7.1 Case Study 1: Cylindrical and Taper Job Cutting

Apart from cylindrical job, various tapered jobs are also produced by WEDM. In WEDM machine, taper cutting is a well-known application to produce parts with complex geometry such as extrusion dies, cutting dies, mould cavities etc. During machining, the wire lag creates deviations in radius and angle of taper in machined parts. This wire lag causes dimensional inaccuracy and loss of tolerances that can lead to the rejection of high value parts or products.

From the beginning, many researchers attempted to improve the accuracy and precision of taper cutting in WEDM. Kinoshita et al. (1997) and Lin et al. (2009) used the wire offset value by measuring the deviation between actual taper profile and theoretical taper profile and this offset value was used in the part program to improve the taper angle accuracy. Plaza et al. (2009) developed a finite element method of wire to predict the actual angle of the wire and this was used to correct the taper angle during cutting in WEDM. Other researchers followed that same strategy. All the past research in this field are oriented towards either by giving the offset value in part program or by predicting the wire position by using software package or by minimizing the cutting force. But, so far no research study has been carried out focusing the influence of wire lag phenomenon on accuracy of taper job.
The required wire lag compensation value for various radii has already been established in the previous chapter. The mathematical expression is as equation (4.50). From the equation it is obvious that for a given machining condition, required wire lag compensation value is inversely proportional to the programmed radius and this value is required to be added with the wire offset value for straight profile cutting and this modified wire offset setting is used in the CNC part program to generate the exact radius of the circular profile. During taper parts cutting, there are basically two different radii present at two ends of the part. So at the two ends, two different wire lag compensation values are used in the part program to compensate the error on account of wire lag and thereby it is possible to achieve the exact taper angle. In this connection it may be pointed out that during taper cutting direction of gap force is not exactly perpendicular to wire and as such model developed in the present research study is not strictly applicable; but for small taper angle the gap force can be reasonably assumed to be perpendicular to wire.

Experimental investigation has been carried out to understand the implication of wire lag compensation for cylindrical and taper jobs. The objective was to produce two cylindrical jobs having diameters 3 mm and 6 mm; and another two taper jobs having 1 mm diameter at smaller end and taper angle are $5^0$ and $10^0$. All the four jobs were produced by WEDM using ‘without wire lag compensation strategy’ and also by using ‘wire lag compensation strategy’. Height of the job in all cases is 23.1 mm and the desired diameters of the taper job at the top and bottom surfaces are shown in the Table 7.1. In all cases the desired surface roughness was less than or equal to 2.5 μm.

To optimize the process, the strategy outlined in the present research study has been utilized. In the present case the optimization problem can be mathematically stated as follows;
Maximize $\text{MRR} = f(\text{Ton}, \text{Frequency}, I_p, \text{WT})$

Subject to $R_a \leq 2.5 \, \mu\text{m}$

$0.8 \leq \text{Ton} \leq 1.2 \, (\mu\text{s})$

$0.032 \leq \text{frequency} \leq 0.053 \, \text{(MHz)}$

$120 \leq \text{Peak current} \leq 220 \, \text{(amp)}$

$780 \leq \text{Wire Tension} \leq 1260 \, (\text{gm})$

To optimize the process for best productivity the technology guideline (i.e. Table 6.15 in chapter 6) developed in the present research study has been utilized. For the present problem it can be seen that the serial no 28 of Table 6.15 indicate the best parameter combination and corresponding surface roughness is $2.48 \mu\text{m}$ and MRR is $25.3 \text{mm}^2/\text{min}$. In this parameters combination, the radial wire lag compensation constant ($K$) is 0.071 and the wire offset is 0.155 mm. In the first case i.e. without adopting wire lag compensation strategy the value of $K$ is of no use and only wire offset value 0.155 mm is used.

