Chapter 2

Wave propagation in porous solid containing liquid filled bound pores and two-phase fluid in connected pores

A mathematical model is considered for wave motion in a porous solid containing liquid-filled bound pores and a connected pore space saturated by two-phase fluid. For the propagation of harmonic plane waves, the model is solved into a modified form of Christoffel equations, which are solved further to define the complex velocities of four attenuated waves in the medium. Three of these waves are longitudinal waves and the one is a transverse wave. Inhomogeneous propagation is considered with a complex specification of slowness vector involving a finite non-dimensional inhomogeneity parameter. The phase velocities and attenuation coefficients are calculated for the inhomogeneous propagation of each of the four attenuated waves in the porous aggregate. A numerical example is studied to analyse the effects of bound liquid film, sharing of connected pore-space, wave frequency, miscibility of pore-fluids and capillary pressure on the phase velocity and attenuation. The incidence of an acoustic wave at the plane boundary of the ocean bottom is studied to calculate energy partition among the acoustic wave reflected in water and the four waves refracted to oceanic crust. The effects of bound liquid film, sharing of connected pore-space between gas and liquid, wave frequency and capillary pressure on energy partition at the interface are studied in the numerical example. In this study, the porous solid refers to composite porous
aggregate containing fluids whereas, a drained frame with bound liquid film is referred
to as porous matrix. For convenience, the two fluid phases in pores are identified as
drilled liquid and gas.

2.1 Field equations for wave motion

Composite porous medium consists of four constituents, i.e. solid grains, bound liquid
film, pore-liquid, pore-gas, which are identified with indices 's', 'α', 'l', 'g', respectively.
Out of the total porosity (f) of the medium, a fraction α is occupied by bound liquid
film and the remaining part (1 − α)f is the connected porosity Φ. Then, the volume
fractions of the constituents are defined as

\[ δ_s = 1 - f, \quad δ_α = αf, \quad δ_l = (1 - \sigma)Φ, \quad δ_g = \sigmaΦ, \]  

(2.1)

where σ is the fraction of gas saturation in connected pore-space. These volume frac-
tions are scaling functions which are used to relate partial and intrinsic values of any
characteristic of the medium. For example, the product ρδ_s defines the contribution of
solid grains in the aggregate density (ρ) of multiphase mixture.

In applying continuum models to treat multiphase media, it is assumed that the
local variables can be replaced by mixture variables averaged over a region, which
is quite large in comparison to grain-size but very small when compared to sample-
size. It is further assumed that for each of the phases, i) partial stress tensors are
symmetric, ii) external body forces are absent, and iii) deformations are infinitesimal.
The gas is assumed to be soluble in liquid but no mass exchange is allowed between
the solid matrix and the twin-phase pore-fluid. Inertial coupling between the mixture
constituents is excluded. Following Garg and Nayfeh (1986), the equations of motion
for the low-frequency vibrations of constituent particles in isotropic porous solid are
given by

\[ (\delta_s + \delta_α)\tau^{(d)}_{ij,l} = (\delta_sρ_s + \delta_αρ_α)\ddot{u}_i - d_l(\dot{v}_i - \dot{u}_i) - d_g(\dot{w}_i - \dot{u}_i), \]
\[ \delta_l\tau^{(l)}_{ij,l} = δ_lρ_l\ddot{v}_i + d_l(\dot{v}_i - \dot{u}_i), \]
\[ \delta_g\tau^{(g)}_{ij,l} = δ_gρ_g\ddot{w}_i + d_g(\dot{w}_i - \dot{u}_i), \]

(2.2)

where the superscript '(d)' is used to denote drained porous solid frame. The particles
of elastic skeleton and bound liquid have the same displacement and pressure. Hence, 
the drained porous matrix is considered a single continuum which behaves viscoelastic 
to wave propagation [Edelman, 1997]. \(\tau\)'s are used to define stresses and \(\rho\)'s are intrinsic 
densities. \(u_i, v_i\) and \(w_i\) denote the components of displacements of the drained solid, 
liquid and gas particles, respectively. The indices (other than \(s, \alpha, l, g\)) can take 
values 1, 2 and 3. A repetition of these indices implies summation. Dot over a variable 
implies partial derivative with time and comma before an index implies partial space 
differentiation.

