Conceptual Design Of A Composite Sleeper

7.0 Unlike several structural components in the field of Civil Engineering, the design of a sleeper in a Railway Track involves precise calculations. The assessment of the various factors that influence the design has been fraught with a number of inponderables such as the dynamic loads, ballast reaction, fatigue and a score of other interrelated items. Obviously the determination of such factors can at best be done on a probabilistic approach, but cannot be dealt with deterministically. Let us examine them in detail vis-à-vis the current world Railway practices and arrive at suitable values which would be our guideline for optimizing the revised design of C.L. Sleepers.

7.1 Loads: One of the most important factors in the design of the railway sleeper is its dimensioning to resist external load. It is possible to evaluate the static load. It is however not easy to estimate dynamic loads, due to inadequate knowledge of the dynamics of track structure, with the complexity of variables like nature of formation, type and depth of ballast, sleeper spacing, type of fastenings, maintenance conditions of track, level, alignment etc. Loads consist of external loads and the ballast reaction which is equal and opposite from underneath.

7.2 Ballast Reaction and its distribution has a very great effect on the movements in the middle section of the sleeper. The elasticity is also different for caked or clean ballast. Any theoretical calculation requires an exact knowledge of the initial configuration of the surfaces supporting the sleeper on the ballast; but this configuration is apt to differ greatly on the track. On the other hand
this is not a static but a dynamic problem so that its theoretical analysis is extremely complex.

7.3 Alternating Stresses: A sleeper is subjected to alternating stresses due to very high vibrations. Cast iron being brittle and at the same time rigid in its characteristics, the exact analysis of fatigue strength of cast iron materials under dynamic loads has limitations. In the earlier days it was considered that the section under rail seat was subject to bending moment with compression at the top, but it was soon found that tensile stresses develop in many sleepers, at the top as well as under the rail seat. This feature is attributed to "reversal of stress created by what is known as 'rebounding action' after the passage of the wheel". Under the wheel load the sleeper deflects and stores up dynamic energy and then the wheel moves off, the sudden release of stress causes a kick-back, the stresses momentarily flying to opposite peaks. It is also a common phenomenon which will be dealt with later, that the vibrations at the sleeper level rise to a high frequency of 1000 cps, causing accelerations up to 100 g. Even elastic fastenings could only have a limited damping of the above vibrations.

7.4 The UIC/CE in their question No. D-71 "Stresses on Sleepers" says further that the theoretical calculations involve the following difficulties:

(a) The strength of materials theory is hardly applicable when the width of a component is large in relation to its length. The 'elementary principles of simple bending' enunciate that (i) Plane transverse sections remain plane and normal after bending; (ii) The material is homogeneous and obeys Hooke's law and the limits of elasticity are not exceeded.
(iii) Every layer of material is free to expand or contract longitudinally or laterally under stress, as if separate from other layers. (iv) Module of direct elasticity has the same value in compression as well as for tensile strains.

Even wood or steel for that matter, obeys the simple theory of bending only within certain working stresses. When the actual stresses increase, as in the case of heavy trafficked routes, any design on the simple theory of bending will be inaccurate.

(b) The mechanical characteristics of the elastic support and the sleeper bottom vary considerably.

(c) Even leaving the above difficulties aside, it is still very tricky to calculate a flexible component resting over its whole surface or part of its surface, on an elastic support.

(d) Finally the elastic theory is only approximate. Thus it will be seen that it is virtually impossible to calculate the stresses occurring on a sleeper. One can design a sleeper with the highest ever loads and a coincidence of the most unfavourable factors. That will only result in a thoroughly uneconomical sleeper. Therefore, traditionally there had been two Schools of thought on this subject.

First School of Thought (credited to British and German Railways).

The above railway systems have designed the sleepers after extensive practical tests/investigations. These relate to the measurement of load at the baseplate level (rail-sleeper junction) with axles of varying weight and at varying speeds. The distribution of ballast reaction is based on certain assumptions, as a result of their trials. Stresses in the sleepers were determined by conventional "Strength of Materials" theory, treating the sleeper as a rigid beam lying on a slightly elastic or rigid bed.
of ballast. The bending moments thus calculated were equated with moments of resistance. In short the sleeper is designed on principles applied to structures subject to static loadings.

