6.1 Introduction

Ellipsometry is a non-destructive, sensitive and highly accurate optical method which is used for different purposes like, measurement of optical constants, testing of surface accuracy, study of corrosion etc. This technique though well known even a century ago was not widely used mainly because the equations used to interpret the measured parameters involved tedious computations. In recent years these difficulties have been resolved by the introduction of automatic ellipsometers, high speed and rather cheap computers and development of better computer programmes for reducing the data. All these things resulted in the quick development of ellipsometry in the 1970's and now the technique is widely used in different areas including industries where the applications include the study of molecular contaminants on the performance of space born optical systems and the monitoring of dielectric layers deposited over silicon chips in semiconductor industry.

The basis of ellipsometry is the characterisation of changes in the state of polarisation of light reflected (transmitted) by an optical surface
(transparent material). The changes in polarisation is described by two angles $\Delta$ and $\psi$ which correspond to the phase difference and amplitude attenuation of the electric field vectors perpendicular and parallel to the plane of incidence. Using these two parameters and the value of angle of incidence one can calculate the optical constants 'n' and 'k' of the material (i.e., the real and imaginary parts of the refractive index). If the surface of the material is covered by a film its thickness can also be calculated, provided the refractive indices of both film and substrate (surface over which the film is formed) are known. An advantage of this method is that $\Delta$ and $\psi$ are not dependent on the absolute light intensities but only on the ratio of intensities. Another important study using ellipsometry is on the nature of surfaces; the variation of ellipsometric parameters can characterise surface features. It is found that $\psi$ is very sensitive to the surface roughness which is relatively insensitive to the thickness of dielectric (e.g. oxide) films on the surface. But $\Delta$ is extremely sensitive to the film thickness and relatively insensitive to the surface roughness. The phase shift $\Delta$ can be used to detect as little as 0.1Å to as much as 5000Å of oxide or a film of contamination on the top of an oxide.
Another point supporting the ellipsometric technique is its capability to be used as a set up for in situ studies of different reactions which many other experimental techniques used in this direction (like electron microscopy) do not have. For example, ellipsometry can be used for the study of reaction taking place at a solid-liquid interface, which cannot be analysed by using an electron microscope since the surface to be analysed using it should be kept in high vacuum. The fact that the area of the surface to be studied can be very small, is another advantage over other methods of analysis. Along with these specialities it is also to be taken into account that this is a nondestructive technique.

6.1a Brief review of earlier works

The studies conducted by various investigators testify to the extent of applicability of this method. Meyer\(^3\) employed ellipsometry for the calculation of surface state densities of semiconductors for which he estimated the dielectric constants from the values of refractive indices. This technique was also adopted by different groups\(^4\),\(^5\) for studying the effect of doping
in different materials. For example, Tyagai et al.\(^4\) studied the morphology of silicon surfaces after doping it with silver while Briska et al.\(^5\) have analysed the effect of doping arsenic. They studied variations due to different arsenic concentrations at different temperatures in different ambient conditions. Hare et al.\(^6\) have investigated the properties of F centres in KCl crystals using ellipsometry. These centres were formed as a result of bombardment by electrons with energy in the range 2—l0KeV. It has been shown that, unlike earlier methods, this technique can give reproducible and consistent results. The study of oxide layer formation on different materials under different conditions is an important area in which this optical technique finds maximum application.\(^7\)-\(^13\). Among these, Allen\(^7\) developed an IR ellipsometer for studying the oxide layer formation by anodisation of aluminium and the instrument operated at 10.6μm. Boggio\(^8\) investigated ellipsometrically the oxidation of (111) face of copper single crystals at 21°C for oxygen pressure from 0.03 to 760 torr and found that a mono-layer of CuO forms on the top of Cu₂O film at high oxygen pressure. Habraken et al.\(^9\) have also analysed
the oxidation of (111) face of copper for which they used Auger electron spectroscopy and LEED along with ellipsometry. The oxygen pressure in this investigation was very low \((10^{-6} - 10^{-4} \text{ torr})\) while the temperature range was quite wide \((23^\circ - 400^\circ \text{C})\). Idozak et al\(^{10}\) have published an algorithm to determine the refractive index and thickness of oxide layers on absorbing substrate. The growth of very thin oxide layers \((15-80\text{Å})\) on polycrystalline silicon has been studied by Horiuchi\(^{12}\) in low oxygen pressure \((10^{-8} \text{ atm.})\) and high temperature range \((850-1050^\circ \text{C})\). Another important study is performed by Belyaev\(^{14}\) in which this technique is used for noting the changes in ellipticity of light after passage through thin homogeneously oriented layers of liquid crystals. The use of this technique in the field of amorphous silicon is recently gaining prominence\(^{15-17}\). Theeten\(^{15}\) has given a good review regarding the use of ellipsometry in connection with vapour phase epitaxy or plasma processing of semiconductors. In this paper it is shown that this surface sensitive, nondestructive real time method can be used to monitor the kinetics of thin film deposition and also to analyse the interface regions between successive layers. Drevillon et al\(^{16}\) used fast real time ellipsometry
for studying the growth of amorphous hydrogenated silicon deposited in low pressure D.C. discharge of silane. Flammé\textsuperscript{17} developed a new method for measuring refractive index absorption index and thickness of thin films using ellipsometry and applied it on amorphous silicon films (with thickness less than 1050 Å) prepared by low pressure chemical vapour deposition technique.

