Chapter-3

Wind Turbine Structural Models

3.1 Wind Turbine Technology

The modern wind turbines work on aerodynamic lift produced by impact of incoming wind on blades. The modeling of wind turbines along with control strategies has been extensively studied as in [5],[7],[8],[18],[56]-[68]. The resulting force due to blade (airfoil) and wind interaction consists of two components i.e. one drag force component parallel to direction of wind flow and the other one perpendicular to the drag component known as lift. The strength of this perpendicular force component is larger than the drag component and helps rotate heavy rotor of wind turbine. These aerodynamic lift based wind turbines are further classified on the basis of spin axis orientation; Vertical axis turbines as shown in Figure 3.1 and Horizontal axis wind turbine (HAWT) in Figure 3.2[7],[18],[28].

![Figure 3.1 Vertical axis wind turbine](image1)

![Figure 3.2 Horizontal axis wind turbine](image2)
In case of vertical axis wind turbine, the rotor shaft is installed vertically along with generators and gearboxes placed near to ground as shown in Figure 3.1 [5],[22],[23],[27],[28]. The horizontal axis wind turbine (HAWT) is shown in Figure 3.2 where the whole energy conversion structure is uplifted in the air on a steel tower. The blade, rotor hub, drive train, generator and pitch angle control are installed inside the nacelle and electronic converter is inside the tower [7]-[11],[22],[23]. Here, turbine rotor/hub is rotating parallel to wind stream due to the aerodynamic lift produced by the pressure difference created by airfoil shaped blade between top edges of blade to center of air foil. Drag force perpendicular to the lift force is also impedes rotor rotation [28].

3.2 Aerodynamics of Wind Turbine

3.2.1 Tip speed ratio

Modern horizontal axis turbines being built now-a-days are of mainly two types; three-blades and more blades. The number of blades is directly linked to the Tip Speed Ratio $\lambda$ (TSR) which is the ratio of blade tip speed to wind speed and is vital factor in wind turbine design is given as below [28]

$$\lambda = \frac{\omega_{tp} R}{V_{wind}}$$  \hspace{1cm} (3.1)

where

$\omega_{tp}$ represents angular velocity or frequency of rotor rotation

$R$ represents radius of the aerodynamic rotor

$V_{wind}$ represents wind speed.

The wind turbine rotating at a high speed appears like a solid wall to wind that may cause oscillations to wind turbine mechanical structure. On the other hand, if the speed of wind
turbine rotor is too slowly, most of the wind shall pass through the gap between the rotor blades. Therefore, wind turbines are designed to have an optimal TSR control to extract maximal power from the wind. Turbulence will be caused in turbine wake when wind speeds over its blades, as a result of which arrival of second blade in the same turbulent air shall not effectively extract power from the wind. However, if the rotor span is adjusted through a proper control then the air hitting each turbine blade would no longer be turbulent. Therefore the TSR is selected to extract maximum power from the wind. The optimum TSR depends on the number of blades of the wind turbine rotor. The fewer the number of blades, the speed of the wind turbine rotor needs to respond quickly to extract maximum power from the wind. A two-bladed rotor has an optimum TSR of around six, a three-bladed rotor around five, and a four-bladed rotor around three [5],[27],[28].

3.2.2 Aerodynamic model

The mechanical power produced by the turbine depends upon swept area $S$ of the air disk formed by turbine blades rotation, power coefficient $C_p$ (which depends upon collective blade pitch angle $\beta$ and tip to speed ratio $\lambda$), air density and wind speed $V$. The expression for power extracted from wind is given by following equation [20],[27],[28] as:

$$P_r = \frac{1}{2}\rho C_p(\beta, \lambda)SV^3$$

(3.2)

Similarly, the torque produced is given as:

$$T_r = \frac{1}{2}\rho C_q R(\beta, \lambda)SV^2$$

(3.3)

Thus, the thrust exerted on the tower $F_r$ is given as:
\[ F_T = \frac{1}{2} \rho C_T(\beta, \lambda)SV^2 \]  

(3.4)

Where, \( C_T \) is the thrust coefficient defined as \( C_T = \left[ 1 - \frac{V_d}{V_u} \right] \) [20], \( V_d \) is upwind velocity \( V_u \) is upwind velocity, \( C_Q \) is torque coefficient \( C_Q = \frac{C_P}{\lambda} \) and \( R \) is the radius of the rotor.

