CHAPTER 4

CONTROL OF SWITCHED RELUCTANCE MOTORS

4.1 INTRODUCTION
The Switched Reluctance Machine (SRM) is a doubly notable pole machine in which torque is created by the propensity of the rotor to move towards the position where the inductance of the exciting stator pole winding is amplified. It has basic, solid and dependable mechanical structure. These features, in addition to the low manufacturing cost and its capability to operate in tough environment over a wide speed range make it an attractive for variable speed drive applications [141]. As of late, SRM has pulled refreshed interests for fast and elite motion control. Be that as it may, there have never been more choices in high valuable position actuators. Furthermore, the characteristics of SRM are highly dependent on its complex magnetic circuit, therefore it is difficult to model, simulate and control them. However, the higher torque ripple compared with conventional machines is a serious drawback that causes vibrations and acoustic noise in SRM. The present chapter targets to developments with SRM for high precision position control applications.

In [142], a micro-step control strategy realized with a fuzzy logic based controller was proposed. Anyhow, the position precision is limited to a predefined step angle. A position control system was presented in [122] but only response characteristics of the position controller in slow ramp-shape command position were analyzed. Basic four-quadrant sensorless compositions for SRM have been described in [94] and [118]. The development of linear motion actuator systems for position control applications were presented in [85] and [168]. The position control system with a 2-D planar SRM was explained in [130]. The controls of SRM require proper synchronism of rotor position
with pulses of phase currents. The larger angle is an obstacle for obtaining high precision position control. However, a proper control strategy that is based on regulation of the controller parameters could compensate this drawback. The PID control offers the simplest and competent solution that is needed for an automatic control system with SRM. The outlining and tuning of a relative necessary differential (PID) controller is somewhat instinctive however the consideration of some propelled control methods could result to a SRM control framework that fulfills the control targets of strength with short and smooth transient reaction.

In the past, either a linear model of SRM [72] or a nonlinear model with predetermined parameters was used for close-loop control [93]. New control techniques using PI and PD have been presented to optimize the steady-state performance of the drive [143]- [154]. A numerical method to calculate the magnetic characteristics for SRM drives is presented in [49] and the model parameters are determined by the 2D least squares method.

Aim of this paper is to develop a high performance control system, that the position precision is augmented by on-line fine tuning the speed controller parameters. The control plant is regarded as a single-input single-output system and the real time estimation of the speed PI parameters is accomplished through a nonlinear compensation look-up table that takes into account the influence of load torque and rotor speed variation. The four-quadrant control scheme of [42] is used that is based on the average torque control method. The real time approach improves the robustness of the control system with high dynamic performance and provides low torque ripple in steady state operation. A prototype experimental system is developed to demonstrate the effectiveness of the proposed control scheme.

4.2. SRM OPERATING PRINCIPLES

The SRM carries extremely non-linear features because of its non-linear flux conduct. The fundamental equations of SRMs are the stator voltage per phase

\[ \mu_j = R_j i_j + \frac{\partial \phi_j}{\partial t} \]  

(4.1)
and the electromagnetic torque production

\[ T_e = \sum_{j=1}^{m} \frac{\partial}{\partial \theta} \int_0^{i_j} \lambda_j \, di \]  

(4.2)

where \( u_j \) is the voltage applied to the terminals of phase \( j \), \( i_j \) is the current of phase \( j \), the winding resistance is \( R_j \), \( \lambda_j \) is the flux-linkage of phase \( j \), \( \theta \) is the mechanical angle, \( T_e \) is the total instantaneous electromagnetic torque and \( m \) is the number of SRM phases.

The electromagnetic torque of an SRM is described by the nonlinear torque-current-angle (\( T-i-\theta \)) data. The magnetic non-linear nature of the machine can be considered through the appropriate modeling of the nonlinear flux-current-angle (\( \lambda-i-\theta \)) characteristics. The machine model consists of look-up tables with the flux linkage \( \lambda(i, \theta) \) and the torque \( T_e(i, \theta) \) expressed as functions of current level \( i \) and rotor position \( \theta \).

