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The study of surface boundary layer attracts lot of attention because most of the human activity is limited within this thin layer of atmosphere. Most of the studies have used the concept of stationary and homogeneity, which led to the assumption of constant heat and momentum flux layer. Results of many experiments show that the variations of these fluxes are not more than 10% within the surface boundary layer.

3.1 Atmospheric Stability

Atmospheric stability is one of the most vital parameters for any kind of simulation process. Most simple form of the definition is as follows:

- Atmosphere in which forces do not affect the vertical displacement of parcel, is called neutral stability.
- Atmosphere that supports the vertical motion of buoyant parcel is known as unstable.
- Atmosphere, which suppresses vertical movement of the parcel - called stable condition.
From the vertical profile of temperature, the state of the atmosphere can readily be determined. When the environmental lapse rate (ELR) is less than dry adiabatic lapse rate (DALR), the atmosphere is said to be unstable. If ELR is greater than DALR, the atmosphere is stable and if it equals neutral. Although lapse rate is best suited to determine the stability, it is difficult to get the measurements of vertical profile of temperature all the time.

Various schemes are developed to determine the atmospheric stability. They can be widely classified into two categories - empirical methods and physically sound methods. Various empirical schemes developed are Giblett (1932), Smith (1951), BNL type (1966), Pasquill (1961), etc. Some of the physically sound methods are Richardson (Ri), Monin - Obukhov length (L), etc.

Pasquill stability classes are one of the widely used stability categorisations. There are six stability classes - extremely unstable ‘A’ to stable class ‘F’. These stability classes are selected according to intensity of turbulence in the atmosphere. The main criteria to determine the stability classes are the insolation, wind speed and cloud cover. Basic properties of surface layer flow assumed (Pasquill and Smith 1983) while formulating the scheme were

- Restriction of large numerical values of the low-level Richardson number to very light winds and
• Well defined diurnal cycle of vertical profile of temperature in the first few meters and its dependence on the state of the sky.

Later, Turner (1964) modified the Pasquill's stability classification scheme by introducing solar elevation angle, cloud cover and cloud heights. A net radiation (NR) index has been developed based on the solar angle. With these changes, the stability classification schemes could be applied to any place and time. The stability classes changed to 1 to 6 corresponding to A to F. Later, Holtzworth added one more class in the stable side. This stability classification scheme is in use presently. Basic steps followed to determine the stability using the Pasquill's stability classification are as follows.

• Calculate the solar angle using the Julian day, local time, latitude and longitude.
• Determine the insolation class number form the table below and modify it by using cloud cover and height to get NR.
• Determine the stability classes according to wind speed and the modified NR.

Solar elevation angle has been calculated by using time of the day, latitude and declination.
Net Radiation Index: The index can be obtained from the following procedure and steps as defined by Turner (1964).

Determine the insolation class based on position of sun with respect to the place of interest. The classification of insolation class based on solar altitude is given below:

<table>
<thead>
<tr>
<th>Solar altitude</th>
<th>Insolation</th>
<th>Insolation class</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha \leq 15^\circ$</td>
<td>Weak</td>
<td>1</td>
</tr>
<tr>
<td>$15^\circ &lt; \alpha &lt; 35^\circ$</td>
<td>Slight</td>
<td>2</td>
</tr>
<tr>
<td>$35^\circ &lt; \alpha &lt; 60^\circ$</td>
<td>Moderate</td>
<td>3</td>
</tr>
<tr>
<td>$\alpha &gt; 60^\circ$</td>
<td>Strong</td>
<td>4</td>
</tr>
</tbody>
</table>

The insolation class number should be modified according to the cloud cover. Irrespective of the time, if sky is overcast, and cloud base height less than 7000 feet (2133.6), the net radiation class falls under zero. During night time, if cloud cover is less than or equal 3 oktas, then value of NR will be -2. If cloud cover is between 3 oktas and 7 oktas, then NR will be -1.

During daytime, following conditions apply:

If cloud cover is between four and 7 oktas; and if cloud ceiling is less than 2133 m (7000 feet), then subtract 2 from corresponding value of insolation table; if cloud ceiling is between 2133 m and 4878 m, then subtract 1 from corresponding value of insolation table.

