INTRODUCTION

Fuzzy subsets:

Among the various paradigmatic changes in science and mathematics, one such change concerns the concept of uncertainty. Identification of this important role of uncertainty by some researchers began the stage of transition from the traditional view to the modern view of uncertainty and such transition is characterized by the emergence of several new theories of uncertainty from probability theory.

An important point in the evolution of the modern concept of uncertainty was the publication of a seminal paper by Lotfi A. Zadeh [35]. In his paper Zadeh introduced a theory whose objects fuzzy sets are sets with boundaries that are not precise. The membership in a fuzzy set is not a matter of affirmation or denial, but rather a matter of degree. This concept is being used and is also found to be more appropriate in solving problems of all disciplines.

The concept of fuzzy sets was initiated by Zadeh in 1965 to represent / manipulate data and information possessing non-statistical uncertainties. It was specifically designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision intrinsic to many problems.

The first publication in fuzzy subset theory by Zadeh (1965 ) and then by Goguen (1967, 1969 ) show the intention of the authors to generalize the classical set. In classical set theory, a subset A of a set X can be defined by its characteristic function \( \chi_A : X \to \{0, 1\} \) is defined by \( \chi_A(x) = 0 \), if \( x \notin A \) and \( \chi_A(x) = 1 \), if \( x \in A \).
The mapping may be represented as a set of ordered pairs \{ (x, \chi_A(x)) \} with exactly one ordered pair present for each element of X. The first element of the ordered pair is an element of the set X and the second is its value in \{0, 1\}. The value "0" is used to represent non-membership and the value "1" is used to represent membership of the element A. The truth or falsity of the statement "x is in A" is determined by the ordered pair. The statement is true, if the second element of the ordered pair is "1", and the statement is false, if it is "0".

Similarly, a fuzzy subset A of a set X can be defined as a set of ordered pairs \{ (x, \mu_A(x)) : x \in X \}, each with the first element from X and the second element from the interval [0, 1] with exactly one ordered pair present for each element of X. This defines a mapping, \mu_A between elements of the set X and values in the interval [0, 1]. That is, \mu_A : X \rightarrow [0, 1].

The value "0" is used to represent complete non-membership, the value "1" is used to represent complete membership and values in between are used to represent intermediate degrees of membership. The set X is referred to as the universe of discourse for the fuzzy subset A. Frequently, the mapping \mu_A is described as a function, the membership function of A, the degree to which the statement "x is in A" is true, is determined by finding the ordered pair (x, \mu_A(x)). The degree of truth of the statement is the second element of the ordered pair.

**Bipolar valued fuzzy subsets:**

In the traditional fuzzy sets, the membership degrees of elements range over the interval [0, 1]. The membership degree expresses the degree of belongingness of elements to a fuzzy set. The membership degree 1 indicates that an element
completely belongs to its corresponding fuzzy set and the membership degree 0 indicates that an element does not belong to the fuzzy set. The membership degrees on the interval (0, 1) indicate the partial membership to the fuzzy set. Sometimes, the membership degree means the satisfaction degree of elements to some property or constraint corresponding to a fuzzy set. In the viewpoint of satisfaction degree, the membership degree 0 is assigned to elements which do not satisfy some property. The elements with membership degree 0 are usually regarded as having the same characteristics in the fuzzy set representation. By the way, among such elements, some have irrelevant characteristics to the property corresponding to a fuzzy set and the others have contrary characteristics to the property. The traditional fuzzy set representation cannot tell apart contrary elements from irrelevant elements. Consider a fuzzy set $A$ “young” defined on the age domain $[0, 100]$, whose membership degree $A(x) = (50 - x)/50$, otherwise 0. Now consider two ages 50 and 95 with membership degree 0. Although both of them do not satisfy the property “young”, we may say that age 95 is more apart from the property rather than age 50. Only with the membership degrees ranged on the interval $[0, 1]$, it is difficult to express the difference of the irrelevant elements from the contrary elements in fuzzy sets. If a set representation could express this kind of difference, it would be more informative than the traditional fuzzy set representation. Based on these observations, Lee [20, 21] introduced an extension of fuzzy sets named bipolar valued fuzzy sets. He gave two kinds of representations of the notion of bipolar valued fuzzy sets.

Fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. There are several kinds of fuzzy set extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval-valued fuzzy sets,
vague sets etc. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval \([0, 1]\) to \([-1, 1]\). Bipolar valued fuzzy sets have membership degrees that represent the degree of satisfaction to the property corresponding to a fuzzy set and its counter-property. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degrees on \((0, 1]\) indicate that elements somewhat satisfy the property, and the membership degrees on \([-1, 0)\) indicate that elements somewhat satisfy the implicit counter-property (see [20, 21]).