If magnetic saturation and mutual coupling effects are neglected, the total instantaneous electromagnetic torque is given by

\[ T_e = \frac{1}{2} \sum_{j=1}^{m} i_j^2 \frac{dL_j(\theta)}{d\theta} \]  

(4.3)

where \( L_j \) is the inductance at the position \( \theta \) and \( dL_j/d\theta \) is the rate change of inductance for phase \( j \) at the position \( \theta \). As the phase current is squared, the torque direction can be inverted if the machine is energized in the negative inductance slope region. In this way, positive and negative torque can be produced by feeding current in one direction.

### 4.3 DEFINING THE FIRING ANGLE CONDITIONS FOR SMOOTH TORQUE POSITIONING CONTROL

The torque pulsations in a SRM are due to the discrete nature of torque production mechanism. The SRM stator phases are independently controlled and the total torque is the sum of the torques generated by each phases.

Torque pulsations are larger at the commutation intervals due to two adjacent phases form stabilized torques. Hence, it is achievable to part currents provides a smooth total current waveform.
As the speed is low in position control, the Switched reluctance motor operates in PWM mode. Soft-chopping control is used in motoring mode while hard-chopping control is applied in braking mode to hold on the load torque and to provide stable operation at zero speed. The turn-on and turn-off angles are online determined through simple formulas, as given in [157] and [126], for providing smooth torque operation and high performance position control.

The relationship between the idealized inductance profile and the motoring and generating flux-linkage, phase current and voltage waveforms of a SRM with PWM current control is shown in Fig. 4.1. The turn-on angle is selected so that the phase current acquires its reference value $i_{ref}$ on the angle $\theta_1$ at which the stator and rotor poles start to overlap and the inductance starts rising [157] and is determined by

$$\theta_{on} = \theta_1 - \theta_0 = \theta_1 - \frac{L_u i_{ref} \omega_r}{V_{dc}} \tag{4.4}$$

The flux-linkage on angle $\theta_1$ is then given by:

$$\lambda_d = L_u i_{ref} \tag{4.5}$$

For providing smooth current transfer between adjacent phases, the firing angles are controlled; $\theta_0$ interval is equal to the half of the de-fluxing period $\theta_{c2}$. Equation (4.6)
From (4.5) and (4.7) it is obtained

\[ \lambda_c = 2L_u i_{ref} \]  

Putting equation (4.8) in (4.3), the equation of the instantaneous torque is as follows:

\[ T_j(t) = \frac{1}{2} \frac{\dot{\lambda}_c^2}{(L_u)^2} \frac{dL_j(\theta)}{d\theta} = \lambda_c^2 K_n(L_j, \theta) \]  

Where \( K_n(L_j, \theta) \) is a function that depends on the variation of the motor inductance with respect to rotor angle.

The turn-off angle for smooth torque PWM motoring operation is determined by

\[ \theta_{off}^M = \theta_{on}^M + \theta_{sk} \]  

The turn-off angle for smooth hard-chopping PWM generating operation is selected at the rotor position that stator and rotor pole corners complete overlap [157]

\[ \theta_{off}^G = \theta'_{iu} \]  

and the turn-on angle is defined by

\[ \theta_{on}^G = \theta'_{iu} - \left( \theta_{sk} + \frac{\theta_{on}^G}{2} \right) \]  

### 4.4 Controller FOR SPEED CONTROL

A simplified SRM control system with speed closed-loop is shown in Figure 4.2. The torque production of the SRM is described by equation (4.2) is nonlinear and hence it should be linearized, if the basic linear system theory is to be used for determining the speed PI control parameters. This can be accomplished by regarding the small-exursion behavior of the system for small displacement of its variables. For this purpose, the flux-linkage \( \lambda_c \) is expressed by the following form

\[ \lambda_c = \lambda_{c0} + \Delta \lambda_c \]  