Second School of thought : (Attributed to French Railways)

The French Engineers have primarily gone on the assumption that
(i) Load is unknown. Hence all subsequent assumptions and hypotheses are highly questionable;
(ii) Fatigue stresses are not known particularly in the case of railway sleepers subjected to alternating stresses caused by high frequency vibrations.
(iii) Concept of "Strength of Materials" calculation for heterogeneous materials subject to dynamic loads is incorrect.

Hence their sleeper design has been evolved on an empirical basis of "trial and error". The sleepers were modified/improved from time to time by trials/tests. Conventional calculations were only used to assist checking of certain results from track or at the test for throwing light on certain failures and to give an idea of what modifications should be made in order to improve the performance.

Based on the above schools of thought, the following loads were evolved by the advanced railway systems for design and installation of concrete sleepers for their high speed track.

7.5 Loads considered in Design

7.5.1 German Railways

(a) Static Loads : (i) On a straight track, a static wheel load of 11t at the rail head with the heaviest German locomotive was found to cause vertical sleeper reaction at the rail seat of about 6 tonnes due to its distribution over the adjacent sleepers through rail and elasticity of bedding; (ii) On curves, the maximum
horizontal wheel pressure was measured to be 12 tonnes acting at the rail head, 35 cm above the sleeper soffit. Allowing for distribution amongst several sleepers, it is estimated that 60% of the reaction i.e. 7 tonnes is carried by each sleeper.

Where lateral forces occur, the magnitude of vertical force is increased on the outer rails and reduced on the inner rails, approaching zero under certain conditions. The force Q shown below is a passive rail pressure exerted by rail to balance the overturning of the sleeper.

(b) Dynamic Loads: To assess the effects of dynamic loads, the German Railways have carried out extensive measurements of (i) the accelerations and frequencies of vibrations of the sleeper under loads and (ii) the actual elastic strains at critical points of the sleeper. Stresses to which sleepers are subjected were assessed separately by both these methods, and a close agreement was observed. It was found that while dynamic stresses on well maintained tracks were not very high, additional stress due to wheel flats, the impacts at joints and track irregularity were quite considerable. To allow for the above effects, the German Railways have adopted a design load of 15 tonnes at each rail seat based on the static load of 6 tonnes, augmented by a dynamic factor of 150% under three conditions of loading shown below:
7.5.2 British Railways: They evolved their design criteria on the basis of a full series of track tests. The stresses developed at various points of sleeper, as also the external loads coming on the rail seats, were actually measured. It was seen that the bending moments calculated with the help of the measured stresses and assessed loads were in close agreement. Based on these tests, the following standards of loading were adopted for their "Type F" concrete sleepers:

(i) Vertical load of 23.4 tonnes at each rail seat;
(ii) Ballast reaction of same magnitude distributed as in the figure below.

One reason for adoption of a higher vertical rail seat load by British Railways seems to be their wider sleeper spacing of 76 cm compared to 63 cm on German and 52 cm for French Railways.

7.5.3. American Rail Roads do not specify any loads of the theoretical calculation of the bending moments. In an entirely different approach, a new sleeper was designed to carry the same maximum moments as a wooden sleeper in track would be required to carry under most adverse conditions with identical spacing of 20". The calculations and measurements of stress with wooden sleepers, spaced 20" apart, indicated that a bending moment of 1,15 tonnes metres at the rail seat was about the maximum that could be developed under service conditions. The calculations also showed that a bending moment of 4.00 tonnes metre was required to break a wooden sleeper (Oak) under
centre bound conditions suggesting that other types of sleepers should also have an equal moment capacity, as a few broken oak sleepers which apparently failed due to centre bound conditions were actually observed in the track. While designing the concrete sleepers, after allowing for an additional impact factor for increased spacing of sleepers, a bending moment of 1.75 t.m at rail seat was assumed for concrete sleeper design. The bending moment of 4.60 t.m at centre posed quite a problem as no sleeper could be designed for such a heavy bending moment. This was however overcome by shaping the central portion of the sleeper like a wedge having its apex downwards in order to relieve the sleeper of ballast pressure at centre.

