CHAPTER 2

DEVICE MODELLING AND ANALYSIS

2.1 INTRODUCTION

The TOF developed in this work is a tunable FP microcavity formed in between two 1D photonic crystal broadband reflectors. The working of the filter is schematically shown in the Figure 2.1 in which both the PC structures can be displaced to change the cavity width and hence the output can be tuned to any frequencies inside the PBG of the PC structures. Silicon is chosen as the base material since it is mechanically robust, electrically conductive and optically transparent to WDM telecommunication windows in addition to the availability of well established silicon micromachining techniques. Each PC consists of three Si layers separated from each other by air layers, enough to give strong reflection due to high refractive index contrast between silicon and air layers.

![Figure 2.1](image)

**Figure 2.1** Schematic of the tunable FP cavity with two movable 1D PC structures as mirrors
This chapter is devoted to the numerical modeling of the filter and the filter geometry and the tolerance for fabrication has been arrived at based on the different analyses performed using this mathematical model. The tunability of the filter by altering different parameters of the defect layer such as dimension, refractive index, angle and position is explored thoroughly and possible tuning mechanisms for the same are also explored.

2.2 MODELING THE TOF

The mathematical model of the filter is constructed using the simple FP interferometer approach in the general framework of TMM since the filter is essentially an FP cavity with two PC broadband reflectors as mirrors. The wave propagation through the PC layers is analyzed using the TMM by assuming that plane waves are entering in to the device normally. Among all the theoretical approaches for numerical simulation of 1D structures, TMM method is the simplest and most computationally efficient method which can also be used for aperiodic structures. Therefore this simple but powerful method has been used for the detailed simulation of the TOF.

2.2.1 The Transfer Matrix Method (TMM)

Kurt Schuster developed thin film optical theory in 1949 and in the following years Florin Abelès extended it to multilayers and formulated the technique for computing the reflection and transmission, called transfer matrix method. This technique has proved to be powerful for the computation of the transfer characteristics of finite periodic as well as random stratified media of different materials of varying physical properties. In the case of a finite periodic stratified media the problem size can be reduced by dividing the entire structure in to identical unit cells as shown in the Figure 2.2, and the overall response can be extracted by multiplying the responses of each unit cell.
Figure 2.2  Unit cell of a periodic stratified medium of materials A and B. Hence the overall response $R$ of the complete structure is given by,

$$R = (R_{AB})^N$$  \hspace{1cm} (2.1)$$

Where $R_{AB}$ and $N$ are the response and number of unit cells respectively. According to TMM the propagation of electromagnetic wave through each layer is described by a $2 \times 2$ matrix, i.e.,

$$
\begin{pmatrix}
  a_{n-1} \\
  b_{n-1}
\end{pmatrix} = M
\begin{pmatrix}
  a_n \\
  b_n
\end{pmatrix}$$

$$\hspace{1cm} (2.2)$$

Where

$$M = \begin{pmatrix}
  \cos \delta & i \eta^{-1} \sin \delta \\
  i \eta \sin \delta & \cos \delta
\end{pmatrix}$$

$$\hspace{1cm} (2.3)$$

is the transfer matrix.

Where $\delta = kn d \cos \phi$

$$\eta = n \cos \phi \hspace{2cm} \text{for s – polarization}$$

$$\eta = n \cos^{-1} \phi \hspace{2cm} \text{for p – polarization}$$
And $k=2\pi/\lambda$, is the wave vector, $n$ and $d$ are the refractive index and thickness of the layer respectively and $\varphi$ is the angle of incidence in the layer. The product of the characteristic matrices of all the layers gives the transfer function of the complete multilayered system, i.e.

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = M_{\text{tot}} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

(2.4)

where $M_{\text{tot}}=M_1.M_2........M_j$

Reflection and transmission coefficients are given by

$$r = \frac{m_{21}}{m_{11}}$$

(2.5)

and

$$t = \frac{1}{m_{11}}$$

(2.6)

respectively, where $m_{21}$ and $m_{11}$ are matrix elements.

Therefore reflectivity and transmissivity of the entire periodic structure can be written as $R=r^2$ and $T=t^2$ respectively. The detailed theoretical formulation of the TMM is given in the appendix A1.

### 2.2.2 Photonic crystal FP cavity

The conventional FP cavities with metal coated mirrors cannot provide large FSR and hence high finesse value as required by an optical component employed in the WDM networks. Multilayered dielectric stacks of differing refractive indices have been explored for the realization of broadband reflectors and most recently PBG exhibited by high contrast DBR often referred as 1D PC is being explored. As described at the beginning of this chapter the optical system analyzed is a microcavity formed in between two 1D PC mirrors which are separately analyzed using TMM described in the previous section. The equations for reflectance and transmittance of this
combined system are formulated using the asymmetric FP approach in which the reflectivity and transmissivity of each mirror can be introduced separately, Yariv and Yeh (2007); hence any nonlinearity caused by the structural deformation of the mirrors can be incorporated into the numerical modeling. This aspect of our device will be discussed later in this chapter.