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If direct measurements of radiation are available, NR can be classified as per following:

<table>
<thead>
<tr>
<th>Insolation (×4.19 W/m²)</th>
<th>Category</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 50</td>
<td>Strong</td>
<td>4</td>
</tr>
<tr>
<td>50 - 25</td>
<td>Moderate</td>
<td>3</td>
</tr>
<tr>
<td>25 - 12.5</td>
<td>Weak</td>
<td>2</td>
</tr>
<tr>
<td>&lt;12.5</td>
<td>Night</td>
<td>0</td>
</tr>
</tbody>
</table>

The stability of the atmosphere can be determined from the table below using the net radiation index and wind speed.
Cloud cover, cloud type, and radiation data of Calcutta (Dumdum station) were collected from India Meteorological Department (IMD) for the corresponding period of MONTBLEX. The observation time of IMD data, however, is not exactly matching with the timings of recording of fast sensors. For convenience of computation, low and medium level clouds together considered for stability calculation.

The atmospheric stability with respect to standard deviation of horizontal wind direction can also be computed. Following table gives the comparison with Pasquill's classification and its relationships to lapse rate.
Richardson while investigating (1920, 1925) the effects of gravity on the suppression of turbulence, derived a ratio of work done against gravitational stability to the energy transformed ($R_i$) from mean to turbulence motion. The $R_i$ can be written as follows:

$$R_i = g \frac{\partial T}{\partial z} + \Gamma \left( \frac{\partial U}{\partial z} \right)^2$$

(3-1)

where $R_i$ is the Richardson number

- $g$ = acceleration due to gravity
- $T$ = mean temperature ($°K$)
- $U$ = mean wind (m/s)
- $\frac{\partial T}{\partial z}$ = temperature gradient

<table>
<thead>
<tr>
<th>Pasquill's stability class</th>
<th>Standard deviation ($\sigma_0$) of horizontal wind direction at 10m</th>
<th>Lapse rate °C/100 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>&gt; 22.5</td>
<td>&lt; -1.9</td>
</tr>
<tr>
<td>B</td>
<td>22.5 - 17.5</td>
<td>-1.9 to -1.7</td>
</tr>
<tr>
<td>C</td>
<td>17.5 - 12.5</td>
<td>-1.7 to -1.5</td>
</tr>
<tr>
<td>D</td>
<td>12.5 - 7.5</td>
<td>-1.5 to -0.5</td>
</tr>
<tr>
<td>E</td>
<td>7.5 - 3.5</td>
<td>-0.5 to +1.5</td>
</tr>
<tr>
<td>F</td>
<td>&lt; 3.5</td>
<td>1.5 to 4</td>
</tr>
</tbody>
</table>

Source: DOE/TIC-27601, p. 591
\[
\frac{\partial U}{\partial z} = \text{wind shear (s}^{-1}\text{)} \quad \text{and} \\
\Gamma = \text{dry adiabatic lapse rate} (0.0098 \, ^{\circ}\text{C/m})
\]

If \( R_i > 0 \), the atmosphere is considered to be stable and \( R_i \) equal to zero is neutral and \( R_i < 0 \), is for unstable condition. Isitzer (1965) proposed a scale to compare with value of \( R_i \) to Pasquill scheme. According to him, value -0.26 of \( R_i \) for type A (class 1 in modified scheme) to, value 0.046 of \( R_i \) for type F (class 6 in modified scheme). At critical \( R_i \) of 0.25, the air flow undergoes transition form turbulent flow to laminar, and as \( R_i \) exceeds 0.25 temperature stratification dampens out the effect of wind shear, producing a flow devoid of the type of random fluctuation, that associated with turbulence (Kaimal J C, 1988).

Main disadvantage of the above stability criteria is the height dependency of \( R_i \). \( R_i \) computed using 62 m tower data at White Sands Missile Range (Hansen, 1966) shows that it can vary from -0.288 (at 2.95 m) to -4.684 (at 55.5 m) within the boundary layer. The dependency of \( R_i \) on the unknown function of height makes it difficult for characterisation of boundary layer.