The use of ellipsometry in the area of corrosion is reviewed by Petit et al\textsuperscript{18}. They show that this method is very much useful in the case of localised corrosion. Another good review in this direction is given by Kruger et al\textsuperscript{19} in which the necessary theory and experimental set up are described in an elaborate manner with due stress on materials like copper, aluminium, titanium iron and steel etc.

Analysis of surface roughness (of thin films and bulk materials) is a field where ellipsometry has a unique place. Works in this field, using this technique are reviewed in detail in chapter VII.

In order to get a clear idea about different types of ellisometers and their specified uses one
has to go through the concerned reviews. Aspenes\textsuperscript{20}, in his review, touches all aspects of spectroscopic ellipsometers. Muller\textsuperscript{21} gives the operating principles and capabilities of automatic ellipsometers. Also he hints about the possible developments. Kinonita et al\textsuperscript{22} describe quick methods to evaluate the refractive index and thickness from the ellipsometric parameters using modern ellipsometers. They also consider the analysis of submonolayers. Hauge\textsuperscript{23} in his recent review gives a good description of modern ellipsometers along with necessary theory. He describes the developments in different branches of ellipsometry.

6.2 Theory

For interpreting ellipsometric data, when polarised light is reflected from or transmitted through bare or filmed substrates, one should use the electromagnetic theory to derive expressions of complex amplitude reflection and transmission coefficients from macroscopic optical properties. A large amount of theoretical work has been done on this topic and detailed discussions can be found in a number of books\textsuperscript{24–26} and review articles\textsuperscript{27–30}. Here a simple structure consisting
of a planar interface separating two homogeneous optically isotropic media (medium-0 and medium-1) is considered. On to this interface a plane polarised light beam is incident at an angle of incidence $\theta_0$. This is illustrated in fig.6.1. This model just enables one to get an idea about the relation between the ellipsometric parameters ($\psi$ and $\Delta$), Fresnel's reflection (transmission) coefficients and the refractive index. In the following treatment, the complex refractive index of an absorbing medium is always taken as

$$N = n - ik$$

(6.1)

where $n$ is called index of refraction (or real part of N) and $k$ the extinction coefficient (or imaginary part of N). Also the expression for the electric vector of an optical plane wave travelling in the positive direction of Z axis in an isotropic absorbing medium, with planes of constant phase and constant amplitude parallel, is given by

$$E = E_0 e^{i(\omega t + \phi)} e^{-i\omega Nz/c}$$

$$= E_0 e^{i(\omega t + \phi)} e^{-i\omega z/c} e^{-\omega k z/c}$$

(6.2)

where $\phi$ is a constant phase angle, $c$ is free space
Fig. 6.1 Oblique reflection and transmission of plane wave at planar interface between two semi-infinite media 0 and 1.
velocity and \( E_0 \) (which is in general complex) defines both amplitude and the polarisation of the wave. From eqn. (6.2) it is clear that the wave amplitude decays exponentially along the direction of propagation at the rate of \( \omega k/c \).