Figure 3.3 illustrates the variation of performance coefficients of \( C_P \) and \( C_T \) with respect to TSR and blade pitch angle \( \beta \).

![Figure 3.3](image)

Figure 3.3 (a) Variations of \( Cp(\alpha, \beta) \) (b) Variation of \( CT(\alpha, \beta) \)

![Figure 3.4](image)

Figure 3.4 Variations of \( C_P \) and \( C_T \) in relation to Tip Speed Ratio \( \lambda \) (Lambda) and pitch angle beta (\( \beta^0 \))

Figure 3.4 shows the variations of \( C_P \) and \( C_T \) with Tip Speed Ratio \( \lambda \) (Lambda) and pitch angle beta (\( \beta^0 \)). It is well known that a turbine cannot extract more than 59% of the
total power in an undisturbed tube of air with the cross-sectional area equal to the wind turbine swept area. This limit is known as Bertz limits and thus the maximum value of the performance coefficient is 0.59 [5],[7],[18],[27],[28]. It was proved that the three-bladed turbines provide better performance such as lower noise, better torque characteristics and speed control as compared to two-bladed wind turbines. Industry is now-a-days growing from MW to multi MW scale with the advent of new technologies required for high capacity wind turbines. Several manufacturers are manufacturing turbines with power rating from 5 MW-10MW [21]-[25].

3.3 Two mass Drive train model

The drive train system for wind turbine coupled to generator shaft can be modeled as a two-mass spring and damper model as shown in Figure 3.5. The model is particularly suitable to analyze wind turbine dynamic behavior during unpredictable wind patterns and sudden grid faults. The drive train model consists of two-mass connected by a flexible shaft of stiffness $K_{shaft}$ and a damper with damping coefficient $B_s$ [18],[20],[23],[27]. The flexible shaft acts as torsion spring connected between two masses, with one mass representing turbine rotor inertia $J_{rotor}$ and the other one generator inertia $J_g$. The gearbox is assumed to have an exchange ratio $1:Z_g$. Both damping and stiffness components are modeled on low speed shaft, whereas high speed generator shaft is assumed to be stiff. It is worth to mention that for direct driven wind turbines with multi-pole synchronous generators, gearbox may be omitted ($Z_g=1$) [7]-[10],[27]. The dynamic equations for the two mass drive chain model are given as [23],[27].

$$J_{rotor} \dot{\omega}_r = T_r(\alpha, \beta, V_r) + \frac{B_s}{Z_g} W_g - K_{\theta_s} \theta_s - (B_{\theta_s} + B_s) \omega_r$$  \hspace{1cm} (3.5)
where \( W_r \) and \( W_g \) are the rotor and the generator speeds, \( B_{hs}, B_{dt}, B_s \) are the damping coefficients of the high-speed shaft, drivetrain, the low-speed shaft respectively with \( B_s \) as equivalent damping constant, \( K_{dt} \) is the drive-train equivalent stiffness coefficient as \( K_{shaft} \) and \( Z_g \) is the transmission gear ratio as shown in Figure 3.5.

### 3.4 Wind Turbine structural model

Wind structural model proposed by Bianchi [27] is used to analyze the impact of wind force on the wind turbine mechanical structural design. The structural model of a wind turbine includes tower and drive train mechanical system described by the equation of motion as:

\[
G \ddot{q} + H \dot{q} + Lq = P(q', q, t, u)
\]  

(3.7)

where \( G, H \) and \( L \) are the mass, damping and stiffness matrices respectively, \( P \) is the vector of forces acting on the system, \( q = [y \quad \xi \quad \beta_r \quad \beta_s]^T \) where, \( y \) is the axial displacement of the nacelle, \( \xi \) is the angular displacement of blade out of the plane of
rotation, $\beta_r$ and $\beta_g$ are angular positions of rotor and generator respectively. After some manipulation, state space model is given as:

$$\dot{x} = \begin{bmatrix} 0_4 & I_4 \\ -G^{-1}L & -G^{-1}H \end{bmatrix} x + \begin{bmatrix} 0_4 \\ G^{-1} \end{bmatrix} P \quad (3.8)$$