In the second case, wire lag compensation strategy has been adopted and the value of $K$ is used to determine the wire lag compensation value ($\varepsilon_{wl}$) using equation no 4.50. This wire lag compensation value was then added with wire offset value and the total wire offset value was included in the part program. The results obtained by using the new compensation technique is very close to the desired diameters and the taper angle and surface finish is also within specified value as shown in Table 7.1 and Table 7.2. Table 7.3 shows errors in diameters without using wire lag compensation strategy and with wire lag compensation strategy. It is clearly observed that the proposed wire lag compensation strategy gives better result; it is thus obvious that high precision job can be produced by adopting the strategy outlined in the present research study with a very good accuracy and by avoiding using costly, time consuming trial and error approach.
### Table 7.1
Comparison of the dimension of the taper job produced using new wire lag compensation strategy and without wire lag compensation strategy

<table>
<thead>
<tr>
<th>Required Taper Angle</th>
<th>Top Diameter (mm)</th>
<th>Bottom Diameter (mm)</th>
<th>Top Angle</th>
<th>Bottom Diameter (mm)</th>
<th>Top Angle</th>
<th>Bottom Diameter (mm)</th>
<th>Surface Roughness (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5°</td>
<td>5.042</td>
<td>1</td>
<td>5.29°</td>
<td>4.916</td>
<td>0.702</td>
<td>5.03°</td>
<td>5.022</td>
</tr>
<tr>
<td>10°</td>
<td>9.146</td>
<td>1</td>
<td>10.35°</td>
<td>9.042</td>
<td>0.709</td>
<td>10.05°</td>
<td>9.128</td>
</tr>
</tbody>
</table>

### Table 7.2
Comparison of the diameter of the cylindrical job produced using wire lag compensation strategy and without wire lag compensation strategy

<table>
<thead>
<tr>
<th>Desired Diameter (mm)</th>
<th>Diameter ‘without Wire Lag Compensation Strategy’ (mm)</th>
<th>Diameter ‘using Wire Lag Compensation Strategy’ (mm)</th>
<th>Surface Roughness (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.882</td>
<td>2.99</td>
<td>2.49</td>
</tr>
<tr>
<td>6</td>
<td>5.942</td>
<td>5.992</td>
<td>2.46</td>
</tr>
</tbody>
</table>

### Table 7.3
Errors in diameters using wire lag compensation strategy and without wire lag compensation strategy

<table>
<thead>
<tr>
<th>Without Wire Lag Compensation Strategy</th>
<th>Using Wire Lag Compensation Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Top Diameter (mm)</td>
<td>Bottom Diameter (mm)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>0.126</td>
<td>0.298</td>
</tr>
<tr>
<td>0.104</td>
<td>0.291</td>
</tr>
</tbody>
</table>
7.2 Case Study 2: Precision Slit Cutting on Steel Pipe

PROBLEM:

This practical problem is related to a collaborative work with H.P. Lab Project-III in association with ESCO Engineering Pvt. Ltd. A narrow slit perpendicular to the axis of a hollow pipe has to be produced. Required widths of the slits are 290μm, 380μm and 500μm with ±10μm tolerance. The bore diameter of the pipes was 100 mm and radial wall thickness of the pipe was 7.84 mm. The required arc length of the slit was 63.5 ± 0.01 mm.

PROCEDURE ADOPTED:

The WEDM has been utilized to generate this high precision slit. But lot of difficulties was faced particularly to achieve the desired arc length (63.5mm). This was basically due to large value of the wire deflection mainly due to high value of gap between wire guide support and job surface (b). Nevertheless, the gap between wire guide support and job surface (b) and as well as job thickness (h) were also varying during machining (Fig. 7.2); which leads to varying value of wire lag with progress of machining. Under this circumstance it was extremely difficult to achieve the specified length of the slit (and indeed this necessity was the driving force behind this research study). Wire deflection for the present machining condition has been analyzed using a separate mathematical model and the technology guideline developed in the present research study has been utilized to generate the high precision slit.
Fig. 7.1 Wire position at a radial distance ‘x’ from the outer surface of the hollow pipe

The outer and inner radius of the pipe are \( r_1 \) and \( r_2 \) respectively as shown in Fig. 7.1. The deflection at the top and bottom surface of the job (i.e. where wire is leaving the job surface) is \( \delta_1 \). To simplify the analysis, deflection in between top and bottom surface of the job \( \delta_2 \) (shown in Fig. 7.2) has not been shown in Fig. 7.1 as it has a very little influence on the comparatively large dimension like job height \( (h) \) and other parameters like travel of wire \( (x) \) and gap between job surface and wire guide \( (b) \).

