Chapter 4

ac susceptibility measurements

This chapter deals with the experimental results obtained from ac susceptibility measurements performed using a home made mutual inductance setup described in Chapter 2. Initially, the methods of analysis were established using the data obtained from sintered Bi-Sr-Ca-Cu-0 (BSCCO) sample, and were later extended to sintered and melt textured Nd-Ba-Cu-0 (NdBCO) samples.

4.1 Introduction

Characterizing a superconducting transition is commonly achieved by measuring the temperature dependence of electrical resistivity. This works very well in the case of homogeneous samples where both the critical temperature $T_c$, and the transition width $\Delta T_c$, are unambiguously defined and can be used as a check for purity. However, in practice, a superconducting transition is associated with a more complicated behavior in the case of granular samples where superconducting grains of higher critical current density ($J_c$) are linked by low $J_c$ regions like grain boundaries, impurity phases that form the intergranular region, with distinct critical parameters ($T_c, H_c, J_c$ etc.). Accordingly, chemical inhomogeneities are known to exist in the case of NdBa$_2$Cu$_3$O$_{7-\delta}$ (Nd-123) superconductor because it forms low $T_c$ solid solution phases of the type Nd$_{1+x}$Ba$_{2-x}$Cu$_3$O$_{7-\delta}$, when processed in air. Formation of these extra phases can be suppressed by processing Nd-123 in reduced oxygen partial pressure. However, there could be some minor amounts of
these phases that form locally in the Nd$_{123}$ matrix. In this case the electrical transition corresponds only to the percolation limit and is not characteristic of the whole volume of the sample. Magnetic characterization techniques offer a higher sensitivity to the individual phases present in the sample. They are also easier because of the absence of problems associated with making electrical contacts.

The magnetic response of conventional superconductors was initially studied by Maxwell and Strongin [1], Ishida and Mazaki [2], Khoder [3] and Hein [4]. ac susceptibility is measured as $\chi = \chi' ~ i ~ \chi''$, where the real part $\chi'$ is the in-phase signal and the imaginary part $\chi''$ is the out of phase signal with respect to the applied ac field. The real part reflects the screening properties expressed as a difference in the energy (associated with the magnetic field) of the sample between the nonsuperconducting and the superconducting states. The imaginary part corresponds to the ac losses that represent the amount of ac magnetic energy converted to heat in the sample.

Bean's critical state model for the magnetization of type-II superconductors, which assumes $J_c$ to be independent of local magnetic field, predicts the existence of only odd-harmonic susceptibilities. Experimental studies on fundamental and higher-harmonic susceptibility were carried out by Ishida and Mazaki [5] in a multi connected low temperature superconductor and similar studies were reported later in HTSCs by many other groups [6-18].
4.2 Temperature variation of nc susceptibility

4.2.1 Polycrystalline NdBCO sample

The temperature variation of fundamental ac susceptibility measured at different ac fields on the polycrystalline (sintered) NdBCO sample is shown in Fig. 4.1. $x'(T)$ has a broad transition, reaching the value of -1 at low temperatures when the Meissner state (complete shielding) is reached. The broadness of the transition is seen to increase with the ac field amplitude, but the dc electrical resistivity measured on the same sample shows a relatively sharp transition at 93 K (Fig. 3.3) with a transition width of 2 K. The electrical transition measures the $T_c$ to be 93 K which is the percolation limit through the high $T_c$ phase and is not characteristic of a transition throughout the whole volume of the sample. The appearance of a peak in the $\chi''$ versus temperature is a very common feature associated with the superconducting transition. The maximum occurs at a temperature $T_m$ which is lower than the critical temperature $T_c$. The full width at half maximum of the peak increases and $T_m$ decreases with increasing applied ac field. The peak in $\chi''$ occurs at the point where the ac magnetic-field just reaches to the center of the sample [19]. The reason for the occurrence of the peak is that if one first considers a temperature far below $T_c$, and $H_{ac}$ is less than the lower critical field of the intergranular region, $H_{c1i}$, the screening current generated by the alternating field is confined to a region near the sample surface, and little or no magnetic flux enters the bulk of the sample. Then $\chi''$ is small or even zero. As the temperature is allowed to increase, and both $J_c$ and $H_{c1i}$ decrease, the field begins to penetrate the sample and the intergranular ac loss ($\propto \chi''$) increases as the magnetic energy absorption increases. This process continues until the field penetrates the sample completely and reaches the center of the sample. As the temperature increases further, less and less of the sample volume remains superconducting as the magnetic flux penetrates more and more into the sample, so that less and less
Fig. 4.1 Temperature dependence of the (a) real part ($\chi'$) and (b) imaginary part ($\chi''$) of the fundamental susceptibility of the sintered NdBa$_2$Cu$_3$O$_{7-\delta}$ for different ac fields at 77 K.
Fig. 4.2. Field dependence of the ac loss peak position for sintered NdBCO sample. Solid line shows the fit-to $H_{ac} = H(o)/(1 - (T_m/T_c))$.

Fig. 4.3. Temperature variation of critical current density, calculated from the peak position of ac loss using Bean's model.
magnetic loss takes place and $\chi''$ decreases, eventually reaching zero at $T_c$. Existence of two loss peaks (Fig. 4.1(b)) in the sample is an indication of granularity of the specimen wherein the grains are coupled by weak links or Josephson-type junctions [10-18]. Here, the peak in $\chi''$ close to $T_c$ is due to the loss in the grains, and can be understood on lines similar to those discussed for the intergranular peak.