In characterizing the optical cavity, we assume that the light experience equal reflectance and transmittance when it incident on either sides of the PC mirrors. The incident light undergoes multiple reflections inside the cavity of refractive index $n$ and width $d$ and the phase accumulated by successive reflections from the first mirror after having a round trip inside the cavity is,

$$\delta = \frac{4\pi}{\lambda} nd \cos \theta$$

(2.7)

Where $\theta$ is the refraction angle inside the cavity. The total reflected and transmitted power will be the superposition of all the reflected and transmitted power at the left and right side of the cavity respectively as shown in the Figure 2.3. In this derivation we have made the following conventions,

- $r_1$ – amplitude reflection of the mirror PC1
- $r_2$ - amplitude reflection of the mirror PC2
- $R_1$ – intensity reflection of the mirror PC1
- $R_2$ - intensity reflection of the mirror PC2
- $t_1$ - amplitude transmission of the mirror PC1
- $t_2$ . amplitude transmission of the mirror PC2
- $T_1$ – intensity transmission of the mirror PC1
- $T_2$ – intensity transmission of the mirror PC2
\[ r_1 \neq r_2, t_1 \neq t_2, R_1 \neq R_2, T_1 \neq T_2, t_1^2 = T_1, t_2^2 = T_2, r_1^2 = R_1 \text{ and } r_2^2 = R_2 \quad (2.8) \]

Figure 2.3 Multiple reflections inside an FP cavity

Let \(a_0\) be the amplitude of the incident light and the amplitude of the total reflected light at the exit plane is given by the superposition of all the successive reflections as shown below,

\[ a_r = a_0 \left[ r_1 + t_1 r_2 e^{i\delta} \left( 1 + r_2 t_2 e^{2i\delta} + r_2^2 t_1^2 e^{3i\delta} + \ldots \right) \right] \quad (2.9) \]

The above equation can be simplified using the Maclaurin series as follows,

\[ a_r = a_0 \left[ \frac{r_1 - r_2 e^{i\delta} \left( r_1^2 + t_1^2 \right)}{1 - r_1 r_2 e^{i\delta}} \right] \quad (2.10) \]

According to Stokes’ relation we have,

\[ R + T = 1 \quad (2.11) \]
And using the conventions made in (2.8), the equation (2.11) can be simplified as,

\[
a_r = a_0 \frac{r_1 - r_2 e^{i\delta}}{1 - r_1 r_2 e^{i\delta}}
\]  

(2.12)

The intensity \( I_r \) is proportional to \( |a_r|^2 \), and after some mathematical manipulation the transmittance of the photonic crystal FP cavity is given by the following equation,

\[
T_{\text{tot}} = \frac{(1 - r_1 r_2)^2 - (r_1 - r_2)^2}{(1 - r_1 r_2)^2 + 4 r_1 r_2 \sin^2 (\delta / 2)}
\]  

(2.13)

Where \( r_1 \) and \( r_2 \) are intensity reflectance of the two PC mirrors respectively. Therefore as a generalized case, we have used the equations (2.13) for the numerical analysis of our device.

2.3  OPTICAL ANALYSIS

The schematic of the optical system is shown in the Figure 2.4, in which each PC broadband reflector consists of three silicon layers (\( n_H \)) as high refractive index medium and two air layers (\( n_L \)) as low refractive index medium. Since this is a free space device light can be coupled through lensed fibers.
Figure 2.4  Schematic of the optical system investigated as a Fabry-Perot TOF consisting of two 1D PC broadband reflectors

The PC structures are essentially a stack of quarter wavelength layers and the basic principle of working is the interference of multiple reflections from each interface.

To achieve the targeted specifications of the TOF meant for WDM applications mentioned in the first chapter, we have designed the FP filter using 1D Si/Air PC’s as mirrors which ensure high reflectivity of more than 0.999 for a wide range of frequencies that can be altered by changing the structural dimensions of the PC; and the filter is implemented in silicon – on – insulator optical bench platform with MEMS combdrive actuators to provide low power fast reconfigurability. The batch processing capability of MEMS microfabrication techniques lowers the cost of the device and ensure small footprint.
Table 2.1 Design specifications of the TOF

<table>
<thead>
<tr>
<th>Properties</th>
<th>Silicon</th>
<th>Air</th>
<th>Cavity</th>
</tr>
</thead>
<tbody>
<tr>
<td>n at 1.6µm</td>
<td>3.474</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>No. of quarter wavelength layer (m)</td>
<td>12</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Layer width (d(µm)=m.λc/n.4)</td>
<td>1.4</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