Where $x = [q^T \quad \dot{q}^T]^T$, $q = [y \quad \xi \quad \beta_r \quad \beta_g]$, with $\theta_s = \beta_r - \beta_g$. The modified state model of the mechanical subsystem by incorporating turbine rotor and generator speeds $w_r$ and $w_g$ is given by [27]:

$$\dot{q} = \begin{bmatrix} 0_3 & I_{3x4} \\ -G^{-1}\hat{L}_{4x3} & -G^{-1}H \end{bmatrix} q + \begin{bmatrix} 0_3 \\ G^{-1} \end{bmatrix} P \quad (3.9)$$

where $q = [\dot{y} \quad \dot{\xi} \quad \theta_s \quad y \quad \xi \quad w_r \quad w_g]$, $-G^{-1} = \begin{bmatrix} -c & b & 0 & 0 \\ b & -a & 0 & 0 \\ 0 & 0 & -ac & 0 \\ 0 & 0 & 0 & -ac/j_g \end{bmatrix}$, $\theta_s = \beta_r - \beta_g$, and

$$-G^{-1}\hat{L}_{4x3} = \frac{1}{ac-b^2} \begin{bmatrix} -cK_r & bK_r & 0 \\ bK_r & -aK_r & 0 \\ 0 & 0 & -K_rac/j_r \\ 0 & 0 & K_rac/j_g \end{bmatrix}$$

where $\{a = m_r + Nm_b, b = Nm_r p_b, c = Nm_r p_b^2, q = [\dot{y} \quad \dot{\xi} \quad \theta_s \quad y \quad \xi \quad w_r \quad w_g]\}$
Table 3.1 Parameter description of the mechanical subsystem model referred to the low speed side of the WECS [27]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_t$</td>
<td>Mass of the tower</td>
<td>$r_b$</td>
<td>Distance from the rotor axis</td>
</tr>
<tr>
<td>$m_b$</td>
<td>Mass of each blade</td>
<td>$F_T$</td>
<td>Lumped force acting on the blade</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of blades</td>
<td>$K_t$</td>
<td>Stiffness of the tower and blade</td>
</tr>
</tbody>
</table>

Also, $\theta_s = \beta_s - (1/Z_s)\beta_a$ is the torsion angle.

![Figure 3.6 Wind structural model of 1.5 MW Wind Turbine](image-url)

Figure 3.6 Wind structural model of 1.5 MW Wind Turbine
State space representation of wind turbine structural model including tower vibrations is given as [27]:

\[
\begin{bmatrix}
\dot{\mathbf{y}} \\
\dot{\theta}_s \\
\dot{z}_s \\
\dot{w}_r \\
\dot{w}_g
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 \\
-\frac{c_K}{(ac-b^2)} & \frac{b^* K_b r_s^2}{(ac-b^2)} & 0 & -\frac{c_B}{(ac-b^2)} & \frac{b^* B_b r_s^2}{(ac-b^2)} & 0 & 0 \\
\frac{b^* K_b}{(ac-b^2)} & -\frac{a^* K_s r_s^2}{(ac-b^2)} & 0 & \frac{b^* B_s}{(ac-b^2)} & -\frac{a^* B_s r_s^2}{(ac-b^2)} & 0 & 0 \\
0 & 0 & -\frac{K_{ac}}{(ac-b^2) J_e} & 0 & 0 & -\frac{B_{ac}}{(ac-b^2) J_e} & \frac{B_{ac}}{(ac-b^2) J_e} \\
0 & 0 & 0 & \frac{K_{ac}}{(ac-b^2) J_e} & 0 & 0 & -\frac{B_{ac}}{(ac-b^2) J_e} & \frac{B_{ac}}{(ac-b^2) J_e}
\end{bmatrix}
\begin{bmatrix}
\ddot{y} \\
\ddot{\theta}_s \\
\ddot{z}_s \\
\ddot{w}_r \\
\ddot{w}_g
\end{bmatrix}
\]