From fig. 6.1 it is quite clear that \( \phi_1 \) is the angle of refraction in medium-1 and the plane of incidence is the plane of the paper. It is assumed that \( N_o \) and \( N_1 \) are the complex refractive indices of medium-0 and medium-1 and the change of refractive index across the interface is abrupt. Now according to Snell's law we have,

\[
N_o \sin \phi_o = N_1 \sin \phi_1 \quad (6.3)
\]

If both media are transparent, \( N_o \) and \( N_1 \) are real numbers, the angles \( \phi_o \) and \( \phi_1 \) are also real and the case is very simple. However, when either one or both media is absorbing, the angles and refractive indices, in general, become complex and the physical picture becomes much complicated.

It is well-known that when incident wave is linearly polarised, with its electric vector vibrating
parallel (p) to the plane of incidence both reflected and transmitted waves will also be similarly polarised with their electric vectors vibrating parallel to the plane of incidence. Similarly, when the incident wave is linearly polarised perpendicular (s) to the plane of incidence, the reflected and transmitted waves are found to be linearly polarised perpendicular to the same plane. Hence the linear vibrations parallel (p) and perpendicular (s) to the plane of incidence are called the 'Eigenpolarisations' of reflection and refraction.

Now one can consider the amplitudes of reflected and transmitted waves in terms of those of the incident wave. For this, an arbitrarily polarised incident wave is considered and its electric vector is resolved into p and s components. Each component is treated separately and the results are combined later. Let $E_{ip}$ and $E_{is}$ represent the p and s components of the complex amplitudes of the electric vectors of the incident wave. Similarly $(E_{rp}, E_{rs})$ and $(E_{tp}, E_{ts})$ be the corresponding components of the electric vectors of the reflected and transmitted waves respectively.
Matching the tangential E and H fields across the interface one gets

$$\frac{E_{rp}}{E_{ip}} = r_p = \frac{N_1 \cos \phi_0 - N_0 \cos \phi_1}{N_1 \cos \phi_0 + N_0 \cos \phi_1}$$ (6.4)

$$\frac{E_{rs}}{E_{is}} = r_s = \frac{N_0 \cos \phi_0 - N_1 \cos \phi_1}{N_0 \cos \phi_0 + N_1 \cos \phi_1}$$ (6.5)

$$\frac{E_{tp}}{E_{ip}} = t_p = \frac{2N_0 \cos \phi_0}{N_1 \cos \phi_0 + N_0 \cos \phi_1}$$ (6.6)

$$\frac{E_{ts}}{E_{is}} = t_s = \frac{2N_0 \cos \phi_0}{N_0 \cos \phi_0 + N_1 \cos \phi_1}$$ (6.7)

These are called Fresnel's complex amplitude reflection (r) and transmission (t) coefficients for p and s polarisations. Snell's law (eqn (6.3)) can be used to change eqns. (6.4)-(6.7) such that these coefficients are dependent on $\phi_0$ and $\phi_1$ only.

In order to study the effect of reflection and transmission (refraction) on the amplitude and phase of
the waves separately one can write the complex Fresnel coefficients as

\[ r_p = \left| r_p \right| e^{i \delta_{rp}} \quad (6.8) \]

\[ r_s = \left| r_s \right| e^{i \delta_{rs}} \quad (6.9) \]

\[ t_p = \left| t_p \right| e^{i \delta_{tp}} \quad (6.10) \]

\[ t_s = \left| t_s \right| e^{i \delta_{ts}} \quad (6.11) \]

Here \( r_p \) gives the ratio of amplitude of the vibrations of electric vectors of reflected wave to that of the incident wave when the latter is polarised parallel to the plane of incidence with similar meanings for \( r_s \), \( t_p \), and \( t_s \). \( \delta_{rp} \) represents the phase shift due to reflection experienced by the electric vector parallel to the plane of incidence with similar meanings for \( \delta_{rs} \), \( \delta_{tp} \), and \( \delta_{ts} \).