The structural dimensions of our device are given in the Table 2.1. In which the layer widths are defined to provide a stopband in the C and L bands of the WDM grid. The silicon and air layer widths taken are a few integer multiples of quarter wavelength because it is too difficult to etch quarter wavelength layers of width of few 100nm with high aspect ratio. The initial cavity width of the device is 4µm which is 10 times the quarter wavelength at \( \lambda_c = 1.6 \mu m \) and can be reduced to tune the transmission wavelength of the device. All the optical analysis is done using these dimensions assuming plane waves incident normally on the device. The material dispersion of silicon is incorporated in the numerical analysis through Sellmeier equation which is as follows.

\[
    n^2 = \varepsilon + \frac{A}{\lambda^2} + \frac{B\lambda_i^2}{(\lambda^2 - \lambda_i^2)}
\]

(2.14)

Where \( \lambda_i = 1.1071 \mu m \), \( \varepsilon = 1.16858 \times 10^4 \), \( A = 9.39816 \times 10^{-1} \) and \( B = 8.10461 \times 10^{-3} \)

The defect mode and the PBG of the Si/air periodic structure will be shifted due to the combined effect of thermo-optic and thermal expansion of silicon layers. The passband of a TOF meant for WDM should be independent of temperature fluctuation, or at least the passband at all the operating temperatures should still transmit the same ITU-channel even though signal loss may increase, so as to reduce interchannel crosstalk. Otherwise these devices will require additional temperature control.
mechanisms which will increase the operational expense per device. Therefore this modeling is crucial in estimating the operating range and complexity of the TOF design for application in the WDM link. The combined effect of thermal expansion and thermo-optic effect are also incorporated in to the simulation using the following equations as reported by Park and Lau (1971) and Padgaongar (2004) respectively.

The change in thickness $\Delta d$ of silicon layer for a change in temperature $\Delta T$ is given by,

$$\Delta d = d_0 \alpha \Delta T$$

Where $\alpha = 4.657 \times 10^{-6} + 2.3503 \times 10^{-9} t - 3.0695 \times 10^{-12} t^2$ at a given temperature $t$ and $d_0$ is the thickness at room temperature. Therefore at a temperature $t$ the thickness of silicon layer is given by the equation,

$$d = d_0 + \Delta d \quad (2.15)$$

Similarly the change in refractive index of silicon $\Delta n$ for a change in temperature $\Delta T$ is given by,

$$\Delta n = \beta \Delta T$$

Where $\beta$ is the thermo-optic coefficient which is equal to $1.8 \times 10^{-4}$, therefore the refractive index $n$ of silicon layer at a given temperature $t$ can be obtained from the relation,

$$n = n_0 + \beta \Delta T \quad (2.16)$$

The reflectance of one PC mirror as a function of number of Si layers is shown in the Figure 2.5. The dimensions given in the Table 2.1 and
equation (2.5) are used for this simulation assuming the angle of incidence $\theta = 0$ i.e. normal incidence.

![Graph showing reflectance vs wavelength for different number of Si layers](image)

**Figure 2.5** Reflectance measured for a single PC mirror as function of number of Si layers

It is evident from the above figure that at least three silicon layers are needed for strong reflection of more than 0.999 and more numbers of layers will increase the reflectivity of the PC mirrors.

The FP cavity is formed by using two PC structures as mirrors and it is clear from the equation (2.13) that the transmission of the FP cavity depends only on the reflection coefficient of the constituting PC mirrors (light absorption of silicon is negligible in the 1250nm-1650nm range). The transmission characteristics of the FP based TOF is shown in the Figure 2.6.
Figure 2.6 Transmission of the FP based TOF

The spectral width of the pass band depends on the reflectivity of the PC mirrors which in turn depends on the number of layers. Therefore at least three silicon layers are needed for each PC mirrors to ensure the TOF with narrow pass band width as per the WDM requirements. Two cases have been analyzed, in the first case the PC structure has 3.4µm lattice constant and 41% filling factor (S1) and in the second case the lattice constant is 2.4µm and the filling factor is 50% (S2).

Figure 2.7 Influence of number of layers on the (a) pass band width and (b) Q-factor
The influence of number of layers of the PC structures on the pass band width and the quality factor (Q-factor) of the FP cavity is depicted in the Figures 2.7(a) and 2.7(b) respectively. The cavity width is tuned to have a resonance at 1550nm wavelength for both cases. Therefore, based on these analyses, we have used three silicon layers for each PC mirrors in our device. Increasing number of layers will lead to transmission loss due to off-normal incidence (this will be discussed in next section) and other fabrication induced structural non-uniformities which broadens the pass band width also (this will be discussed in the section 2.3.6). Therefore three silicon layers for each PC provide a better tradeoff between the pass-band width and insertion loss requirements for the CWDM networks in which the channel spacing is 20nm and the filter pass band width can be wider to accommodate frequency drift of the transmitter due to ageing and temperature variation and the temperature dependence of the filter pass band itself, and short enough to reduce the inter channel crosstalk.