Darcy’s law relates viscous dissipation to the motion of gas and liquid particles 
relative to the pore-walls. The assumption of Poiseuille flow, necessary for this law, 
breaks down if the frequency exceeds a certain value. The present work is specifically 
restricted to low frequency such that viscous-fluid dissipation does not depend on 
frequency. Following Garg and Nayfeh (1986), dissipation coefficients for liquid \((d_l)\) 
and gas \((d_g)\) are defined as follows:

\[
d_k = \nu_k \delta_k^2 / (\Xi_k), \quad (k = l, g),
\]

where \(\nu_k\) and \(\Xi_k\) define the viscosity and the relative permeability of fluid phase \(k\). \(\Xi\) 
denotes the intrinsic permeability of the porous medium.

Garg and Nayfeh (1986) employed the concepts from the theory of interacting 
continua [Bedford and Drumheller, 1983] to formulate separate constitutive models 
for different constituents of porous medium. In terms of intrinsic stress tensors and 
densities, the constitutive relations for porous matrix, liquid and gas are defined as follows:

\[
(\delta_s + \delta_{\alpha}) \tau_{ij}^{(d)} = (\lambda_{11} u_{k,k} + \lambda_{12} v_{k,k} + \lambda_{13} w_{k,k}) \delta_{ij} + \mu_p \left( u_{i,j} + u_{j,i} - \frac{2}{3} u_{k,k} \right),
\]

\[
\delta_l \tau_{ij}^{(l)} = (\lambda_{21} u_{k,k} + \lambda_{22} v_{k,k} + \lambda_{23} w_{k,k}) \delta_{ij},
\]

\[
\delta_g \tau_{ij}^{(g)} = (\lambda_{31} u_{k,k} + \lambda_{32} v_{k,k} + \lambda_{33} w_{k,k}) \delta_{ij},
\]

where \(\delta_{ij}\) is Kronecker symbol. The elastic constants \(\lambda\)'s, derived from the elastic 
moduli of the constituents are

\[
\lambda_{3k} = \mathcal{H}_{3k} / \left[ 1 - \eta_0 (\mathcal{H}_{32} - R \mathcal{H}_{33}) \right],
\]

\[
\lambda_{2k} = \mathcal{H}_{2k} + \lambda_{3k} \eta_0 (\mathcal{H}_{22} - R \mathcal{H}_{23}),
\]