Obukhov length \( (L) \) is a useful stability parameter. It is a surface layer scaling parameter. Ratio of height \( Z \) over \( L \) can be expressed as
where \( U_s \) = frictional velocity

\( T \) = temperature

\( k \) = Von karman constant

The negative sign is introduced so that \( z/L \) has the same sign of \( R_i \). This quantity is more useful than \( R_i \), because its vertical variation is given in the equation. It also implies that within the surface layer, the effect of varying height and stability are interchangeable. The Businger-Dyer-Pandolfo results (Businger1966; Pandolfo, 1966) state that in unstable atmosphere, \( R_i \) will be equal to \( z/L \) and in stable atmosphere the relationship become,

\[
\frac{z}{L} = \frac{R_i}{(1 - 5R_i)}
\]

Convective Velocity Scale (\( w^* \))

Field observations, laboratory experiments, and numerical modeling studies show that the large turbulent eddies in the CBL have velocities proportional to the convective velocity (\( w^* \)) scale (Wyngaard, 1988).
Convective velocity scale can be estimated (Wyngaard, 1988) using the following equation.

\[
\frac{g H z}{\rho c_p T} \]

Atmospheric stability is determined using various methods presented in the next chapter.

To understand the sensitivity of stability computed using different methods on air quality, a simple air quality simulation under rural atmosphere has been made. For the air quality simulation, typical Gaussian distribution of pollutant from an elevated stack of a power plant has been used. The Gaussian plume equation in the general form may be expressed as:

\[
\chi = \frac{Q}{2\pi \sigma_y \sigma_z} \exp \left( -\frac{y^2}{2\sigma_y^2} \right) \left[ \exp \left\{ -\frac{\left( z - h_e \right)^2}{2\sigma_z^2} \right\} + \exp \left\{ -\frac{\left( z + h_e \right)^2}{2\sigma_z^2} \right\} \right] 
\]

where \( Q \) is the source strength, \( \sigma_y \) and \( \sigma_z \) are the dispersion coefficients, \( h_e \) is the effective stack height and \( U \) is the wind speed at stack level.

Chemical transformations with exponential decay and physical removal by settling are not used in the model. Ground level

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concentrations have been calculated corresponding to a single time combination stability (under various methods) and wind speed. Concentrations of pollutant at different atmospheric stabilities on the down wind direction were computed show the difference and presented in the next chapter.

The parameters that affect the distribution pattern hey are

(i) Stability, which is the main parameter that determines the diffusion coefficients. Briggs diffusion equations are used to find the vertical and horizontal diffusion parameters.

(ii) Wind speed at stack level, which inturn affects the concentration distribution again in two ways - in the determination of effective stack height and as a dilution factor (the term of mean wind is in the denominator) in the Gaussian equation.

Brigg's(1975) plume rise formula is used in the model for computing the effective plume height

\[ \Delta h = \frac{1.6 F^{1/3} x^{2/3}}{U} \]

(3-6)

where \( \Delta h \) is the plume rise, \( F \) is the buoyancy flux parameter, \( x \) is the distance from the stack, and \( U \) is the wind speed at stack level.
Buoyancy fluxes $F$ have been calculated as

$$F = \left(\frac{g}{T_s}\right) \times V_s \times r_s^2 \quad (3-7)$$

where $T_s$, and $V_s$ are the exit gas temperature, exit gas velocity of the plume and $r_s$ is the radius of the stack at outlet.

$$\Delta T = T_s - T_a \quad (3-8)$$

Plume centre line is assumed to be horizontal at the height of final rise above source.

### 3.2 Heat and Momentum Fluxes

Surface layer parameters such as energy fluxes, frictional velocity, and turbulent kinetic energy are studied using fast and slow response data collected from Kharagpur.

Determination of these turbulent exchanges between earth and the atmosphere is one of the primary concerns in micrometeorology. Number of methods have been devised with varying degrees of sophistication. Some of the methods are as follows.

- Surface Drag Method
- Energy Balance Method
- Bulk Transfer Method
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- Aerodynamic Method
- Profile Method
- Eddy Correlation Method

Surface Drag Measurements is the only direct method to measure the shearing stress on a sample surface representing uniform terrain condition. The drag forces experienced on the sensor are measured using strain gauges and electrochemical transducers. Operation of drag plate needs lot of skill and experience. Such measurements are taken in the Kansas field program 1968 (Arya, 1988). The surface energy balance estimates sensible heat flux at the earth surface as, \( R_N = H + H_L + H_G \), where \( R_N \) is the net radiation \( H \) and \( H_L \) are the sensible and latent heat fluxes and \( H_G \) is the ground heat flux. Net radiation and \( H_G \) can be measured directly. By either Bowen's ratio or any other method \( H \) can be determined.

