filter and adaptive filter receivers like RLS, LMS filter. In Figure 2.14 Bit Error Rate for POMF receiver is compared to Matched filter receiver. For the SNR range between 4 dB to 8 dB, the POMF receiver performs better than the Matched Filter receiver. At high SNR, the performance of POMF receiver is close to that of the Matched Filter receiver. The corresponding curve for LMS, RLS and Kalman Filter are plotted for comparison. From the simulation results, the POMF receiver performs better than adaptive and Kalman Filter for SNR range between 4 dB to 8 dB. The results are summarized in Table 2.2.

From these observations, the relative improvement in performance of the POMF over the MF detector is increased with increasing SNR and predominant for large signal size. For increasing value of SNR the relative improvement in performance of POMF detector
increases. In this regime the POMF receiver outperforms both Matched Filter and adaptive filter receivers.

Table 2.2 Error rate comparison of POMF Receiver with Matched Filter, Kalman, RLS and LMS Filters

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>Filters</th>
<th>POMF</th>
<th>MF</th>
<th>LMS</th>
<th>RLS</th>
<th>Kalman</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>0.001400</td>
<td>0.046595</td>
<td>0.083871</td>
<td>0.065233</td>
<td>0.074552</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.000605</td>
<td>0.037679</td>
<td>0.067822</td>
<td>0.052751</td>
<td>0.060286</td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>0.000260</td>
<td>0.029806</td>
<td>0.053651</td>
<td>0.041729</td>
<td>0.047690</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6.46*10^{-5}</td>
<td>0.023007</td>
<td>0.041413</td>
<td>0.032210</td>
<td>0.036811</td>
<td></td>
</tr>
<tr>
<td>6.5</td>
<td>1.05*10^{-5}</td>
<td>0.017279</td>
<td>0.031103</td>
<td>0.024191</td>
<td>0.027647</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1.72*10^{-5}</td>
<td>0.012587</td>
<td>0.022657</td>
<td>0.017622</td>
<td>0.020139</td>
<td></td>
</tr>
</tbody>
</table>

2.10 CONCLUSION

In this chapter, a modified receiver has been designed referred to as POMF receiver which is based on orthogonal constraints. The Matched Filter (MF), RLS, LMS, Kalman Filter (KF) receivers are shown to be the special cases of POMF receiver. In this section several different interpretations of the POMF receiver are developed which provide further insight into its properties. The results suggest that the receiver output shows a steep probability in signal detection and a well-drop probability in the error prediction of the signal. Next a comparative analysis with Matched, Kalman Filter and adaptive filter receivers can be performed over Projected Orthogonal Matched Filter (POMF) receivers. The analysis of the POMF receiver extended to calculate the Bit Error Rate levels and the results suggest that the Bit Error Rate levels are inferior over a wide range of channel parameters than other type of receivers.