7.5.4 French Railways believe, as stated earlier that the design is one that needs to be made on an empirical basis. Their design, as such has been progressively improved after observing the sleepers on track for nearly 20 years, the calculations serving only as a rough guide in the early stages. For initial dimensioning of concrete sleepers, the following external loads were adopted:

(i) Rail reactions of 6.3 t (based on a static load of 10 tonnes), increased by 35% to 8.4 tonnes for allowing for the effects of the dynamic load; the positive bending moment (+ 0.75 t.m) under rail seats ascertained by this is multiplied by a Safety Factor of 2 (+ 1.5 t.m);

(ii) A lateral reaction of 7 tonnes based on theoretical calculations and supported by the results of field investigations. Necessary modifications were later incorporated to improve the resisting moment capacity of the sleeper based on its actual behaviour in track.
7.5.5 Indian Railways: Before the advent of concrete sleepers, there were three types of sleepers viz. wooden, cast iron and steel in use. As regards the cast iron and wooden ones, no conventional design calculations exist. Rough dimensioning of the sleeper sizes were arrived at in the initial stages of evolution, which underwent a process of improvement by trial and error and after gaining experience from the field. As regards the wooden sleeper, the earlier assumption was that the entire wheel load is carried by one sleeper only and no allowance was made for conditions under dynamic loading. The loading conditions are indicated below:

\[ P = \text{Wheel load} \]
\[ G = \text{Gauge} \]
\[ P_1 \text{ and } P_2 = \text{ballast reactions/ unit areas} \]
\[ l_1 = \text{packing length} \]
\[ l_2 = \text{unpacked length of the sleeper} \]

For wooden sleepers, the values that had been assumed are as follows:

\[ P = 11.25 \text{ tons} \]
\[ P_1 = 0.29 \text{ ton/inch} \]
\[ l_1 = 39.30 \text{ inches} \]
\[ l_2 = 29.24 \text{ inches} \]

But subsequently when the design of concrete sleepers was done indigenously, the experience of other countries substantiated by field experiments in India set the guideline and the following criteria have been adopted based on German Sleeper design, on the sole consideration that Germans have a considerable experience in this field and the maximum percentage of concrete sleeper in the track is still available on their system.
The criteria assumed are as follows:

7.5.6 UKG / GEF Studies * D.7.2 *: The above studies bring out the difficulty in theoretically working out the stresses in railway sleepers, and suggest a statistical envelope that can be determined of the bending stresses (on the basis of a statistical evaluation of field trial results) characterising the various types of sleepers in the track and simple models can be visualised of the distribution of the ballast pressure, so that the maximum bending stresses observed during the tests can be reproduced. After extensive trials on continental systems, they had evolved a formula as follows:

\[ Q_n = \text{Nominal wheel load} \]

\[ R = \text{Reaction force exerted on the sleeper by the rail} \]

\[ A = \text{Dynamic mean ratio} \]

\[ A = \frac{\text{Mean value of } R}{\text{Mean value of } Q} \]

\[ d = \text{distance from the centre of rail to the end of sleeper} \]

\[ e = \text{distance from centre of rail to the end of sleeper reaction area} \]

\[ y = \frac{d - e}{2} \]

\[ \theta = \text{Coefficient for increase in wheel load } Q_n \text{ due to dynamic effect} \]

\[ x = \text{Coefficient for increase in sleeper reaction due to dynamic effect} \]
\( \varphi_1 = \text{Coefficient in increase in } B,M(\text{of the sleeper below the rail seat due to dynamic effects)} \)

\( \varphi_2 = \text{Coefficient in increase in } B,M \text{ (Mc under the centre of sleeper due to dynamic effect)} \).

