CHAPTER 2

FINITE ELEMENT METHOD (FEM)

2.1 INTRODUCTION
Modeling the electrical machines is vital and essential since it saves money and time; practically to build and make 30 to 40 distinctive motor designs cost money, time, and efforts also. The finite element method (FEM) is trustworthy when it manages with electromagnetic design. The electrical machines are modeled by the two-dimensional finite element method. SRM modeling has been used ever since the motor was in existence. The finite element method (FEM) is generally considered to be the preferred approach to figure out the static phase flux linkage and torque inside the motor. Different methods to translate static model into dynamic model have also been proposed. Among these, the most widely referred methods are the look-up table based approach of Stephenson and Corda [75], and the analytical logical expression based approach of Torrey [123]. The look-up table approach can be very precise but generally slow in simulation due to the retrieval process of the huge tabular data base middle point interpolation routines. To interpolate the data points to improve the accuracy of look-up table approach, Cubic spline technique has been suggested. B-spline based flux linkage analytical expression with respect to phase current is proposed to reduce the data base of look-up table approach, but data point interpolation with respect to rotor position is still necessary [153].

During the past twenty years, the number-based computation of magnetic fields has slowly become a standard in electrical machine design. At the same time, the amount of power electronics linked with electrical machines has continuously increased. The model of converters and electrical machines has normally been carried out discretely, but the expectations for improved productivity and performance at lower cost pushes
the product development activities towards a combined design process. Especially in large drives and variable-speed drives, both machine and converter must be individually fitted to work together and in that way assures the best possible performance for the application. In the field of complex engineering design problems, the numerical creation is tiresome and normally not possible by investigative methods. Hence the use of numeric techniques is attractive and useful. Finite element method FEM, which is very powerful tool for getting the number-based solution of a wide range of engineering problems. The basic idea is that a body or structure may be divided into smaller elements of finite dimensions called "Finite Elements". The original body or structure is then assembled as a finite number of joints called "Nodes" or "Nodal Points". The equations of balance for the whole structure or body are then got by combining the balance equation of each element such that the uninterrupted is made sure of at each node[18][47][61][62][63][100].

The first major finite element code for general use was NAS-TRAN developed for NASA by the MacNeal-Schwendler Corporation and Computer Sciences Corporation in the mid 1960s. In the late 1960s and into the 1970s, the use of the finite element method required the utilization of a large mainframe computer. With these machines, it was possible for designers and analysis engineers to use finite element analysis as part of their work without necessarily relying on the support of a finite element specialist. Applications of the finite element method can be divided into two categories, depending on the nature of the problem to be solved. In the primary category all the issues are known as equilibrium problems or time-independent problems. These are steady-state issues whose solution regularly requires the determination of natural frequencies and methods of vibration of solids and fluids. In the second category is the multitude of time-dependent or propagation problems of continuum mechanics [18][48][61][62][63][83][95][100][104].

The use of finite element method is very popular in the designing of electromechanical and electromagnetic devices. The mathematical algorithm and general conditions of the finite element method has a history of about fifty years. The components used were triangle, linear two dimensional (x-y) plane components. The modeling process
involves much more than filling out data records. A good physical understanding of the
device and an appreciation of the engineering aspects of the problem are needed.
Electromagnetic devices such as electrical machines, transformers, waveguides, and
antennas have their behavior governed by the electromagnetic fields. These fields
follow Maxwell's equations; therefore, in order to be able to estimate performance
characteristics, it is essential, in the course of design of these devices to solve the
Maxwell equations describing the field. Differential or, integral form of the Maxwell
equations has made electromagnetic field computations a heavily mathematically
oriented field of study. Before the development of the computer, available resources
mathematics had to solve electromagnetic equations, using solution ideas such as series
of expansion separation of variables, Bessel and Legendre polynomials, transform
Laplace, and the like [48][83][95][100][104][107][117][140][164].