It is seen from Fig. 7.1 arc AD (= \( S/2 \)) makes an angle \( \theta_2 \) at centre ‘O’ and here ‘S’ is the required arc length (63.5 mm). From Fig. 7.1

\[
OC = r_1 \cos \theta_1 = r_2 \cos \theta_2
\]

(7.1)
Then putting the value of $\theta_2$ (in radian) the above expression can be rewritten as follows

$$\cos \theta_1 = \left(\frac{r_2}{r_1}\right) \cos\left(\frac{s}{2r_2}\right)$$

(7.2)

From the above equation the value $\theta_1$ can be estimated as follows

$$\theta_1 = \cos^{-1}\left\{\left(\frac{r_2}{r_1}\right) \cos\left(\frac{s}{2r_2}\right)\right\}$$

(7.3)

The vertical length $AB=h$ is half of the machining height. Therefore it is essential to determine this height $h$. From the Fig.7.1 it is obvious that

$$h = CA - CB$$

$$\Rightarrow h = r_2 \sin \theta_2 - r_1 \sin \theta_1$$

(7.4)

Now putting the value of $\theta_1$ and $\theta_2$ in the above equation;

$$h = r_2 \sin \frac{s}{2r_2} - r_1 \sin[\cos^{-1}\left\{\left(\frac{r_2}{r_1}\right) \cos\left(\frac{s}{2r_2}\right)\right\}]$$

(7.5)

By using the above expression $h$ can be estimated for a given machining condition.

To determine the value of gap between job surface and wire guide $(b)$ it is necessary to determine the value of $AE= y$. From Fig.7.1 ‘$y$’ may be expressed as follows;

$$y = r_2 \left\{1 - \sin\left(s/2r_2\right)\right\}$$

(7.6)

Thus, $b = y + c$

(7.7)

Where, ‘$c$’ is the clearance between wire guide and top and bottom most extreme surfaces of the job.
The travel of the wire from the surface of the pipe towards the axis of the pipe is \( CD = x \). From Fig.7.1 ‘\( x' \) may be expressed as follows;

\[
x = OD - OC
\]

\[
\Rightarrow x = r_2 - r_2 \cos \theta_2
\]

\[
\Rightarrow x = r_2 [1 - \cos (\frac{s}{2r_2})]
\]  \( (7.8) \)

The travel of the wire guide will be more than the travel of the wire \( (x) \) for a given arc length \( (S) \) due to wire deflection \( \delta \) as shown in Fig.7.2. To determine the travel of the wire guide towards the axis of the pipe; it is necessary to determine wire lag for this particular geometry of the job. The deformation pattern is shown in Fig.7.2. The upper half portion of the deformed wire is shown in Fig.7.3; ABCD is the actual wire position during machining. The portion AB and CD are straight line as there is no gap force and from B to C wire bends due to sparking with a radius of curvature ‘\( R' \). The gap force intensity is i.e. gap force per unit length is ‘\( q' \) is assumed to be constant. Thus, total force acting on the upper half of the wire is ‘\( qh' \). Here, \( \delta_1 \) is the wire lag at the top and bottom surface of the job and \( \delta_2 \) is the wire lag within the job and \( \theta \) is the slope of the wire with respect to vertical axis passing through two wire guides.