It is obvious from eqns. (6.4)-(6.7) that the Fresnel complex amplitude reflection \( r \) and transmission coefficients have different values for
p- and s-polarisations. Hence due to reflection (or transmission) the relative amplitude and phase relationship between these two components will be changed. It is also clear that if the incident light wave is polarised in an arbitrary state (other than p or s states) the polarisation of the reflected and transmitted waves will be changed due to the difference in absolute values or the angles (or both) of the Fresnel coefficients of reflection and transmission.

In reflection ellipsometry one measures the states of polarisation of incident and reflected waves and determines the ratio (\( \ell \)) of complex Fresnel reflection coefficients for p and s polarisations as

\[
\ell = \frac{r_p}{r_s} \tag{6.12}
\]

Using eqns. (6.8) and (6.9) one can rewrite the above eqn. as

\[
\ell = \frac{|r_p|}{|r_s|} e^{i(\delta p - \delta rs)}
\]

\[
= \tan \psi e^{i\Delta} \tag{6.13}
\]
Hence, $\psi$ and $\Delta$ represent the differential changes in amplitude and phase respectively due to reflection by the component vibrations of the electric vector parallel and perpendicular to the plane of incidence.

Substituting values of $r_p$ and $r_s$ from eqns. (6.4) and (6.5) in eqn (6.12) one gets,

$$N_l \cos \phi_o - N_0 \cos \phi_1 \over N_l \cos \phi_o + N_1 \cos \phi_1 \times N_0 \cos \phi_o + N_l \cos \phi_1 \over N_0 \cos \phi_o - N_1 \cos \phi_1$$

This on simplification using Snell's law (eqn.(6.3)) gives the following expression

$$\frac{N_l}{N_0} = \sin \phi_o [1 + (\frac{1-\ell}{1+\ell})^2 \tan \phi_o] \quad (6.15)$$

Hence $N_l$ the refractive index of medium 1 can be computed provided one has $\ell$ and $\phi_o$.

6.2a Ellipsometric evaluation of $\ell$

In this section, the theoretical aspects of the measurement of ellipsometric parameters $\psi$ and $\Delta$ and the computation of $\ell$ and the refractive index from
them are considered. For this, the most general ellipsometric set up consisting of Polariser, Compensator, System and Analyser (the PCSA arrangement) is taken into account and is shown in fig.6.2. The light waves (which is well collimated, unpolarised and monochromatic) from the source L is rendered plane polarised by the polariser P after which it passes through the compensator C. The emergent beam from C is reflected by the optical system, S which is under investigation and the reflected beam then comes to the detector D, after passing through the analyser A. Here the optical system S is assumed to be having its eigenpolarisations along the orthogonal coordinates X and Y (ie., in the case of a reflecting film, the p and s polarisations will be coinciding with X and Y coordinates). Also the orientations of polariser, compensator and analyser around the beam axis are specified by the azimuth angles P, C and A respectively in fig.6.2. For the polariser and analyser, azimuths P and A define the orientation of their transmission axes (ie., the directions of transmitted linear eigenpolarisations) while for the compensator the azimuth C defines orientation of its fast axis (ie., the direction of the
Fig. 6.2 The Polariser-Compensator-System-Analyser (PCSA) arrangement.

L - Light source; D - Detector.
fast linear eigenpolarisation). All these azimuths P, C and A are measured from the direction of the X linear eigenpolarisation of the optical system, S and the angles are taken to be positive in anticlockwise direction when looking against the direction of propagation of the beam. Here the light beam is described by its Jones vector and the optical elements by their Jones matrices \(^{31}\) since, by this method, one can easily follow the state of polarisation of the light beam as it progresses through the components of the ellipsometer.

The notation used in the following treatment can be explained in a simple way as follows. The superscripts denote the coordinate system with respect to which the Jones vector or matrix is referenced. In the subscript the first letter denotes the optical component while the second shows whether the beam is its input or output. For example \(E_{\text{fs}}^{\text{CO}}\) is the Jones vector (representing the electric field) of the light beam at the output of the compensator, in its fast slow principal frame of reference (The frame of reference is the coordinate system in which the Jones matrix of an optical component is diagonal). Another important
matter to be considered is the change of coordinate system, which becomes a necessary matter when the light beam after emerging from a system enters into a new one. This is accomplished by using the rotation matrix $R(\alpha)$ and the counter rotation matrix $R(-\alpha)$ and is written as

$$
R(\alpha) = \begin{bmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{bmatrix}
$$

(6.16)

The effect of the polariser is to be considered first. The output of this system will be plane polarised light which can be denoted as,

$$
E_{PO}^{te} = A_c \begin{bmatrix} 1 \\ 0 \end{bmatrix}
$$

(6.17)

Here, $A_c$ contains information regarding the intensity and absolute phase of the wave emergent from the polariser. The superscript 'te' denotes the 'transmission-extinction' principal frame of reference.