The loss peak is found to shift towards lower temperatures as the ac field strength is increased. The origin of such a shift can be understood as follows. The pinning force at temperature $T_m$ has a value such that Abrikosov vortices just reach the center of the sample. To attain the same situation at a higher ac field amplitude, strong pinning forces are required. Therefore, the maxima of $x''$ shift to lower temperatures with increasing $H_{ac}$. The field dependence of the loss peak positions allows a qualitative estimate of the pinning force density. Figure 4.2 shows the variation of the peak temperatures of $x''$ for the intergranular and intragranular regions, $T_{mi}$ and $T_{mg}$ respectively, with the ac field amplitude. The intergranular peak position is found to vary as $H_{ac} \propto (1 - T_{mi}/T_c)$. Variation in the intragranular peak position is found to follow the relation $H_{ac} \propto (1 - T_{mg}/T_c)$, which is unusual since the pinning force in the grains is proportional to $H^2_{cg}$ for Abrikosov vortices, where $H_{cg}$ is the thermodynamical critical field of the grains [20]. The expected behavior is $H \propto (1 - (T_{mg}/T_c)^2)$ for the grains [20]. By applying Bean’s critical state model, it is possible [21] to use the field dependence of loss peak temperature ($T_{mi}$ or $T_{mg}$), to determine the temperature dependence of the critical current density close to $T_c$. According to the model, the critical current is a consequence of the gradient of the flux lines that exist as the flux is driven into the superconductor. For a granular superconductor there are both intergrain $J_{ci}$, and intragrain, $J_{cg}$, components given by $H^* = J_{ci}d/2, H^*_g = J_{cg}R_g$, where $H^*$ and $H^*_g$ are the fields for which flux just reaches the center of a slab of thickness $d$ and the center of a grain of radius $R_g$, respectively.
Figure 4.3 clearly shows that $J_{c_g} \gg J_{c_i}$, as expected. $J_{c_g}$ is very high due to the strong pinning of flux within the grains. The rapid shift in the intergranular loss ($\chi''$) peak position with respect to $H_{ac}$ represents a strong temperature and field dependence of $J_{c_i}$ for the intergranular region, indicating that there are no strong pinning along the boundaries of the grains. The drastic variation in $\chi''$ grain peak position with $H_{ac}$ is unusual since grains exhibit larger pinning forces when compared to intergranular region [20]. But in this particular sample, the grain loss peak position is also significantly field dependent which might have its origin in the solid solution formation as evidenced by broad diamagnetic transition.

4.2.2 Melt textured NdBCO samples

In this section we present the results of temperature variation measurements of ac susceptibility on various multidomain bulk melt textured NdBCO samples, which were prepared as described in section 3.4.2. The sizes of the samples used for the study are 4 mm x 4 mm x 12 mm.

Figure 4.4 shows the ac susceptibility measured on various samples, viz. Nd-123 with 0, 10, 20, 30 and 40 mol% Nd-422, which are here referred to as Nd-0, Nd-10, Nd-20, Nd-30 and Nd-40 respectively. All these samples were processed in high purity Argon atmosphere and the rate of cooling through peritectic temperature ($T_p$) was 1 °C/h (see Fig. 3.6). Temperature variation of $\chi'$ shows single sharp diamagnetic transition around 92 K with a transition width of $\sim 2$ K and that of $\chi''$ shows a narrow peak for all the samples. These observations are indicative of suppression of solid solution phase formation in melt textured NdBCO samples.

The Nd-40 sample processed in commercial Argon atmosphere (referred to as Nd-40-S)
Fig. 4.4 Temperature dependence of measured fundamental susceptibility, (a) $\chi'$ and (b) $\chi''$ of various melt textured Ni23-Nd422 composites. Solid lines are guide to the eye.
shows a broad diamagnetic transition at 90 K with a transition width of 8 K and a broad \( \chi'' \) peak, indicating a distribution of \( T \)'s. This could be due to either oxygen deficiency or existence of solid solutions. Even though the samples were annealed in oxygen atmosphere for a long time, there was no change in the transition width suggesting the formation of solid solutions to be more likely. Since the samples processed in high pure Argon atmosphere showed sharp diamagnetic transition at 92 K, the purity of Argon seems to be an important parameter during processing.