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\[ \lambda_{ik} = \mathcal{K}_{ik} + \lambda_{3k} \sigma_0 (\mathcal{H}_{12} - R \mathcal{H}_{13}) ; \quad (k = 1, 2, 3) , \]
\[ R = \delta_\nu \rho / (\delta_\nu \rho) , \quad \eta_0 = \alpha_h / (\sigma \Phi) , \]
where \( \alpha_h \) is Henry’s constant [Garg and Nayfeh, 1986] to represent the mixing of two pore-fluids. For immiscible pore-fluids, i.e., \( \alpha_h = 0 \), \( \lambda_{ik} \) are reduced to \( \mathcal{K}_{ik} \), which are expressed as follows:
\[ \mathcal{K}_{11} = K_r \left[ (1 - \Phi) + \Phi \eta_1 \right] , \]
\[ \mathcal{K}_{12} = -K_r \Phi \eta_2 , \quad \mathcal{K}_{13} = -K_r \Phi \eta_3 , \quad K_r = (\delta_\alpha K_\alpha + \delta_s K_s) / (1 - \Phi) , \]
\[ \mathcal{K}_{21} = -K_r \Phi \left[ \sigma \xi_1 + (1 - \sigma) \eta_1 \right] , \quad \mathcal{K}_{22} = K_r \Phi \left[ -\sigma \xi_2 + (1 - \sigma) (1 + \eta_2) \right] , \]
\[ \mathcal{K}_{23} = K_r \Phi \left[ -\sigma \xi_3 + (1 - \sigma) \eta_3 \right] , \]
\[ \mathcal{K}_{31} = K_g \Phi \sigma (\xi_1 - \eta_1) , \quad \mathcal{K}_{32} = K_g \Phi \sigma (\xi_2 + \eta_2) , \quad \mathcal{K}_{33} = K_g \Phi \sigma (1 + \xi_3 + \eta_3) , \]
\[ \xi_1 = (1 - \sigma) \eta_1 (K_g - K_i) / K , \quad \eta_1 = (1 - \Phi) K_r \beta / [1 - \Phi \beta K_r - (1 - \Phi) \beta K_c] , \]
\[ K_c = [K_i K_g + (1 - \sigma) \gamma K_i + \sigma \gamma K_g] / K , \quad K = (1 - \sigma) K_g + \sigma K_i + \gamma , \]
\[ \beta = [1/K_r - (1 - \Phi) / K_d] / \Phi , \]
\[ \xi_2 = (1 - \sigma) [\eta_2 (K_i - K_g) + K_i] / K , \quad \eta_2 = \eta_1 (1 - \sigma) K_i (K_g + \gamma) / (K_i K) , \]
\[ \gamma = (1 - \sigma) K_{cap} \]
\[ \xi_3 = (1 - \sigma) [\eta_3 (K_i - K_g) - K_g] / K , \quad \eta_3 = \eta_1 \sigma K_g (K_i + \gamma) / (K_i K) , \]
where \( K_{cap} \) is equivalent bulk modulus for macroscopic capillary pressure [Garg and Nayfeh, 1986] and \( K_d \) is the bulk modulus for drained porous solid. \( K_j \) denotes the bulk modulus of phase \( j (= \alpha, s, l, g) \). \( K_c^{-1} \) represents the effective compressibility of mixture of pore-fluids and \( K_c \) reduces to \( K_i \) when \( \sigma = 0 \). The present work is specifically restricted to the propagation low frequency harmonic waves so as to follow Poiseuille flow. As a result, the capillary pressure in pores is assumed to be independent of frequency and hence a constant \( K_{cap} \).

In Kelvin-Voigt model of linear viscoelasticity, the elastic solid element and viscous fluid element are assumed to be parallel and thus subjected to same strain. Then, an effective elastic modulus of the composite is obtained as the sum of partial values of the modulus for different constituents [Wong and Bollampally, 1999]. Following Edelman (1997), the time-dependent rigidity modulus \( \mu_p \) of viscoelastic porous frame relates to the rigidity \( (\mu_s) \) of solid grains as follows:
\[ \mu_p = \delta_s \mu_s + \delta_\alpha \mu_\alpha \frac{\nu_\alpha}{Re_\alpha} \frac{\partial}{\partial t} , \quad (2.5) \]
where \( \nu_\alpha \) is the dynamic shear viscosity and \( Re_\alpha \) is acoustic Reynolds number for bound
liquid film. Edelman (1997) has defined this number as $1/Re_\alpha = \epsilon^2$. Non-dimensional parameter $\epsilon$ is expressed in terms of fluid viscosity, bulk modulus, density and medium permeability [Nikolaevskiy, 1990; Maksimov et al., 1994].

In terms of the displacement components, the equations of motion (2.2) are expressed as follows:

\[
(\lambda_{11} + \mu_p/3)u_{j,ij} + \lambda_{12}v_{j,ij} + \lambda_{13}w_{j,ij} + \mu_p u_{i,ij} = (\delta_s \rho_s + \delta_\alpha \rho_\alpha)\ddot{u}_i + (d_l + d_g)\dot{u}_i - d_l \dot{v}_i - d_g \dot{w}_i,
\]

\[
\lambda_{21}u_{j,ij} + \lambda_{22}v_{j,ij} + \lambda_{23}w_{j,ij} = \delta_l \rho_l \ddot{v}_i + d_l \dot{v}_i - d_l \dot{u}_i = 0,
\]

\[
\lambda_{31}u_{j,ij} + \lambda_{32}v_{j,ij} + \lambda_{33}w_{j,ij} = \delta_g \rho_g \ddot{w}_i + d_g \dot{w}_i - d_g \dot{u}_i = 0.
\]