\[
= \frac{1}{ac-b^2}
\begin{bmatrix}
0 & 0 & 0 & cN-bNr_b & 0 & 0 & \mathbf{F}_r \\
0 & 0 & 0 & -bN+aNr_b & 0 & 0 & \mathbf{T}_r \\
0 & 0 & ac & 0 & \mathbf{T}_r \\
0 & 0 & 0 & -ac & 0
\end{bmatrix}
\]

(3.10)

The axial acceleration of the nacelle is as follows:

\[
\ddot{y}(t) = \frac{1}{ac-b^2}\left\{-cK_{y}(t) + bK_{b}r_{s}^{2}\ddot{z}(t) - cB_{y}\ddot{y}(t) + bB_{b}r_{s}^{2}\dddot{z}(t)\right\} + F_{r}(t)(cN-bNr_{b})
\]

(3.11)

Angular acceleration out of the plane of rotation is given by:

\[
\dddot{\varphi}(t) = \frac{1}{ac-b^2}\left\{bK_{y}(t) - aK_{b}r_{s}^{2}\dddot{z}(t) - bB_{y}\ddot{y}(t) - aB_{b}r_{s}^{2}\dddot{z}(t)\right\} + F_{r}(t)(-bN+aNr_{b})
\]

(3.12)

\[
\ddot{w}_r = \frac{1}{ac-b^2}\left\{-K_{s}\frac{ac}{J_r}\dddot{\theta}_s - ac\frac{B_s}{J_r} - bB_{r}w_{r} + \frac{acB_{r}w_{r}}{J_r} + T_{s}\left\{\frac{ac}{J_r}\right\}\right\}
\]

(3.13)

\[
\ddot{w}_g = \frac{1}{ac-b^2}\left\{K_{s}\frac{ac}{J_g}\dddot{\theta}_s + ac\frac{B_g}{J_g} - \frac{acB_gw_{g}}{J_g} - T_{g}\left\{\frac{ac}{J_g}\right\}\right\}
\]

(3.14)
Equations (3.11) and (3.12) show the axial hub and blade vibrations, whereas Equations (3.13) and (3.14) show the generator and wind turbine rotor speeds respectively.

### 3.4.1 Reduced ordered structural model of HAWT

To explore wind turbine structural model and performance characteristics under different wind speed conditions, a simulation model HAWT was constructed using MATLAB®. The structural model in [20],[27] is based on flap wise movement of blade $\xi$ and axial vibrations of tower $y$. The state space equations for structural model of HAWT were given as [20]:

$$
(m_t + Nm_b)\ddot{y} + Nm_br_b\ddot{\xi} + B_t\dot{y} + K_t y = NF_t
$$

(3.15)

$$
Nm_br_b\ddot{y} + Nm_br_b^2\ddot{\xi} + B_br_b^2\dot{\xi} + K_br_b^2\dot{\xi} = Nr_t F_t
$$

(3.16)

where, $r_b$ is the radius of the blade at which the lumped thrust force $F_t$ is applied, $y$ is the fore-aft bending displacement of the tower and $\xi$ is the flap-wise angular displacement of the blades. $B_t$ and $B_b$ are the equivalent damping coefficients, $K_t$ and $K_b$ are the equivalent stiffness coefficients of the tower and a blade, respectively. The relative wind speed is defined as $V_e = V_w - \dot{y} - \dot{\xi}r_b$, where $V_w$ denotes the absolute wind speed.