The statistical estimates of the various approximate maximum values are derived from the following formulae:

Dynamic wheel load \( Q_M = \phi \cdot Q_N \)

Dynamic sleeper reaction = \( R_M = \phi x A \cdot Q_N \)

Dynamic bending moment under the rail \( M_r = \varphi_1 R_M V_2 \)

Dynamic bending moment of centre of rail \( M_C = \varphi_2 \cdot \frac{W_r x \text{ Stiffness } R \text{ at centre}}{\text{ Stiffness HI at rail seat}} \)

The various coefficients have the following values

\( \phi = 1.5 \text{ for } V = 140 \text{ km/h and } 1.75 \text{ for } V = 200 \text{ km/h} \)

\( x = 1.32 \Rightarrow \varphi_1 = 1.6, \varphi_2 = 1.2 \)

7.6 The specialist committee set up by UIE/ORE under questions D-22 65 and D-71 have gone further into the problems of design and production of sleepers thoroughly over a number of countries. It is their considered opinion that :-

(i) Theoretical design methods are of limited use for design of sleepers and the calculations can at best serve as rough guidelines for initial dimensioning of the sleeper designs to arrive at a starting point.

(ii) Extensive field trials with a large number of sleepers under actual field conditions are a must and the only way to arrive at the designs suitable for particular locations.

(iii) Such trials have to be spread over long gestation periods of several years and :
(iv) Improvement in design is a continuous process.
It is also worthwhile quoting the opinions of a few eminent engineers in this respect.

Opinions of Leading Experts All Over The World

(1) Roger Sommerville of "French Railways": "Railway sleepers, unlike bridges and other railway engineering structures, cannot be designed solely from mathematical formulae. To calculate the various parts of a structure, the external forces and the reaction on the bearing area must be known. In a railroad sleeper, dynamic forces are transmitted from the rail to the sleeper, but the reaction of the ballast depends on a number of factors, particularly the flexibility of the sleeper and the condition of the ballast, which cannot be precisely determined. Whatever means are used for design a sleeper must, therefore, be evaluated on the basis of service tests, where the modifications in the design can be carried out while in track."

(11) A Tool of the "German Railways"

"Based on thorough theoretical investigations and practical studies carried out, the German Railways decided to use concrete sleepers in 1949. General designs were developed such as B-58, B-60, & B-52 during 1945-1955 and large-scale experiments with nearly 3 million sleepers had further resulted in a design B-58 in 1955 which was modified to B-60 in 1959. By the end of 1969, nearly 22 million sleepers were on track then a new design B-70 came into the field with the advent of speeds up to 250 kmph. B-75 was evolved during 1975. All the above go to prove that design is only a continuous process that has to rely more heavily on experience than on theoretical considerations."
(iii) D.H. Goold of "British Railways".

"Railway sleepers especially concrete type have been the subject of experimental installations on British for over half a century but many failures mar the history of this development. Sometimes the failures related to the design of the sleepers, sometimes to inadequate manufacture, sometimes to inadequate maintenance and sometimes to the inadequacy of the rail fastening systems."

7.7 Ballast and Formation Pressures: The primary object of providing ballast, sub-ballast, or blanket under the track is to distribute the heavy pressures transmitted by dynamic train loads at high speeds to the formation through the track structure and the ballast media and to reduce the pressure on the formation to such an extent as the soil at the formation level is able to withstand without excessive deformation or failure, so that the track structure is maintained perfectly. The details of studies carried out by RLYS have already been discussed at para 5.6 along with a table showing the speeds permissible for various rolling stock on formation pressure criterion.

7.7.1 The problem in our case primarily arises as to the determination of a proper size of the C.I. plated shell which will transmit the pressures to the formation for all expected future train loads. On a perusal of the design of track foundations all over the world railways, the following formulae are in vogue for calculation of maximum pressures in the formation (p_{max}):

(1) Formula based on elastic theory:

\[ p_{\text{max}} = \frac{2}{\pi DL} \text{ load on sleeper} \]

where \( L \) = The effective bearing lengths of sleeper under each rail,

assumed as 75 cm for B.G. track and 68 cm for metre gauge track under wooden, steel trough and concrete (monoblock) sleepers.