Bulk Transfer Method is an indirect method using mean winds and temperature. The fluxes can be calculated by the following equation.

\[
\tau = -\rho C_D U^2 \\
H = \rho C_F C_H U (\theta - \theta_0)
\]

(3-9)  
(3-10)
where $C_D$ and $C_H$ are the drag and heat transfer coefficient respectively and $U$ and $\theta$ are the mean wind and potential temperatures.

\[
C_D = k^2[\ln(z/z_0) - \Psi_m(z/L)]^{-2} \tag{3-11}
\]
\[
C_H = k^2[\ln(z/z_0) - \Psi_m(z/L)]^{-1} [\ln(z/z_0) - \Psi_h(z/L)]^{-1} \tag{3-12}
\]

where $\Psi_m$ and $\Psi_h$ are the M-O similarity functions.

In the Aerodynamic Method, mean differences of velocity and temperature between two heights are utilised to find the fluxes. If mean wind and temperature are available at more than two heights in surface layer, the profile method can be employed.

**Eddy Correlation Method:** This is one of the most reliable methods and is a direct measurement of turbulent exchanges of momentum and heat in the atmosphere. The fluctuation in velocity and temperature reflect the fluxes of momentum and heat. Vertical fluxes of momentum and heat over homogeneous terrain can be written as

\[
\tau = -\rho \overline{u w'} \tag{3-13}
\]
\[
H = \rho c_p \overline{\theta' w'} \tag{3-14}
\]

where $\theta'$ and $w'$ are the turbulent part of the variables. Such data required high frequency sampling, accurate levelling of the
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instrument, orientation, and high degree of maintenance. Above requirements have kept away the eddy correlation method from being widely used, except in special research projects such as MONTBLEX. Measurements during MONTBLEX were at a frequency of 8 Hz. These data are used to determine the fluctuations in these parameters and the fluxes.

3.3 Simulation of Vertical Profiles

Structure of PBL is very complex due to the combined effect of turbulence, friction and Coriolis force. It has been realised that the PBL is a critical factor in producing mesoscale whether systems such as convective storms, land and sea breezes, thermal boundaries and other local circulations (Pielke and Maharer, 1975; Ogura and Chen 1977; Ulanski and Garstang, 1978; McNidar and Pielke 1981; Pielke 1981). Because of the large fluxes of heat, moisture and momentum that take place in the surface layer, one (high-resolution) dimensional model should be tested before extending to two or three dimensional models. One dimensional model was developed by the Diertele (1979) for the homogeneous modified terrain conditions, by including the heating due to divergence of the long and short wave radiation and energy conservation equation for the air-earth interaction, including radiative, sensible and latent heat fluxes. Bornstein (1975) has incorporated these changes for predicting the soil heat flux in his URBMET model. In the formulation, different governing
equations with an explicit finite difference technique to obtain the mean values are given below.

\[
\frac{\partial \xi}{\partial t} = f \frac{\partial \xi}{\partial z} + \frac{\partial^2 \xi}{\partial z^2} \left( \frac{K}{M} \right) \tag{3-15}
\]

\[
\frac{\partial \varphi}{\partial t} = f(u_\varphi - u) + \frac{\partial}{\partial z} \left( K_M \frac{\partial \varphi}{\partial z} \right) \tag{3-16}
\]

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left( K_H \frac{\partial \theta}{\partial z} \right) - \frac{1}{\rho_M C_p} \frac{\partial Q_N}{\partial z} \tag{3-17}
\]

\[
\frac{\partial q}{\partial t} = \frac{\partial}{\partial z} \left( K_q \frac{\partial q}{\partial z} \right) \tag{3-18}
\]

\[
u = \frac{\partial \psi}{\partial z} \tag{3-19}
\]

\[
\zeta = \frac{\partial u}{\partial z} = \frac{\partial^2 \psi}{\partial z^2} \tag{3-20}
\]

Vorticity and stream function approach used in the present one-dimensional study for convenience. Following relationship between temperature and potential temperature has been used:

\[
\theta = T + \Gamma z. \tag{3-21}
\]

The eddy diffusivities \( K_M, K_H \) and \( K_q \) (set equal to \( K_H \)) are specified using the interpolation formula of O'Brien (1970):

\[
K(z) = K(H^*) - \left[ \frac{z-H^*}{H^*-h} \right]^2 \times (K(h) - K(H^*) + (z-h) x
\]

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where the following values were assumed

\[ K(\text{H}^*) = 103 \text{ cm}^2 \text{ sec}^{-1}, \text{ z} \geq \text{H}^* \]  

\[ \text{and} \quad \text{H}^* = U^*/f. \]

The net radiation term in the thermal energy equation for PBL (3-17) is only for pollutant free atmosphere in the present study, which therefore restricts the generality of the model.