2.3.1 Influence of incident angle

The reflectance and transmittance of the PC mirrors are functions of wavelength $\lambda$ and incident angle $\theta$. Therefore the location of the PBG of the PC structure inside the optical spectrum is determined by the angle of incidence and has been analyzed using TMM method described in the section 2.2.1. The incident angle is swept between 0 and $90^0$ and the input wavelength range tested is from 1200nm to 2000nm. We have used the dimensions given in the Table 2.1 for this simulation. The simulation results for both S and P polarized light is shown in the Figures 2.8 (a) and 2.8 (b) respectively,
Figure 2.8  PBG of S-polarized (a) and p-polarized (b) light estimated for different incident angles

In both the figures the black region represents the absence of eigenmodes i.e. the presence of a PBG and the white area represents the presence of eigenmodes. It can be seen that the PBG is blue shifted as the angle of incidence is increasing and also the shift is polarization dependent at higher incident angles. Both the polarizations have same PBGs for a range of incident angle of up to 10 degree. Therefore the PC structures act as broadband reflectors for a wide range of incident angles. Now let us look at the influence of incident angle on the resonance of the FP cavity formed by the PC structures. The resonant transmission of the FP cavity is dependent on the angle of refraction inside the cavity, which in turn determined by the angle of incidence through Snell’s law. The off-normal incidence of light with the PC filter causes resonant mode frequency shift and hence transmission loss at the resonant frequency at normal incidence. The frequency shift of the cavity mode inside the PBG for both p-polarized and s-polarized light is plotted in the graph shown in the Figure 2.9 (a) and (b) respectively.
Figure 2.9  Frequency shift of the resonant modes within the PBG for (a) p-polarized and (b) s-polarized light

The resonant transmission of the FP cavity at 1550nm for varying incident angle (-15° to +15°) is plotted in the graph shown in the Figure 2.10. The calculation is performed using the dimensions given in the Table 2.1.

Figure 2.10  Transmission loss at the resonant wavelength 1550nm for varying incident angle and number of layers in the PC structures

The transmission loss increases with increasing number of layers for a given angle of incidence and it is clear from the above plot that a system
contains three layers for each PC mirrors, the off normal incidence introduces large transmission loss.

Since this is a free space device placed in between two lensed fibers for coupling the light in to and out of the device, the Gaussian light emerged from a fiber diverges in the free space and introduces off-normal incidence with the PC structure, as shown in the Figure 2.11.

![Figure 2.11 The Gaussian beam emerging out of a fiber collimator diverges in the free space and introduces off-normal incidence with the PC structure](image)

A Gaussian beam can be described as a function of radius \( r \) and propagation distance \( z \):

\[
E_{(r,z)} = A_0 \exp \left( -r^2 \left[ \frac{1}{w^2} - \frac{jk}{2R} \right] - j[kz + \phi] \right)
\]

(2.17)

where \( A_0 \) is the amplitude of the beam at the centre, \( w \) is the radius of the beam where the amplitude drops to 1/e, \( R \) is the radius of curvature of the
surfaces of constant phase and z is the axial distance measured from the beam
waist. The wave number is k and \( \phi \) is the phase change described by,

\[
\tan \phi = \frac{\lambda z}{\pi w_0^2}
\]

(2.18)

\( w_0 \) is the radius of the beam at its waist. We calculate the beam radius as a function of propagation according to:

\[
w^2 = w_0^2 \left( 1 + \left( \frac{z}{z_0} \right)^2 \right)
\]

(2.19)

Where \( z_0 \) is the Rayleigh distance and it determines the distance over which the beam retains it width and curvature. i.e.

\[
z_0 = \frac{\pi w_0^2}{\lambda}
\]

(2.20)

The divergence of the incident beam as a function of propagation distance is shown in the Figure 2.12.
In Figure 2.12, the black lines represent the beam divergence of a cleaved single mode fiber with beam radius of 5µm at the beam waist. Beams with large radius widens slower than the beams with small radius at the beam waist. We have seen that off-normal incidence deteriorate the performance of the filter and therefore, fiber collimators with large spot size at the beam waist is desired. Since the collimated beams are wider, deep etched high aspect ratio PC structures are needed for the realization of the TOF. The divergence angle of a Gaussian beam emerging from a fiber collimator can be calculated from the equation,

$$\theta_0 = \frac{\lambda}{\pi w_0}$$  \hspace{1cm} (2.21)
Therefore for, a beam diameter of 25µm the divergence angle is less than 0.6° at the beam waist (refer to Figure 2.13), i.e. the light incident on the filter at an angle of 0.6°. For this angle the transmission loss is less than 2dB for a structure having three layers in the PC lattice, which is as shown in the Figure 2.14.