(2.6)

### 2.2 Harmonic plane waves

To seek the harmonic solution of system of equations (2.6), for the propagation of plane waves, the displacement components are written as follows:

\[
(u_j, v_j, w_j) = (S_j, L_j, G_j) \exp \{i\omega(p_k x_k - t)\}, \quad (j = 1, 2, 3),
\]

(2.7)

where $\omega$ is angular frequency and $(p_1, p_2, p_3)$ make the slowness vector $p$. The vectors $S = (S_1, S_2, S_3)^T$, $L = (L_1, L_2, L_3)^T$ and $G = (G_1, G_2, G_3)^T$ define, respectively, the polarizations for the motions of the drained solid, liquid and gas particles in the porous medium. Substituting (2.7) in (2.6) yields a system of nine equations, given by

\[
\left[ \left( \lambda_{11} + \frac{\mu_p}{3} \right) p^T + (\mu_p \Lambda - \rho_0) I \right] S \\
+ \left( \lambda_{12} p^T + \frac{i}{\omega} d_l I \right) L + \left( \lambda_{13} p^T + \frac{i}{\omega} d_g I \right) G = 0,
\]

(2.8)

\[
\left( \lambda_{21} p^T + \frac{i}{\omega} d_l I \right) S + \left( \lambda_{22} p^T - \rho_1 I \right) L + \lambda_{23} p^T G = 0, \quad \rho_1 = \delta_l \rho_l + \frac{i}{\omega} d_l,
\]

(2.9)

\[
\left( \lambda_{31} p^T + \frac{i}{\omega} d_g I \right) S + \lambda_{32} p^T L + \left( \lambda_{33} p^T - \rho_2 I \right) G = 0 \quad \rho_2 = \delta_g \rho_g + \frac{i}{\omega} d_g,
\]

(2.10)

where $p^T$ denotes the transpose of $p$, $\Lambda = pp^T$ and $\rho_0 = \delta_s \rho_s + \delta_\alpha \rho_\alpha + \frac{i}{\omega} (d_l + d_g)$. $I$ is identity tensor. Note that similar to $\rho_j$, $(j = 0, 1, 2)$, the rigidity modulus $\mu_p$ defined in (2.5) for viscoelastic porous frame is also a complex valued function of frequency $\omega$. 

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The equations (2.9) and (2.10) of this system are solved into two relations, given by

\[ \mathbf{L} = \mathbf{AS}, \quad \mathbf{A} = l_0 \mathbf{I} + \frac{(l_2 - s_2 l_0) \Lambda + (l_1 - s_1 l_0)}{s_2 \Lambda^2 + s_1 \Lambda + s_0} \mathbf{p} \mathbf{p}^T, \quad (2.11) \]

\[ \mathbf{G} = \mathbf{BS}, \quad \mathbf{B} = g_0 \mathbf{I} + \frac{(g_2 - s_2 g_0) \Lambda + (g_1 - s_1 g_0)}{s_2 \Lambda^2 + s_1 \Lambda + s_0} \mathbf{p} \mathbf{p}^T, \quad (2.12) \]

which relate the polarisations (or displacements) of particles of solid, liquid and gas phases in the porous medium. Using these relations in (2.8) yields a system of three equations, given by

\[ [\gamma_1 \mathbf{p}^T \mathbf{p} + \gamma_2 (\Lambda \mathbf{I} - \mathbf{p}^T \mathbf{p})] \mathbf{S} = \mathbf{0}, \quad (2.13) \]

which are the Christoffel equations for propagation of harmonic plane waves in the porous medium saturated by two miscible fluids and containing a bound liquid film. Coefficients used in various relations are defined as follows:

\[ \gamma_1 = \left( \lambda_p + \frac{r_2 \Lambda^2 + r_1 \Lambda + r_0}{s_2 \Lambda^2 + s_1 \Lambda + s_0} \right) \Lambda - \rho_p, \quad \gamma_2 = \mu_p \Lambda - \rho_p, \]