Long wave radiative flux is a function of the vertical distribution of temperature, water vapour, and carbon dioxide. The two gases are the two most important absorber-emitters of long wave radiation in the lower atmosphere (Elsasser, 1942). While distributions of temperature and water vapour are obtained from (3-16) and (3-17) respectively a uniform constant concentration of 310 ppm is used for carbon dioxide. Following Swan (1972), the downward flux of long wave radiation to a give depth z consists of the sum of the long wave emission transmitted from layers above z, plus the amount from above the boundary layer transmitted to level z. This sum can be represented by

\[ Q_{\downarrow}(z) = \int_{\omega^*}^{\omega} \sigma T^4(\omega') d\varepsilon(\omega - \omega') + Q_{\downarrow}(H) [1 - \varepsilon(\omega_z)]. \]  

where \( \omega' \) is a dummy variable of integration, \( d\varepsilon(\omega_z - \omega') \) is the differential emittance for the layer \( (\omega_z - \omega') \), \( Q_{\downarrow}(H) \) is the
downward flux at the top of the boundary layer, and where \( \omega' \) is assumed to be zero at the top of the boundary layer.

Optical path lengths for water vapour and carbon dioxide through the layer \( z_1 \) to \( z_2 \) are computed from

\[
\omega_{\text{H}_2\text{O}} = \rho_m \int_z^{z_2} q d
\]

(3-26)

and

\[
\omega_{\text{C}02} = 3.14 \times 10^{-4} \int_z^{z_2} d z
\]

(3-27)

Previous work by Atwater (1970) has shown that it is not necessary to correct for pressure and temperature effects on emissivity.

Wave length integrated emissivities used in the model for water vapour are taken from Atwater (1970). They were derived by Atwater from the data of Kuhn (1963), who found that they are in close agreement to heating rates predicted by more detailed computations of Davis and Viezee (1964). Following Kodratyev (1969), the corresponding expression for \( \text{CO}_2 \) is

\[
\varepsilon_{\text{CO}_2} = [1-\exp(-0.3919 \omega_{\text{CO}_2})]
\]

The total effect of \( \text{H}_2\text{O} \) and \( \text{CO}_2 \) is given, following Atwater (1970), as

\[
\varepsilon_{\text{H}_2\text{O}, \text{CO}_2} = \varepsilon_{\text{H}_2\text{O}} + 0.185 \varepsilon_{\text{CO}_2}
\]
Similarly, upward long wave radiant flux to level \( z \) is given as

\[
Q_{L+}(Z) = \int_{\omega_\text{m}}^{\omega_z} \sigma T^4(\omega') d\epsilon (\omega' - \omega_z) \\
+ \epsilon, \sigma T^4 \left[ 1 - \epsilon \left( \omega_1 + \omega_z \right) \right] \\
+ (1 - \epsilon) \left[ 1 - \epsilon \left( \omega_1 + \omega_z \right) \right] \int_{0}^{\omega_z} \sigma T^4 \omega' \ d\epsilon (\omega_1 - \omega')
\]  

(3-28)

where \( \omega_1 \) is the total optical depth for the planetary boundary layer, and the first term is radiation emitted by the layer below level \( z \). The second term represents radiation emitted by the surface upwards through the intervening layer to level \( z \), while the third term represents radiation reaching level \( z \) after being emitted by the entire atmosphere and then undergoing simple reflection at the earth’s surface. Finally, the net upward long wave flux at level \( z \) is given by

\[
Q_N(Z) = Q_{L+}(Z) - Q_{L-}(Z)
\]  

(3-29)