Figure 2.13  Divergence angle of the beam emerging from a GRIN lens as a function of beam width (diameter) at its waist

Figure 2.14  Transmission loss of the filter for incident angle ranging from -0.6° to +0.6°
Therefore low transmission loss can be achieved by using collimators with wide beams and the device under test (DUT) should be kept at the working distance or at the beam waist. We used AR coated micro GRIN lensed fibers of more than 25µm beam size (MC 01) from Shinko microelectronics (Japan) and hemispherical ball lensed fibers from LaseOptics (USA) for the light coupling to and out of the devices.

2.3.2 Influence of layer thickness (filling fraction)

The influence of silicon layer thickness within the lattice period of 3.4µm on the PBG of the PC structure is analyzed. The 2-D PBG map of the broadband reflectors for different filling ratio is plotted in the Figure 2.15.

The periodicity of the PC broadband reflector a is 3.4µm. Therefore the silicon layer thickness is calculated from, \( d_{Si} = a \times f \). Where \( f \) is filling fraction and the air layer thickness can be calculated as, \( d_{Air} = a - d_{Si} \)

![Figure 2.15 2D PBG map for different filling fraction](image)

The white area indicates the absence of electromagnetic eigenmodes and hence represents the PBG (Photonic Band Gap). The region where eigen modes are allowed is indicated in blue colour. The 41% filling ratio (marked by a vertical black line) for a period of 3.4µm is chosen for the
design because it provides a stop band of width 100nm in the 1500nm-
1600nm wavelength range; besides, from the fabrication point of view this
filling ratio enables high aspect ratio etching of Si layers by ensuring the PBG
within the desired wavelength range.

2.3.3 Influence of defect layer

The PBG is a range of frequency for which the PC is opaque and a
highly localized transmission can be triggered inside this PBG by breaking
the periodicity of the PC or introducing a defect layer inside the PC lattice,
which is often called as defect mode of the PC. The defect mode can be tuned
to any frequencies inside the PBG by altering the parameters of the defect
layer such as thickness and refractive index and the defect mode intensity can
be varied by changing the defect layer position inside the PC lattice, this will
be discussed later in this chapter.

The graph plotted in the Figure 2.16 (a) depicts the defect mode transmission
inside a PBG of 1500nm-1600nm range. The corresponding defect layer
width is 3.88µm and the PC layer dimensions used for this simulation are
taken from the Table 2.1. The Figure 2.16(b) shows the tuning of defect
modes to different frequencies inside the PBG by varying the defect layer
width. In this simulation the defect layer width is decreased from an initial
value of 4µm to a final value of 2µm as proposed for the TOF realized in this
work. This gap tuning can be achieved by electrostatic actuation using comb
drives. Since the two parallel PC mirrors around the defect layer induce
parallel plate effect, the initial defect layer width is set to 4µm to reduce the
direct actuation caused by the parallel plate effect which leads to nonlinear
actuation, and this aspect of the device will be discussed in the next chapter.
Further increasing the initial defect layer width result to small tuning efficiency. The tuning efficiency \((d\lambda/dl, \text{ where } l \text{ is the defect layer width})\) in the 4µm-3.5µm (shown in grey scale in the Figure 2.16(b)) region is 40% i.e. the defect layer needs to be varied by 2.5nm for 1nm wavelength shift. In the second 3.5µm-3µm and the third 3µm-2.5µm regions the tuning efficiency is 50% (i.e. 2nm defect layer change for 1nm wavelength shift) and 65% (i.e. 1.54nm defect layer change for 1nm wavelength shift) respectively. A low wavelength tuning sensitivity implies requirement of larger power for actuation whereas high sensitivity implies requirement of stringent control over displacement and hence finer control of actuation voltages. It can also be seen that simultaneous tuning over different stop bands is possible if the initial cavity width is 4µm (shown in grey scale in the Figure 2.16(b)). Therefore the 4µm initial cavity is a better trade-off between tuning efficiency, sensitivity of wavelength tuning and simultaneous tuning.

### 2.3.4 Influence of non-verticality

The three major requirements, which the PC layers must meet for the device to have envisaged optical properties, are...
- Perfect parallelism between the PC layers
- Accurate dimension (CD uniformity)
- Smooth surfaces

Side wall angles alter the optical thickness the light encounters from top to bottom of the PC layers and hence the transmitted light propagates in different directions; this introduces spectral losses and resonant wavelength shift from the designed value. Figure 2.17 shows the schematic of the PC with non-vertical layers due to high aspect ratio DRIE,

![Figure 2.17 Schematic showing the device with non-vertical PC layers due to etch angle \( \theta_e \).](image)

