Chapter – XII

On the Least Remainder Division Method
INTRODUCTION

In order to make the division process more operative and practical, in the previous chapter, we have proposed a method based on the postulate restricting the partial quotient \( q_i \) such that

\[
\frac{x_i}{d} - 1 < q_i < \frac{x_i}{d} + 1 ,
\]

where \( d \) is the leading digit of the divisor and \( x_i \) the partial dividend. This method has its own merit, academic as well as operational, since the restriction imposed on the \( q_i \) is mathematically viable and not totally adhoc similar to that of Parthasarathi and Jhunjhunwala (preprint) who restrict the domain of the partial remainder \( r_i \) such that

\[
-\frac{1}{2} d \leq r_i \leq \frac{1}{2} d
\]

(12.2)

It is true our result in the previous chapter and that of Parthasarathi and Jhunjhunwala (preprint) are in excellent agreement in respect of the quotient \( Q \), though they differ on the pattern of \( x_i, r_i \) and \( q_i \).

As far as Parthasarathi and Jhunjhunwala (preprint) is concerned its practicality is more but owing to the silence on its justification it can be accepted only on the pretext that it suits and confirms the findings. Though our approach in the previous chapter serves our purpose but there is a doubt whether the students at primary standard appreciate the inequality and then choose the minimum integer \( q_i \) from the negative rational numbers. Knowing fully the limitations of
these methods another simpler one suitable to the needs of the students is very much wanting and this note is devoted to this cause.

**Procedure**

Following the meaning of the symbols $X$, $D$, $Q$ and $R$ of our chapter 10, we have

\[ \frac{X}{D} - 1 < Q < \frac{X}{D} + 1 \] \hspace{1cm} (12.3)

Using the division algorithm, we derive

\[ \frac{X}{D} - \frac{R}{D} = Q \] \hspace{1cm} (12.4)

Considering the left part of (12.3), above (12.4) gives

\[ \frac{X}{D} - \frac{R}{D} > \frac{X}{D} - 1 \]

\[ \Rightarrow \quad \frac{X}{D} > -1 \]

\[ \Rightarrow \quad -R > -D \]

\[ \Rightarrow \quad R < D \]

Similarly, (12.4) with the right part of (12.3) derives

\[ R < -D \]

Thus we get

\[ -D < R < D \]
or in terms of the partial quantities this becomes

\[-d < r_i < d\]  \hspace{1cm} (12.5)

We employ above to solve \(X = 6250, \ D = 539\)

Here \(d = 5\) and hence the restriction on \(r_i\) is

\[-5 < r_i < 5\]

i.e., \(r_i = 0, \pm 1, \pm 2, \pm 3, \pm 4\)  \hspace{1cm} (12.6)

we have

\[
\begin{array}{cccccc}
8 & 32 & 3 & 30 & 25 \\
5 & 4 & 1 & 6 & 2 & 5 & 0 & 0 & 0 \\
1 & 3 & 2 & 2 & 0 & 0 \\
1 & 1 & 6 & 6 & 5 \\
\end{array}
\]

\[\Rightarrow \ Q = 1111.6165 = 11.595\]

The technique works satisfactorily but there is a major lacuna in this method – the choice of \(r_i\) and hence that of \(q_i\) is not unique. For example consider the second stage: \(x_2 = 8\).

Corresponding to this (11.5) or (11.6) gives \(r_2 = 3, -2\)

Thus partial remainder: \(3, \ 2\)

\[\uparrow \ \uparrow\]

Partial quotients: \(1, \ 2\)
The question is how to achieve the uniqueness of the choice of \( r_2 \) i.e., 3 or \( \bar{2} \) to avoid arbitrariness? Here we have selected 3 but one can also select \( \bar{2} \). Therefore, we have to evolve some process which terminates the arbitrariness in the method. The inequality (12.5) does possess an inherent potential to resolve the dilemma. It is easy that

\[
(-d, d) = (-d, 0) \cup (0, d)
\]

Thus instead of (12.5), we consider two inequalities

\[
\begin{align*}
-d < r_i < 0 & \quad \text{(12.7a)} \\
0 \leq r_i < d & \quad \text{(12.7b)}
\end{align*}
\]

Now we postulate: choose \( q_i \) s.t.

\[
\begin{cases}
    r_i \in (-d, 0) & \text{for } x_i < 0 \\
    r_i \in [0, d) & \text{for } x_i \geq 0
\end{cases}
\]

The postulation is natural and amounts to

(i) Positive dividends \( \rightarrow \) positive remainder and positive quotients

(ii) negative dividends \( \rightarrow \) negative remainders and negative quotients

The very idea of the least remainder is not at all required and should be buried with due honours.

Now let us come to the objection which was raised earlier in case \( x_2 = 8, \ r_2 = 3, \bar{2} \)
Here $x_2 > 0 \Rightarrow r_2 > 0$

Thus $r_2 = \overline{2}$ is rejected and we select $r_2 = 3$. With this the detailed calculations present the following:

\[
\begin{array}{cccccccc}
8 & 32 & 3 & 24 & 24 & 28 \\
5 & 4 & \overline{1} & 6 & 2 & 5 & 0 & 0 & 0 \\
1 & 3 & 2 & \overline{3} & 4 & \overline{4} & \overline{3} \\
1 & 1 & 6 & 0 & \overline{4} & \overline{4} & \overline{5}
\end{array}
\]

$\Rightarrow Q = 11.60 \overline{4} \overline{4} \overline{5} = 11.59555$

We hope the students at large will be benefited as the arbitrariness for the estimation of the remainders has been taken care of. However, we suggest that this method and also the other methods for fast calculations should be employed if they are demanded or necessity arises. One must not be crazy for these methods. These methods should be brought into action as and when the general conventional methods become cumbersome, time consuming or inconvenient. If the problem can easily be solved by trial methods it should be done so and one need not insist on a particular method since our aim is to make computations simpler and not to tax the boys with intricacies.

For example suppose

\[
X = 741900, \quad D = 742
\]
The division can be performed trivially such as

\[
\frac{X}{D} = \frac{742000 - 100}{742} = \frac{742000}{742} - \frac{100}{742}
\]

\[
= 1000 - \frac{100}{742} = 999 + 1 - \frac{100}{742}
\]

\[
= 999 + \frac{742}{742} - \frac{100}{742}
\]

\[
= 999 + \frac{642}{742}
\]

⇒ \(Q = 999, \ R = 64\)

Again we stress that the method may not be very convenient in some problems having \(d = 1, 2\) etc. It is noteworthy that the problem cited in (3) gets easy solutions done by proximity \(-d\) for \(d = 2\).

The detailed sums are as follows:

\[
\begin{array}{cccccccccccc}
2 & 4 & 2 & 0 & 1 & 1 & 10 & 4 & 8 & 6 & 6 & 8 & 4 \\
2 & 4 & 1 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
1 & 0 & 0 & 0 & 4 & 1 & 1 & 2 & 2 & 3 & 3 & 1
\end{array}
\]

\(Q = 999.5867769\)