The short wave flux at the surface is obtained after accounting for the absorption of short wave energy by permanent gases (CO\(_2\), O\(_2\), and O\(_3\)) and water vapour in the atmosphere, as well as for scattering due to dust. Diurnal variation of solar flux at the top of the atmosphere is governed by the following expression:

\[
\cos Z = \cos \Phi \cos \delta + \sin \Phi \sin \delta \cos H_0.
\]  

(3-30)

The transmission function for the permanent gases \( t_\text{g} \) is given as
\[
I_R = 0.485 + 0.515 \gamma, \\
\text{where}
\]
\[
\gamma = 1.041 - \frac{0.160}{\cos Z} \left[ 0.949 \frac{P}{P_0} + 0.051 \right]^{1/2} \\
(3-31)
\]

These expressions, originally given by Kondratyev (1969), were modified by Atwater and Brown (1974) to account for forward molecular scattering. The transmission function for water vapour used in the present study is from Sasamori et al (1972), and is given as

\[
T_{H_2O} = 1 - [0.11 (6.31 \times 10^4 + \omega_{H_2O} \text{sec} Z)^{0.3} - 0.0121] \\
(3-32)
\]

where the term in brackets represents absorption by water vapor \(A_{H_2O}\). Hence, the amount of solar energy back scattered by water vapour molecules has been ignored. Finally, the transmission functions for dust used in the present model is that of Houghton (1954), which assumes a globally averaged dust concentration and given as

\[
I_D = 0.95 \text{sec} Z \\
\text{With these expressions, the net solar flux at the surface is given by}
\]
\[
Q_R = (1 - A) R_0 \cos Z (I_R - A_{H_2O}) I_D, \\
(3-33)
\]

where the solar constant \(R_0\) is \(1.353 \times 10^6 \text{ergs cm}^{-2} \text{sec}^{-1}\).
Long wave heating is obtained from

\[
\left( \frac{\partial T}{\partial t} \right)_{lw} = -\frac{1}{\rho_m C_p} \left[ \frac{\partial}{\partial z} \left( \cos Z \cdot A_{HI,O}(z) \right) \right]
\]

(3-34)

The net heating of the atmosphere at a given level will result from divergence of the total radiative flux at that level. Total radiative heating is divided into long-and short wave components, i.e.,

\[
\left( \frac{\partial \theta}{\partial t} \right)_{RAD} = \left( \frac{\partial T}{\partial t} \right)_{RAD} = \left( \frac{\partial T}{\partial t} \right)_{LW} + \left( \frac{\partial T}{\partial t} \right)_{SW}
\]

(3-35)

where the equality on the left follows from (3-20)

Constant Flux Layer

The lower layer is assumed to be an equilibrium surface boundary layer (SBL) of 25m in depth, in which the fractional deviation of the stress is small, and thus in which fluxes are assumed constant with height. In this layer analytical expressions are used to specify vertical profiles of wind velocity, potential temperature, and specific humidity. According to Monin-Obukhov similarity theory, described by Monin and Yaglom (1971), vertical gradients of these particular quantities are functions of the atmospheric stability parameter \( z / L \), where the Monin-Obukhov length \( L \) given as

\[
L = \left( \frac{Q_M}{\rho_M} \right)^{3/2} \frac{\theta_m}{k \cdot g Q_H / \rho_m C_p}
\]

(3-36)
In a non-neutral SBL, vertical gradients can be parameterised by

\[
\frac{\partial U}{\partial z} = \frac{U_*}{k_o(z + z_o)} \phi_M \left( \frac{z + z_o}{L} \right)
\]

(3-37)

\[
\frac{\partial \theta}{\partial z} = \frac{\theta_*}{k_o(z + z_o)} \phi_H \left( \frac{z + z_o}{L} \right)
\]

(3-38)

\[
\frac{\partial q}{\partial z} = \frac{q_*}{k_o(z + z_o)} \phi_q \left( \frac{z + z_o}{L} \right)
\]

(3-39)

The empirical functions of Businger et al. (1971) are used for \( \phi_M \), \( \phi_H \), and \( \phi_q \). The von Karman constant \( K_o \) is chosen such that \( \phi_M(0) \) is unity, thus giving the logarithmic profile for neutral conditions. According to Businger et al., this value is 0.35, as opposed to the more commonly used value of 0.4. Note that \( \phi_q \) is assumed to be the same as \( \phi_H \).

