LITERATURE REVIEW

2.1.1 As cited in Reference [6] around the middle of fifteenth century coaches appeared which had the passenger compartment suspended on leather straps which were attached to supports. This may be the first suspension used. The first American air spring patent was issued to a JW Hoogland in 1861 and the reference is made in the 'Scientific American' issue of July 20, 1961 that the inventor "is satisfied that he had overcome all the difficulties encountered in the use of air springs". However vehicles using air suspensions appeared around 1950.

However hardly any technical literature is available on 'air-springs' before 1958. In SAE Transactions of 1958 five papers [1,2,3,4,5] were published describing the use of air suspensions on four different cars. Layout of suspension mechanisms, their advantages and testing have been described in these papers. However no theoretical analysis has been presented.

2.2.1 Mc Farland et al[2] state "It has long been known that low spring rates would assist greatly in giving a good boulevard ride, but under normal circumstances the low rate has been considered by many as giving unstable highway ride. Air Suspensions have characteristics which overcome this latter objection and *wed* these presumed opposing characteristics. This has led to the introduction of Air-Poise suspension on the 1958 Buick".

2.2.2 Authors have described the suspension arrangement. One 'height control valve' on the front and two 'height control valves' on the rear axle were used. An air compressor driven by the engine is used. Airspring embodies a diaphragm of 2-ply nylon cord design, rubber coated inside and outside. This member fits into outer rim of a metal container. The inner ring of the diaphragm is clamped to the top of a piston-
like member, which in turn has a flat bearing at its lower end contacting a spherical bearing located in the lower frame. At normal design height the front air springs have an area of \( 25 \text{ sq.in.} \) and the volume of \( 300 \text{ cu.in.} \) and the rear springs have an area of \( 12 \text{ sq.in.} \) and a volume of \( 235 \text{ cu.in.} \). Operating pressures which vary with load are in the neighbourhood of 100 p.s.i. for a 5 passenger load depending on the model/car. These in combination with linkages result in a wheel rate of 71 lb. per in. front and 81 rear or less depending on the car model. Wheel rate varies as shown in the figure 2.1.

![Wheel rate variation](image)

Variation is due to the equation \( PV = K \), change in the size of the inner piston and outer container diameter and slight effect due to cord angle. This change in wheel rate permits a soft boulevard ride and increased ride stability on rough roads.

These sizes have resulted in a frequency of 43 cpm front and 52 cpm rear, regardless of load with the car in the vicinity of design height. To increase roll stability the roll centre at the rear wheels has been raised slightly.

2.2.3 Control system comprising of various height
control and relief valves have been described in the paper. Following advantages of the air-suspension have been brought out:

1. Car is at the same height at all times regardless of passenger load.
2. Ride is soft on boulevard, yet controllable on rough roads.
3. Ride is level with no perceptible pitch.
4. Frequency of front and rear suspensions is substantially constant regardless of load.
5. Ability of suspensions to remain level at constant height keeps headlights on road.
6. Car can be raised to cope with unusual clearance conditions.

2.3.1 W.S. Berry[2] has described air springs used on Rambler car of American Motors Corp. It is pointed out that airsprings can maintain the height of the car regardless of load and can give a low rate for boulevard-type ride and high rates for rough roads which produce maximum axle movement. Since the rear springs were most affected by extreme loads air-springs were used on rear axle. The rolling lobe air spring was used. Air springs were used along with the steel coil springs. With the car empty, 20 psi pressure exists in the airspring and each airspring supports about 100 lb. while the steel spring supports 500 lb. In loaded station wagon air pressure is about 100 psi. Levelling valves are used for height control and use of ball check restrictions prevents any loss of air during bump. Pressure limiting valves are used to limit lower pressure. Running the engine for approximately 1/2 min supplies ample air pressure for normal operation of car. Nylon cords covered with neoprene is used for rolling lobe with a burst pressure of 520 psi.

2.3.2 Dynamic load characteristics for 27.5 psig, 55 psig, and 85 psig are given which show increasing dynamic rate.
with compression. Hence there is no bump-through with these air springs. Design permits 4-in travel in compression and 4-in travel in expansion.

Author claims that many thousands of miles of testing have shown that airspring is direct and effective solution to maintain height regardless of load. A rising rate curve is used to minimize any tendency for rear axle to bump through on rough roads. The increase in rate improves the handling and stability of the vehicle when heavily loaded. Reliability and ease of service is achieved through by combining airspring with coil spring.

2.4.1 O'Shea has explained 'The Ford Approach to Air Suspension' in reference [3]. He points out that the trend towards lower vehicles has placed a sharp limitation on the use of conventional steel springs with lower rates due to available wheel travel space limitation. In addition with appreciably lower rate leaf springs torque windup becomes a problem. Since steel springs are essentially constant rate devices, satisfactory cushioning at the compression and rebound extremities becomes increasingly difficult.

Features selected for the design & development of air suspension system are :-

1. Maximum improvement in riding comfort on all types of road.
2. Steering and handling characteristics equivalent to standard suspension cars.
3. Continuous 2-speed automatic height and leveling control.
4. Harshness and road noise reduction.
5. A practical, trouble-free system requiring minimum maintenance and adoptable to conventional production and service requirements.

2.4.2 System is divided into the following functional subsystem :-

1. Air supply system.
2. Control System.
3. Air Springs.
4. Suspension Structural Components.

Details of the subsystems are presented in the paper. It has been brought out that static and dynamic load-deflection curves for air-springs bear a strong general resemblance to each other, but the dynamic rates in the normal ride range are very much higher than the static rates. Cord angle in the fabric layer of the rubber diaphragm and the slope of the piston have great influence on load-deflection curves. Vertical cords with straight piston give almost an ideal curve but strength and folding characteristics of the diaphragm prohibit their use. An hour-glass shape piston with diagonal cords gave the best solution. It is observed that pressure differential between air-spring and reservoir is also important for good spring characteristics under rough road conditions.

It is concluded that with air springs the rear-seat ride is remarkably free from pitch and there is a noticeable reduction in harshness and road noise. There is also asthetic gain as car height remains constant.

2.5.1 Hansen et al[4] have described 1958 'chevrolet' air-springs. They have called it as 'Level Air Suspensions' or a 'constant frequency ride'. As the authors put it using air as a medium, they have developed a suspension with self-dampering qualities for a gentle boulevard ride, as well as the proper absorption characteristics for high impact forces encountered on secondary roads.

General description of the system is given in the paper. Height of the car is controlled at three points. In order to keep unrestricted cross flow between two rear springs from interfering with the vehicle's handling, a 0.020-in orifice is included in the line.
A single convolution rubber-bellow is used as spring. Authors point out that the slope or rate of the load deflection curve differs depending on whether the spring is cycled at ride-frequencies or tested in static condition.

Hysteresis of air-springs has been shown which amounts to damping in the spring. This reduces work of shock absorber. (Figure 2.2)

2.5.2 It is also mentioned that effective area variation is a function of the cord angles in the bellows, the moulded shape of the bellows, the piston contour, the linkage arrangement and to a small extent, the pressure in the bellow. Curve of effective area (defined as load/pressure) against wheel travel is as shown in the figure 2.3.

---

**Figure 2.2**

**Hysteresis curve**

**Figure 2.3**

**Load deflection curve**
Near design height suspension frequency is almost independent of load.

In endurance test no failure occurred for 100,000 full stroke cycles. Hence reliability of air springs is well established.

It is concluded that Air-suspension is probably the greatest advancement in passenger-car springing.

2.6.1. R.W. Perkins[5] has described 'New-Matic ride' an air-spring system developed by General Motors Corp. Air-springs operate at 100 psi. Spring volume is 295 cu.in. in front and 240 cu.in. in the rear.

Other details of mechanism, assembly and testing have been given. Advantages of airsprings have been brought out which are similar to the previous papers.