However, the solution of electromagnetic field in the inside of even trivial devices
employing these methods is a rather lengthy and cumbersome procedure. Moreover, it
happens often that no resolution is possible without resorting to rather intense
simplifying presumptions concerning device geometry, current or charge distributions
etc. Fortunately, with the advent of the digital computers and the subsequent advances
in computing power, storage devices, as well as developments in numerical techniques,
it is now possible to use simple numerical approximation schemes to solve large-scale
problems within reasonable time limits[46][103][107][165].

2.2 COUPLED FIELD-CIRCUIT PROBLEMS
The coupled field-circuit problems are studied from the viewpoint of electrical
machines and converters. The main field of interest is the coupling of two-dimensional
finite element analysis with the circuit and control equations. In the early 1980's,
concept for such coupling was developed for modeling voltage-supplied electrical
machines. Including external circuits with power electronics was presented widely
during the late 1980's and early 1990's [120]. However, most of the studies concerned
rather simple geometries and circuits, because the calculating facilities were limited and
most of the authors had to develop the program codes themselves. Together with the

23
increasing computing power and development of the software, the complex difficulty of the modeled systems has also increased. The recent trend is to model large systems as a whole, including electro-magnetic, thermal fields, and dynamic and control systems. However, there is still a lot of work ahead to accomplish this goal and the coupling structure need to be studied further [34][120].

The usual approach is the magnetic vector possible creation with each element and solid conductors. The each element conductors, sometimes referred as stranded conductors, consist of many turns of thin wire carrying the same current. In order to simplify the analysis, the eddy currents in one conductor are not taken into account, but a constant current density is assumed. In the solid conductors, or conductors, eddy current represent a significant part of the total excitation and they cannot be left out from the analysis. The number-based solution of the coupled problem is generally completed directly or indirectly. The distinction lies in, whether the field and circuit equations are resolved at same time progressively. When the time constants in the sub-domains vary significantly from each other, it is benefit providing to disconnect the domains and use diverse time steps. Another major advantage is that the disconnected models can be built separately by the experts in different fields. Many types of coupled problems have been classified on the basis of physical, number-based or geometrical coupling. While consideration of the coupling between magnetic fields and electrical circuits, the coupling is physically strong, this means that they cannot be thought separately without causing a significant error in the analysis. Though, they can be analyzed circuitously in the case of diverse time constants [34][120].

2.2.1 Numerical Methods
In the time-stepping analysis of FEM-based non linear differential equations, the solution process needs methods for modelling the time-dependence, handling the non–linearity and solving the resulting system of equations [5]. The simple difference methods, like backward Euler, Galerkin or Crank-Nicholson, are the most commonly used methods for simulations of time-stepping. While these use results from two adjoining time steps, there are also many multi-step methods to accomplish numerical integration over many time steps and providing higher correctness. When processes of
substantially very different time scales are coupled together, the problem is mathematically believed as stiff. Most of the multi-step strategies usually fail for this kind of issues, but the implicit difference methods converge [5][60][96].

An iterative scheme is required for solution of non-linear equation. The traditional Newton-Raphson method, with its so many moderations, is used broadly for this reason, as well as the block iterative Picard methods. In order to improve the convergence, the cycle is often damped by relaxation procedures. The final system of equations arising from the finite element method is usually symmetric and positive definite. The equation of system is indefinite and often ill-conditioned when coupled with field circuit problems. This must be checked in selecting appropriate method for preconditioning and factorization. [77][81][82].