The effect of etch angle on the transmission of the complete device has been studied by assuming the etch angle is uniform for all layers which form the PC structure. The direction in which the light propagates after each layer is predicted geometrically by calculating the angle of incidence and angle of refraction for each layer as follows (refer to Figure 2.18),
Figure 2.18 Schematic representation of the trace of the angle of refraction through the wedged PC layers

The incident angle at the first interface (air/Si) will be equal to the etch angle $\theta_e$ i.e. $\theta_i = \theta_e$ provided the input light travels normal to the front plane of the device, shown as dotted line in the Figure 2.18 and the corresponding refraction angle through the first Si layer can be calculated using Snell’s equation. The light encounters opposite wedge at the second interface (Si/air) and hence the incident angle can be written,

$$\theta_{i1} = 2\theta_e - \theta_r$$

(2.22)

Where $\theta_r$ is the refraction angle in the first Si layer and the corresponding refraction angle in the first air layer is given by,

$$\theta_{r1} = \sin^{-1}\left[\sin(2\theta_e - \theta_r) \times \frac{n_{Si}}{n_{air}}\right]$$

(2.23)

At third air/Si interface the wedge is different from the second interface and hence the incident angle is given by,

$$\theta_{i2} = \theta_{r1} - 2\theta_e$$

(2.24)
The corresponding refraction angle is then given by,

\[ \theta_{r1} = \sin^{-1} \left[ \sin(\theta - 2\theta_e) \times \frac{n_{Si}}{n_{air}} \right] \]  

(2.25)

For the successive interfaces the incident and corresponding refraction angles can be calculated recursively using equations (2.46) and (2.48) as shown in the Figure 2.18. The spectral response of the complete device for varying etch angle is shown in the Figure 2.19 for both polarization,

![Figure 2.19](image)

Figure 2.19  Spectral response of the device simulated for different etch angle; (a) P-polarization and (b) S-polarization

It is clear that after a certain value of etch angle (7.8° for both polarization in the Figure 2.19) there is no transmission through the device since the light completely walks off from the device due to the etch angle. The transmission loss of the TOF at the resonant wavelength 1550nm for etch angle from -2° to +2° is plotted in the Figure 2.20.
The etch angle introduces large transmission losses in the output spectrum of the TOF. Therefore the PC layers should be extremely vertical, for the TOF to have optical properties required by the WDM networks. Since the best practical aspect ratio is less than 1:200, the best possible etch angle will be $\tan^{-1}(1/200)=0.29^0$, Lipson et al (2007). This is far from the optical requirement of $0^0$ for minimum transmission loss, so care must be taken during fabrication to ensure PC layers with vertical sidewalls.

### 2.3.5 Influence of position of cavity inside the PC lattice

In our device the FP cavity is formed by introducing a planar defect at the center of a continuous 1D Si/air PC. We have explored the influence of the position of this defect layer on the transmittance of the system. Zi et al (1998) has cited that the transmission of the defect mode is due to the coupling between the eigen modes of the defect layer and those at the band edges of the constituent photonic crystals. In Figure 2.21 the intensity of the defect mode at the peak wavelength 1552nm is plotted for different defect layer position inside the PC lattice. The corresponding defect layer width is 3.88$\mu$m. We have analyzed three cases in which the position of the defect layer is changed inside the PC lattice and observed that the defect mode intensity is maximum when the defect layer is at the center of the PC lattice.
Figure 2.21  Intensity of the defect mode is maximum (blue line) when the defect layer is at the center of the PC lattice

Case 1 – The defect layer is placed after the first silicon bar as shown in the Figure 2.22(a). The black line in the Figure 2.21 represents the defect mode at this position and the transmission level is -15.41dB.

Case 2 – The defect layer is placed after the second silicon bar as shown in the Figure 2.22(b). The red line in the Figure 2.21 represents the defect mode at this position and the transmission level is -5.37dB.

Case 3 – The defect layer is placed exactly at the center of the PC lattice as shown in the Figure 2.22 (c). The blue line in the Figure 2.21
represents the defect mode at this position and the transmission level is 0dB (i.e. maximum transmission).

The coupling between the eigen modes of the defect layer and those at the band edges is strong when the defect layer is at the center of the PC lattice and is weak when the defect layer is far from the center. This causes the defect mode intensity variation.

2.3.6 Influence of geometrical chirping

The geometrical chirping is the non-uniform thickness variation of the PC layers caused by the fabrication induced critical dimension non-uniformity as shown in the Figure 2.23. This leaves the system as an asymmetric FP cavity with PC mirrors of different spectral characteristics and hence we used equation (2.13) for numerically simulating the spectra of the system.

Figure 2.23 Thickness variation of the PC layers due to CD non-uniformity (a) ideal PC layers and (b) geometrically chirped PC layers with central defect layer
The dotted lines in the Figure 2.23(b) indicate the layer thickness expected in an ideal PC (Figure 2.23(a)) and H and L represent the higher (silicon) and lower (air) refractive index materials respectively. The fabrication induced CD non-uniformity and geometrical chirping will be discussed in detail in the next chapter. For the numerical simulation we have introduced a 10% asymmetry in the filling factor of one of the PC halves say PC2 and the other half has the same dimension as given in the Table 2.1. The dimensions of the symmetric and asymmetric FP cavities used for the numerical analysis are given in the Table 2.2.