$$
egin{align*}
\begin{bmatrix}
(m_t + Nm_b) & Nm_br_b \\
-Nm_br_b & m_br_b^2
\end{bmatrix}
\begin{bmatrix}
\ddot{y} \\
\ddot{\xi}
\end{bmatrix} +
\begin{bmatrix}
B_t & 0 \\
0 & B_br_b^2
\end{bmatrix}
\begin{bmatrix}
\dot{y} \\
\dot{\xi}
\end{bmatrix} +
\begin{bmatrix}
K_t & 0 \\
0 & K_br_b^2
\end{bmatrix}
\begin{bmatrix}
y \\
\xi
\end{bmatrix} =
\begin{bmatrix}
N \\
0
\end{bmatrix}
\begin{bmatrix}
F_t \\
r_b
\end{bmatrix}
\end{align*}

(3.17)

We define $M$ as

$$
M = 
\begin{bmatrix}
(m_t + Nm_b) & Nm_br_b \\
-Nm_br_b & m_br_b^2
\end{bmatrix}
\quad \left| M \right| = (m_t + Nm_b)m_br_b^2 - Nm_b^2r_b^2
$$

Therefore, $M^{-1}$ can be written as:
\[
M^{-1} = \frac{1}{|M|} \begin{bmatrix}
    m_b r_b^2 & -Nm_b r_b \\
    -m_b r_b^2 & (m_b + Nm_b)
\end{bmatrix}
\]

Equation (3.17) can be rewritten in terms of \( M^{-1} \) as:

\[
\begin{bmatrix}
    \dot{y} \\
    \dot{\xi}
\end{bmatrix} = M^{-1} \begin{bmatrix}
    N & 0 & F_t \\
    0 & 0 & F_t
\end{bmatrix} M^{-1} \begin{bmatrix}
    B_i & 0 & 0 \\
    0 & B_b r_b^2
\end{bmatrix} \begin{bmatrix}
    \dot{y} \\
    \dot{\xi}
\end{bmatrix} M^{-1} \begin{bmatrix}
    K_i & 0 \\
    0 & K_f r_b^2
\end{bmatrix} \begin{bmatrix}
    y \\
    \xi
\end{bmatrix}
\]

(3.18)

\[
\begin{bmatrix}
    \dot{y} \\
    \dot{\xi}
\end{bmatrix} = \begin{bmatrix}
    m_b r_b^2 N - Nm_b r_b^2 \\
    -m_b r_b N + (m_b + Nm_b) r_b
\end{bmatrix} F_t
- \begin{bmatrix}
    m_b r_b^2 B_i - Nm_b B_b r_b^3 \\
    -m_b r_b B_i + (m_b + Nm_b) B_b r_b^2
\end{bmatrix} \begin{bmatrix}
    \dot{y} \\
    \dot{\xi}
\end{bmatrix}
- \begin{bmatrix}
    m_b r_b^2 K_i - Nm_b K_f r_b^3 \\
    -m_b r_b K_i + (m_b + Nm_b) K_f r_b^2
\end{bmatrix} \begin{bmatrix}
    y \\
    \xi
\end{bmatrix}
\]

(3.19)

The above expression Equation (3.19) is used to study the analysis of dynamic vibrations induced in the blade and tower due to the impact of frequently changing wind patterns. The force exerted on the blade airfoil will result in bending of blade and tower that are described by the acceleration coefficients of \( y \) and \( \xi \). A study has been carried out to examine the effect of wind force on blade and tower structure of 1.5MW wind turbine through MATLAB/ Simulink® model under different wind conditions. The parametric values for 1.5MW wind turbine [21] are as indicated in table 3.2. The minimum rated speed of generator is selected as 880 rpm, the turbine speed as 10 rpm, maximum generator speed as 1800 rpm and the maximum rotor speed as 20.5 rpm. The gear ratio of 87.965 with generator efficiency of 95 percent to 100 percent has been used. The minimum pitch angle has been chosen to be 2° as an optimal angle [21]. Two case studies have been implemented in simulation software to analyze the performance of 1.5MW wind turbine.
3.4.2 1.5MW wind turbine model

To study the effects of wind gust on the wind turbine structure, a modified 1.5MW wind turbine Simulink model has been used. In the modified test model, Equation (3.19) is incorporated to examine the vibration modes of blade and tower under sudden rise in wind speed.