A minimum positive value for the Monin-Obukhov length is necessary since the profiles for wind, temperature, and moisture in the stable regions are asymptotic to what Wyngaard (1973) has referred to as 'z-less stratification.' As \( L \) approaches zero, \( z/L \) becomes infinite, and the variables become independent of height for levels greater than \( L \), where turbulent fluxes tend to zero. To overcome this problem in a numerical model it is necessary to impose the following two constraints:
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- whenever \( z/L \) exceeds 2, \( z/L \) is reset to 2, and
- Eddy diffusivities are not allowed to fall below \( 10^3 \) cm\(^2\) sec\(^{-1}\).

Grid and Boundary Conditions

The one-dimensional version of the interlaced grid of Fromm (1964) computes vorticity and stream function values at grid locations vertically displaced by one-half of a grid interval from locations of the velocity components. Studies by Clarke (1970) and Taylor and Delage (1971) discuss the superiority of variable grid spacing in achieving high resolution near the surface.

Boundary conditions adopted for this simulation study is presented below.

At \( z = 0 \)

\[
\begin{align*}
u &= \frac{\partial \psi}{\partial z} = 0 \quad (3-40) \\
v &= 0 \quad (3-41) \\
n &= g(t) \quad (3-42) \\
\theta &= f(t), \quad (3-43)
\end{align*}
\]

where \( g(t) \) and \( f(t) \) are discussed below.

At \( z = h \)

Continuity of \( U, \theta, q, \frac{\partial U}{\partial z}, \frac{\partial \theta}{\partial z}, \frac{\partial q}{\partial z} \) \quad (3-44)
At $z = H$

$$u = \frac{\partial \psi}{\partial z} = u = -\frac{1}{\rho_m f} \frac{\partial p}{\partial y} \quad (3-45)$$

$$\frac{\partial u}{\partial z} = \frac{\partial^2 \psi}{\partial z^2} = 0 \quad (3-46)$$

$$v = 0 \quad (3-47)$$

$$\theta = \text{const.} \quad (3-48)$$

$$q = \text{const.} \quad (3-49)$$

In order to obtain the function $f(t)$, it is assumed that the earth's surface is a black body radiator and that the instantaneous energy balance at the soil surface can be written as

$$Q_R + Q_L + Q_H + Q_E - Q_S = 0, \quad (3-50)$$

where all components of energy balance are taken as positive if directed towards the surface and negative otherwise. As discussed by Bornstein (1968), this equation states that a net daytime gain of energy through radiation at the earth-atmosphere interface results in a turbulent transfer of heat $Q_H$ to the atmosphere conduction of heat $Q_S$ to the surface and evaporation $Q_E$. During the night, a net loss of energy through radiation at the interface results in decreased evaporation or condensation, turbulent transfer of heat from the atmosphere, and conduction of heat from deep layers to the surface.
The net incoming long and short radiative fluxes are described in transition layer. Long-wave radiation emitted by the surface $Q_{L\uparrow}$ is related to surface temperature by the Stefan-Boltzmann law equation

$$Q_{L\uparrow} = \frac{E_s \sigma T_f^4}{s} \tag{3-51}$$

After substituting for $Q_{L\uparrow}$, for $Q_H$, for $Q_E$, and for $Q_s$, becomes

$$\{Q_R + Q_L - e_s \sigma T_f^4 + \rho_m C_p u \cdot \theta_s +$$

$$\rho_m L_E u \cdot q + \lambda \frac{\partial T_s}{\partial z} + L_E \rho_s D_w \frac{\partial h_s}{\partial z} \} = 0 \tag{3-52}$$

In order to obtain the function $g(t)$, it is assumed that there is a surface moisture balance between the atmospheric surface water vapour flux $w$ and the soil surface moisture flux $w_s$, i.e.,

$$w + w_s = 0, \tag{3-53}$$

where the atmospheric water vapour flux is given by

$$w = -\rho_m u \cdot q \tag{3-54}$$

Substituting for $w_s$ leads to

$$\rho_m u \cdot q + \rho_s \frac{\partial h_s}{\partial z} + \rho_{wfr} \frac{\partial T_s}{\partial z} + \rho_{wK} w = 0, \tag{3-55}$$

Solutions of the two non-linear partial differential equations with two unknowns, i.e., moisture and temperature, are obtained by
using a Newton-Ralphson double iterative technique (Carnahan et al., 1969).