On cast iron and twin block RGG sleepers, length of each block may be taken

\( D = \text{Depth of ballast in cm.} \)

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The sleeper load should be calculated taking dynamic augment into consideration.

\[ P_{\text{sleeper}} = F_s \cdot \frac{4}{\frac{R}{64 \, \text{N} \cdot \text{L}^2}} \quad \text{where } P \text{ is wheel load} \quad S \text{ is sleeper spacing} \quad \text{and } R \text{ is track modulus} \]

(2) Talbot formula \( \text{ (Used in U.S.A.) } \)

\[ P_{\max} = \frac{P_a \times 16.8}{h \times 1.25} \]

Where \( P_a \) = Mean pressure at the bottom of the sleeper
\( h \) = Thickness of ballast in inches.

The mean intensity of pressure at the bottom of the sleeper \( (P_a) \) is obtained by dividing the total load on sleeper by the total area of the bottom surface of the sleeper \( \text{viz.} \, \text{eff. length} \times \text{breadth} \)
for wooden, steel, trough and concrete (nominal) sleeper and area of two blocks for cast iron and RCC sleepers.

(3) Schramm's formula \( \text{ (Used in Germany) } \)

\[ P_{\max} = \frac{1.5 \times P}{3 \left( \frac{L\,d}{b} + b \right)} \cdot h \cdot \tan \alpha \]

Where \( P \) = Total load on one sleeper
\( L \) = Length of sleeper in cm
\( S \) = Track gauge in cm
\( b \) = Breadth of sleeper in cm
\( h \) = Depth of ballast in cm
\( \alpha \) = Angle of distribution of pressure through ballast (normally 45°)

(4) Japanese National Railways \( \text{ (JNR) } \)

\[ P_n = \frac{58}{10 + d \times 1.35} \times P_0 \]

where \( d \) = depth of ballast in cm

\( P_n \) = formation pressure in kg/cm\(^2\) and
\( P_0 \) = pressure intensity on sleeper (kg/cm\(^2\))
(5) Boussinesq's Equation

\[ p_n = \frac{6}{\pi} \frac{q_0}{h^2} \]

where \( p_n \) = max. pressure in kg/cm²

\[ q_0 = \frac{Ra \cdot Ab}{2} \]

where

\( p_a \) = average pressure under the sleeper (kg/cm²)

\( Ab \) = bearing area of sleeper in cm²

\( h \) = depth of ballast in cm

The studies conducted by CIR have shown that the single layer elastic theory provides an adequate means of calculating formation stress for practical engineering purposes as a more sophisticated approach is not justified due to high degree of scatter involved in the sleeper support condition. It has also been found that the stresses in track formation materials are directly proportional to the magnitude of load on sleeper and are not influenced by the rate of application of load and hence the results from quasi-static test can be extrapolated directly to the dynamic condition. Sparrow & Terry have also stated that the magnitude of vertical and horizontal stresses is independent of the rate of loading. Hence the determination of stresses under dynamic load considering static load is justified.

From the accepted level of permissible pressure on the Indian railway track, the sleeper surface contact area can be worked out as per the various formulas given above with the following assumptions:

- \( P = \) max. load on sleeper = 15.2 tonnes (as per design assumptions)
- Depth of ballast = 9" (20 cm)
- Permissible pressure = 7200 lbs/ Sq.ft (3.5 kg/cm²)
(i) Elastic Theory ... 2177 Sq. cm
(ii) Talbot formula ... 3994 sq. cm
(iii) Schram's formula ... 1968 Sq. cm
(iv) Japanese National Railways Formula ... 2966 Sq. cm
(v) Boussinesq's equation ... 3869 Sq. cm.

As may be seen from above, for the same thickness of 25 cm ballast, the bearing area needed for a sleeper to restrict the max. possible formation pressures upto 3.5 kg/ cm² as given by various formulae vary considerably from one another. In India as ascertained by extensive tests by NRC, the "Elastic theory formula" propounded above has generally given comparable results between theoretical and experimental results. As such the bearing area of sleeper at the rail seat can be kept somewhere near 2000 - 2200 cm².