Where \( \lambda_{c0} \) is the steady-state value of \( \lambda_c \) and \( \Delta \lambda_c \) is the small excursion of the flux-linkage about its steady-state value \( \lambda_{c0} \). Substituting equation (4.13) into (4.9), it is obtained

\[ T_j(\lambda_c) = T_j(\lambda_{c0}) + \Delta T_j(\lambda_c) \]  

where
the least displacement expression for torque. Thus, the equation (4.15) explains the vibrant nature of the SRM for smallest displacements of the flux-linkage about a steady-state operating point.

The transfer function of the speed closed-loop with respect to small-displacement of speed command is given as follows

\[ G_s(s) \equiv \frac{\Delta \omega_r(s)}{\Delta \omega_r^*(s)} = K_1 \frac{s^2 K_i / K_p}{s^2 + K_1 s + K_2} \]  

(4.16)

where

\[ K_1 = \frac{(2K_p K_n \lambda_{co})}{J} \]  

(4.17)

And

\[ K_2 = \frac{(2K_i K_n \lambda_{co})}{J} \]  

(4.18)

To find out \( K_p \) and \( K_i \) in order to meet the organize objectives, a easy and clear-cut design process is followed. One of the most common control goals are the frequency bandwidth \( \omega_n \) and the damping ratio \( \zeta_c \) of the speed control loop. The transfer function of a small oscillation of an input signal around its steady state value is given by

\[ G_s(s) \equiv \frac{\omega_r(s)}{\omega_r^*(s)} = \left( \frac{\omega_n^2}{\delta} \right) \frac{s + \delta}{s^2 + 2\zeta_c \omega_n s + \omega_n^2} \]  

(4.19)

From (4.16) and (4.19), and for given values of \( \omega_n \) and \( \zeta_c = 1/\sqrt{2} \), the speed PI controller gains are determined by

\[ K_i = \frac{J \omega_n^2}{2K_n \lambda_{co}} \]  

(4.20)

and

\[ K_p = \frac{\sqrt{2} K_i}{\omega_n} \]  

(4.21)
From the above equations it is concluded that the speed PI controller gains depend on the frequency bandwidth $\omega_n$ and the function $K_n(L_j, \theta)$. Therefore, the $K_p$ and $K_i$ are adapted according to $K_n(L_j, \theta)$, instead of keeping them constant, consequently characteristics of the speed control loop will be same at all rotor positions. This means that, the speed PI parameters are adjusted according to the speed and load torque conditions, in order to meet the control objectives and thus it provide high performance of SRM.

The information for the nonlinear variation of the phase inductance with respect to rotor angle included in the function $K_n(L_j, \theta)$ can be given to the controller using a look-up table. This can be determined with a combination of offline experiments and pre-calculated data obtained from the numerical analysis of the SRM model.

![Figure 4.2. Speed control closed-loop of SRM.](image)

### 4.5. CONTROL STRATEGY

For deriving a low-complexity model of the SRM that is suitable for speed and position control, the machine dynamics are classified into groups with regard to their time constants. Therefore, a cascade control approach with two closed-loop controllers for speed and position is used.

Fig. 4.3 illustrates the general block diagram of the developed SRM speed and position control system. The phase current is controlled with a hysteresis-band PWM control method that is incorporated into the "Switch pulse “block.
For the speed control, a proportional-integral (PI) controller has been adopted to provide quick transient response and to ensure zero steady-state error. Standard form of speed PI controller is given by

$$G_{pi}(s) = K_{ps} \left( 1 + \frac{1}{T_D s} \right)$$  \hspace{1cm} (4.22)

An anti-windup protection is used in order to avoid low-frequency oscillations that may lead to instability [53]. This is realized by applying inner feedback to the integral action of the speed PI controller.