**ac susceptibility measurements at different ac fields applied along the longer axis of the samples.**

Figure 4.5 shows the temperature variation of ac susceptibility measured on Nd-0 sample at various ac fields. The maximum applied ac field strength was 68 Oe. The \( \chi' \) curve is found to become slightly broader with increasing \( H_{ac} \) values, but no kink or shoulder appeared in the transition region of the curve. For all \( H_{ac} \) values, measured \( \chi'' \) exhibited only a single peak as opposed to the behavior of the sintered material (see Fig. 4.1). As \( H_{ac} \) is increased, features like an increase in the broadness of \( \chi'' \), an increase in the peak height followed by saturation, and a shift in the peak temperature towards lower temperatures are observed. The decrease of ac loss peak height with decrease of \( H_{ac} \) is attributed to reversible fluxoid motion and is discussed in section 4.3. Figures 4.6, 4.7, 4.8 and 4.9 show the measured temperature variation of the in phase and out-of-phase components of ac susceptibility on Nd-10, Nd-20, Nd-30 and Nd-40 samples at various fields respectively. Similar features are seen in these samples as well, but with relatively sharper diamagnetic transition and sharper peaks in \( \chi'' \) when compared to the stoichiometric Nd-123 (Nd-0). However, in the case of Nd-40 sample alone, a shoulder appeared at low temperature beside the \( \chi'' \) peak. The shifts in the peak positions are less dependent on the applied field. Due to strong pinning of vortices, in the melt textured samples, the peak in \( \chi'' \) appears at a higher temperature, and its position is only slightly
Fig. 4.5 Temperature dependence of measured fundamental susceptibility, (a) \( \chi' \) and (b) \( \chi'' \) of the melt textured multidomain Nd-0 superconductor for different ac fields. Solid lines are guide to the eye. Peak value of the \( \chi'' \) can be seen to be reducing with decrease of \( H_{ac} \), which is attributed to reversible fluxoid motion of a pinned vortex lattice.
Fig. 4.6 Temperature dependence of measured fundamental susceptibility, (a) $\chi'$ and (b) $\chi''$ of the melt textured multidomain Nd-10 superconductor for different ac fields. Solid lines are guide to the eye.
Fig. 4.7 Temperature dependence of measured fundamental susceptibility, (a) \( \chi' \) and (b) \( \chi'' \) of the melt textured multidomain Nd-20 superconductor for different ac fields. Solid lines are guide to the eye.
Fig. 4.8 Temperature dependence of measured fundamental susceptibility, (a) $\chi'$ and (b) $\chi''$ of the melt textured multidomain Nd-30 superconductor for different ac fields. Solid lines are guide to the eye.
Fig. 4.9 Temperature dependence of measured fundamental susceptibility, (a) $\chi'$ and (b) $\chi''$ of the melt textured multidomain Nd-40 superconductor for different ac fields. Solid lines are guide to the eye. Development of a second loss peak in $\chi''$ is due to anisotropy in pinning.
affected by the changing field amplitude due to a weak field dependence of \( J_c \). This is analogous to the behavior of the intragrain peak in the sintered samples \([22]\). The field dependence of the peak position is found to follow the empirical formula,

\[
H_{ac} = H(0)(1 - (T_m/T_c)^2)^4
\]  

(4.2.1)

and is shown in Fig. 4.10; here \( H(0) \) is a constant and is related to the pinning force density at \( T = 0 \). As \( H_{ac} \) is increased, the penetrated flux front just reaches the cylinder axis at the value \( H_{ac} = H_p \), where \( H_p = J_c R \), this situation corresponds to a maximum in the dissipative component \( \chi'' \). In the critical state models, \( J_c(H) \) is assumed to vary as \( K/H_0 \) and \( K/(H + H_0) \) by Bean and Kim respectively, where \( K \) is the pinning force density and is dependent only on the temperature. In the limit \( H \to 0, J_c(H = 0, T) = K(T)/H_0 \). This implies that the temperature dependence of \( J_c(H = 0, T) \) is just that of pinning force density \( K(T) \). Muller et al.\([22]\) reported \( K(T) \sim (1 - (T/T_c)^2) \) for YBCO sintered rods, leading to \( J_c(T) \sim (T_c - T)^2 \). For single crystals of YBCO, Wu and Sridhar \([23]\) reported \( K(T) \sim (1 - (T/T_c)^2)^2 \). The parameter \( H(0) \) is tabulated for all the samples in Table 4.1. Estimated bulk critical current density \( J_c \) at 88 K from Eqn. 4.2.1 is also tabulated. The critical current density is found to have increased as the Nd-422 content in the sample is increased up to Nd-30 sample and decreased for Nd-40 sample. If the melt textured samples were to exhibit weaklink nature we would observe two peaks in \( \chi' \) with the position of the low temperature peak being strongly affected by the external field and also two different diamagnetic transitions in \( \chi' \) as in the case of sintered samples. Absence of a second peak and an observation of a sharp diamagnetic transition indicate that melt textured NdBCO samples prepared by the above route do not have significant amount of weaklink regions inside the sample.
Fig. 4.10 Field dependence of the ac loss peak position for various melt textured samples. Solid line shows the fit to $H_{ac} = H(0) \left(1 - \frac{T_m}{T_c}\right)^2$.

Fig. 4.11 Temperature dependence of $\chi''$ when the field is applied perpendicular to the length of the sample showing two peaks. When the field was applied along the length of the sample, a major sharp peak was observed (Fig. 4.9) with a minor peak.
Table 4.1 Fit parameter $H(o)$ in Eqn. 4.2.1.

<table>
<thead>
<tr>
<th>$H(o)$</th>
<th>$J_c$ (88 K)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A/cm^2$</td>
</tr>
<tr>
<td>Nd-0</td>
<td>$1.31 \times 10^6$</td>
</tr>
<tr>
<td>Nd-10</td>
<td>$1.72 \times 10^7$</td>
</tr>
<tr>
<td>Nd-20</td>
<td>$2.39 \times 10^7$</td>
</tr>
<tr>
<td>Nd-30</td>
<td>$1.42 \times 10^8$</td>
</tr>
<tr>
<td>Nd-40</td>
<td>$1.05 \times 10^8$</td>
</tr>
</tbody>
</table>

It is seen in Fig. 4.9 that there is an additional peak (shoulder) in the $\chi''$ peak which is also weakly field dependent. In order to understand the origin of this, ac susceptibility is recorded when the field is applied perpendicular to the longer axis of the sample. Figure 4.11 shows the data that exhibits two peaks in $\chi''$. The most important aspect in the data on the imaginary components is that the boundary between the two loss peaks becomes indistinct as the ac magnetic field increases to $>15$ Oe. This phenomenon is entirely different from the case of sintered NdBCO sample, and also from the results of Shinde et al [24]. In the sintered case (Fig. 4.1), the two separate peaks always stayed apart as $H_{ac}$ increased from about 1 to 68 Oe and the distance between the maxima of the two loss peaks became larger and larger. Here (Nd-40 sample), the widths of the peaks have increased as the ac magnetic field is increased, but the maximum width remains less than 2 K when the field is increased from 0.45 Oe to 68 Oe. The peak temperature $T_m$ moves to low temperatures when ac field increases, but the total displacement is only by 1 K when the field changes between 68 to 0.45 Oe. These observations are an indication of a weak field dependence of the loss, which is nearly the same for both the peaks and is
On the contrary, for a weak-link junction, the critical current density at temperature $T$ is given by [25],

$$J_c(T) = \frac{\pi \Delta(T)}{2eR_n} \tanh\left[\frac{\Delta(T)}{2K_BT}\right]$$

(4.2.2)

where $R_n$ is the tunneling resistance per unit area of the junction in the normal state and $2\Delta(T)$ is the energy gap. For $T$ close to $T_c$, $J_c(T) \propto (1 - T/T_c)$. Since the observed field dependence is much weaker, it can be concluded that the two peaks are not associated with the loss at weaklinks.