From Fig.7.3 it can be written as

\[
\sin \theta = \frac{\delta_1}{b}
\]

\[
\Rightarrow \theta = \frac{\delta_1}{b} \quad (as \ the \ value \ of \ \theta \ is \ small) \quad (7.9)
\]

Now considering equilibrium of the portion of the wire BC as shown in Fig.7.3 the following equation is obtained:

\[
2T \sin \frac{\theta}{2} = qh
\]  \( (7.10) \)
Where, \( T \) is the tension of the wire and noting the fact that \( \theta \) is small and also by putting the value of \( \theta \) in the above expression, it becomes:

\[
T \times \frac{\delta_1}{b} = qh 
\]  
(7.11)

Rearranging the above expression \( \delta_1 \) may be expressed as follows;

\[
\delta_1 = \frac{qhb}{T} 
\]  
(7.12)

Using the above expression one can determine \( \delta_1 \).

From Fig.7.3 it is seen that

\[
R \sin \theta = h 
\]

\[\Rightarrow R \theta = h \quad \text{(as the angle } \theta \text{ is small)}\]

\[\Rightarrow R = \frac{h}{\theta} \]  
(7.13)

Again from Fig.7.3 \( \delta_2 \) can be expressed as follows;

\[
\delta_2 = OC - OP 
\]

\[\Rightarrow \delta_2 = R(1 - \cos \theta) \]

\[\Rightarrow \delta_2 = R \frac{\theta^2}{2} \quad \text{(as for small value of } \theta , \cos \theta \approx 1 - \theta^2 / 2)\]  
(7.14)

Now substituting the value of ‘\( R \)’ and \( \theta \) from equations (7.19) and (7.13) the above expression becomes;

\[
\delta_2 = \frac{h}{2} \times \frac{\delta_1}{b} 
\]  
(7.15)

Now putting the value of \( \delta_1 \) in the above expression;

\[
\delta_2 = \frac{h^2q}{2T} 
\]  
(7.16)
Therefore the total wire lag \((\delta)\) will be as follows
\[
\delta = \delta_1 + \delta_2 = \frac{qhb}{T} + \frac{h^2q}{2T}
\] (7.17)

Now using this equation one can estimate wire lag within the hollow pipe. It is also interesting to note that the total wire deflection is independent of the value of ‘\(t\)’.

Required travel of the wire guide for the specified dimension of the slit in pipe can now be expressed as follows;
\[
x' = x + \delta = r_2 \left[1 - \cos\left(\frac{s}{2r_2}\right)\right] + \left[\frac{qhb}{T} + \frac{h^2q}{2T}\right]
\] (7.18)

In case it is not possible to achieve the desired slit width in single pass cutting then multi-pass cutting was adopted with appropriate wire offset setting. The parameter setting for this trim cutting operation is readily available in the manual supplied by the wire machine builders.

In the present case the arc length of the slit is 63.5mm and external radius \((r_2)\) and internal radius \((r_1)\) of the hollow pipe are 57.84 mm and 50 mm respectively. Thus ‘\(h\)’ may be calculated as follows;
\[
h = r_2 \sin \left(\frac{s}{2r_2}\right) - r_1 \sin\left[\cos^{-1}\left(\frac{r_2}{r_1}\right)\cos\left(\frac{s}{2r_2}\right)\right] = 19.65 \text{ mm}
\]
and gap between job surface and wire guide has been calculated using the following expression;
\[
b = y + c = r_2 \left\{1 - \sin\left(\frac{S}{2r_2}\right)\right\} + 2.37 = 30.03 \text{ mm}
\]

Thus in all cases thickness of the job being machined is \((2h=) 39.3\text{ mm}\) and gap between job surface and wire guide is 30.03 mm at the end of machining operation. In the first case slit width was 290\(\mu\)m. Under this circumstances objective is to maximize material removal rate while maintaining the kerf width less than or equal to 290\(\mu\)m. Using the
Pareto chart for height 39.3 mm, optimum parameter setting was selected as shown in Table 7.4 as follows

\[ T_{\text{on}} = 0.8 \, \mu s, \quad \text{Frequency} = 0.053 \, \text{MHz}, \quad I_p = 120 \, \text{amp}, \quad W_T = 1260 \, \text{gm}; \]