Before examining the effect of the compensator one has to change the reference coordinate system, from the transmission-extinction principal frame of the
polariser to the 'fast-slow' principal frame of the compensator. This is achieved by using the rotational matrix \( R(P-C) \) as,

\[
E_{CI}^{fs} = R(P-C) E_{PO}^{te}
\]

\[
= A_C \begin{bmatrix}
\cos(P-C) & -\sin(P-C) \\
\sin(P-C) & \cos(P-C)
\end{bmatrix} \begin{bmatrix}
1 \\
0
\end{bmatrix}
\]

\[
= A_C \begin{bmatrix}
\cos(P-C) \\
\sin(P-C)
\end{bmatrix}
\]

(6.18)

The compensator is having a slow-to-fast complex amplitude transmittance given by,

\[
\ell_c = T_C e^{i \delta_c}
\]

(6.19)

This equation shows that the component of the electric vector incident on the compensator parallel to its slow axis is retarded by \(-\delta_c\) and is attenuated in amplitude by \(T_C\) relative to the component parallel to its fast axis. Its Jones matrix can be written as

\[
T_{CI}^{fs} = K_C \begin{bmatrix}
1 & 0 \\
0 & \ell_c
\end{bmatrix}
\]

(6.20)
where $K_C$ accounts for the equal attenuation and phase shift along the fast and slow axes. To get the Jones vector of the output beam, $E_{CI}^{fs}$ should be multiplied by $T_C^{fs}$ which gives,

$$
E_{CO}^{fs} = T_C^{fs} E_{CI}^{fs}
$$

$$
= K_C A_C \begin{bmatrix} \cos(P-C) \\ \ell_C \sin(P-C) \end{bmatrix} \tag{6.21}
$$

In order to bring the Jones matrix to the X-Y principal frame of the optical system $S$, one has to perform a coordinate counter rotation by an angle $C$ and can be performed as

$$
E_{CO}^{xy} = R(-C) E_{CO}^{fs}
$$

$$
= K_C A_C \begin{bmatrix} \cos(P-C) - \ell_C \sin C \sin(P-C) \\ \sin C \cos(P-C) + \ell_C \cos C \sin(P-C) \end{bmatrix} \tag{6.22}
$$

Now one can note the modifications of the polarisation caused by the optical system, $S$. Since there is no other systems in between the compensator and the
system, $S$ it can be taken that,

$$E_{CO}^{xy} = E_{SI}^{xy}$$  \hspace{1cm} (6.23)$$

Hence the Jones vector of the beam at the output of $S$ is given by the equation,

$$E_{SO}^{xy} = T_{S}^{xy}E_{SI}^{xy}$$  \hspace{1cm} (6.24)$$

In the above equation the Jones matrix of the system, $T_{S}^{xy}$, is diagonal since it is assumed to have orthogonal linear eigenpolarisations, parallel to $X$-$Y$ coordinate axes and the matrix can be written as

$$T_{S}^{xy} = \begin{bmatrix} V_{ex} & 0 \\ 0 & V_{ey} \end{bmatrix}$$  \hspace{1cm} (6.25)$$

where $V_{ex}$ and $V_{ey}$ represent the eigenvalues of the $X$ and $Y$ linear eigenpolarisations. Substituting the values of $E_{SI}^{xy}$ and $T_{S}^{xy}$ in eqn.(6.24) we get,

$$E_{SO}^{xy} = K_C A_C \begin{bmatrix} V_{ex} [\cos C \cos (P-C) - \ell_c \sin C \sin (P-C)] \\ V_{ey} [\sin C \cos (P-C) + \ell_c \cos C \sin (P-C)] \end{bmatrix}$$

$$\hspace{1cm} (6.26)$$
This beam, coming from S has to pass through the analyser A which can be rotated around the beam axis. To study the effect of the analyser on the polarisation of the beam, one has to make a coordinate rotation from the optical system's X-Y principal frame to the analyser's t-e principal frame as

$$E_{Al}^{te} = R(A)E_{Al}^{xy}$$  \hspace{1cm} (6.27)