The B.C. sleeper base is 900 x 300 mm and the metre gauge 700 x 250 mm. Hence for an economical design, a base size of 750 x 300 mm ( 2250 cm² area ) has been finally kept in our conceptual design which will imply covering the load and pressures expected in service.

7.6 Tie Bar Assembly: The present arrangement of tie-bar cotter assembly has been criticised by several people that it is too flexible and does not add sufficient rigidity to the central portion of the sleeper. But one must bear in mind the following factors:-
(i) In the case of a twin block ECC sleepers, the tie bar which is rigidly fixed to two B.C.C. blocks makes the sleeper difficult to handle manually and hence recourse must be had to mechanical relaying methods, which prove to be costly. (ii) The sleeper needs very careful handling right from the factory base till it is laid on track. In fact, it is commonly experienced on the French Railway Track that the sleepers do get bent at the tie bar at centre, thereby causing difficulties in maintenance of track geometry parameters, which assumes great importance.
in high speed track. (iii) Once the tie bar becomes rigid, the forces exerted by the centre bound track do cause a vertical uplift on the tie bar and it has, therefore, been specified that a central ridge has to be kept at the centre to keep the tie bar free from contact with the ballast.

Hence, it has been decided to keep the tie bar cotter assembly the same in the revised design as was existing, earlier.

7.9 Depth of the Cast Iron Bowl: As mentioned earlier in Chapter III, the obvious deficiency in present sleeper form is the shallow depth, which causes lateral bending and high order vibrations. A sleeper primarily needs depth to keep vertical rigidity. At the same time too high a sleeper will cause columnar instability and may cause brittle fracture in the parent metal. Further, the sleeper, once it becomes too high would involve heavy lifting of the track which leads to difficulties in relaying on electrified sections. After extensive trials with the foundry shops of the Arakam Workshops, Southern Railway, the height of the sleeper below the rail seat has been kept as 175 mm.

7.10 Design of Elastic Fastenings: Any modern sleeper cannot incorporate features of a rigid fastening for rail-sleeper fixtures. A double elastic fastening consisting of a spring clip with a rubber pad is synonymous with the advances in the modern track. The choice of a suitable fastening for the new sleeper design however is fraught with the following factors:

(i) In India, the only fastening which has been fully developed for large scale manufacture is the traditional Panirial / Elastic Rail clip to IS 9's Drawing No. 7/1892. No other fastening has been commercially developed which can be adopted straightway for our new design.
(ii) The top of the sleeper base under the rail needs a very simple structural change to accommodate the above clip.

(iii) Elastic rail clip as developed on Indian Railways is a fully tested fastening in the high speed track and its behaviour and characteristics are quite satisfactory.

(iv) Several firms in India have come into the market to undertake manufacture of such a fastening in large quantities. Any other fastening, several of which are under trial stage in Railways, will involve long periods of gestation for perfecting their design. In view of the above the elastic rail clip with a rubber pad forming a double elastic fastening is proposed to be adopted.

7.11 Composite action between C.S.I. Shell & bitumen fill : Composite action in structures is normally considered as the "interaction of different structural elements and may be developed using either different or similar structural materials". The principle of composite action in the stressed thin "continuum" type of structure is to consider combing the different elements of the structure into one complex but integrated continuum in order to resist loads and external environmental forces. Instead of assigning each structural member a single isolated and specific task, the continuum attempts to use composite action by uniting all the structural member by means of proper connections, thus creating a single structural member that acts as a multidirectional load-carrying element in the total structure. In order to ensure such interaction in the case of our sleeper a natural bond has to develop between the metallic shell and bitumen fill inside, by a proper design of a shear connector. The purpose of the shear connector in the composite sleeper would be to ensure the following:

(a) The rail seat load is taken up by the metallic portion underneath which is transmitted to the ballast through the bitumen fill. The ballast pressure exerted is then transferred peripherally to the metallic shell so as to achieve membrane stresses.

(b) The transference as above has to be ensured throughout its life by a proper bond at the interface without any possibility of a slip between the inelastic and visco-elastic media.