The melt textured sample consists of several domains which do not necessarily have the same orientation [26]. As a result, ac magnetic response can be resolved as c and ab components. The above two observed peaks can be ascribed to $\chi''$loss, associated with the c and ab components. In other words, the two peaks are caused by anisotropy of pinning in the ab (basal) plane and along the c-axis. Resolution of ab and c components is observed as shown in Fig. 4.9. Better resolution of the two peaks, when $H$ is applied perpendicular to the longer axis of the sample is observed. This is analogous to the observations made by Couach et al. [27] in Y-123 single crystals, where they obtained multiple peaks when the field was applied in the ab (basal) plane and a single major peak when $H$ was parallel to c. Since the high $T_c$ superconductors are layered compounds, in a large sized multidomain sample, misorientation of domains can cause two peaks due to the anisotropy in pinning. The pinning strength along the ab plane is stronger than that parallel to c-axis. Therefore for a well aligned melt textured sample with slightly misoriented domains, the peak at higher temperature is associated with the ab component.
4.3 Reversible fluxoid motion

In the melt textured samples (Figs 4.5-4.9), the peak value of the imaginary $\chi''$ is found to decrease remarkably with decreasing ac magnetic field, accompanied by a shift of its position to higher temperatures. The former behavior contradicts the prediction of the critical state model in which the flux pinning is assumed to be completely irreversible. According to Bean’s critical state model [28], $\chi''$ in low frequency range is

$$\chi'' = \frac{4\mu_0 H_m}{3\pi J_c d}; \quad H_m \leq J_c d/2$$

(4.3.1)

$$= \frac{\mu_0 J_c d}{\pi H_m (1 - \frac{J_c d}{3H_m})}; \quad H_m > J_c d/2$$

(4.3.2)

where $d$ is the sample size. Thus $\chi''$ takes a maximum value of $3/4\pi = 0.239$, at $H_m = 2/3J_c d$. The ac loss calculated using the Bean’s model predicts that the height of the loss peak is constant. But the observed height of the ac loss peak increases with the applied ac field. A similar observation was made in conventional multifilamentary wires with very fine superconducting filaments [29]. Since all the measurements here were carried out at a low frequency of 33 Hz, the flux line lattice is stable without any creep and the deviation from the critical state model could be due to the reversible fluxoid motion inside the pinning potentials as proposed by Campbell [30]. The reversible ac penetration depth ($\lambda_c'$) is the characteristic shielding length of a pinned vortex lattice and hence takes larger values for higher temperatures and for more weakly pinned superconductor. It can be expressed in terms of $J_c$ as $\lambda_c' = (Bd_i/\mu_0 J_c)^{1/2}$ where $B$ is the magnetic flux density and $d_i$ is the interaction distance representing the radius of the averaged pinning potential. Since the theoretical calculations for fundamental harmonic susceptibilities based on Campbell’s reversible fluxoid motion can be done only numerically and are complicated, some approximate formulae have been proposed [31] as

$$\chi'_r = \frac{-\mu_0 H_p}{[1 + 3(2\lambda_c'/d)^2]H_p + H_m}$$

(4.3.3)
where \( H_p \) is the full penetration field and equals \( J_c d / 2 \mu_0 \) the case of \( A' \ll d \), the critical state model describes the phenomenon approximately. In order to discuss the temperature dependence of \( \chi' \) and \( \chi'' \), we assume the temperature dependence of \( J_c \) and \( \lambda_c' \). As the temperature approaches \( T_c \), \( J_c \) is reduced to zero and \( \lambda_c' \) diverges. Here we simply assume,

\[
J_c(T) = J_c(0)(1 - t^2)^m \quad \text{and} \quad \lambda_c'(T) = \lambda_c'(0)(1 - t^2)^{-n}
\]

(4.3.5)

where \( t = T / T_c \) is the reduced temperature and \( m \) and \( n \) are positive constants. The calculated \( \chi'_c \) and \( \chi''_c \) versus temperature at various applied fields are respectively shown in Fig. 4.12, where it has been assumed that \( J_c(0) = 8 \times 10^7 \text{ A/cm}^2 \), \( \lambda_c(0) = 0.11 \mu\text{m} \), \( d = 2.5 \mu\text{m} \), \( m = 4 \) and \( n = 0.5 \). The results shown in Fig. 4.9 agree quite well with the theoretical calculations as shown in Fig. 4.12. Figure 4.13 shows the maximum value of measured \( x''_m \), \( i.e., \chi''_m \) versus the applied field obtained from Fig. 4.9 as data points, and the solid line represents the theoretical fit to Eqn. (4.3.4). From this it can be concluded that the observed decrease in the peak value with decrease of ac field could be well described by considering reversible fluxoid motion.