Taking that $E_{Al}^{xy} = E_{Al}^{xy}$ and substituting for $R(A)$ the above equation becomes,

$$\begin{bmatrix} E_{Al,t} \\ E_{Al,e} \end{bmatrix} = \begin{bmatrix} (\cos A E_{Al,x} + \sin A E_{Al,y}) \\ (-\sin A E_{Al,x} + \cos A E_{Al,y}) \end{bmatrix}$$  \hspace{1cm} (6.28)

(In eqn.(6.28) the third term in the subscripts denote the corresponding coordinate axis). The Jones matrix of the analyser is given by the expression

$$T_A = K_A \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$  \hspace{1cm} (6.29)

where $K_A$ represents the amplitude and phase changes experienced by the transmitted linear eigenpolarisation.
Then the electric vector at the output of the analyser is obtained as,

\[ E_{AO}^{te} = T_A E_{AI}^{te} \]  \hspace{1cm} (6.30)

Substituting for the value of \( T_A \) from eqn.(6.29),

\[ E_{AO}^{te} = K_A \begin{bmatrix} E_{AI,t}^* \\ 0 \end{bmatrix} \]  \hspace{1cm} (6.31)

Now the intensity of the detected signal \( I_D \) is given by the expression,

\[ I_D = K_D (E_{AO}^+ E_{AO}) \]

\[ = K_D K_A K_A^* (E_{AI,t}^* E_{AI,t}) \]

\[ = K_D |K_A|^2 |E_{AI,t}|^2 \]  \hspace{1cm} (6.32)

where \( K_D \) is a real factor which depends on the intensity profile of the light beam and nature of the photodetector. From eqn.(6.28) it can be easily be
written as,

\[ E_{AI,t} = \cos A \, E_{AI,x} + \sin A \, E_{AI,y} \quad (6.33) \]

But the values of \( E_{AI,x} \) and \( E_{AI,y} \) are given by eqn.6.26. Substituting these values in eqn.(6.33) one gets

\[ E_{AI,t} = K_A \begin{Bmatrix} \sqrt{V_x} \cos A \begin{bmatrix} \cos C \cos(\phi - C) - \ell_c \sin C \sin(\phi - C) \end{bmatrix} \\ \sqrt{V_y} \sin A \begin{bmatrix} \sin C \cos(\phi - C) + \ell_c \cos C \sin(\phi - C) \end{bmatrix} \end{Bmatrix} \]

(6.34)

Now eqn.(6.32) can be written in a more convenient form as

\[ I_D = G \, \mathbf{L} \, \mathbf{L}^* = G \, |\mathbf{L}|^2 \quad (6.35) \]

where \( G = |A|^2 |K|^2 |K_A|^2 K_D \) (6.36)

and \( \mathbf{L} = \sqrt{V_x} \cos A \begin{bmatrix} \cos C \cos(\phi - C) - \ell_c \sin C \sin(\phi - C) \end{bmatrix} \]

\[ + \sqrt{V_y} \sin A \begin{bmatrix} \sin C \cos(\phi - C) + \ell_c \cos C \sin(\phi - C) \end{bmatrix} \]

(6.37)

The above result has been obtained by analysing the state of polarisation at different points along the
path of the beam. The same result can be achieved by using the fact that the effect of a large number of devices placed in the path of the light beam is given by the resultant Jones matrix which is obtained by multiplying the Jones matrices of different devices taking into account of their arrangement and coordinate transformation. Applying this, the Jones vector of the output of the analyser is given by the expression

\[ E_{AO} = T_e^{t} R(A) T_{s}^{xy} R(-C) T_{c}^{fs} R(P-C) E_{P0} \tag{6.38} \]