### Table 2.2 PC layer thicknesses of symmetric and asymmetric FP cavities

<table>
<thead>
<tr>
<th>Type</th>
<th>PC name</th>
<th>Si (µm)</th>
<th>Air (µm)</th>
<th>Si (µm)</th>
<th>Air (µm)</th>
<th>Si (µm)</th>
<th>Cavity (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric</td>
<td>PC1</td>
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<td>2</td>
<td>1.4</td>
<td>2</td>
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<td>4</td>
</tr>
<tr>
<td></td>
<td>PC2</td>
<td>1.4</td>
<td>2</td>
<td>1.4</td>
<td>2</td>
<td>1.4</td>
<td>4</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>PC1</td>
<td>1.4</td>
<td>2</td>
<td>1.4</td>
<td>2</td>
<td>1.4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>PC2</td>
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<td>1.666</td>
<td>1.734</td>
<td>1.666</td>
<td>1.734</td>
<td>4</td>
</tr>
</tbody>
</table>

The theoretical analysis illustrates two different properties of the PC arising from the asymmetric geometrical variations. First one is the large frequency range with minimum transmission except for the defect modes which appear inside this stop band and the second one is the intensity variation of the defect modes at different frequencies inside the stop band.

2.3.6.1 Large free spectral range (FSR)

A comparison between the transmission spectra of asymmetric and symmetric FP cavities is shown in the Figure 2.24. The 10% asymmetry introduced in to the filling factor of one of the PC halves induce large frequency range with minimum transmission compared to the symmetric FP
cavity in which both the PC halves are identical, i.e. same filling factor and layer thicknesses.

![Figure 2.24](image)

**Figure 2.24** FSR of FP cavities with (a) 10% asymmetry between the PC layers and (b) symmetric PC layers

The reason for this phenomenon is illustrated in the Figure 2.25 shown below,

![Figure 2.25](image)

**Figure 2.25** Large FSR due to the asymmetry between the PC halves, red (PC1), blue (PC2) and black (PC1+PC2)

The normalized transmittance of PC1 (red line), PC2 (blue line) and the combined system (PC1+PC2, black line) for the asymmetric FP cavity is depicted in the Figure 2.25. Since the two halves of the combined system have different lattice properties, they posses distinct spectral characteristics and the pass band of PC1 lies inside the stop band of PC2 and vice versa resulting in a large spectral range with minimum transmission.
2.3.6.2  Intensity variation of the defect mode

In an ideal PC, with central defect layer and geometrically symmetric structures on either side of the defect layer, the normalized transmission intensity of the defect mode is unity and is independent of the defect mode frequency (refer to Figure 2.26(a)). In the case of geometrically chirped 1D PC structures, in which the PC halves are asymmetric with respect to the central defect layer, the intensity of the defect mode varies with defect mode frequency. A comparison between the intensity spectra of symmetric and asymmetric FP cavities is shown in the Figure 2.26 (a) and (b) respectively.

Since the defect layer is surrounded by two PCs of different filling factors i.e. both the halves are geometrically asymmetric or chirped, they have different spectral responses. The band edge transmission of one PC half say PC1 is suppressed in the combined spectrum due to which it falls in to the stop-band of the other half say PC2 and the defect mode transmission is enhanced by the coupling between the eigen modes of the defect and those at the band edges of the constituent photonic crystals.

![Figure 26](image)

**Figure 26**  Intensity spectra of symmetric (a) and asymmetric (b) FP cavities for varying defect lengths

It can be seen from the Figure 2.26(b) that the defect mode intensity varies across the band gap and drops to minimum at the center of the band
edges and it appears as a notch in the transmission spectra as marked by circle. This is discussed using the Figure 2.27,

![Figure 2.27 Intensity map of the defect mode showing maximum and minimum levels](image)