For the appearance of reversible fluxoid motion, the size of the superconducting region in which the current flows uniformly should be smaller than the size of the bulk sample [31]. Microstructure of the Nd-40 sample shows domains consisting of Nd-123 platelets of average size 2-3 \( \mu\text{m} \) separated by 0.2 \( \mu\text{m} \) size gaps. At close to \( T_c \), these gaps prevent the flow of uniform transport current through the sample and the size of the superconducting specimen, in which current flows uniformly, becomes the size of platelets. The platelet size is comparable or less than the Campbell's reversible ac penetration depth \( \lambda'_c \) and hence the reduction in ac loss peak.
Fig. 4.12 Variation of $\chi'_r$ and $\chi''_r$, calculated from the Eqns. (4.3.3) and (4.3.4) for the Nd-40 sample having an average platelet size of 2.5 $\mu$m. Good agreement with the experimental data (Fig. 4.9) can be seen.
4.4 Bulk-pinning hysteresis loss

One of the crucial superconducting properties of importance for applications as power transmission cables operating at elevated temperatures is to have low ac losses at the operating conditions. The loss per cycle per unit volume $W_v$ in an ideal, pinning free, type-II superconductor depends strongly upon the frequency. In the presence of pinning, magnetic hysteresis plays a significant role, and if hysteretic losses dominate, the loss per cycle $W$ becomes independent of frequency. For $J > J_c$, the rate of dissipation per unit volume can be written as [32],

$$JE = E^2/\rho_f + J_c E$$

where $E$ is electric field, and $\rho_f$ is flux flow resistivity.

The first term on the right hand side represents the eddy-current, flux flow or viscous losses. The second term, which describes the pinning losses, gives the rate of heat generation per unit volume near the pinning centers that impede vortex motion. The hysteretic losses dominate when $E \ll \rho_f J_c$, i.e, when the electric field generated by vortex motion is at a relatively low level.

The energy dissipation $W_v$, per unit volume per cycle can be calculated as the area enclosed by the $B-H$ loop for $H(t) = H_{ac} \cos(\omega t)$.

$$W_v = \int BdH = \pi \chi'' \mu_o H_{ac}^2$$

(4.4.1)

It is important to recognize, from the orthogonality relations of the trigonometric functions, that Eqn. (4.4.1) remains unchanged inspite of higher harmonic components.

$$i.e., W_v = \pi \chi_1'' \mu_o H_{ac}^2$$
Fig. 4.13 Plot of ac loss peak height with ac field strength, obtained from the Fig. (4.9b). Solid line is a fit to Eqn. (4.3.4).

Fig. 4.14 The ac loss per unit volume per cycle of Nd-40 sample at 77 K and 33 Hz, obtained from the imaginary component. $\chi''$. Solid line is a fit to Eqn. (4.4.2).
The above equation shows that only the fundamental signal carries the energy loss information. Measuring the out-of phase component of either fundamental ($\chi''$) or flat band ($\chi''_f$) ac susceptibility would enable the determination of the bulk hysteresis losses in HTSCs.

The ac loss per unit volume per cycle of sample Nd-40, determined from the measured out-of phase component of flat band susceptibility, is shown in Fig. 4.14 as a function of ac field. The solid line in the figure shows the loss estimated by Bean’s critical state model which for small ac fields, is given by,

$$W_v = \frac{5H^3_{ac}}{6\pi J_c d}$$

(4.4.2)

It can be seen that the experimental data is well described by the above expression. Here, since the applied ac field is low, the ac loss is confined only to the surface of the sample and it increases with the ac field following the cubic law.

4.5 Universal behavior of ac susceptibility

There are many reports on the analysis of the temperature dependence of $\chi$ based on critical state models [28,33-35]. Chen et al. [36] have suggested a way to determine the intergranular $J_c$, as a function of temperature and the local field from ac susceptibility measurements. Subsequently, there are many reports on the temperature dependence of average $J_c$ obtained for a \textit{YBa}_2\textit{Cu}_3\textit{O}_7 (YBCO) sample [36-38] and for the superconducting phases of (\textit{Bi,Pb})-\textit{Sr}-\textit{Ca}-\textit{Cu}-\textit{O} (BSCCO) system [39,40]. Ishida and Goldfarb [41] reported a detailed experimental study of the field (both ac and dc) and temperature dependences of the harmonic susceptibilities of a YBCO superconductor and showed that their results agree well with the predictions of Ji et al. [42]. Kim et al. [43,44] have
reported a study of the dependence of $\chi'$ and $\chi''$ on applied ac and dc fields using a modified critical state model, assuming a critical current density $J_c(h) = H^{-2}$, where $h$ is the local field. Yamaguchi et al. [45] have analyzed the frequency and ac field dependence of $\chi''$ versus $H_{dc}$ curves measured for a $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ single crystal, on the basis of the thermally assisted flux flow model. Shatz et al. [46] derived universal expressions for the harmonic susceptibilities of type-II superconductors, using critical state models with a general dependence of the critical current on the magnetic field. The basic assumption of their analysis is that the alternating field acts as a small perturbation to the steady field, i.e. $H_{ac} \ll H_{dc}$, since the flux profile in the regions of the slab affected by the ac field is then approximately linear with a slope $S = \pm 4\pi J(H_{dc})/c$. This slope is related to a parameter $\delta = H_{ac}/(Sa)$ which is a measure of the extent of penetration of the alternating field into the slab. Here, $a$ is the half-thickness of the slab. For a given field dependence of $J_c$, one can arrive at a universal curve: $\chi_m$ versus $\delta$ (here $\delta$ absorbs the dependence of $\chi_m$ on ac, dc fields and the temperature) by choosing an appropriate expression for $\delta$.