2.7.1 Bank[6] in his paper titled 'Some ABC's of Air-spring Suspensions for Commercial Road Vehicles' discusses fundamental principles of air springs as used on commercial vehicles. Factors which can favourably affect the success of a suspension are listed which include different costs, weight saving, damage to cargo etc. The basic advantages of pneumatic suspensions are listed as:

1. Low natural frequency for a soft ride and lower shock inputs.
2. Constant natural frequency throughout normal vehicle load range.
3. One suspension height for all load conditions.
4. Better controlled shock forces near the end of the suspension compression travel.

Few other advantages have also been listed.
2.7.2 Basic concepts of air springs have been discussed and convoluted type and reversible sleeve type springs are compared. Advantages of using higher operating pressure have been brought out. Figure 2.4 shows graph of natural frequency against pressure. It can be seen that natural frequency sharply reduces with pressure in lower pressure region. Then the reduction with pressure is very small. Effect of pressure on cost/KN force is also demonstrated graphically.

\[\text{Natural frequency Vs Pr.}\]

It is said that using high operating pressure air-spring diameter may fall to 170 to 250 mm for most of the current commercial vehicles.

Dynamic load deflection curves for a low height, moderate operating pressure, rolling sleeve type spring
Comparison of springs

Fig 25

It is seen that higher pressure gives lower natural frequency, lower shock forces, and higher energy absorption. Following conclusions are drawn:

1. Increasing the pneumatic spring assembly height gives greatest improvement in shock control.

2. Increasing pressure gives improvement in shock control for the same height.

Operational problems with air spring and some current design concepts have been discussed.
2.8.1 Hirtreiter[7] has described the very basic principles and application of airspring in this paper. Rate of an Air-spring is derived as
\[ \text{Rate} = \frac{n p A}{V} \]
where \( n \) = polytropic exponent of gas.
\( A \) = effective area.
\( p \) = absolute pressure.
\( V \) = volume.
showing that rate can be reduced by using sulphur hexafluoride (\( n = 1.09 \)) in place of air (\( n = 1.38 \)).

Steps in the manufacture of airsprings have been given and types of airsprings and their characteristics described.

Likely advantages of airsprings and precautions in their design are given in details.

2.9.1 Hirtreiter[8] has given a design procedure for Air-springs. To begin with the author points out 'Air-springs provide practically frictionless action, adjustable load capacity, simplicity of height control, controllable spring rate and excellent life. They can be used in both light and heavy suspension applications.'

Effective area of an airspring is defined as the nominal area found by dividing the load by the gas pressure within the spring.

Air springs have three spring rates, viz.
1. Static or isothermal rate.
2. Dynamic or adiabatic rate. It is 40 to 50 percent higher than the static rate.
3. Polytropic rate. This is most common at natural vibration frequency of 60 cpm or less.

2.9.2 Two basic type of air springs are the bellows, and the piston - with numerous variations of each. If two air springs of the same load carrying capacity are compressed by equal amount, the air pressure rise and increase in effective area of
the bellow-type will be much greater than in the piston type. Hence they have high spring rates and natural frequencies (105 to 180 cpm).

The bellow spring has a relatively high lateral rate which is equal to its axial rate in the single and two-lobe versions; it is also stable as a column. However it requires high expansion - reservoir volume for good isolation. For a given load capacity, the over-all diameter of a bellow spring is approximately 12 percent higher than the other types. Pipe connecting bellow to reservoir must be atleast 25 mm diameter for general sizes to get lower spring rates. Bellow type air springs have a zero effective area when fully extended; this area increases to a maximum as the spring is deflected to its fully compressed position.

Piston-type air springs use a piston which is attached to the inner head of a reversible flexible member. There are two sub-types. In the reversible diaphragm spring the piston head usually passes through the opposite head of the flexible member. In the reversible-sleeve type, the piston head travels within the flexible member and does not pass through the opposite head. Their natural frequencies are from 40 to 100 cpm.

Comparison of various types of bellows for various characteristics has been given as shown in the table below.

<table>
<thead>
<tr>
<th>Bellows</th>
<th>Reversible Diaphragm</th>
<th>Reversible-Sleeve Rolling Lobe</th>
<th>Restrained Rolling Lobe</th>
<th>Lat.-Restrained Rolling Lobe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life</td>
<td>Excellent</td>
<td>Excellent</td>
<td>Excellent</td>
<td>Excellent</td>
</tr>
<tr>
<td>Isolation</td>
<td>Good</td>
<td>Very good</td>
<td>Very good</td>
<td>Very good</td>
</tr>
<tr>
<td>Capacity</td>
<td>Good</td>
<td>Good</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>Size</td>
<td>Large</td>
<td>Small</td>
<td>Small</td>
<td>Small</td>
</tr>
<tr>
<td>Reservoir required</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Strength</td>
<td>Good</td>
<td>Very good</td>
<td>Excellent</td>
<td>Excellent</td>
</tr>
<tr>
<td>Design flexibility</td>
<td>Fair</td>
<td>Good</td>
<td>Excellent</td>
<td>Excellent</td>
</tr>
<tr>
<td>Lateral rate/vertical rate</td>
<td>1 to 1</td>
<td>1 to 1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

SPRING RATE

Fig 2-6
Reversible diaphragm type is noted for its low natural frequency. Its volume can be changed by modifying the upper retainer. Its almost equal vertical and horizontal deflection characteristics makes it useful for mounting machine and equipment.

Reversible - sleeve rolling type are very versatile with low lateral rate. It has a wide static load range and different spring rates can be obtained by choosing various piston contours.

'Restrained Rolling Lobe' type has a nonexpandable sleeve over the flexible member.

Other types of air springs have also been described.

2.9.3 Design Procedure

The effective diameter of the piston type spring may be assumed to be equal to one-half the sum of the piston diameter and the outside diameter of the spring. The area calculated with the diameter is multiplied by 0.9 to obtain a very close approximation of the effective area. This area multiplied by air pressure gives load capacity at the design height. For natural frequencies author has given formulae for different arrangements. Natural frequency can be varied by adding to the working volume or by changing piston contour.

When transient loads are encountered it is sometimes desirable to use height control valves with built in time delay.

A typical design example has been given.

In addition it is suggested:
Use air springs
1 For automatic levelling or height control.
2 When natural frequency must be constant for varying load.
3. When large deflections for low natural frequency is needed and space is limited.
4. For light weight.
5. For good isolation when height is limited to 300 mm.

Don't use them:
1. Where high spring rates are required.
2. Where springs cannot be protected against splashing hot metal.
3. When torsional as well as linear forces are present.
4. Where available space is long and narrow.
5. Where low cost is the major consideration.
6. Where vertical space is limited to less than 75 mm.
7. Where gas source is not available for automatic replenishment and system must go unattended for more than six months.

2.10.1 Design of pneumatic springs has been considered in more details by Burkley and Myers in the paper 'Design and Validation of Variable Rate Pneumatic Springs'[9]. Unique characteristics of pneumatic springs are given as:

1. Variable spring rate over a wide load range.
2. Nearly frictionless action.
3. Outstanding noise and vibration isolation.
4. Wide range of load-carrying ability.
5. Constant levelling or levelling on demand.
7. Engineered control of spring rate.
8. Low maintenance operation.

Bellows and Rolling Lobe are two types in com-
mercial use. For bellows external reservoirs are needed to reduce the natural frequency. They have advantage of minimum collapsed height. Bellows are in convolute configuration.

The design of rolling-lobe spring utilizes a flexible member in which the side wall forms a rolling meniscus on the piston. A wide range of specific load deflection requirements can be met by tailoring the piston contour. Hence it does not require an external reservoir to reduce the spring rate. But they have virtually no lateral stability and hence an encapsulating cylinder is required.

Though the foregoing information is more or less contained in the previous papers it must be mentioned that this paper is oriented to pneumatic spring design itself and not to suspension mechanism.