The eqns. (6.35) and (6.37) show that

\[ I_D = f(P,C,A,E_c,V_{ex},V_{ey}) \tag{6.39} \]

which means that the detected signal intensity is a function of (i) the azimuth angles of polarizer compensator and analyser (ii) the slow-to-fast relative complex amplitude transmittance of the compensator and (iii) the complex eigenvalues \( V_{ex} \) and \( V_{ey} \) of the optical system to be analysed. In both photometric and null ellipsometric systems, one tries to gather information about \( V_{ex} \) and \( V_{ey} \) by proper use of eqn. (6.39).
6.3a Null ellipsometry

This is based on finding a set of azimuth angles for the polariser, compensator and analyser (P, C, A) to make the intensity of light falling on the detector to be zero. This condition can be represented as

\[ I_D = 0 \]  \hspace{1cm} (6.40)

for which the necessary condition is

\[ L = 0 \]  \hspace{1cm} (6.41)

Substituting this condition in eqn.(6.37) one gets

\[ 0 = V_{ex} \cos A [\cos C \cos (P-C) - \ell_c \sin C \sin (P-C)] \]

\[ + V_{ey} \sin A [\sin C \cos (P-C) + \ell_c \cos C \sin (P-C)] \]

Or

\[ \frac{V_{ex}}{V_{ey}} = - \frac{\sin A [\sin C \cos (P-C) + \ell_c \cos C \sin (P-C)]}{\cos A [\cos C \cos (P-C) - \ell_c \sin C \sin (P-C)]} \]  \hspace{1cm} (6.42)

Or

\[ \ell = - \frac{\tan A [\tan C + \ell_c \tan (P-C)]}{1 - \ell_c \tan C \tan (P-C)} \]  \hspace{1cm} (6.43)

where \( \ell = \frac{V_{ex}}{V_{ey}} \)  \hspace{1cm} (6.44)
which is the same constant defined in eqns. (6.12) and (6.13). If one gets the values of one set of 'nulling angles' \( P, C \) and \( A \) and also the value of slow-to-fast relative complex amplitude transmittance \( \epsilon_c \), the value of \( \epsilon \) can be easily calculated. Using \( \epsilon \) and the angle of incidence \( \phi_o \), the refractive index of the material (which forms the optical system \( S \)) can be computed for which the expression in eqn. (6.15) may be used.

6.3b Photometric ellipsometers

In this type of ellipsometers, as the name indicates, the detected intensity is never zero. Actually here the variation of the detected light flux as a function of the azimuth angles, phase retardation or angle of incidence is noted. For different intensity values, different equations (of the type given in eqn. (6.45)) can be formed and by solving them the value of \( \epsilon \) and hence the refractive index can be calculated.

\[
I_D = G \left| V_{ey} \right|^2 \left| \epsilon \cos A \cos C \cos (P-C) - \epsilon_c \sin C \sin (P-C) \right| \\
+ \left| \sin A \sin C \cos (P-C) + \epsilon_c \cos C \sin (P-C) \right|^2
\]

(6.45)
Eqn. (6.45) is directly obtained from eqn. (6.35) when $V_{ex} = \ell V_{ey}$ is substituted.

There are two types of photometric ellipsometers; one is dynamic and the other is static. Both differ in the way by which the parameters $P, C, A$ and $\varphi_c$ are varied. In dynamic photometric ellipsometers, some of these parameters are periodically varied with time and the detected signal intensity is analysed using Fourier analysis. But in the static photometric ellipsometers the intensity of the signal is measured at predetermined fixed values of $P, C, A$ and $\varphi_c$.

6.3b.1 Dynamic photometric ellipsometers

A very brief note on this type of ellipsometers is included here merely to complete the description of methods of evaluation of $\ell$ and hence the refractive index. In this type, as stated earlier, one or more optical parameters is modulated and the detected signal is Fourier analysed. Depending upon the parameter or combination of parameters selected for modulation there are large number of possibilities and here the most important and recent systems are considered.
The first one is the Rotating Analyser Ellipsometer (RAE). This has either polariser-compensator-system-analyser or polariser-system-compensator-analyser arrangement. Keeping other elements (P,C) set at fixed azimuths, the analyser is rotated at a constant angular velocity \( \omega \) and the intensity of the detected signal is Fourier analysed. The detailed description regarding its working, theory, method of computation and accuracy are given in several recent publications\(^{32-37}\).