2.2.2 Modeling by Field and Circuit Equations
In the finite element model of an electrical machine, the magnetic field is energized by the currents in the coils. However, it is often more appropriate to model the feeding circuit as a voltage source, which leads to the combined solution of the field and circuit equations. Primarily, time harmonic formations using complex variables were represented for sinusoidal supply. Then time-stepping simulation was acquire from in order to model random voltage waveforms or transients. The phase windings in the stator and rotor are usually modeled as filamentary conductors, and the rotor bars in cage induction machines or damper windings in synchronous machines are modeled as solid conductors with eddy currents [82][86]

2.2.3 Coupling with External Circuits
Adding external circuits is comparatively simple, since it just needs only new elements into the circuit equations of the windings. For this purpose, many authors have presented general methods, in which any circuit models composed of resistors, inductors, capacitors, diodes or other thyristor, MOSFET etc can be connected with the electromagnetic model of the electrical machine. The mathematical creations for the circuit equations are usually based on loop currents or nodal voltages, but most of the formulation combines both approaches. The main reason for this is that the currents of
conductors with filament and inductances, as well as the voltages of solid conductors and capacitances, are the most natural selections for unknown variables in the coupled formulation, and therefore result in the lowest possible number of equations. A generalized formation for coupling two-dimensional finite element analysis with solid conductors with filament using sinusoidal voltage or current sources has been represented. Additional methodologies for time-stepping analysis have been formed to allow resistive and inductive elements in the external circuit. The unknown variables of the formulation were the magnetic vector potential, current in the filamentary conductors and inductors, and voltage drop over the solid conductors. Many authors have considered the field-circuit coupling from the circuit theoretical point of view. The methods presented, were based on the state-space approach, where the inductor currents and capacitor voltages were considered as the unknown variables in the circuit model. Wang [126] formulated the field equations to represent a multi-port circuit element, which was coupled to the electric circuit by the currents and voltages of the filamentary and solid conductors [35][59][166][167][169].

2.2.4 Coupling with Power Electronics

The simulation of power electronics together with electrical machines can be performed in various ways. The easiest approach is to define the supply voltage waveform with respect to time or position and utilize this pre-characterized supply in the simulation. However, modeling the real interaction between the electrical machine and the converter also demands models for the semiconductors. Normally, the switching elements are signified in the circuit model as binary-valued resistors, the value of which relies on the condition of the switch. A difference is often made between diodes and externally controlled switches because of the differences in defining the switching instant. In the simulation of diodes, the time step must be changed to fit the switching instants in order to prevent negative overshoots in the current. For the externally controlled switches, synchronization of the time steps is simple, since the switching instants are already known in advance [146][151][152].

The diodes were shown as binary valued resistors and the time steps were picked by rate of change in the magnetic properties and switching instant of the rectifier. A field-
circuit simulation of a load commutated inverter supplying a perpetual magnet engine has been introduced. Switches were demonstrated as binary valued resistors, and the converter operation was isolated into conduction and commutation order. The resistance and inductance values in the phase were changed by conditions of the switches. The strategy was created further and the state-space methodology was put into utilization. To develop a general procedure strategy by utilizing a programmed system to build the state-space condition for arbitrary circuit topologies and demonstrated method of the technique by simulating a fly-back converter with a saturated transformer. Linear movements and forces was included for modeling contactors and at the end method was extended for rotating machines by taking polyphase structure and rotational movements into account. [19][24][29][30][31][147][148][149][168].

2.3 FINITE ELEMENT MODEL FOR ELECTRICAL MACHINES
In the method of the electrical machine, the magnetic field in the iron core, windings and air gap is solved by the two-dimensional finite element method and coupled with the voltage equations of the stator and rotor windings. The resulting equations are solved by a time-stepping approach, while the Newton-Raphson iteration is utilized for handling the nonlinearities [1][2][51]

2.3.1 Maxwell’s equations
The magnetic field in an electrical machine is represented by Maxwell’s equations:

\[
\begin{align*}
\nabla \times H &= J \\
\n\nabla \times E &= -\frac{\partial B}{\partial t}
\end{align*}
\]

(2.1)

(2.2)

Where:

H is the magnetic field strength
J is the current density
E is the electric field strength
B is the magnetic flux density.
It is presumed that the polarization and displacement currents are insignificant due to low frequencies used with the electrical machines. Therefore, those components are omitted from equation (2.1) and the analysis is referred to as quasi-static.