\[ \lambda_p = \lambda_{11} + \frac{4}{3} \mu_p + l_0 \lambda_{12} + g_0 \lambda_{13}, \quad \rho_p = \rho_0 - \frac{i}{\omega} (l_0 d_l + g_0 d_g), \]

\[ r_2 = \lambda_{12} (l_2 - l_0 s_2) + \lambda_{13} (g_2 - g_0 s_2), \quad r_0 = \frac{i}{\omega} \left[ d_l (l_1 - l_0 s_1) + d_g (g_1 - g_0 s_1) \right], \]

\[ r_1 = \lambda_{12} (l_1 - l_0 s_1) + \lambda_{13} (g_1 - g_0 s_1) + \frac{i}{\omega} \left[ d_l (l_2 - l_0 s_2) + d_g (g_2 - g_0 s_2) \right], \]

\[ s_2 = \lambda_{22} \lambda_{33} - \lambda_{23} \lambda_{32}, \quad s_1 = - (\lambda_{33} \rho_1 + \lambda_{22} \rho_2), \quad s_0 = \rho_1 \rho_2, \]

\[ l_2 = \lambda_{23} \lambda_{31} - \lambda_{21} \lambda_{33}, \quad l_1 = \lambda_{21} \rho_2 + \frac{i}{\omega} (\lambda_{23} d_g - \lambda_{33} d_l), \quad l_0 = (1 - \frac{i \omega}{d_l} \delta_l \rho_l)^{-1}, \]

\[ g_2 = \lambda_{12} \lambda_{32} - \lambda_{22} \lambda_{31}, \quad g_1 = \lambda_{31} \rho_1 + \frac{i}{\omega} (\lambda_{32} d_l - \lambda_{22} d_g), \quad g_0 = (1 - \frac{i \omega}{d_g} \delta_g \rho_g)^{-1}. \]

In terms of velocity \( V \), slowness vector is defined as \( \mathbf{p} = \mathbf{N} / V \) such that \( \mathbf{NN}^T = 1 \) and \( \Lambda = 1/V^2 \). The complex vector \( \mathbf{N} \) represents the directions of propagation and attenuation of a wave in dissipative porous medium. In terms of \( \mathbf{N} \) and \( V \), the Christoffel equations (2.13) are expressed as

\[ [\gamma_1 \mathbf{N}^T \mathbf{N} + \gamma_2 (\mathbf{I} - \mathbf{N}^T \mathbf{N})] \mathbf{S} = \mathbf{0}. \quad (2.14) \]
The non-trivial solution for Christoffel equations is ensured by vanishing the determinant \(\gamma_1 \gamma_2^2\) of the matrix \(\gamma_1 N^T N + \gamma_2 (I - N^T N)\). This condition translates into two equations as follows.

The first one (i.e. \(\gamma_1 = 0\)) implies that

\[
(-s_0 \rho_p)V^6 + (s_0 \lambda_p + r_0 - s_1 \rho_p)V^4 + (s_1 \lambda_p + r_1 - s_2 \rho_p)V^2 + (s_2 \lambda_p + r_2) = 0. \tag{2.15}
\]

Three roots of this complex cubic equation (in \(V^2\)) define the complex velocities \((V_j, \ j = 1, 2, 3)\) of three attenuating waves in the dissipative porous medium. Then, the polarization vector \((S_1, S_2, S_3)\), corresponding to equation (2.14), is calculated to be parallel to \(N\) and hence the three waves identified with velocities \(V_1, V_2\) and \(V_3\) are longitudinal waves. The third wave arises from the presence of second fluid phase in pores and is an addition to Biot’s theory.

Another equation (i.e. \(\gamma_2 = 0\)) yields

\[
\rho_p V^2 - \mu_p = 0, \tag{2.16}
\]

which implies a dispersive wave with frequency-dependent complex velocity \(V_4 = \sqrt{\mu_p/\rho_p}\). The corresponding polarization vector \((S_1, S_2, S_3)\), is represented through a singular matrix \((I - N^T N)\). So, the polarization vector may be parallel to a column (or, row) vector of this symmetric matrix. This defines the direction of polarisation in a plane, which is normal to the propagation vector \(N\). This implies that the attenuated wave with velocity \(V_4\) is a transverse wave.