The next one is the Polarisation Modulated Ellipsometer (PME). In this, the state of polarisation of the light beam at a suitable point in its path is modulated in a prescribed fashion so that information on the optical system under investigation is retrievable from harmonic analysis of the resulting time-varying detected intensity. Here also, there are several possibilities, depending upon the position and method of modulation. A convenient arrangement is the one proposed by Jasperson\(^{38,39}\) in which the sequence of the optical components is polariser (P), modulator (M), the optical system under investigation (S) and analyser (A). This PMSA arrangement can be considered to be the same as the conventional PCSA
arrangement in which the compensator's relative retardation $\xi_c$ is periodically modulated as a function of time. The main advantages of this ellipsometer are that all optical components remain stationary and it allows very high speed of measurement.

6.3b.2 Static photometric ellipsometers

The theory of this type of ellipsometers will be dealt in detail here since it is this type which is constructed and used for the present work. Since no compensator is required one can consider the polariser-system-analyser (PSA) arrangement. The detected signal is a function of azimuth angles $\psi$ and $\Delta$ only and this is directly obtained by setting $C = 0$ and $\xi_c = 1$ in eqn. (6.45) and it becomes,

$$I_D = G|V_{ey}|^2 |\xi_1 \cosA \cos\psi + \sinA \sin\psi|^2$$

$$= G|V_{ey}|^2 |\tan\psi e^{i\Delta} \cosA \cos\psi + \sinA \sin\psi|^2$$

(6.46)

On further simplification eqn. (6.46) becomes,

$$I_D = F'[i-\cos 2\psi(\cos 2A + \cos 2\psi) + \cos 2A \cos 2\psi$$

$$+ \sin 2\psi \cos \Delta \sin 2A \sin 2\psi]'$$

(6.47)

(where $F'$ is a constant).
If \( I_{D1}, I_{D2} \) and \( I_{D3} \) represent the detected intensities for three different settings \((P_1A_1)\) (\(P_2A_2\)) and \((P_3A_3)\) of the polariser and the analyser, three equations of the type in eqn.(6.47) can be formed. \( F' \) can be avoided by dividing any two of the equations by the third one. Using the remaining two equations the two unknown quantities \( \psi \) and \( \Delta \) can be obtained.

In the above method, nothing is stated regarding the choice of the settings of the polariser and analyser and is left arbitrary. However, to make the computations still easier, one can select \( \pi/4 \) as the azimuth of the polariser and \( +\pi/4, 0 \) and \( -\pi/4 \) to be the three analyser settings for which the corresponding intensity values are \( I_{D1}, I_{D2} \) and \( I_{D3} \).

From eqn.(6.47) one gets,

\[
I_{D1} = I_D(\pi/4, -\pi/4) = F'(1 - \sin 2\psi \cos \Delta) \quad (6.48a)
\]

\[
I_{D2} = I_D(\pi/4, 0) = F'(1 - \cos 2\psi) \quad (6.48b)
\]

\[
I_{D3} = I_D(\pi/4, \pi/4) = F'(1 + \sin 2\psi \cos \Delta) \quad (6.48c)
\]

From these three equations two simpler equations
can be formed as

\[ I_{D1} + I_{D3} = 2F' \] (6.49a)

\[ I_{D3} - I_{D1} = 2F' \sin 2\psi \cos \Delta \] (6.49b)

Using eqns. (6.48b), (6.49a) and (6.49b) one gets

\[ \psi = \frac{1}{2} \cos^{-1}\left[\frac{(I_{D1} - 2I_{D2} + I_{D3})}{(I_{D1} + I_{D3})}\right] \] (6.50)

\[ \Delta = \cos^{-1}\left[\frac{1}{2} \sin 2\psi \frac{(I_{D3} - I_{D1})}{(I_{D1} + I_{D3})}\right] \] (6.51)

Knowing \( \psi \) and \( \Delta \), the value of \( \ell \) and hence the value of the refractive index can be computed.

An ellipsometer working on this principle has been constructed and its details are given in chapter II. Using this set up the nature of the surface changes due to heating on silver films has been studied. The experimental details along with the relevant theory are given in chapter VII.
REFERENCES