Using the reluctivity, we have the material equation

\[ H = v B \]  \hspace{1cm} (2.3)

Where \( v \) represents a material-dependent, possibly nonlinear function of the magnetic field. The magnetic vector potential \( A \) defines the magnetic flux density as:

\[ B = \nabla \times A \]  \hspace{1cm} (2.4)

and the substitution of (2.4) and (2.3) into (2.1) gives the fundamental equation of the vector potential formulation for magnetic field

\[ \nabla \times (v \nabla \times A) = J \]  \hspace{1cm} (2.5)

The two-dimensional model is based on the assumption that the magnetic vector potential and current density have only \( z \)-axis components and their values are determined in the \( xy \)-plane as shown below:

\[ A = A (x, y) \, e_z \]  \hspace{1cm} (2.6)

\[ J = J (x, y) \, e_z \]  \hspace{1cm} (2.7)

Where, \( e_z \) denotes the unit vector in the \( z \)-axis direction. As a result, equation (2.5) becomes:

\[ - \nabla \cdot (v \nabla A) = J \]  \hspace{1cm} (2.8)
2.3.2 Source of the Field:

Although the two-dimensional analysis will be utilized, let us first consider a general case. The current density on the right-hand side of equation (2.5) can be determined from the material equation:

\[ J = \sigma E \]  

(2.9)

Where \( \sigma \) is the conductivity. Combining (2.2) with (2.4) gives

\[ \nabla \times E = -\frac{\partial \psi}{\partial t} \times A \]  

(2.10)

This is satisfied by defining the current density as:

\[ J = -\sigma \frac{\partial A}{\partial t} - \sigma \nabla \psi \]  

(2.11)

Where, \( A \) represents the electric scalar potential.

2.3.3 Material Properties

The magnetic properties of the laminated iron core are modeled by the relativity \( v \), which is a single-valued nonlinear function of the flux density \( B \), thus excluding the effect of magnetic hysteresis from the analysis. Since the eddy current is greatly reduced by the laminated structure, the conductivity is set to zero in laminated iron core. The shaft and pole shoes, which are usually made of alloy steel, are modeled as conductive iron with a non-linear magnetization curve. Resulting from the analysis above, the magnetic field in different materials can be presented in the form:

\[ \nabla \cdot (\nu \nabla A) = \begin{cases} 
0 & \text{in air and laminated iron} \\
Nw i_w / Sw & \text{in phase winding} \\
-\sigma \frac{\partial}{\partial t} A + \frac{\sigma u_b}{i_b} & \text{in rotor bars} \\
-\frac{\partial}{\partial t} A & \text{in conductive iron}
\end{cases} \]  

(2.12)
2.3.4 Stator Windings

The computational method of the electrical machine can be highly enhanced by coupling the circuit equations of the stator windings with the two-dimensional field equation (2.12). The equations of circuit, the dependence between voltage and current is resolved and the circuit quantities are added with the magnetic field by method for flux linkage. Likewise, the end-windings outside the region of core are demonstrated by incorporating an additional inductance in the circuit model.

\[ U_b = I_b \int_{s_b} \frac{\partial A}{\partial t} \, ds + R_{b} i_b + L_{bc} \frac{di_b}{dt} \]  

(2.13)

Where \( R_b \) denotes the resistance of the bar including the end region. All the rotor bars are associated by short circuit rings in both closures of the rotor core. This is taken into account by defining the end-ring resistance \( R_{sc} \) and the end-ring inductance \( L_{sc} \)

\[ u_{sc} = R_{sc} i_{sc} + L_{sc} \frac{di_{sc}}{dt} \]  

(2.14)

Where, \( u_{sc} \) and \( i_{sc} \) are vectors of voltage and current in the end-ring that connects the bars to each other.