The polarisation vector \(S\) defines the polarisation of solid particles in the porous medium. Polarisations of the liquid and gas particles in porous solid are calculated from the relations (2.11) and (2.12), respectively. For convenience in discussion, the three longitudinal waves with velocity order \(\Re(V_1) > \Re(V_2) > \Re(V_3)\) are named as \(P_I, P_{II}, P_{III}\) waves, respectively. The lone transverse wave is identified as S wave.

### 2.2.1 Reduced cases

- For \(\delta_\alpha = 0\), we have \(f = \Phi, \ \mu_p = \delta_s \mu_s\) and the mathematical model represents the wave motion in porous medium without any bound liquid film.
• For $d_g = 0$, the mathematical model represents the absence of viscosity in one of the pore fluids, i.e. gas. This implies that $g_0 = 0$ and hence cubic equation (2.15) is simplified with reduced expressions for involved coefficients.

• The substitution of $d_g = d_l = 0$ implies the saturation of porous matrix with non-viscous fluids. The expressions derived are simplified with $l_0 = g_0 = r_0 = 0$, $\rho_p = \rho_0 = \delta_{\alpha} \rho_\alpha + \delta_s \rho_s$ and $s_0 = \delta_l \delta_l \rho_l \rho_g$. The cubic equation (2.15) reduces to (for $\lambda_p = \lambda_{11} + \frac{4}{3} \mu_p$)

$$-s_0 \rho_0 V^6 + (s_0 \lambda_p - s_1 \rho_0) V^4 + (\lambda_p s_1 + \lambda_{12} l_1 + \lambda_{13} g_1 - s_2 \rho_0) V^2 + (\lambda_p s_2 + \lambda_{12} l_2 + \lambda_{13} g_2) = 0. \quad (2.17)$$

• For immiscible pore fluids, we have $\alpha_h = 0$ and then the elastic tensor $\lambda_{ij}$ reduces to $\mathcal{H}_{ij}$.

• For the absence of capillary pressure, i.e. $K_{cap} = 0$, we have $\gamma = 0$ and the elastic tensor $\lambda_{ij}$ will be symmetric now. One of the roots of the cubic equation (2.15) will be zero and hence it represents the propagation of only two longitudinal waves (analogous to Biot’s theory).

• To define the model for single pore-fluid (i.e. liquid), the substitution of the gas share $\sigma = 0$, in absence of capillary pressure, above yields $\eta_3 = \mathcal{H}_{13} = \mathcal{H}_{23} = \mathcal{H}_{33} = \mathcal{H}_{31} = \mathcal{H}_{32} = 0$. This implies that only four elastic constants ($\mathcal{H}_{11}$, $\mathcal{H}_{22}$, $\mathcal{H}_{12} = \mathcal{H}_{21}$, $\mu_p$) define the elastic character of the wave motion (i.e., Biot’s theory without inertial solid-fluid coupling).

### 2.2.2 Propagation and attenuation

Propagation of an attenuated wave in a dissipative medium is defined through a complex slowness vector. The real and imaginary parts of this slowness vector are termed as propagation vector and attenuation vector, respectively. The difference in the directions of its propagation vector and attenuation vector defines the inhomogeneity of the
attenuated wave. Hence, a general plane attenuated wave in a dissipative medium is considered to be an inhomogeneous wave [Borcherdt, 1982]. A finite, non-dimensional inhomogeneity parameter is preferred [Sharma, 2008] to define the inhomogeneous propagation of attenuated waves with complex slowness vector \( p \) written as follows:

\[
p = \frac{1}{c} \left[ \hat{n} + ib\hat{n} + id\hat{m} \right],
\]  