2.10.2 Author defines the spring rate as the difference in reaction from 10 mm below design height and 10 mm above design height as shown in fig. 2.6.

\[
\text{Spring rate} = \frac{F_{10\text{mm}} - F_{-10\text{mm}}}{20 \times 10^{-3}}
\]

Polytropic constant mass process is expressed as...
\[
\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^\frac{n-1}{n} = \left(\frac{V_1}{V_2}\right)^{n-1}
\]

Law for air suggested is \( p \cdot u^{1.38} \) = constant. The suspended structure and its several pivot joints contribute to spring rate.

Factors affecting the spring rate are

1. Polytropic gas exponent.
2. Pressure.
3. Effective area.
4. Basic Spring Volume.
5. Piston Contour (rolling lobes)

\[
K = \frac{n \cdot p \cdot A^2}{V} + p \frac{dA}{dx}
\]

A is the effective area.

An empirical formula for the effective area for rolling lobe air spring is given as

\[
A_e = \frac{T T \{(D+d)/2\}^2}{4} \times (9)
\]

\( D \) = flexible member working diameter.
\( d \) = piston diameter.

Spring rate can be reduced by using an external heat sink which is difficult.

Each factor affecting the spring rate has been discussed in some detail. Variation of spring rate and effective area for three different piston shapes has been shown.

Author mentions that plumbing of spring to reservoir will introduce losses and hence damping. However he has suggested to limit the orifice size to 20 mm ID which is contrary to present thinking where damping (orifice) is considered as advantageous.
Validation of dynamic spring rate by extensive testing is recommended.

Parameters affecting the load/stroke curve are given viz.

Total test stroke length.
Percent of compression and extension of total stroke.
Input or exciting frequency.
Test stroke programme (sine wave etc.)
Design height.
Piston Geometry.

Hence mutual understanding between the manufacturer and the customer is recommended. Testing procedure as followed in the Goodyear laboratory is given.

2.11.1 (R.D. Cavanaugh[10] has brought out some results regarding 'Air suspensions and servo-controlled Isolation Systems' in Shock and Vibration Handbook which can be regarded as the summary of work in the field till that time. Formulae for stiffness of single acting (air pressure on one side of the piston) and double acting (air pressure on both sides of the piston) air springs have been derived.

For constant area of cross-section single acting spring

\[ K = \frac{dF}{db} = \frac{n p_i A_i^2}{V_i} \left[ \frac{1}{n V_i} \right]^{n+1} \]

and for a double acting spring,

\[ K = \frac{A_2 dp_2 - A_1 dp_1}{db} \approx \frac{n p_i A_i^2}{V_i} + \frac{n p_2 A_2^2}{V_2} \]
2.11.2 A double acting damped spring has also been considered. Both the sides of the cylinder are connected to respective tanks through capillaries to introduce the dampings. Compressible flow of air is analysed by considering the rate of change of energy which occurs in the individual cylinder and tank volume. The rate at which the energy leaves the cylinder is found by multiplying the total energy content per unit weight of gas by the weight rate of flow. Total energy content consists of the internal energy of the gas at a particular temperature and the energy gained or lost by the gas as a result of its compression or expansion. This is expressed as rate of change of enthalpy.

\[
\frac{dH}{dt} = WC_T - p_0 = \frac{dU}{dt} + \frac{dE}{dt}
\]

\[ H = \text{enthalpy}, \quad U = \text{internal energy}, \quad E = \text{work done on the gas}, \quad W = \text{Flow rate of gas}.\]

This finally reduces to the equation

\[
WtB = \frac{q}{RT_0} \left[ \frac{V}{V_t} \frac{dp}{dt} + p \frac{dv}{dt} \right]
\]

Similarly capillary flow is considered to give

\[
Wt = \frac{qCr}{2RT_0} \left[ \frac{p_1^2 - p_2^2}{1} \right]
\]

It may be noted that these two equations have been used by most of the authors.

From the above two equations, stiffness of the double acting damped pneumatic spring is shown to be

\[
K = \frac{2n_p A^2}{V_c} \left[ \frac{1}{1 + \frac{V_t}{V_c}} \right] \left[ \frac{1}{1 + \frac{V_t}{n_v p_i}} \right]
\]

where \( n \) = adiabatic constant.

\( p_i \) = mean pressure in the two cylinders.
\[ V_t, V_c = \text{tank and cylinder volumes respectively.} \]
\[ C_r = \text{Capillary constant.} \]
\[ D = \frac{d}{dt} \text{operator.} \]

For sinusoidal input motion, transmissibility is shown to be

\[
T = \left( 1 + \frac{2 \frac{C}{C_c} \frac{w}{w_n}}{\left( 1 + \frac{2}{N+1} \frac{w^2}{w_n^2} \right) + \frac{2 \frac{C}{C_c} \frac{w}{w_n}^2 \left( 1 - \frac{1}{N} \frac{w^2}{w_n^2} \right)}{1 + \frac{2}{N+1} \frac{w^2}{w_n^2}} } \right)^{1/2}
\]

where
\[
\frac{V_t}{ncr \rho_l} = \frac{2C}{C_c w_n}, \quad K = \frac{2np_i A^2}{V_t}, \quad \omega_n^2 = \frac{K}{m}, \quad \frac{V_t}{V_c} = N
\]

2.11.3 For very low frequency motion, the pressure developed when the piston is displaced from its equilibrium position is a function of the combined volume of the cylinder and tank. The stiffness expression then becomes,

\[
K_0 = \frac{2 np_i A^2}{V_c + V_t}
\]

For high frequencies the tank is cut off,

\[
K_\infty = \frac{2 np_i A^2}{V_c}
\]

For frequencies less than 3 Hz process is almost isothermal. When the passage between the tank and cylinder is large

\[
C_r = \infty, \quad \frac{C}{C_c} = 0
\]
For \( C_r = 0 \), \( C/C_c = \infty \).

\[
\omega_0 = \left( \frac{2np_iA^2}{(Vt+V_c)m} \right)^{1/2} = \omega_n\sqrt{\frac{N}{N+1}}
\]

Both these cases can give resonance conditions. Where the transmissibility curves for these two natural frequencies cut each other is a point through which all curves with values of \( C_r \) between zero and infinity must pass. This frequency is given by the expression

\[
\omega_c = \omega_n \left( \frac{2N}{2+N} \right)^{1/2}
\]

2.12.1 Soliman and Ardabili [11] presented fundamental analysis to show that by introducing capillary resistance between the pneumatic spring and receiver, air suspension unit will inherently have a 'dual' natural frequency. Thus it is capable of providing optimum attenuation of both continuous disturbing forces and shock loading. This damping is called as "Transient pressure feed back".

Stiffness \( K = \frac{d(pqA)}{d6} = Ai\left(\frac{\partial P}{\partial 6}\right)_l + Pq_i \left(\frac{\partial A}{\partial 6}\right)\)

\[
\frac{\partial P}{\partial 6} = P_iV_i\cdot \frac{n}{V_i^{n+1}} \left(\frac{\partial V}{\partial 6}\right)
\]

\( \delta \) is the relative displ. of their isolator.

Assuming \( V_1 = V \) and \( \frac{\partial V}{\partial 6} \) and \( \frac{\partial V}{\partial 6} \) vi
taking rubber stiffness $K_r$ into account.

\[
K = \frac{n p_1 A_l}{V_1} \left( \frac{\partial V_i}{\partial \delta} \right) p g l \left( \frac{\partial A}{\partial \delta} \right) + K_r
\]

This stiffness is represented as three springs in parallel.

Rate of change of pressure for the isolator is derived as

\[
\frac{d p}{d \delta} = \frac{n p_i \left( \frac{\partial V_i}{\partial \delta} \right)}{V_i \left[ (1 + \Pi v) + \Pi v \Pi T D \right]}
\]

where $\Pi r = \frac{V_r}{V_i}$ and $\Pi l = \frac{V_i}{n B P l}$

\[
B = \frac{W t^3}{12 l^2}
\]

capillary resistance.