(c) Sleeper suffers a progression wave i.e., the uplift/depression under the passage of wheel loads. The shear connector has to prevent separation at right angles to the interface.
Various types of shear connectors have been used to resist longitudinal shear and uplift. They are rigid, flexible, bond type high strength friction grip bolts and some employ epoxy glueing between the two components. Broadly they can be either 'rigid' or 'flexible'.

The former is the 'bar like' heavy type, while the latter is the 'stud and channel' type. Bar-like connectors are limited to shear transfer in one direction while the flexible stud types can resist and transfer shear in any direction.

The author had made two alternative designs viz., one with the rigid shear connector in the form of a radial rib at the four corners of the shell extending from top to bottom and the other with stud like projections. The latter idea had to be abandoned due to:

(a) Casting of studs in the C.I. shell proved to be problematic in the design of moulds

(b) Damages due to manual handling were heavy

(c) For effective shear connection, a large number was needed, which makes the design rather unworkable in mass manufacture.

Accordingly the former type after extensive trials proved to be satisfactory and was adopted in the final design.

7.12 The most vital question that affects the soundness and overall economy is that of the weight of the sleeper. "What should be the optimum weight of a sleeper?" In the words of the celebrated German Engineer J. Eisenmann 72 "For speeds up to 250 mph, the peak values of ballast pressures must be limited to 0.25 N/m² with a ballast depth of 30 cm which will ensure a fairly uniform life behaviour of track.

For an axle loading of 150 to 200 kN for such speeds, the track structure should consist of 60 kg/m UIC rails, 2.6 to 2.8 m long sleepers with a width of 22 to 30 cm pitched at 60 cm centres. Such a track should have heavily weighed sleepers to provide a track panel of 600 to 700 kg/m mass which will ensure lesser ballast deterioration due to
lifting of track panel in front and behind an axle 
A mass of 600 kg has been advocated above, and a perusal of the standards prevalent abroad would indicate that the foreign systems are slowly advocating heavier and heavier sleepers at higher speeds and the sleeper spacing also progressively gets reduced to increase mass per unit length. In fact a sleeper contributes three to four times the weight of the rail per metre, which from cost point of view favours heavier sleepers than heavier rails.

7.12.1 The problem of weight of sleeper in conjunction with other components of the track structure could be solved by a two stage two degree of freedom model as follows:

The above shows a diagrammatic representation of the track structure starting from the rail and ending at the formation.

Let

\[ W_1 = \text{Weight of rail vibrating per rail seat in (kg)} \]
\[ W_2 = \text{Weight of sleeper per rail seat} \]
\[ C_1 = \frac{1}{3} \text{weight of one sleeper (kg)} \]
\[ K_1 = \text{Stiffness of the elastic clip (kg/cm)} \]
\[ K_p = \text{Stiffness of the rubber pad (kg/cm)} \]
$K_2$ = Stiffness of the ballast bed (kg/cm)

$C_1$ = Coefficient of damping of the rubber pad

$C_2$ = Coefficient of damping of the ballast bed

$$K_2 = \frac{P_1}{Y_0} = \frac{4\sqrt{3}}{185.2} \times 10^4 \text{ kg/cm}$$

where $I$ = moment of inertia of the rail (cm$^4$)

$K$ = Equivalent track modulus (kg/cm/cm)

$F_o$ = Amplitude of force applied on the rail due to the rail acceleration.

$x_1$ = Amplitude of forced interaction in rail

$x_2$ = Amplitude of forced vibration in sleeper

Equating the forces acting on $W_1$ and $W_2$, we have

$$\frac{W_1x_1}{2} + \frac{W_2x_2}{2} + C_2 (x_1-x_2) + (k_1+k_2)(x_1-x_2) = F_o \sin wt$$

and

$$W_2x_2 = C_1(x_1-x_2) + (k_1+k_2)(x_1-x_2) + k_2x_2 + C_2 x_2 = 0 \quad \ldots \quad 7.12.1(1)$$