$\delta$ is given below for various models in the linear regime:

**Bean's model**

$\delta = H_{ac}/H^*$

**Anderson - Kim model**

$\delta = 2H_{ac}H_{dc}/H_p^2$

**Powerlaw model**

$\delta = (2 - \gamma)H_{ac}h/H_p^{(2-\gamma)}$

**Exponential model**

$\delta = H_{ac}\exp(H_{dc}/H_0)/H_0(\exp(H_p/H_o))$

where $\gamma$ is a constant. $H^*$ and $H_p$ denote the full penetration fields in Bean’s model and the other field dependent models respectively. Fundamental $\chi'_m$ and $\chi''_m$ are calculated
in terms of $\delta$ as given by [46]

$$\chi'_m = -1 + \delta/2 \quad \delta < 1 \quad (4.5.1)$$

$$\cdot -((6/2 - 1)(8/\pi - 1/2) + (\delta/2 + 2/3\delta - 2/3) \cos \alpha/\pi) \quad \delta > 1 \quad (4.5.2)$$

Where $Q = \sin^{-1}(1 - 2/6), \quad \pi/2 < \alpha < 3\pi/2$

$$\chi''_m = 2\delta/3\pi \quad \delta < 1 \quad (4.5.3)$$

$$= 2(3 - 2/\delta)/(3\pi\delta) \quad \delta > 1 \quad (4.5.4)$$

The experimental intergranular matrix susceptibility is extracted from the measured susceptibility using the following equations, [36]

$$X' = f_g \chi'_g + (1 - f_g)\chi'_m \quad (4.5.5)$$

$$\chi'' = f_g \chi''_g + (1 - f_g)\chi''_m \quad (4.5.6)$$

where $f_g$ is the effective volume fraction of the grains, $\chi'_m$ and $\chi''_m$ are the components of intergranular matrix susceptibility, $\chi'_g$ and $\chi''_g$ are that of measured susceptibility. Here, it is assumed that the grain susceptibility $\chi'_g = -1$, at temperatures well below the transition temperature. Shatz et al. have demonstrated the universality of the third-harmonic susceptibilities from the measured temperature dependence of susceptibility at different dc fields, in a polycrystalline sample of YBCO.

Here, we initially determined the flux profile in the BSCCO sample at different dc fields using a phase sensitive detection method suggested by Campbell [47,48] and identified the regime where linearity is observed. Further details of flux profiles are discussed in Chapter 5. Then suitability of different models describing field dependence of $J_c$ have been examined through a fit of the field dependence of $\chi'_m$ and $\chi''_m$ to Eqs. 4.5.1-4.5.4 derived from individual models. Universality of the fundamental intergranular
matrix susceptibility obtained for sintered $Bi_{1.2}Pr_{0.3}Sr_{1.5}Ca_2Cu_{3}O_y$ superconductor is first demonstrated. Subsequently the universal behavior of the fundamental ac susceptibility measured for melt textured NdBCO (Nd-40) superconductor is examined from an analysis of the dc field and temperature dependences of the measured susceptibility, $\chi(H_{dc}, H_{ac})_T$ and $\chi(T, H_{dc})_H_{ac}$ respectively. Here the magnitude of ac field is chosen to be much less than $H_{dc}$ so that the variation of critical current density over the ac field amplitude is a constant.

4.5.1 Field dependence of ac susceptibility in BSCCO superconductor

The ac susceptibility $\chi'$ and $\chi''$ are measured as a function of dc field ($H_{dc}$) in the range of 0 to 80 Oe, which was superimposed by an ac field ($H_{ac}$) at fixed temperatures 77 K and 85 K. The plots of $\chi'$ and $\chi''$ as a function of $\log_{10}(H_{dc})$ at 77 K and 85 K are shown in Fig. 4.15 and Fig. 4.16 respectively. The position of $\chi_{\text{max}}$ shifts towards lower dc fields as the ac field amplitude is increased. For $H_{ac} \ll H_{dc}$, where the flux profile in the regions of the slab affected by the alternating field is nearly linear, $\chi''(H_{dc})$ curves show a single peak while for ac fields higher than 96 A/m an anomalous behavior (a dip) is observed. $\chi'$ is found to show a dip at $H_{ac} = H_{dc}$, which is attributed to an increase of average $J_c$ of the sample due to the effective field being zero during a short time. To determine the experimental intergranular matrix susceptibility $\chi_{m}(H_{dc}, H_{ac})_T$ from Eqns. 4.5.5 and 4.5.6, we need to know the $f_g$ value. Firstly, the components $\chi'_m$ and $\chi''_m$ of experimental intergranular matrix susceptibility are extracted from the measured susceptibility using equations 4.5.5 and 4.5.6, and by choosing appropriate $f_g$ that satisfies the equations, as described in Ref. 39. Then, $\chi_m$ is calculated from the values of $\chi'_m$ and $\chi''_m$. Figure 4.17 shows the experimental $\chi'_m$ and $\chi''_m$(discrete points) along with the theoretical matrix susceptibility derived from the Kim's $^{48}$ power - law and the exponential models as a function of the field. From the figure it can be seen that the
Fig. 4.15 The dc field dependence of measured fundamental ac susceptibility, (a) $\chi'$, and (b) $\chi''$ of the $\text{Bi}_{1.2}\text{Pb}_{0.3}\text{Sr}_{1.5}\text{Ca}_{2}\text{Cu}_{3}\text{O}_{y}$ (BSCCO) slab for different ac fields ($H_{ac}$) at 77 K.
Fig. 4.16 The dc field dependence of measured fundamental ac susceptibility, (a) $\chi'$, and (b) $\chi''$ of the BSCCO slab for two different ac fields ($H_{ac}$) at 85 K.
Fig. 4.17 Field dependence of (a) real and (b) imaginary components of intergranular matrix susceptibility \((x'_m, x''_m)\) at 77 K determined from the measured susceptibility (Fig. 4.15) is shown as discrete points. The theoretical fits to different models are shown as solid and dashed lines.
power-law with $7 = -0.4$ and $H_p = 000 \text{ A/m}$ gives the best fit. Sharpness of $\chi''_m$ peak increases as one goes from Kim's model to exponential model. $\delta(H_{dc}, H_{ac})_T$ values are determined from the experimental $\chi_m$, using equations 4.5.1-4.5.4 and are shown in Fig. 4.18 as discrete points. Field dependence of $\delta$ is found to be described well by