(2.18)

where the propagation direction \( \hat{n} \) and an orthogonal unit vector \( \hat{m} \) identifies the propagation-attenuation plane. The vector \((b/c)\hat{n}\) defines the attenuation along the direction of propagation. Hence, it represents the homogeneous propagation and its contribution \(b/c\) to is termed as homogeneous attenuation. The other vector \((d/c)\hat{m}\) in (2.18) then represents the contribution of inhomogeneous (evanescent) propagation of wave. The vector sum of these two orthogonal vectors defines total attenuation with magnitude \(\sqrt{b^2 + d^2}/c\). Also for \(d = 0\), the attenuated wave propagates as homogeneous wave, which implies that inhomogeneous propagation of the attenuated wave is represented through the deviation of \(d\) from zero. Hence, \(d\) is termed as inhomogeneity parameter and its magnitude represents the strength of the inhomogeneous wave. For known values of propagation direction \(\hat{n}\), orthogonal direction \(\hat{m}\) and inhomogeneity parameter \(d \in [0, 1)\), we use \(p = N/V, \ NN^T = 1\) and \(h = V^2\) to obtain

\[
c^2 = -2b\frac{|h|^2}{\Im(h)}, \]

\[
b = \frac{\Re(h)}{\Im(h)} + \sqrt{\left(\frac{\Re(h)}{\Im(h)}\right)^2 + 1 - d^2},
\]

\[
N = \frac{(1 + ib)\hat{n} + id\hat{m}}{\sqrt{1 - b^2 - d^2 + i2b}}.
\]  

(2.19)

The effect of wave inhomogeneity (i.e. \(d\)) on attenuation may be observed through an attenuation coefficient defined by

\[
\zeta = \omega\sqrt{b^2 + d^2}/c.
\]  

(2.20)

For the homogeneous propagation (i.e. \(d = 0\)) of an attenuated wave, we have

\[
c = \frac{|V|^2}{\Re(V)}, \quad b = -\frac{\Im(V)}{\Re(V)}, \quad N = \hat{n}, \quad \zeta = \frac{b}{c^2}.
\]  

(2.21)
With \( b = 0 \), the slowness vector \( \mathbf{p} = (\mathbf{n} + ud\mathbf{m})/c \) defines a wave with attenuation direction orthogonal to its propagation direction, i.e. evanescent wave. This requires a real \( V \) and defines a constant value \( \sqrt{1 - c^2/V^2} \) for \( d \).

### 2.3 Liquid-porous solid boundary (Figure 2.1)

Purpose is to model the oceanic crust as a porous solid considered in this study. Hence, the porous solid half-space (Medium \( M_2 \)) is considered to be in contact with a non-viscous liquid so as to represent an ocean bottom (Medium \( M_1 \)). For propagation confined to \( x_1-x_3 \) plane, the plane \( x_3 = 0 \) is the ocean bottom and \( x_3 \)-axis is directed along the depth of oceanic crust, as shown in figure 2.1.

#### 2.3.1 Reflection and refraction

An acoustic wave through ocean water is incident at its interface with saturated porous solid bottom. Let \( \theta_0 \) be the angle that incident wave makes with the normal to interface. This defines its slowness vector \( \mathbf{p} = (\sin\theta_0, 0, \cos\theta_0)/v_0 \), where \( v_0 \) denotes the velocity of incident wave. This incidence results in the refraction of four attenuated waves to the dissipative oceanic crust along with a reflected acoustic wave into the ocean water. According to Snell’s law, the horizontal slowness \( p_1 = \sin\theta_0/v_0 \) will be same for the reflected and four refracted (\( P_I, P_{II}, P_{III}, SV \)) waves. The vertical slowness values for these refracted waves, in present geometry, are defined as

\[
q_j = \sqrt{1/V_j^2 - p_1^2}, \quad \Im(q_j) > 0, \quad (j = 1, 2, 3, 4). \tag{2.22}
\]

Due to the incidence through a loss-less fluid medium (i.e., real \( p_1 \)), the attenuation vectors of the refracted waves are always perpendicular to the interface. In other words, each refracted wave is an inhomogeneous wave with inhomogeneity angle equal to the angle that its propagation direction makes with normal to the interface.
Figure 2.1: Geometry of the problem.
2.3.2 Displacements and stresses