Rewriting these equations we have:

$$x_1 + \frac{C_1}{W_1} (x_1-x_2) + \frac{(k_1+k_2)}{W_1} \cdot \delta \cdot (x_1-x_2) = F_o \frac{g}{W_1} \sin wt$$

and

$$x_2 = \frac{C_1}{W_2} (x_1-x_2) + \frac{(k_1+k_2)}{W_2} \cdot \delta \cdot (x_1-x_2)$$

$$+ \frac{k_2}{W_2} \cdot x_2 + \frac{C_2}{W_2} \cdot x_2 = 0$$

or

$$\begin{align*}
\frac{C_1}{W_1} \cdot x_1 + \alpha (x_1-x_2) &= \beta \sin wt \\
\frac{C_1}{W_2} \cdot x_2 - \gamma (x_1-x_2) + \delta x_2 + \eta x_2 &= 0
\end{align*} \quad \ldots \quad 7.12.1(2)$$

where

$$\begin{align*}
\alpha &= \frac{C_1}{W_1} \\
\beta &= \frac{F_o g}{W_1} \\
\gamma &= \frac{(k_1+k_2)}{W_2} \\
\delta &= \frac{k_2}{W_2} \\
\eta &= \frac{C_2}{W_2}
\end{align*}$$
The solution of the above equation would consist of the "complementary function" and the "particular integral". The complementary function consists of free vibrations which would be quickly damped away and are of little interest. The steady state vibrations are given by the particular solution of equations 7.12.1 (2) on the form:

\[ X_1 = L \cos \omega t + M \sin \omega t \]
\[ X_2 = N \cos \omega t + P \sin \omega t \]

Substituting these values in equations (2) we have the following equations to determine \( L, M, N \) and \( P \):

\( (-W^2 + \alpha) L + \omega^2 N - \alpha M - P = 0 \) ... 7.12.1. (3)
\( -\omega^2 L + (\omega^2 + \alpha) M + \omega^2 N - \alpha P = \beta = 0 \) ... 7.12.1. (4)
\( -\alpha M - \beta \omega L + (\omega^2 + \gamma + \delta) N + (w) + (\gamma + \omega^2) P = 0 \) ... 7.12.1. (5)
\( \delta \omega L - \gamma N + (-\delta - \gamma + \omega^2) P = 0 \) ... 7.12.1. (6)

Amplitude of forced vibrations of the rail

\[ X_1 \text{ max } = \sqrt{L^2 + M^2} \]
and that of sleeper

\[ X_2 \text{ max } = \sqrt{N^2 + P^2} \]

How far \( \frac{x_1 \text{ (max)}}{x_2 \text{ (max)}} \) can be controlled to optimum values is a function of various structural constants mentioned above. Table 1 gives typical values of

\[ \frac{x_1 \text{ max}}{x_2 \text{ max}} \]

for different track structures combinations.

It may be seen for sleepers bearing lesser weight at rail seat and all other factors remaining constant, the
dampening ratio varies from 3.703 to 4.675 only whereas for heavier
sleepers this may go upto 15, thereby indicating that the shear
weight of the sleeper does play an important part. A normal
dampening value of 10,0 is preferable since normally the vibrations of
an inferior track may extend upto 15 to 20 cm thereby inducing net
movement of 1.5 to 2.0 cm at the sleeper level which is within the
normal manufacturing tolerances permitted in the various components.
It may be seen that from a twin block concrete sleeper to that of a
monoblock sleeper the reduction in the ratio \( \frac{N_2}{N_1} \) is not
concomitant with the increased weight of sleeper \( 235 \text{ kg for a monoblock
as against } 215 \text{ kg for at two block}. \) Hence a weight of 100 to 110 kg
at the rail seat would be ideal for achieving levels of amortising
vibrations at the speeds prevalent in our B.C. tracks and with this in
view, a sleeper design has been conceived with the following features:

- A cast iron shell weighing about 60 kg
- A bituminous fill to the specification mentioned at para 6.6 = 40 kg
- A tie bar with 4 cotters
- Elastic rail clips & rubber pads to standard designs
- Composite action between 6.1 shell to the bituminous fill
  rendered by the corner ribs (peripheral projections acting as shear
  connectors).