Another important phenomenon observed from this analysis is the broadening of the pass band width, i.e. with 10% asymmetry between the two constituting PC’s the pass band width widens to 6nm compared to 1nm in the case of symmetric FP. In the Fig. 2.27, shaded green region represents the map of the normalized transmission of defect mode as the defect layer thickness between PC1 and PC2 is swept from 3.7\(\mu\)m to 4\(\mu\)m in steps of 1nm, blue and red lines represent transmission of PC2 and PC1 respectively, black line represents the transmission difference between PC1 and PC2. It is can be concluded that the transmission intensity of defect modes is maximum at frequencies for which the two PC halves have equal transmittance and minimum at frequencies for which the two PC halves have large variation in the transmittance and it appears as a notch in the defect mode transmission spectrum. Therefore this study reveals that the geometrical asymmetry in the structure with respect to the central defect layer leads to (i) wider stop band ranges with suppressed band-edges (ii) causes large variation in the transmission intensities of the defect modes and (iii) effectively shifts the tuning range of the device.
Therefore the reflectivity of the mirrors in an FP cavity should be the same for the operating wavelength to ensure envisaged optical performance. Since the reflectivity of the PC mirrors forming the FP cavity is mainly determined by the dielectric layer thickness, it is of utmost important to ensure uniform layer thickness for the two PC mirrors during the fabrication process. The simulations with asymmetry, (even 1% asymmetry, corresponding to 40nm deviation from the original thickness of the silicon layer) show that reflectivities of the two PC mirrors differ from each other. Therefore the CD non-uniformity within the PC mirrors should be less than this value to ensure envisaged optical performance of the TOF.

2.4  TUNING THE PHOTONIC CRYSTAL FP CAVITY

In this section we discuss about different techniques which can be used to tune the cavity modes of the photonic crystal FP cavity. Various techniques using thermo-optic and electro-optic effects and cavity width variation (gap tuning) are explored for the realization of tunability in 1D PC based FP cavities. Our target specifications are listed below,

1. Wide tuning range of more than 70nm in the C and L bands (EDFA gain bandwidth)
2. Low power consumption
3. Fast reconfiguration (tuning) of up to 10KHz
4. Simple and compact

2.4.1  Thermo-optic tuning

Materials with high thermo-optic coefficient are used to realize thermally tunable optical filters. The thermo-optic effect of the host matrix or infiltrated materials with high thermo-optic effect compared to the host matrix can be exploited for this purpose. An FP cavity of α-silicon (amorphous
silicon) deposited by PECVD has been reported by Hohlfeld et al (2004). The α-silicon has high thermo-optic coefficient of 3.25×10⁴/K and they used Si₃N₄/SiO₂ quarter wavelength layers as DBR mirrors (1D PC structures). They reported a tuning range of 36.5nm with a tuning efficiency of 51.7pm/K and the tuning speed is several milliseconds. Therefore thermal tuning suffers from slow tuning speed and narrow tuning range.

2.4.2 Electro-optic tuning

Like thermo-optic tunable filter, either the host matrix or the cavity can be defined by an electro-optic material. Materials with large electro-optic coefficient are desired for these type of tunable filters. Fast tuning speed of few nanoseconds can be achieved by this technique. Zhu et al (2009) has proposed an electro-optically tunable 1D PC based on LiNbO₃, since it has large electro-optic coefficient of 30.9pm/V. They have used a LiNbO₃ layer as resonant cavity and applied voltage across this cavity to change the refractive index. They applied voltage between -30KV and 30KV and theoretically predicted a tuning efficiency of 1.358nm/KV. Therefore this type of tuning requires high voltage of the order of KV and the tuning range is very less.

2.4.3 Electrostatic Gap tuning

This is the widely explored technique for the realization of wavelength tuning in 1D PC based FP filters. A small change in the cavity width cause large resonant wavelength shift inside the stop band or the PBG. MEMS electrostatic actuation is the most viable and hence widely explored tuning mechanism for the gap tuning of the cavity width. With this tuning mechanism all the anticipated target specifications mentioned above can be achieved. The objective in this case is to achieve the required tuning with voltages less than 5V.
2.5 CONCLUSION

The geometry of the PC mirrors has been optimized based on the different optical analyses carried out using the numerical model of the TOF based on FP approach in the general framework of TMM. The optical analysis shows that three silicon layers are required for each PC mirrors to have strong reflectivity simultaneously meeting low loss operation in case of off-normal incidence. The optical analysis also shows that PC layer verticality is crucial for the low loss operation of the device. The FP arrangement in which the cavity placed at the center of the PC lattice has maximum intensity for the resonant transmission. The geometrical asymmetry between the PC mirrors should be less than 1% to reduce the transmission loss and hence tight control of layer thickness should be ensured during the fabrication process steps. The optical analysis revealed that high tuning efficiency can be achieved by gap tuning of the cavity mode, the typical tuning efficiency is 40% i.e. 250nm cavity width variation for 100nm wavelength range ensuring simultaneous tuning over different stop bands. This can be achieved by low power MEMS electrostatic actuation and the MEMS based architecture of the TOF will be discussed in the next chapter.

The PBG layer design specifications arrived at is as follows:

- Si layer thickness : 1.4 microns
- Air layer thickness : 2 microns
- No. of silicon layers in each PC half : 3
- Maximum cavity width : 4 micron
- Defect width range : 3.5 micron-4 micron
- Side wall angle : <0.1°
- CD non-uniformity : <40nm
- Angle of incidence of input beam : <0.6°