Table 3.2 Technical parameters of 1.5MW wind turbine [21]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_s$ (Swept area)</td>
<td>3167m$^2$</td>
<td>$J_{rs}$ (Rotor inertia)</td>
<td>2962444 Kg m$^2$</td>
</tr>
<tr>
<td>$R$ (Turbine rotor radius)</td>
<td>35m</td>
<td>$J_{rg}$ (Generator Inertia)</td>
<td>53.036 Kg m$^2$</td>
</tr>
<tr>
<td>$N_b$ (no. of blades)</td>
<td>3</td>
<td>$P_s$ (Power rating)</td>
<td>1.5 MW</td>
</tr>
<tr>
<td>$Z_s$ (Gear ratio)</td>
<td>87.9</td>
<td>$m_t$ (Mass of tower)</td>
<td>1230000 Kg</td>
</tr>
<tr>
<td>$r_r$ (Effective radius of the blade)</td>
<td>29m</td>
<td>$m_b$ (Mass of blade)</td>
<td>3912 Kg</td>
</tr>
<tr>
<td>$K_t$ (Stiffness of the tower)</td>
<td>509871 N rad/m</td>
<td>$\omega_b$ (natural frequency of blade)</td>
<td>3.38 rad/s</td>
</tr>
<tr>
<td>$K_b$ (Stiffness of each blade)</td>
<td>44692 N rad/m</td>
<td>$\omega_{t1}$ (first tower frequency)</td>
<td>2.0724 rad/sec</td>
</tr>
<tr>
<td>$B_t$ (Damping coefficient of tower)</td>
<td>40068.29 N rad/m</td>
<td>$\rho$ (air density)</td>
<td>1.2231 Kg/m$^3$</td>
</tr>
<tr>
<td>$B_b$ (Damp.coefficient of blade)</td>
<td>126.55 N rad/m</td>
<td>$h$ (Hub height)</td>
<td>84m</td>
</tr>
</tbody>
</table>

3.4.3 Case study 1: Wind speed variations from 15 m/s to 17 m/s

Here, an ideal step increase varies from 15 m/s to 17 m/s at time $t = 150$ sec was applied. The simulation results for accelerations and vibrations produced in the blade $\xi$ and tower $\gamma$ are as shown in Figure 3.7. From the simulation results, it is observed that tower gets displaced from its axis about 0.18m to 0.16m, and tip of the blade gets displaced about 1.6m to 1.2m. This is due to instant rise in wind speed variation pitch angle to $23^\circ$ so as to reduce the impact of wind speed. Figure 3.8 shows the effect of step rise in wind gust on different parameters such as shaft torque, pitch angle, generator speed, electrical power output and turbine rotor torque. Figure 3.8 shows that rise in generator shaft to 8596Nm
from 7955Nm results in rise of electrical power from 1.468MW to 1.472MW, rise in generator speed to 1849 rpm.

Figure 3.7 Blade and tower oscillations due to lumped force $F_t$

Figure 3.8 1.5MW Wind turbine variations in generator and turbine parameters
from 1760 rpm and increase in turbine rotor to \(7.5 \times 10^5\) Nm from \(7 \times 10^5\) Nm. The pitch angle control provides automated adjustment of pitch angle over wind speed variations to stabilize the power output from wind turbine.

3.4.4 Case Study 2: Random signal response of wind speed variation from 10 m/s to 11 m/s

In this case, a random signal is generated to model the continuously changing wind speed pattern. The unpredictable behavior of wind speed is made through uniform random number block of MATLAB. The acceleration mode of blade \(\ddot{z}\) (m/s\(^2\)) as zeta\(**\) and tower as \(\ddot{y}\) (m/s\(^2\)) corresponding to fluctuating wind speed are as shown in Figure 3.9.

Figure 3.9 Blade and Tower oscillations due to lumped force \(F_t\)

Figure 3.9 shows the vibrations induced in the blade and tower structure. The tower gets displaced from its axis by 0.17 m and was vibrating with the wind speed fluctuations. The blade gets displaced from its position by 1.8 m approximately and vibrated as per wind speed variations. Figure 3.10 shows the variations in generator shaft torque, electrical...
power, generator speed, turbine rotor speed to randomly changing wind speed within 10 m/s to 11 m/s. It is observed from Figure 3.10 that pitch does not change instantly and remains unaffected. One of the reasons may be small variation in wind speed that is unable to actuate pitch control block.