$$\delta = (2 - \gamma)H_{dc}^{(1-\gamma)}H_{ac}/H_{p}^{(2-\gamma)}$$  \hspace{1cm} (4.5.7)

derived based on the power law model with $7 = -0.4$, for small ac fields. The full penetration fields $(H_p)$ at 77 K and 85 K are obtained as 600 A/m and 309 A/m. Theoretical $\delta(H_{dc}, H_{ac})_T$ are seen to deviate from the experimental data at higher ac fields, since the Eqs. 4.5.1-4.5.4 are valid only in the regime of linear flux profile in the slab. As a result, at higher ac fields the experimental $\chi_m$ cannot be analyzed in terms of the universal model of Shatz et al..

The universal curve is then obtained by plotting the measured $\chi_m(H_{dc}, H_{ac})_T$ as a function of $\delta$ (determined from the fit parameters using Eq. (4.5.7)) and is shown in Fig. 4.19, as the four curves with $T= 77 \text{ K and } 85 \text{ K}$.

4.5.2 Temperature dependence of ac susceptibility in BSCCO superconductor

The temperature dependence of ac susceptibility $\chi(T, H_{dc}, H_{ac})$ was measured in the temperature range from 77 to 120 K. A small ac field at 33 Hz and a high dc field were chosen to be applied, such that $H_{ac} \ll H_{dc}$ for the flux profile in the slab to be linear. $\chi'(T)$, and $\chi''(T)$ measured at different dc fields and a constant ac field of 17 A/m are shown in Figs. 4.20(a) and 4.20(b). The experimental $\chi_m(T, H_{dc}, H_{ac})$ is determined from Eqs. 4.5.5 and 4.5.6 and $\delta(T, H_{dc}, H_{ac})$ is then determined using equations 4.5.1-4.5.4. Figure 4.21 shows the T dependence of $\delta$ at different $H_{dc}$ as discrete points. Power law model
Fig. 4.18 Field dependence of $\delta$ extracted from $\chi_m(H_{dc})$ data obtained at 77 K and 85 K for various ac fields (discrete points). Theoretical fits of the $S$ to power-law model (Eqn. 4.5.7) are shown as lines. Inset shows deviation of fit from the data at higher ac fields.
Fig. 4.19 A number of curves plotted for experimental $\chi_m$ as a function of $\delta$, using both the measured field and temperature dependences of $\chi_m$, are shown to demonstrate universality of fundamental susceptibility in BSCCO sample.
is chosen to define the relationship between \( J_c \) and \( H_{dc} \). The variation of \( H_p \) with \( T \) is assumed \([49,50]\) to be

\[
H_p(T) = \frac{\beta}{(1 - \left(\frac{T}{T_c}\right)^2)^{\alpha} \left[1 - (T/T_i)^2\right]}\]

where \( \beta \) is a constant. Strong dependence of \( \delta \) on \( T \) suggests that \( \delta(T) \) can be expressed as

\[
\delta(T) = \frac{P_o}{\left[1 - (T/T_c)^2\right]\left[1 - (T/T_i)^2\right]^n}\]

(4.5.8)

Here, the \([1 - (T/T_c)^2]\) term arises from the temperature dependence of pinning force, which is field independent, and varies slowly with temperature. The second term is derived from an assumed \([49]\) temperature dependence of critical current, and \( T_i \) is the irreversibility temperature. Our data is found to fit well for \( n = 1 \). The values of the best fit for \( P_o, T_i, \) and \( T_c \) from the curves obtained for different \( H_{dc} \) are shown in Table 4.2 and the corresponding theoretical curves of \( S(T) \) calculated from Eqn. 4.5.8 are shown in Fig. 4.21 as solid lines. The universal curve is then obtained by plotting the measured \( \chi_m(T, H_{dc})H_{ac} \) against the \( \delta(T) \) determined from the fit parameters and is shown in Fig. 4.19 as the curves for different \( H_{dc} \).

Table 4.2 Fit parameters in Eqn. 4.5.8.

<table>
<thead>
<tr>
<th>( H_{dc} ) A/m</th>
<th>( P_0 )</th>
<th>( T_i ) K</th>
<th>( T_c ) K</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>0.021</td>
<td>100.86</td>
<td>103.4</td>
</tr>
<tr>
<td>240</td>
<td>0.023</td>
<td>99.87</td>
<td>103.2</td>
</tr>
<tr>
<td>358</td>
<td>0.024</td>
<td>97.48</td>
<td>102.9</td>
</tr>
<tr>
<td>477</td>
<td>0.030</td>
<td>97.29</td>
<td>102.8</td>
</tr>
<tr>
<td>716</td>
<td>0.044</td>
<td>96.08</td>
<td>102.3</td>
</tr>
<tr>
<td>962</td>
<td>0.046</td>
<td>94.71</td>
<td>101.6</td>
</tr>
<tr>
<td>1320</td>
<td>0.046</td>
<td>92.96</td>
<td>101.4</td>
</tr>
<tr>
<td>1782</td>
<td>0.053</td>
<td>91.78</td>
<td>101.0</td>
</tr>
<tr>
<td>2410</td>
<td>0.059</td>
<td>91.01</td>
<td>100.7</td>
</tr>
</tbody>
</table>
Fig. 4.20 Temperature dependence of measured fundamental susceptibility, (a) $\chi'$ and (b) $\chi''$ of the BSCCO slab for different dc fields ($H_{dc}$), with $H_{ac} = 17$ A/m.
Fig. 4.21 Temperature dependence of $\delta(T, H_{dc})_{H_{ac}}$ extracted from $\chi_m(T)$ data, derived using measured susceptibility shown in Fig. 4.20 for various dc fields, is shown as discrete points. Theoretical fits of the $\delta$ to Eq. (4.5.8) are shown as lines.
4.5.3 Field variation of \( \chi \) susceptibility in melt textured NdBCO superconductor