For position control, a proportional-differential (PD) controller has been used to compensate the delay effect of the integration applied in the speed control loop and to suppress the overshoot in the position response. The standard form of the position PD controller is

$$G_{pd}(s) = K_{pp} (1 + T_D s)$$  \hspace{1cm} (4.23)

The derivative term of the position control gives a high control signal when a step change of the reference or disturbance happens. To prevent this impulse of the control signal, the filtering remedy is most commonly adopted and the differentiator is cascaded with a low-pass filter [138].

Hence, the position controller becomes

$$G_{pd}(s) = K_{pp} \frac{T_D s}{1 + \left( T_D / \beta \right) s}$$  \hspace{1cm} (4.24)
where $\beta$ is constant. To improve the set-point tracking performance, a low-pass filter is included in the proportional term. This filter improves the regulation performance at steady-state and advances the load disturbance rejection.

The feedback system with speed and position controllers is shown in Figure 4.4. The transfer function $G_p(s)$ depicts the SRM and the power converter with the current control.

**Figure 4.4** Model configuration of the speed and position controllers

### 4.6. DESIGN AND IMPLEMENTATION OF THE CONTROL SYSTEM

Fig. 4.5 illustrates the Simulink diagram of the SRM drive that is used in simulations. Figs. 4.6 (a) and (b) represent the Simulink diagrams of the speed and position controllers of the SRM drive, respectively. The block “Firing angles” determines the command turn-on and turn-off angles, $\theta_{on}$ and $\theta_{off}$, respectively, through conditions presented in Section III. The block “Switch pulses” generates the control pulses for the switches of SRM power converter.

**Figure 4.5** Simulink diagram of the SRM drive with altering speed PI controller.
The block “Adaptive Speed PI” determines the reference phase current $i_{\text{ref}}$ through the speed error $\left( \omega^* - \omega_r \right)$. The gains $K_p$ and $K_i$ of the PI controller are adapted according to load torque and rotor speed and they are determined from expressions (20) and (21), respectively. The nonlinear function $K$ that considers the variation of phase inductance with respect to rotor angle is modeled by a look-up table.

**4.7. EXPERIMENTAL RESULTS**

A four-phase, 1-hp, 8/6 SRM drive was used to validate the developed position control system. The transient responses of the SRM drive to small and large rotation angle step commands are exhibited in Figs. 4.7 and 4.8, respectively. The mechanical load torque is 1 Nm. The desired position is obtained without overshoots the controller reacts very fast. The values of the turn-on and turn-off angles are significantly changed when transition between motoring and braking operation is occurred.
Note that, for the small rotation angle of 30, the conduction of only three successive phases is needed (Fig. 4.7). This justifies the difficulty in the controller design for achieving robust position tracking response. During the steady-stat operation, only one of the SRM phases is conducting to hold on the load torque at zero speed. Fig. 4.9 illustrates the transient response of the SRM drive to an abrupt load torque demand. It can be seen that the electromagnetic torque increases immediately to sustain the load torque. Throughout the transient operation, highest movement of the rotor is less than 8 degrees and the settling time is less than 0.2 s.

Figure 4.7: Transient reaction of the SRM drive to minute rotation angle step command [00 → 300 → 600].
Figure 4.8. Transient response of the SRM drive to a rotation angle, step command (1 revolution) to (3 revolutions).
4.8. CONCLUSIONS

In this chapter, PI and PD controllers for SRM position control are presented. The parameters of the speed PI controller are on-line adjusted according to the load torque and rotor speed. The analysis of the SRM control system is conducted using a small displacement version of the non-linear electromagnetic torque model. The four quadrant control plan depends on the normal torque control strategy and it is fit for keeping up the torque swell at a satisfactory level over a wide speed range. Hence, it provides high performance and precise position control of the SRM drive. The proposed control scheme has been applied on a 4-phase, 8/6 SRM and several experimental results are presented.