![Graphs showing various parameters](image)

**Figure 3.10** 1.5 MW Wind Turbine variations in generator and turbine parameters

### 3.4.5 Case study 3: Wind speed variations from 9 m/s to 11 m/s on 5MW wind turbine

The wind turbine structural model study was explored for large rating wind turbine i.e. 5MW wind turbine. The parametric values for 5MW wind turbine were adapted from [20] and the modified wind turbine Simulink model used the state space model as per Equation (3.19). Figure 3.11(a) shows the blade and tower oscillations [66] as the blade
is displaced from 1.2m to 1.57m and tower axis shifted from 0.58m to 0.78m. Figure 3.11(b) shows the performance parameters of 5 MW wind turbine namely shaft torque, electrical power output, generator and wind speed variations.

Figure 3.11 (a) Blade and tower oscillations of 5MW wind turbine due to lumped force $F_t$.

Figure 3.11(b) Wind turbine variations in generator and turbine parameters.
3.5 Simplified Model of Wind Turbine for frequency support

The conventional WECS usually have no support for system frequency and with no contribution to system inertia. In past, WECS were unable to regulate their power output so as to participate in system frequency support under sudden fall and rise in frequency levels. Due to large penetration of high rated wind generating units in utility market, there is an increase in net generation. However, total system inertia participating in frequency control is decreased below adequate level that helps stabilize the system frequency under unbalance conditions. Therefore, further penetration of WECS in power system utility demands inertial contributions to support system frequency regulation [143],[176]-[179].

In past decades the research remained focused on voltage, active and reactive powers controls applicable for synchronous generator based conventional power plants, fixed and variable speed based micro grids. With the large penetration of wind based micro-grids interconnected to steam and hydro based units, the blackouts were caused due to frequency fall below the permissible limits calls for frequency control. There was limited research dedicated to frequency and inertial control as in [174], [176]-[179] for earlier turbines (such as fixed-speed turbines) proposed to utilize wind power output as reserve margin by preventing the wind turbines from supplying their maximum available power in normal situations. Recently, the research focused on extracting kinetic energy stored in the rotor shaft of variable speed DFIG based wind turbine to support frequency regulation [174], [117], [179]. A wind turbine inertial control model was developed to control the power delivered by DFIG based turbine as well as kinetic energy has been extracted which was utilized to support frequency regulation under disturbed grid conditions in [181],[182]. The block diagram of such a control system is as shown in Figure 3.12. Fixed and variable speed wind turbines have a significant amount of kinetic energy stored in rotating mass of their rotor and blades much similar to conventional
steam and hydro based synchronous generators. The variable speed DFIG based wind turbine has the ability to control active and reactive power over wide wind speed range as compared to the fixed speed WECS [174], [175], [179].

Figure 3.12 DFIG Inertial Emulation Control [179]

The active power delivered by variable speed WECS to the grid under steady state operation depend upon the wind speed and the power can be dynamically controlled to an extent by use of stored mechanical energy. This result due to the capability of variable speed DFIG based wind power systems to work at asynchronous speeds, and the wind energy gets transferred efficiently for a given wind speed with reduced mechanical stress. DFIGs generally have slip ring rotor with three phase winding wound on the stator as well as rotor frames. The stator of winding of DFIG is directly connected to the grid, whereas the rotor is supplied via a power electronic converter. Stator’s three phase ac winding set up net a flux of $\frac{3}{2}\Phi_{ph}$ rotating at the angular speed that can be written as; [29],[175]
\[
\frac{W_{st}}{P_{st}} = \omega_{mec} \pm \frac{W_{ro}}{P_{ro}}
\]  
(3.20)

where, \( P_{st} \) and \( P_{ro} \) denote number of stator and rotor poles respectively, \( W_{st} \) denotes system frequency that equals sum of the angular velocity of mechanical rotation (\( \omega_{mec} \)) and rotor current frequency (\( \omega_{ro} \)). The amount of kinetic energy released from the shaft of the wind turbine when speed reduces is given by \( \Delta E_k \) as [174],[179].