The fundamental \( \chi' \), and \( \chi'' \) are measured on Nd-40 melt textured sample as a function of dc field \( (H_{dc}) \) in the range 0 to as large as 10 KOe at fixed temperature 77 K and are shown as discrete points in Fig. 4.22. Even though the applied maximum dc field is 10 KOe, we could not see the complete ac loss peak \( (x'') \) in the sample, since the flux has not yet penetrated the sample completely. The grain fraction in the melt textured sample can be considered as 1, since weaklinks are suppressed in the sample. The theoretical susceptibility calculated from the Kim's critical state model, is shown as solid lines and is found to agree well with the experimental data. \( \delta(H_{dc}, H_{ac}) \) values are determined from the experimental \( \chi \), and are shown in Fig. 4.23 as discrete points. Field dependence of \( \delta \) is found to be described well by,

\[
\delta = 2H_{dc}H_{ac}/H_p^2
\]

(4.5.9)
derived based on Kim's model for small ac fields superimposed on large dc fields. The full penetration field \( H_p \) is obtained as \( 2.2 \times 10^5 \) A/m.

The universal curve is then obtained by plotting the measured \( \chi(H_{dc}, H_{ac}) \) as a function of \( \delta \) (determined from the fit parameter using the above equation) and is shown in Fig. 4.24, as the four curves with different ac fields.

4.5.4 Temperature variation of \( \chi \) susceptibility in melt textured NdBCO superconductor

The temperature dependence of \( \chi(T, H_{dc}) \) was measured in the temperature range from 77 to 100 K. A small ac field at 33 Hz and a high dc field were chosen to be applied such that \( H_{ac} << H_{dc} \) for the flux profile in the slab to be linear. \( \chi'(T) \),
Fig. 4.22 The dc field dependence of measured fundamental susceptibility, (a) $\chi'$, and (b) $\chi''$ of the melt textured Nd-40 slab for different ac fields ($H_{ac}$) at 77 K. Data points are experimental data and the solid lines are fits to Kim's critical state model.
Fig. 4.23 Field dependence of $\delta$ extracted from $\chi(H_d)$ data shown in Fig. 4.22 obtained at 77 K for various ac fields (discrete points). Theoretical fits of the $b$ to Kim's model (Eq. (4.5.9)) are shown as lines.
Fig. 4.24 A number of curves plotted for experimental $\chi$ as a function of $\delta$, using both the measured field and temperature dependences of $\chi$, are shown to demonstrate universality of fundamental susceptibility in melt textured NdBCO sample.
and \( \chi''(T) \) measured at different dc fields and a constant ac field of 2 Oe are shown in Fig. 4.25. \( b(T, H_{dc}) \) is then determined using the Eqs. 4.5.1-4.5.4. Figure 4.20 shows the T dependence of \( f_i \) at different \( H_{dc} \) as discrete points. Kim’s model is chosen to define the relationship between \( J_c \) and \( H_{dc} \). The variation of \( H_p \) with \( T \) is assumed to be 

\[
H_p(T) = H(0)((1 - (T/T_c)^2)(1 - (T/T_i)^2)^4).
\]

Strong field dependence of \( \delta \) on \( T \) suggests that \( b(T) \) can be expressed as

\[
\delta(T) = P I((\chi - (T/T_c)^2)(1 - (T/T_i)^2)^4)
\]

Using the values of the best fit for \( P_o \) and \( T_i \), the corresponding theoretical curves of \( S(T) \) calculated from the above Eqn. 4.5.10 are shown as solid lines. The universal curve is then obtained by plotting the measured \( \chi(T,H_{dc})H_{ac} \) against the \( \delta(T) \) determined from the fit parameters and is shown in Fig. 4.24 as the curves for different \( H_{dc} \).

Thus the universal behaviour of ac susceptibility could be demonstrated well in sintered BSCCO as well as melt textured NdBCO samples, which are widely different in their properties. Since the magnitude of bulk critical current density is different by two orders of magnitude, correspondingly the ac susceptibility measurements were chosen to be conducted under different dc field ranges like 0 to 80 Oe for BSCCO with low \( J_c \), and 0 to 10 KOe for melt textured NdBCO with large \( J_c \).
Fig. 4.25 Temperature dependence of measured fundamental susceptibility, (a) $\chi'$ and (b) $\chi''$ of the NdBCO slab for different dc fields ($H_{dc}$), with $H_{ac} = 2$ Oe.
Fig. 4.26 Temperature dependence of $\delta(T, H_{dc})$ extracted from $\chi(T)$ data, derived using measured susceptibility shown in Fig. 4.25 for various dc fields, is shown as discrete points. Theoretical fits of the $\delta$ to Eqn. (4.5.10) are shown as lines.
References