Hence 'Mechanical Impedance' of the isolator works out to be

\[
Z^D = \frac{d F}{d \delta} = \frac{n A l p_i \left( \frac{\partial V_i}{\partial \delta} \right) (1 + j \Pi v + j w \Pi r \Pi f)}{V_i \left( 1 + \Pi v + j w \Pi r \Pi f \right)} + p g l \left( \frac{\partial A l}{\partial \delta} \right) + K_r
\]

\[
= \frac{Z_2 (1 + 2 j \Pi r \Pi r w/\omega_0)}{(1 + \Pi v) + 2 j \Pi v \Pi v w/\omega_0} + Z_r + Z_1
\]

When $B$ is large it is a case of infinite damping and $T_\Pi = 0$. When the cross-sectional area is very small it is a case of zero damping.

Following notation is used.

Reference frequency $\omega_0 = \left( \frac{Z_r + Z_1}{m} \right)^{1/2}$

Reference damping $C_0 = 2 N m \omega_0$

where $N = \frac{Z_2}{Z_1 + Z_1}$
Coefficient of damping \( C = \frac{Z^D_2}{T_i} \)

Damping ratio

\[ \eta = \frac{C}{C_0} = \frac{1}{2} \frac{T_i}{\omega_0} \]

It is shown that the effect of surge tank is to add an impedance \( Z^D_3 \) in series to impedance \( Z^D_2 \) of the undamped system.

\[ Z^D_3 \approx Z^D_2 \left( \frac{1}{T_i} + j\omega T_i \right) \]

This impedance \( Z^D_3 \) can further be split up as an impedance \( Z^D_2 \) in parallel with a viscous damper of having damping coefficient \( jZ^D_2 \omega_0 \).

Entire system is therefore modelled as shown in the figure 2.7 which also shows the effect of frequency on amplitude.

Fig 2.7

2.12.2 For very low frequency vibration the pressure developed in the isolator is the function of the combined volumes of the pneumatic isolator and the surge tank. This is equivalent to infinite value of \( B \) and damping is very small. However why damping is small is not clear.

At high frequencies \( B = 0 \), and again damping is very small.
Hence damping is effective only over a range of frequencies. Resonance can occur at these two extreme values of B thus indicating the dual frequency character of the system.

Absolute transmissibility \( T = \frac{Y_0}{u_0} \) for these two cases is given as

\[
T_{A_0} = \frac{(1 + \Pi v + N)}{(1 + \Pi v + N) - \left(1 + \Pi v \right) \left(\frac{w}{w_0}\right)^2}
\]

\[
T_{A_\infty} = \frac{1 + N}{(1 + N) - \left(\frac{w}{w_\infty}\right)^2}
\]

thus indicating two resonance frequencies.

As shown all curves must pass through one point. This has been called 'Common Transmissibility Frequency' \( \omega_c \)

\[
\left(\frac{\omega_c}{\omega_0}\right)_A = \left(\frac{1 + N}{(1 + N + \Pi v) + N/2 \Pi v}\right)^{1/2}
\]

This indicates that for an optimum system resonant frequency should coincide with \( \omega_L \). This gives,

\[
\omega_{opt} = \left[\frac{(1 + \Pi v + N)^2}{(1 + \Pi v + N) + 3/4 \Pi v N} \left(1 + \Pi v \right) \left(\frac{w}{w_0}\right)^2\right]^{1/2}
\]

\[
\Pi_{opt} = \left[\left(1 + \Pi v \right) \left(1 + \Pi v + N + 3/4 \Pi v N\right)\right]^{1/2} / \left[2 (1 + N) \Pi v\right]
\]

\[
(T OP L)_{res} = \frac{2 (1 + \Pi v + N)^{1/2} \Pi v}{N \Pi v}
\]

2.12.3. From the graphs given in the paper, it is seen that transmissibility reduces with stiffness ratio N and volume ratio \( \Pi v \). However volume ratios greater than
10 do not have much effect on further transmissibility reduction. Optimum damping ratio reduces with stiffness and volume ratio.

Excellent agreement exists between the analytical and experimental results. Experimental data are:

\[ V_1 = 790 \text{ in}^3, \quad V = 30 \text{ in}^3, \quad P = 12 \text{ psig}, \quad m = 124 \text{ lb}. \]
\[ N = 0.69, \quad (\frac{\partial A}{\partial y})_l = 11.9 \text{ in}, \quad \frac{\partial V_1}{\partial y} = 11 \text{ in}^2, \quad K_f = 60 \text{ lbf/in}. \]

Compression at low frequencies was not found strictly adiabatic. Dual frequency was noted.

Selection of parameter 'N' is not suitable to deal with cases of constant area.

2.13.1 Gee-Clough and Waller have described an improved 'Self-Damped Pneumatic Isolator' in \([12]\). A system is described which uses the fact that low frequency pressure fluctuations are transmitted more readily through a capillary tube than high frequency fluctuations. Isolator properties are modified in such a way that its isolation characteristics for both shock forces and steady harmonic forces through out the frequency range are very good.

In the conventional systems the measures which have to be taken to avoid resonance and reduce peak transmissibility are precisely those which lead to poor isolation against shock forces and higher transmissibility in the high frequency zone.

A dual-frequency system having two different effective stiffnesses can be used to avoid limitations of conventional system. Transmissibility curve for dual-frequency system is as shown in fig. 2.8.

At low frequencies of operation the high stiffness spring is employed. At the switch-over frequency the stiffness of the spring is switched to its lower value. Thus resonance is avoided and peak transmissibility is reduced to nearly one. However
the system needs a switching device. Switching device is avoided by using a surge tank through a capillary resistance to a conventional pneumatic isolator. But the dual-frequency properties of this system are the wrong way around, since it has a low effective stiffness at low frequencies and high effective stiffness at high frequencies.

Transmissibility of “dual frequency” isolator with no damping.

Fig 2.8

2.13.2 A system described in this paper to avoid these shortcomings consists of two pneumatic isolators connected by a capillary tube. Upper isolator supports the machine to be isolated while the lower isolator is grounded. In between two blocks is inertia mass. Additional springs are used to support inertia block. (Figure 2.9)
Two isolators are identical. Hence in static case entire weight of the inertia block is carried by springs while machine is supported by two isolators in series.

At low frequencies pressure fluctuations in top isolator are immediately carried to the lower isolator. Hence pressure fluctuations do not affect the inertia-block. Two isolators act in series and system behaves as a single degree of freedom system.

At very high frequencies the capillary tube is essentially choked and pressure fluctuations are not transmitted to lower isolator. The system now behaves as a two-degree-of-freedom system.

Generally two natural frequencies of the two-degrees-of-freedom system will cover the natural frequency of the single-degree-of-freedom system. Figure shows transmissibility curves for two cases i.e. for single-degree-of-freedom system (zero damping in capillary) and two-degrees-of-freedom system (infinite damping, capillary choked). For any actual system transmissibility curves will lie somewhere in the shaded area. (Figure 2.10)
Damping factor is defined as
\[
\xi = \frac{\omega_n t_1}{2}, \quad t_1 = \frac{V_{20}}{\nu B P}
\]

For zero damping \( \xi = 0 \),
\[
B = \text{Capillary Constant.}
\]
\( P = \text{Initial Pressure.} \)

\[
T_1 = \left( \frac{1}{(1 - \frac{\omega_n^2}{\omega_p^2})} \right)^{1/2}
\]

and
\[
\omega_n^2 = \frac{\nu p_{10} C^2}{(V_{10} + V_{20}) M^2}
\]

For \( \xi = \infty \),
\[
\omega_n^2 = \frac{\nu p_{10} C^2}{V_{10} (1 + V) M^2}
\]
\[ T_2 = \left( \frac{(C+V)[M^{-2}(1+V)+V]^2}{[V^{-2}(w/w_n)^4 \cdot (V^{-2}(1+V)+V+M^{-2}(1+V)^2)(w/w_n)^2 + (M^{-2}(1+V)+V(1+V))^2]} \right)^{1/2} \]

where \( M = \frac{M_2}{M_1} \), \( V = \frac{V_2}{V_1} \), \( w_n^2 = \frac{K}{M_1} \), \( w_n^2 = \frac{w_n^2}{w_n^1} \)

At high frequencies,

\[ T_1 \approx \frac{1}{(w/w_n)^2} \]

\[ T_2 \approx \frac{(1+V)[M^{-2}(1+V)+V]}{V^{-2}(w/w_n)^4} \]

\[ T \approx \frac{V^{-2}(w/w_n)^2}{(1+V)(M^{-2}(1+V)+V)} \]

With increase in value of \( V \), higher natural frequency increases, while the lower decreases. With \( M \) both the frequencies increase while with \( \infty \) lower frequency decreases and higher frequency remains constant. Optimum conditions are approached if the parameters are so chosen that the highest natural frequency of the two-degree-of-freedom system is only slightly higher than the frequency of one-degree-of-freedom system. Lower of the two frequencies should be as low as possible.