\[
E_{ko} = \frac{1}{2} J\omega_{mec0}^2
\]  
(3.21)

\[
E_{ko} - E_{k1} = \frac{1}{2} J\omega_{mec0}^2 - \frac{1}{2} J\omega_{mec1}^2
\]  
(3.22)

\[
\Delta E_k = E_{k0} \left( 1 - \frac{\omega_{mec1}^2}{\omega_{mec0}^2} \right)
\]  
(3.23)

\( \Delta E_k \) is dependent on wind speed that varies between zero to 1.0 per unit and \( \omega_{mec1} \) should not be smaller than the \( \omega_{mec(min)} \) as rotational speed of the DFIG-based wind turbine. The instantaneous power extracted from the wind turbine should not exceed allowed maximum. These constraints can be formulated as:

\[
E_{ko} = f(\text{Wind speed})
\]  
(3.24)

\[
0 \leq E_{ko} \leq 1.0 \text{ pu} = f(\text{Wind speed})
\]  
(3.25)

\[
\omega_{mec(min)} \leq \omega_{mec1}
\]  
(3.26)

\[
E_{k0} + \Delta E_k \leq E_{k(max)}
\]  
(3.27)

Figure 3.12 shows the controllers for DFIG-based wind turbines that maintain it at optimal speed to produce maximum power. The error signal is obtained via comparison of reference signal \( \omega_{m,\text{ref}} \) and measured mechanical angular speed of the rotor \( \omega_{m,\text{meas}} \),

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and is fed to the PI controller to provide a power set-point \( \dot{P}_p \). The power set-point is fed to the power converter control unit to control torque and active power through generator rotor current control. An additional control signal \( \Delta \dot{P}_f \) as a function of the deviation in the power system frequency and rate of change of the grid frequency is also added. The emulated inertia of this additional control signal depends on the derivative \( D \) and proportional \( P \) parameters of the controller [175]. This additional control loop gets activated when grid frequency exceeds certain limits to support system frequency as primary loop control and utilized to set the torque demand. With a drop in system frequency, the set point torque gets increased and rotor slows down releasing kinetic energy. The power reference point \( \Delta \dot{P}_{fw} \) has two components; \( \Delta \dot{P}_f \) - additional reference point based on frequency changes and \( \Delta \dot{P}_w \) - based on optimum turbine speed as a function of wind speed, and are given as [29],[177]

\[
\Delta \dot{P}_f = -K_{D_f} \frac{df}{dt} - K_{P_f} \Delta f
\]

\[
\Delta \dot{P}_w = -K_{P_w}(\dot{w}_{ref} - w_{m,meas}) + K_{P_w} \int (\dot{w}_{ref} - w_{m,meas})dt
\]

\[
\Delta \dot{P}_{fw} = \Delta \dot{P}_f + \Delta \dot{P}_w
\]

here, \( K_{D_f} \) and \( K_{P_f} \) are derivative and proportional control action gains respectively.

Considering the two components of the DFIG power set-point Equation (3.30), \( \Delta \dot{P}_w \) changes relatively slow as compared to frequency derivation power set-point \( \Delta \dot{P}_f \). At \( t = 0 \) the \( \Delta \dot{P}_w \) is assumed zero. Considering instant conversion of set point to power by converter, we assume \( \Delta P_{NC} = \Delta \dot{P}_{fw} \) and thus;

\[
\Delta \dot{P}_{fw} = \Delta \dot{P}_f + 0
\]

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This chapter presents the mathematical relations for aerodynamic of wind power, lumped force impact on the wind turbine structural. The MATLAB simulation model was developed for both 1.5MW and 5MW wind turbine to study impact of wind gust on tower and blade structures as well as mechanical vibrations induced in the tower. The importance of pitch control to reduce these mechanical vibrations is also discussed. The simplified model of wind turbine is obtained for inertial control of DFIG based wind turbine to support system frequency under grid frequency disturbances.