The phase windings in the stator consist of several coils connected in series and distributed in several slots in the stator core. When the number of positively arranged coil sides is \( N_{pos} \) and the quantity of negatively oriented coil sides is \( N_{neg} \). Integration of the current density over all the coil sides in a phase winding provides a voltage equation

\[ u_w = I_w \left[ \sum_{n=1}^{N_{pos}} N_{wn} \int_{s_{wn}} \frac{\partial A}{\partial t} \, ds - \sum_{n=1}^{N_{neg}} s_{wn} \frac{\partial A}{\partial t} \, ds \right] + R_w i_w + L_{we} \frac{di_w}{dt} \]  

(2.15)

where \( I_w \) represents the length of the coils in the core region, \( N_{wn} \) is the number of turns in the coil side \( n \) and \( S_{wn} \) is the cross section area of the coil side \( n \). Voltage \( u_w \) is apply to the whole winding and current \( i_w \) flow across all coils that belong to phase winding. Resistance \( R_w \) include all coils and the end region outside the iron core. \( L_{we} \) is the inductance outside the centre region. Several different methods can be utilized in the numerical solution of the magnetic field equation (2.12), such as reluctance networks,
the boundary element method, finite difference method or finite element method. In this work, the numerical analysis is based on the finite element method (FEM). The two-dimensional geometry is covered by a finite element mesh, consisting of first or second-order triangular elements.

2.4 MOTION AND ELECTROMAGNETIC TORQUE

Except a constant speed is presumed, the movement of the rotor throughout time steps is resolved from the equations of motion:

\[ J \frac{d\omega_m}{dt} = T_e - T_L \]  \hspace{1cm} (2.16)
\[ \omega_m = \frac{d\theta_m}{dt} \]  \hspace{1cm} (2.17)

Where \( J \) represents moment of inertia, \( \omega_m \) represents angular speed and \( \theta_m \) is the angular position of the rotor. \( T_e \) represents electromagnetic torque and \( T_L \) represents load torque. The new place of the rotor is resolute at the beginning of each time step and a new net is created in the air gap. The electromagnetic torque is determined by the virtual work principle

\[ T_e = \frac{\partial}{\partial \theta} \left( \int_{\Omega}^H \left( \int_{H} B \cdot dH \right) d\Omega \right) \]  \hspace{1cm} (2.18)

where the integration area covers only the air gap. The implementation for finite element analysis follows the approach presented by [60], in which the virtual movement is determined by means of a coordinate transformation matrix without altering the air-gap mesh

2.5 ANSOFT SOFTWARE & FINITE ELEMENT ANALYSIS

Figure 2.1 shows the flow charts of the general procedure for solving the electrostatic or electromagnetic problems. The general procedure summarized below can be used to create a model of a 2d structure for computing the electric or the magnetic fields. This general procedure is to create and solve models of 2d structures; select the type of electric or magnetic field solver, and then select the desired solver. Depending on the
Maxwell 2d package, different electric or magnetic field solvers may be available. Select the type of model to be created, choose drawing, a menu appears, choose XY plane to create a Cartesian model, where the 2d model represents the XY cross-section of structure that extends infinitely long in the z-direction. Select RZ plane to create an asymmetric model, where the 2d model depicts the cross-segment that is revolved around an axis of symmetry; produce the geometric model of the structure. Choose define model, and from the menu that appears:

- Choosing the model to make (or change) the individual protests that make up the 2d cross area of the device for which fields are to be computed. Assign materials to objects in the structure. Picking set up materials to determine the material characteristics of articles, (for example, relative permittivity, and relative penetrability).

- Defining the desired sources (electromagnetic excitations) and boundary conditions for the model Choosing set up boundaries to describe the behavior the electric or magnetic field at object interfaces and the edges of the problem region. Compute other quantities of interest during the solution process. Quantities comprise forces, torques, matrices or flux linkage. Selecting to set up executive parameters and from the options that appears; selecting matrix to calculate a capacitance, inductance, impedance, admittance or conductance matrix for conductors in the structure. Choosing force to compute the force on selected objects due to the electric or magnetic field in the structure.