Absolute transmissibility expression is given as

\[ T = \left( \frac{[V(1+V)(1-%})^{2} \cdot (w/w_n)^2]^{2} + [2(C/w_n)(1+V)(M^{-2}(1+V)+V)] \left( (1-(w/w_n)^2) \right]}{(1-(w/w_n)^2) \cdot (V(1+V)(1-%})^{2} \cdot (w/w_n)^2 \right) \left( 2 \cdot (w/w_n)(V(1-%})^{2} \right) \left( (1-(w/w_n)^2) \right)^{2} \right)^{1/2} \]

2.13.3 It is seen that to lower the transmissibility
when \( \omega/\omega_n = 1/\sqrt{n} \) needs large value of \( n^2 \) and a small value of \( V \), while at \( \omega/\omega_n = 1 \) to reduce transmissibility needs a larger value of \( V \). Thus a compromise is necessary. It is observed that high values of \( n^2 \) increases isolation at high frequency which is contradictory to the effect of damping in a single degree of freedom system.

Graphs showing the effects of \( V \), \( M \) and \( n^2 \) on the transmissibility have been presented in the paper. High value of \( V \) decrease transmissibility but also increases higher of the two natural frequencies. Thus again a compromise is demanded. \( M \) and \( n^2 \) have very little effect on high frequency transmissibility. In low frequency region low value of \( M \) and high value of \( n^2 \) is desirable. For a particular values of \( V \) and \( M \) there is a value of \( n^2 \) above which the system characteristics are hardly altered at all.

2.14.1 Reference [13] deals with the optimization of Pneumatic Vibration Isolation System for Vehicle Suspension. A Two-Degrees of Freedom linear system has been considered. Mathematical model is as shown in the figure 2.11.

![Mathematical model of a vehicle suspension system](image-url)
Expression for self-damped pneumatic isolator stiffness has been derived and is stated as

\[
K_e = \frac{kq + kr + kd}{1 + N} \left[ \frac{(1+N)(1/N + tD) + 1}{1 + N + tD} \right]
\]

where \( t = \frac{d}{dt} \)

\[
K_a = Pq \frac{\Phi(A) \frac{\partial}{\partial h}}{\beta} \frac{\partial A}{\partial h}
\]

\[
K_d = \frac{n \frac{V_j}{V_c}}{v} \frac{\partial V}{\partial n}
\]

\[
K_r = \text{rubber stiffness of isolator.}
\]

\[
\Pi_V = \frac{V_{\text{tank}}}{V_{\text{isolator}}}
\]

\[
N = \frac{kd}{kq + kr} \quad \frac{\Pi}{1 + N} = \frac{V_i}{n \frac{V_j}{V_c}}
\]

It can be noted that this is the same expression as given in reference [11].

2.14.2 With this value of isolator stiffness and assuming sinusoidal excitation (\( z D = j \omega \)) expressions for body transmissibility, wheel transmissibility and relative transmissibility between the body and the wheels have been derived. Effect of tire damping is neglected.

Two undamped natural frequencies for the case \( \Pi r \) (i.e. when passage between the isolator and the surge tank is made very large) are found to be

\[
\frac{\omega_n^2}{\omega_0^2} = \left[ \frac{B_0^2 (1 + N + \Pi V + N) + \mu \tau^2 (1 + N) (1 + \Pi V)}{2 \mu (1 + N) (1 + \Pi V)} \right]^{1/2}
\]

where

\[
B_0 : (1 + \mu) (1 + \Pi V + N) + \mu \tau^2 (1 + N) (1 + \Pi V)
\]

\[
\alpha = M_2 / M_1 \quad \tau = \frac{\omega_n^2}{\omega_0} \quad \omega_n^2 = (k/M_2)^{1/2}
\]
Transmissibility curves for two extreme values of damping (\( \omega_0/2 \)) zero, infinity intersect at three points, first point corresponding to \( \omega\omega_0 = 0.777 \). Frequency of those points through which all transmissibility curves will pass has been called as invariant frequency (Figure 2.12).

\[
\omega_0 = \left[ \frac{kr + ka + kd}{M_1} \right]^{1/2}
\]

First invariant frequency ratio is given by
\[
\omega/\omega_0 = \left( \frac{Q \sqrt{Q^2 - 4BZ}}{2B} \right)
\]

where
\[
B = 1 + N + \Pi v + N \Pi v / 2
\]
\[
Q = (1 + N + \Pi v) (1 + 1/\mu) + B^2
\]
\[
Z = (1 + N + \Pi v)^2
\]

This result can be compared with the dual frequency character mentioned in reference [11]. There for a single-degree of freedom system, system had one invariant frequency while here there are three invariant frequencies for a 2 DOF system.

Absolute transmissibility of the body is a function of the following parameters:

- Mass ratio,
- Input frequency ratio,
- Resonant frequency ratio,
- Damping ratio of the pneumatic isolator,
- Stiffness ratio \( N \),
- Volume ratio \( \Pi v \),
- Tire damping ratio \( \eta \).

Effect of \( \eta \) is very small generally.
2.14.3 Optimization has been carried out on digital computer by changing a particular variable. Flow chart of the process has been given. Optimization consists of finding values of N, \( \Pi_V \) & \( \mu \) so as to make equal resonant transmissibilities at three invariant frequencies.

Optimum resonant transmissibility decreases with the increase in stiffness ratio N, increases with the mass ratio \( \mu \) and decreases with increasing volume ratio \( \Pi_V \). However larger values of \( \Pi_V \) have less effect in decreasing transmissibility & similar behaviour is with N. Optimum value of resonant frequency ratio \( \kappa \) decreases with mass ratio \( \mu \) but N has very little effect on \( \kappa \). Optimum damping ratio \( \xi \) decreases with \( \mu \) but increases with N.

Analysis has been verified by experiments and experimental results are in good agreement with theory. Three common points, of invariant frequency are clearly seen.

2.15.1 In \([14]\) Esmailzadeh has given a design procedure for Pneumatic Isolators for road vehicles which can be thought as continuation to his work given in \([13]\). A smaller surge tank is replaced by a variable volume pneumatic isolator. Isolators are connected by a capillary. Figure 2.13 shows a line diagram as suggested for a vehicle.

Expressions for the stiffness of each isolators have been derived. Derivation is similar to the one given in \([15]\). Using these stiffness values equations for absolute values of transmissibilities of two masses (body and wheels) are derived. Expression for relative transmissibility between the vehicle body and wheels is also given and it is shown that it depends on mass ratio and resonant frequency ratio.
2.15.2 Expression for the resonant frequencies for closed capillary (infinite flow resistance) is shown to be

\[
\left( \frac{\omega}{\omega_n} \right)_{12}^{1/2} = \left[ \frac{B + Q}{2} \right]^{1/2}
\]

where

\[
B = 1 + \frac{2}{\nu} + \frac{2}{\mu}, \quad Q = (\nu^2 + \frac{1}{\nu})^{1/2}
\]

\( \mu \) = Mass ratio of wheel to body
\( \nu \) = resonant frequency ratio
\( \varphi \) = \text{resonant frequency of lower isolator}
\( \psi \) = \text{resonant frequency of main isolator}
In this case absolute transmissibility of the body never becomes zero but absolute transmissibility of wheels becomes zero at antiresonance.