A bipolar valued fuzzy set \(A^* = < A^+, A^- >\) is redefined by \(A^+(x) = (50 - x)/50\) if \(0 \leq x \leq 50\), otherwise 0 and \(A^-(x) = (50 - x)/50\) if \(50 \leq x \leq 100\), otherwise 0 for the fuzzy set \(A\) “young”. The negative membership degrees indicate the satisfaction extent of elements to an implicit counter-property (e.g., old against the property young). This kind of bipolar valued fuzzy set representation enables the elements with membership degree 0 in traditional fuzzy sets, to be expressed into the elements with membership degree 0 (irrelevant elements) and the elements with negative membership degrees (contrary elements). The age elements 50 and 95, with membership degree 0 in the fuzzy set \(A\), have 0 and a negative membership degree in the bipolar valued fuzzy set \(A^*\), respectively. Now it is manifested that 50 is an irrelevant age to the property young and 95 is more apart from the property young than 50, i.e., 95 is a contrary age to the property young (see [20, 21]).

In the definition of bipolar valued fuzzy sets, there are two kinds of representations, so called canonical representation and reduced representation. We use the canonical representation of a bipolar valued fuzzy sets. Let \(X\) be the universe of discourse. A bipolar valued fuzzy set (BVFS) \(A^*\) in \(X\) is defined as an object of the
form $A^* = \{ < x, A^+(x), A^-(x) >/ x \in X \}$, where $A^+: X \rightarrow [0, 1]$ and $A^-: X \rightarrow [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element $x$ to the property corresponding to a bipolar valued fuzzy set $A^*$ and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element $x$ to some implicit counter-property corresponding to a bipolar valued fuzzy set $A^*$. If $A^+(x) \neq 0$ and $A^-(x) = 0$, it is the situation that $x$ is regarded as having only positive satisfaction for $A^*$ and if $A^+(x) = 0$ and $A^-(x) \neq 0$, it is the situation that $x$ does not satisfy the property of $A^*$, but somewhat satisfies the counter property of $A^*$. It is possible for an element $x$ to be such that $A^+(x) \neq 0$ and $A^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of $X$.

**Multi fuzzy subsets:**

Multi fuzzy set theory is an extension of theories of fuzzy sets, L-fuzzy sets and intuitionistic fuzzy sets. Multi fuzzy sets means multi dimensional membership functions. In 2010, Multi fuzzy sets was introduced by Sabu Sebastian and T. V. Ramakrishnan [29, 30]. After that Multi fuzzy sets have been developed by some authors. The multi fuzzy set $A$ in $X$ is defined as an object of the form $A = \{ < x, A_1(x), A_2(x), \ldots, A_i(x), \ldots >/ x \in X \}$, where $A_i : X \rightarrow [0, 1]$ for all $i$. Complete characterization of many real life problems can be done by multi fuzzy membership functions of the objects involved in the problem.

Rosenfeld [11] started fuzzification of various algebraic concepts by his paper Fuzzy groups. Vasantha kandasamy.W.B., well explained the fuzzy algebraic structure with applications in [33]. In [8], [9] and [10], Atanassov.K.T. has defined and
discussed the importance of intuitionistic fuzzy sets and their applications. Palaniappan.N & K.Arjunan in [24], [25], [26] have discussed the operations such as homomorphism, anti-homomorphism in fuzzy, anti-fuzzy subgroups as well as fuzzy and anti-fuzzy ideals and intuitionistic fuzzy subgroups which gave an essential foundation to study these operations and structures throughout our research work. The concept of bipolar valued fuzzy sets was introduced by Lee [20], [21]. Anitha.M.S, K.L.Muruganantha Prasad & K.Arjunan [2], [3] have defined and discussed about the Bipolar valued fuzzy subgroups of a group. In this thesis, bipolar valued multi fuzzy subsemiring of a semiring, bipolar valued multi fuzzy normal subsemiring of a semiring and bipolar valued multi fuzzy ideal of a semiring are defined and discussed.

Outline of the thesis:

In chapter I, we have introduced all the basic definitions and important results to develop the thesis.

In chapter II, we introduce the concept of bipolar valued multi fuzzy subsemirings of a semiring and their properties, operations are discussed.

In chapter III, we introduce the concept of bipolar valued multi fuzzy normal subsemirings of a semiring and their properties, operations are discussed.

In chapter IV, the basic definitions and properties of \((\alpha, \beta)\)-level subsets of bipolar valued multi fuzzy subsemirings of a semiring are discussed. Using these concepts, some results are established.

In chapter V, the bipolar valued multi fuzzy ideals of a semiring, bipolar valued multi fuzzy normal ideals of a semiring and the basic definitions and properties
of $(\alpha, \beta)$-level subsets of bipolar valued multi fuzzy ideals of a semiring are discussed.

Using these concepts, some results are established.