- Choosing the torque to compute the torque on selected objects due to the electric or magnetic field in the structure. Choosing core loss to compute the core loss for a system of objects. Choosing flux linkage to compute a value for the flux linkage across the line. Choosing current flow to compute the current flow across lines. Enter refinement criteria for the various field solvers and specify whether an adaptive analysis should be performed. Choosing set up options to enter this information (in most cases, accept the defaults). To compute fields over a two dimensional space, Maxwell 2d first creates a finite element mesh that divides the structure into thousands of smaller regions.
• The field in each sub-region (element) can then be represented with separate polynomial. In an adaptive analysis, the field simulator automatically refines the field solution in regions where error is highest. Optionally, it can be refining the model’s finite element mesh manually to increase the density of the mesh in areas of interest.

• For transient problems, define the motion parameter of the objects in the model. Choose set options setup to describe the motion parameters. Compute the desired field solution and any requested parameters (force, torque). by choosing solve to generate the solutions, after the solutions are completed, the following step has to be follow:

• Choosing the post process to display contour, shaded, and arrow plots of the electromagnetic field patterns and to manipulate the corresponding field solutions. Mathematical operations are allowing computing any quality of interest that can be derived from the basic electromagnetic fields. Choose solutions at the top of the executive commands window to view the final results from any force, torque, flux linkage, current flow, or matrix computation. In general, the commands in this procedure must be chosen in the sequence listed.

Figure 2.2 shows the details of a finite element mesh in which more than 15 thousands elements are used in ten passes to represent the cross-section of a switched reluctance motor. The mesh is refined in the regions where the flux density is expected to be high; where there is rapid spatial variation of the field; and in the air-gap. The stator coil-side is represented by a simple geometrical shape. For accurate work it is important to try to reproduce the exact cross-section of the coil, and even of each conductor within it, especially for the calculation of the total flux-linkage of the coil. The total flux-linkage is the sum of the flux-linkage of all the individual loops of wire, and these are generally not equal because the flux density varies considerably across the cross-section of the slot, particularly in the unaligned position.
Select solver and drawing plane.

Draw geometric model and (optionally) identify grouped objects.

Assign material properties.

Assign boundary conditions and sources.

Compute other quantities during solution

Yes

Request that force, torque, capacitance, inductance, admittance, impedance, flux linkage, core loss, conductance, or current flow be computed during the solution process.

No

Set up solution criteria and (optionally) refine the mesh.

Generate solution.

Inspect parameter solutions; view solution information, display plots of fields and manipulate basic field quantities.

Figure 2.1: Sequence of Solving Problem
The accuracy of finite-element software depends on the skill of the user and on the nature of the problem. The choice of the angular displacement of the rotor ($\Delta \theta$) is very important to determine the accuracy and the time of the simulation.

![Finite Element Method Mesh for 3 Phases, 6/4 Poles SRM](image)

**Figure 2.2: Finite Element Method Mesh for 3 Phases, 6/4 Poles SRM [129].**

### 2.6 CONCLUSION

The Finite Element Method (FEM) can be utilized to take care of any issue that can be defined as a field issue. It can deliver precise and solid results when planning for electromagnetic devices. FEM can be used by utilizing diverse PC programming. It is an important outline device, if it is utilized accurately and can spare cash, materials and time. FEM is an exceptionally helpful apparatus in the arrangement of electromagnetic issues.
The advancement of programming items has expanded drastically in the most recent 20 years. Finite element investigation would not be the place it is today if PCs had not multiplied and turned out to be speedier and less costly to a degree nearly mind-boggling. FEM can create exact and consistent predictions of the device parameters, and the reliable and accurate model. Its answers depend on an exact representation of the issue and right examination methods. The configuration of electromagnetic devices requires accurate count of the design parameters.