Similarly for zero damping (when capillary cross section is very high) expressions for resonant frequencies are given and it is shown that absolute transmissibility of the body becomes zero when

$$\frac{w}{w_{n1}} = \left[ \frac{2+(2+N) / N}{(2+(2+N) / N)} \right]^{1/2}$$

where $N = \text{stiffness ratio of isolator} = \frac{K_d}{(K_a + K_r)}$

This is an interesting result in the sense that for a spring-mass two degrees of freedom system absolute transmissibility will never become zero. Similarly antiresonance frequency for wheels is given by

$$\text{war} = \left[ \frac{2+(2+N) / N}{2(1+\mu^2)(1+N)} \right]^{1/2}$$

Graphs showing variation of transmissibilities with other parameters viz. damping, mass ratio, resonant frequency ratio and frequency (excitation) are given.

Experimental set up has been described. Good correlation between the theoretical and experimental results is obtained particularly at low excitation frequencies. At high frequencies very little damping is observed.

2.15.3 An optimization computer routine is given. It has been found that stiffness ratio $N$ does not have any optimum value but larger $N$ gives better isolation. Following observations can be made from the graphs given.

1. Optimum damping ratio decreases with mass ratio.
2. Optimum frequency ratio $\gamma$ decreases with mass ratio.

3. Minimum resonant transmissibility decreases with stiffness ratio and increases slightly with mass ratio.

It is concluded that very good isolation can be achieved by a compact self-damped pneumatic isolator over a broad range of excitation frequency. Isolation of delicate instruments can be achieved by this arrangement. For vehicles lower variable volume pneumatic isolator which is secured to the wheel is in contact with the road and acts as a pressure sensor. Any relative motion between the wheel and the road is picked up by the sensor and signalled in the form of transient pressure feed back to top isolator.

However practical application of this device looks difficult.

2.16.1 In [15] Esmailzadeh has analysed 'servovalve-controlled pneumatic suspensions' which is an active vibration isolator. A self-damped pneumatic isolator coupled to a servovalve is considered as a suspension system for a ground vehicle as shown in the figure 2.14.

The results computed from the dynamic analysis of the servo-controlled pneumatic isolator are compared with those obtained experimentally.

2.16.2. Dynamic analysis is mainly based on the following two equations. Equation of mass rate flow from the isolator to surge tank

\[
\Delta m = \frac{1}{RT} Pq D(\Delta V) \frac{1}{nRT} Vq D(\Delta p) + \frac{v_t}{nRT} D(\Delta p) \left( 1 + \frac{v_t D}{n pq cr} \right)
\]
where $\Delta$ denotes small changes in the quantities, $n$ is adiabatic constant. $C_r$ is capillary coefficient. $q$ denotes quiescent stage and $D$ is operator $d/dt$.

Second equation is for the flow of gas through a servovalve.

Combining these two equations rate of change of pressure in the isolator with respect to change of its height is shown to be

$$\frac{\Delta p}{\Delta h} = \frac{s_{d}D/Aq + ky(\Delta x/\Delta h)(1+\tau_{n}D)}{(1+\tau_{d})(1+\tau_{n}D)+\Pi_{v}LD}$$

where $s_{d}$ is stiffness due to volume change.

$\tau_{d}$ Spool valve time constant.

$Aq$ area of isolator.

$ky$ mass rate flow constant of valve.

$\tau_{n}$ time constant for capillary.

$\Pi_{v}$ Volume ratio of tank to isolator.

$x$ is spool displacement.
Similar equation is given for the rate \( \Delta p/\Delta x \).

Stiffness of the isolator is considered as composed of three parts, viz. due to change in effective area, change in pressure and rubber stiffness.

2.16.3 Relations between sprung mass and unsprung mass displacement for a constant spool valve displacement and between isolator and valve displacement for a constant unsprung mass displacement have been derived. These define the dynamic behaviour of the servovalve controlled, self damped pneumatic isolator.

The system has been analysed as a closed loop system under derivative and proportional feed back control. Expression for the closed loop transfer function i.e. for the ratio of sprung mass and unsprung mass displacement is given.

Since the input energy increases with the increase in forcing frequency the force generated by the servovalve must be proportional to the disturbing frequency that is the feed back signal from the sprung mass must be proportional to the time derivative of the displacement of the sprung mass. This is achieved by derivative feed back. This derivative feed back increases resonance frequency due to increased damping but does not impair the isolator effectiveness at high frequencies.

Proportional feed back is necessary for isolation at lower disturbing frequencies. This is done by sensing the displacement of the sprung mass.

Increase in the derivative feedback gain improve the response of the system over the entire frequency range while proportional feedback gives improvement only in the lower frequency range.
Expressions for the absolute transmissibility of sprung and unsprung mass have been given. It is shown that resonant transmissibility increases with proportional feed back.

Experimental set up has been described and good correlation between the experimental results and theoretical linearized analysis is obtained. Analogue computer simulation has been done to take care of non-linearities in the system.

It is concluded that active isolator is very effective at all frequencies including resonant frequency.

2.17.1 An interesting application of airsprings is described by Waller and Benham in [16]. A fatigue machine mounted on airsprings has been installed at Imperical College, London. Not only did the machine have a wide range of operating speeds but there was also a requirement for the maximum practicable insulation from single shocks generated during static testing as well as vibrations. Airsprings also provided for automatic levelling. Machine works in two ranges of frequencies 160 to 1000 cpm upto 20 ton load and 5 to 100 cpm upto 40 ton load.

In order to provide the maximum isolation from shock loads when the specimen is fractured it was decided not to include any additional damping devices. Considering the operational speed ranges the mounting was designed to have two stiffness characteristics; softer for the higher speed range and somewhat stiffer one to avoid resonance with the low speed range but which will still cushion the impact loading.

Standard lorry trailer air springs were used. Springs were connected by 2 in diameter pipe to mild steel receivers which when connected reduced the frequency from 180 cpm to 110 cpm. A butterfly valve was used in the pipeline for this purpose. Additional horizontal springs were provided by cantilever plates. Levelling valves are provided to maintain foundation level between ± 1/16 in. Records show that the floor is almost completely isolated from the machine shocks and vibrations.
2.18.1 Paper by Bachrach & Rivin [17] deals with the effect of design parameters on spring stiffness, damping and frequency response for a single sided damped pneumatic spring. As the authors point out they have concentrated on component performance rather than the conventional studies based on spring-mass-damper systems.

Model consists of a piston moving inside the cylinder and the cylinder is connected to a surge tank through a capillary flow resistance. Derivation is based on two basic equations viz.

1. Mass flow in a cylinder

\[
\frac{dm}{dt} = \frac{1}{Rl} \left[ \frac{V}{n} \frac{dp}{dt} + p \frac{dv}{dt} \right]
\]


\[
\frac{dm}{dt} = \frac{ncr}{Rl} (p_1 - p_2)
\]

where \( p_1 \) & \( p_2 \) are pressures on either side of capillary and \( p_i = (p_1 + p_2) / 2 \) initial pressure.

Using the notation \( K_c = npis^2 / V_c \) = stiffness of airspring without auxiliary tank.

\[
N = \frac{V_t}{V_c} \text{ volume ratio of tank to cylinder, } \omega_0 = \frac{ncr p_i}{V_c}
\]

and using operator notation \( D = j\omega \) for sinusoidal forcing function stiffness of airspring (self damped) is shown to be

\[
K = K_c \frac{1 + N(j\omega / \omega_0)}{1 + N + N(j\omega / \omega_0)}
\]

Author has expressed \( K \) in terms of \( N \) and \( C_r \) to bring out their effect.