For the above parameter setting gap force ‘2qh’ has been calculated using the equation (5.26)

\[ 2qh = 19.45 \]

From the above equation the value of ‘qh’ is obtained as

\[ qh = 9.725 \]

Using this value of total gap force wire deflection was calculated using the following expression;

\[ \text{Total wire lag}, \quad \delta = \frac{qh b h}{T} + \frac{h^2 q}{2T} = 0.38 \, \text{mm} \]

Again travel of the wire (x) was calculated using the following equation;

\[ x = r_i [1 - \cos(\frac{s}{2r_i})] = 8.50 \, \text{mm} \]

Thus the wire guide was programmed to travel the following length from job surface (x');

\[ x' = x + \delta = 8.50 + 0.38 \, \text{mm} = 8.88 \, \text{mm} \]

The predicted slit/kerf width was 290 µm (Table 7.4) and the actual kerf width obtained after machining was 294 µm which is close to the predicted value and also well within tolerance limit of the slit width specified by the customer.

In other two cases, widths of the slit were 380µm and 500µm. The required slit width was achieved by subsequent trim cutting operation by controlling the wire offset parameter in trim cutting operation. It was
seen that the measured dimension of the slit length produced by the WEDM process is well within the specified range i.e. 63.5 ± 0.01mm in all cases.

Thus it is observed that the proposed methodology is very powerful and appropriate for high precision profile generation in WEDM.

Fig. 7.2 Geometry of the deflected wire during slit cutting
Fig. 7.3  Free body diagram of the upper half of the wire while machining of hollow pipe
Table 7.4  Technology chart with increasing order of MRR

<table>
<thead>
<tr>
<th>SL. No.</th>
<th>Pulse on time (µs)</th>
<th>Frequency (MHz)</th>
<th>Peak current (amp)</th>
<th>Wire Tension (gm)</th>
<th>Wire Offset (mm)</th>
<th>MRR (mm²/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8</td>
<td>0.053</td>
<td>120</td>
<td>1260</td>
<td>0.145</td>
<td>12.44</td>
</tr>
<tr>
<td>2</td>
<td>0.9</td>
<td>0.053</td>
<td>120</td>
<td>1260</td>
<td>0.146</td>
<td>13.86</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.053</td>
<td>120</td>
<td>1260</td>
<td>0.147</td>
<td>16.99</td>
</tr>
<tr>
<td>4</td>
<td>1.1</td>
<td>0.053</td>
<td>120</td>
<td>1260</td>
<td>0.148</td>
<td>19.78</td>
</tr>
<tr>
<td>5</td>
<td>1.2</td>
<td>0.053</td>
<td>120</td>
<td>1260</td>
<td>0.149</td>
<td>21.6</td>
</tr>
<tr>
<td>6</td>
<td>1.2</td>
<td>0.053</td>
<td>150</td>
<td>1260</td>
<td>0.15</td>
<td>22.84</td>
</tr>
<tr>
<td>7</td>
<td>1.2</td>
<td>0.053</td>
<td>200</td>
<td>1260</td>
<td>0.151</td>
<td>25.11</td>
</tr>
<tr>
<td>8</td>
<td>1.2</td>
<td>0.053</td>
<td>200</td>
<td>1140</td>
<td>0.152</td>
<td>25.52</td>
</tr>
<tr>
<td>9</td>
<td>1.2</td>
<td>0.053</td>
<td>200</td>
<td>1020</td>
<td>0.153</td>
<td>25.7</td>
</tr>
<tr>
<td>10</td>
<td>1.2</td>
<td>0.053</td>
<td>200</td>
<td>900</td>
<td>0.154</td>
<td>25.94</td>
</tr>
<tr>
<td>11</td>
<td>1.2</td>
<td>0.053</td>
<td>200</td>
<td>780</td>
<td>0.156</td>
<td>26.27</td>
</tr>
</tbody>
</table>