Loss factor \( \eta \) has been defined as \( K_{\text{real}} / K_{\text{imaginary}} \) and it is shown that \( \eta \) max \( \approx \frac{2N}{\sqrt{N+1}} \) at \( \frac{\omega}{\omega_0} = \frac{\sqrt{N+1}}{N} \).
i.e. maximum loss factor is a function of $N$, volume ratio and $\omega_0$, which itself depends on capillary coefficient $C$, stiffness at maximum loss factor is shown to be

$$\frac{K_I}{K_C} = \frac{1}{\sqrt{N+1}} = \frac{1}{N(\omega/\omega_0)}$$

Locus of $\eta_{\text{max}}$ is given as $1/2(\omega/\omega_0)$ and maximum stiffness occurs when $\omega/\omega_0 = \infty$ and its magnitude is $|K| = K_C$

Minimum stiffness occurs at $\omega/\omega_0 = 0$ and its magnitude is $|K| = K_C$

2.18.2 Authors have also presented the analysis of a compound spring where a coil spring of stiffness $K_s$ is in parallel with the airspring as described above. Expressions for the loss factor and total stiffness have been given. For the case when $K_s >> K_c$

$$\eta_{\text{max}} = \frac{kC}{2Ks} \frac{N}{N+1} \quad \text{when } \omega/\omega_0 \sim \frac{N+1}{N} \quad \therefore \eta_{\text{max}} \leq \frac{kC}{2ks}$$

It can be seen that for $K_s >> K_c$, $\eta_{\text{max}}$ does not depend on the volume of auxiliary tank and for $N > 2$, the frequency at which $\eta_{\text{max}}$ occurs is independent of $N$. $\eta_{\text{max}}/K_c$ reaches an asymptotic value of 0.5 as $N$ increases. Thus a compound spring behaves quite differently from the simple air spring.

2.19.1 Literature so far reviewed was concerned with airspring as used in steady state vibration isolation. Hundal in his paper [18, 19] has paid attention to airsprings mainly as shock isolators. Shock problems can be divided in two groups: (i) transient displacement of base and (ii) transient forces within the equipment. Reference [18] deals with the first type of shock considering equipment as a rigid body.

...44/-
A pneumatic damper in parallel with a linear spring is considered as shown in the figure 2.15.

**SYMmetric Passive Pneumatic Shock Isolator System.**

Following two inputs have been considered for the base.

\[ \ddot{u} = A \sin\left(\frac{\pi t}{t_f}\right), \quad t < t_f \quad \text{(half sine pulse)} \]
\[ \ddot{u} = A, \quad t < t_f \quad \text{(rectangular pulse)} \]
\[ \ddot{u} = 0, \quad t > t_f \]

Initial conditions are all zero and adiabatic process is assumed. Equation of state, adiabatic expansion and conservation of mass equation are used in the derivation.

For mass flow-through orifice equations used are

\[ m \bar{d} \co c_3 \left(\frac{\bar{p}_d}{\bar{p}_u}\right)^n [1 - (\frac{\bar{p}_d}{\bar{p}_u})^{n-1}]^{1/2} \frac{\bar{p}_u}{\sqrt{\vartheta u}} \text{ for } \frac{\bar{p}_u}{\bar{p}_d} < 1.894 \]
\[ = \co a c_2 \frac{\bar{p}_u}{\sqrt{\vartheta u}} \text{ for } \frac{\bar{p}_u}{\bar{p}_d} > 1.894 \]

where \( a \) is orifice area, \( \co \) is orifice coefficient, subscripts \( d \) and \( u \) denote downstream and upstream conditions. \( \vartheta \) is temp.
\[ C_2 = \left[ \frac{n}{R \left( \frac{n+1}{2} \right)^{\frac{n+1}{n-1}}} \right]^{\frac{1}{2}} \]
\[ C_3 = \left[ \frac{n}{R \left( n-1 \right)} \right]^{\frac{1}{2}} \]

'\( n \)' is adiabatic constant.

Equations of motion along with the above mentioned equations are transferred into dimensionless quantities to get more general solutions rather than the specific ones.

Equations being nonlinear fourth order Runge-Kutta scheme is employed in the solution. For identical response in either directions, initially piston should be in the middle positions.

In the absence of the damper, ratio of mass acceleration to input acceleration for constant input acceleration works out to be 2 as maximum. Hence only those parameter values which reduce this ratio to less than 2 are considered.

2.19.3 For constant input accelerations it is seen that
1) Output acceleration is a strong function of \( K \) and \( S \) but a weak function of \( M \). Here
   \[
   K = \frac{K \ell}{\text{post}} \quad S \text{ is piston area.}
   \]
   \[
   S = c_0 c_2 \frac{Ra}{\sqrt{\text{IA}}} \quad M = \frac{mA}{P_0 b} \quad A \text{ is input acceleration.}
   \]
2) Maximum acceleration of mass approaches the value of input acceleration as \( K \) approaches zero.
3) Minimum value of output to input acceleration ratio occurs when \( S \) is in the range of 0.4 to 0.5.
4) Minimum value of acceleration ratio can be given as
   \[
   \min(X_{\text{max}}) = 1.03 + 0.483K - 0.117K^2 + 0.0103K^3
   \]
   in the range 0.1 < \( K \) < 0.5.
2.19.4 For rectangular pulse it is observed that maximum value of the acceleration ratio increases monotonically with pulse duration till it reaches a constant value. M does have an effect in this case on maximum displacement. For half-sine pulse after first maximum curve exhibits a minimum at acceleration ratio of unity and approaches this value as pulse duration increases.

Following design procedure is suggested:

1. Mass and base acceleration are given.
2. From maximum allowable cylinder diameter find piston area.
3. Take $K \geq 3M$.
4. If this gives higher acceleration use larger pressure and piston diameter.
5. Find stiffness of spring and length from value $K$ finally fixed.
6. Find orifice area from parameter $S$ since $S$ is to be between 0.4 to 0.5 for best results.

2.20.1 In [19] Hundal has extended his analysis [18] of 'damped-pneumatic-spring as shock isolator' to the case when the shock input is a constant velocity pulse such that

\[ \dot{u} = u_0 \quad \text{for} \quad 0 \leq t \leq t_0 \]
\[ = 0 \quad \text{for} \quad t > t_0 \]

Model is similar to the one in [18] except that the linear spring is absent. Rest of the solution follows the same procedure and solution is obtained in dimensionless variables numerically.

Undamped spring is obtained by making orifice area equal to piston area. Solution shows that maximum piston displacement at the end of the expansion stroke occurs at $t = t_0$. The maximum payload acceleration is at the end of compression stroke and its value depends on pulse duration and mass.

For small values of $T_0 = \frac{u_0 t_0}{L}$ which is a...
case of shock loading, analytical solution is given with certain approximations. Dimensionless displacement $D = d/L$ at the end of compression stroke is given by the equation.

$$\left(1 + D \frac{n}{m}\right)^{\frac{1}{n}} = \left(1 + T_0 \right)^{\frac{1}{n}} + \left(1 - 1\right) \left(D \frac{m}{n} - T_0 \right) = 0$$

By 'least-square-fit' technique, relation can be expressed as

$$MX = 0.001 + 1.38 T + 0.62 T^2$$

This shows that to minimize acceleration, $T_0$ must be made minimum.

2.21.1 In [20] Hundal has given a literature review of 'Pneumatic Shock Absorbers & Isolators'. Six papers on the subject are reviewed. This shows that very little work has been done in the area. Most of the papers reviewed have also been reviewed in the present work.

2.21.2 In [21] Eshleman and Rao have discussed 'The Response of Mechanical Shock Isolation Elements to High Rate Input Loading'. Following elements are studied:-

1. Helical Coil Spring.
2. Ring Spring.
3. Friction Snubber
4. Liquid Spring.
5. Pneumatic Spring.

Isolators were modelled as massless, flexible damped elements to study the gross response of the isolated mass to high rate input and also as distributed mass and elasticity elements to predict surging of isolators.

Local response of the shock isolator system was calculated numerically with time varying boundary conditions and using central difference equations.

Two pneumatic springs in parallel were tested...
at 60 psi with the 960-lb mass. Length of the spring was 11.25 in. Following data was obtained for this spring.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring constant</td>
<td>270 lb/in.</td>
</tr>
<tr>
<td>Viscous damping</td>
<td>4.7 lb-sec/in</td>
</tr>
<tr>
<td>Coulomb damping</td>
<td>3 lb</td>
</tr>
<tr>
<td>Input displacement</td>
<td>5.04 in</td>
</tr>
<tr>
<td>Duration</td>
<td>285 µsec.</td>
</tr>
</tbody>
</table>

A 138g input acceleration induced a 0.76g acceleration in the isolated mass thus giving 99% isolation. No surge loading of the isolator was observed. Good agreement between analytical and experimental results was obtained.

It was noted that non-linearity was important but this spring was not analysed as distributed mass system.

2.22.1 In [22] Fox and Steiner have analysed the 'Transient Response of Passive Pneumatic Isolators'. Schematic Isolator is shown in the figure 2.16. It is a pendulum type pneumatic isolator. An orifice plate separates the load chamber from the damping chamber. Volume of the damping chamber is constant. Analysis is based on the following two equations. First equation is the mass-flow rate equation.

$$ \frac{dn}{dt} = \frac{1}{RT} \left( P \frac{dv}{dt} + \frac{v}{K} \frac{dp}{dt} \right) $$

where $n =$ moles of gas.

$K =$ adiabatic constant.

This equation is applied to both load and damping chamber. Second equation is for the flow of gas through an orifice of cross-sectional area $S.$
\[
\frac{dn}{dt} = \frac{q_2^5}{a m} \left[ 2 P_1 q_1 \left( \frac{k}{k-1} \right) J \left( \frac{P_2}{P_1} \right) \right]^{1/2}
\]

where \( J \left( \frac{P_2}{P_1} \right) = \left( \frac{P_2}{P_1} \right)^{2/k} - \left( \frac{P_2}{P_2^*} \right)^{k+1} \)

\( q_1 \) and \( q_2 \) are gas densities at pressures \( P_1 \) and \( P_2 \) such that \( P_1 \) is greater than \( P_2 \). \( Gm \) is mass per mole of the gas.

It can be seen that author has used a different equation for mass-flow as all other authors have used a capillary flow equation.

2.22.2 Setting \( Y = \frac{dx}{dt} \), a system of first order equations is formed.

\[
\begin{align*}
\frac{dY}{dt} &= \frac{P_L A}{m} - g \\
\frac{dX}{dt} &= Y \\
\frac{dP_D}{dt} &= \frac{P_L K}{n_o - n_L} \frac{dn_D}{dt} \\
\frac{dn_L}{dt} &= \left( \pm 1 \right) q_2^5 \left( 2 P_1 q_1 (K/K-1) J \left( \frac{P_2}{P_1} \right) \right)^{1/2} \\
\frac{dP_L}{dt} &= \frac{P_L K r (\frac{dn_L}{dt} - \frac{n_L A(U-Y)}{V_o + A(U-X)})}{n_L} 
\end{align*}
\]

Equations are solved by using Runge-Kutta fourth order integration scheme. For convergence of solution integration increment was reduced to 1 millisecond. This required further reduction for higher orifice areas.

A constant velocity \( U \) was assumed for \( 0 \leq t \leq 0.1 \)

...50/—
and then the velocity is zero. Thus a step function with finite initial gradient is assumed.

For undamped system pressure and displacement are in phase. But with damping due to orifice a phase difference occurs. Frequency and damping vary with displacement. As displacement approaches zero, damping also approaches zero and frequency is that associated with a large orifice.

2.22.3 Experiments are conducted for 12 different orifice area and for each area 4 different volume ratios are considered. Peak Acceleration decreases with volume ratio $V_D / V_L$. It is minimum for an area ratio $(S/A)$ of nearly 0.0015.

Damping ratio is calculated by

$$\xi = \left[ \frac{2 \pi}{\log \frac{X_2}{X_1}} \right]^2$$

The authors felt reasonably justified in adding 2% critical damping to the calculations after analyzing the time history response of an undamped isolator.

Response characteristics are as shown in the figure 2.17.

These correlate very well with the experimental results. Higher volume ratios give higher frequencies and higher damping.

An ideal property of the pneumatic isolators is their dynamic equivalence with different supported weights. As long as the ratio $P_L A/m$ remains constant, the system will exhibit identical characteristics.
2.23.1 This literature survey will remain incomplete if mention is not made of the report [23] 'The use of Pneumatic Active Suspensions to Improve Lateral Rail Vehicle Ride Quality' published by US Department of Transportation under the 'University Research Program'. Report deals with the pneumatic active suspensions for lateral suspensions in particular but covers almost all aspects of the pneumatic suspension system including passive, semiactive and active systems. Following aspects are covered in the report.

(i) Chapter 1 describes lateral rail suspension system under considerations. Various mechanical details are given.

(ii) Chapter 2 deals with the modelling of active suspension system. Formulae for required compressor work are given. Mass flow through valve is discussed.
Expression for fluid capacitance and mechanical capacitance have been derived taking into account flexibility of spring walls. Capacitance is defined as

\[ \dot{p} = \frac{1}{C} \dot{\dot{W}} \]

(iii) Chapter 3 presents method of simulation of random input. Actual (measured) disturbance is constructed by nine sine waves ranging three octaves from 0.31Hz to 16 Hz of different amplitudes and phases so that their acceleration spectral density matches with the actual measured spectral density.

Various 'Firestone' airsprings and pneumatic cylinders of various diameters are evaluated and compared with ideal actuators for attenuation of vibrations and power requirements. Evaluation is analytical. 4 in bore cylinder gave the best compromise between isolation and compressor power requirements.

Similarly valves with various peak flow capacities were considered. A 40 SCFM gave the best compromise.

(iv) So far analysis was carried out by considering proportional control valve. But for the volume of air to be handled they are very expensive and are not easily available. Hence an on-off controller (solenoid valves) with lead compensation and hysteresis has been proposed.
Control is achieved through a feed-back circuit. A simulation program was written and is solved by Runga-Kutta technique.

'Bang-bang' type pneumatic suspensions are analysed in Chapter 5. It is observed that lateral r.m.s. acceleration could be reduced by increasing velocity or acceleration feed back gains, at the expense of larger r.m.s. stroke. Decreased spring stiffness gave better acceleration response but higher power consumption. Damping coefficient had very little effect on the performance. System performance for various actuator diameter, valve orifice area and opening and closing time delays is considered.

Chapter 6 of the report describes airsprings as semiactive systems. Damping force is controlled by modulation of the damping orifice. An extremely fast orifice modulating device is assumed on an ideal semiactive system. Control strategy is to modulate the damping force to be equal to the force of a skyhook damper. Damper applies force only when

\[ y_c (y_c - y_t) > 0 \]

where \( y_c \) and \( y_t \) represent truck and car velocity respectively. For other cases force is zero or locking force. Control law used for the damper force was

\[ F_d = k y_c \]

where \( k \) is feedback gain. Analysis show that increasing feed back gain increases attenuation at lower frequencies which has adverse effect at higher frequencies. High frequency oscillations are seen in actuating force.
Chapter 7 describes the experimental verification of the simulations discussed so far. Firestone airsprings with solenoid valve is used. Step response of valve-spring system was similar to the first order system. 4 inch cylinder was also tested. Experimental results agreed with the theory.