Displacement components in the fluid medium with density $\rho_f$ and acoustic velocity $v_0$ are written as follows:

\[
\begin{align*}
  u_1^{(0)} &= v_0 p_1 \left[ \exp \{ i\omega (p_1 x_1 + q_0 x_3 - t) \} + f_5 \exp \{ i\omega (p_1 x_1 - q_0 x_3 - t) \} \right], \\
  u_3^{(0)} &= v_0 q_0 \left[ \exp \{ i\omega (p_1 x_1 + q_0 x_3 - t) \} - f_5 \exp \{ i\omega (p_1 x_1 - q_0 x_3 - t) \} \right],
\end{align*}
\]

(2.23)

where $f_5$ denotes the amplitude of reflected wave relative to incident wave. $p_1 = \sin \theta_0/v_0$ and $q_0 = \cos \theta_0/v_0$ define horizontal and vertical slowness values of acoustic waves in $x_1$-$x_3$ plane. The normal stress in the inviscid liquid medium is given by

\[
\sigma_{33}^{(0)} = \rho_f v_0^2 \left( u_{1,1}^{(0)} + u_{3,3}^{(0)} \right).
\]

(2.24)

The displacement of solid particles in the porous medium due to the presence of four refracted waves is expressed as follows:

\[
\begin{align*}
  u_j &= \sum_{k=1}^{4} f_k S_j^{(k)} \exp \{ i\omega (p_1 x_1 + q_k x_3 - t) \}, \quad (j = 1, 3),
\end{align*}
\]

(2.25)

where $f_k$ are the excitation factors for refracted waves relative to incident wave. The complex vector $(S_1^{(k)}, 0, S_3^{(k)})$ defines the polarisation and phase shift of the motion of solid particles for four refracted waves $(k = 1, 2, 3, 4)$. The corresponding displacements of the liquid and gas particles can be calculated from the relations (2.11) and (2.12) using the wave-specific values of the matrices $A$ and $B$.

2.3.3 Boundary conditions

The displacements components for the propagation of plane harmonic waves are given by the equations (2.23) and (2.25). The equations (2.4) and (2.24) relate the stress components in two media to the corresponding displacements there. The surface pores at the ocean bottom are assumed to be sealed so as to prevent the discharge of pore-fluids into the overlying water body. Then, the five boundary conditions appropriate for this interface $x_3=0$ are given by

\[
\begin{align*}
  &i) \quad (\delta_s + \delta_a) \tau^{(d)}_{31} = 0,
\end{align*}
\]

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\[ ii) \quad (\delta_s + \delta_\alpha) \tau_{33}^{(d)} + \delta_l \tau_{33}^{(l)} + \delta_g \tau_{33}^{(g)} = \tau_{33}^{(0)}, \]

\[ iii) \quad (\delta_s + \delta_\alpha) u_3 + \delta_l v_3 + \delta_g w_3 = u_3^{(0)}, \]

\[ iv) \quad \dot{v}_3 - \dot{u}_3 = 0, \]

\[ v) \quad \dot{w}_3 - \dot{u}_3 = 0. \]  

The above boundary conditions are satisfied through a system of five linear inhomogeneous equations in \( f_1, f_2, f_3, f_4 \) and \( f_5 \). This system of equations is given by

\[ \sum_{k=1}^{5} F_{ik} f_k = g_i, \quad (i = 1, 2, \ldots, 5), \]  

(2.27)

where the coefficients \( F_{ik} \) and \( g_i \) are expressed as follows:

\[ F_{1k} = q_k S_1^{(k)} + p_1 S_3^{(k)}, \]

\[ F_{2k} = (\lambda_{11} + \lambda_{21} + \lambda_{31} - 2\mu_p / 3)(p_1 S_1^{(k)} + q_k S_3^{(k)}) \]

\[ + 2\mu_p q_k S_3^{(k)} + (\lambda_{12} + \lambda_{22} + \lambda_{32}) A_{ij}^{(k)} p_i^{(k)} S_j^{(k)} \]

\[ + (\lambda_{13} + \lambda_{23} + \lambda_{33}) B_{ij}^{(k)} p_i^{(k)} S_j^{(k)}, \]

\[ F_{3k} = (\delta_l A_{31}^{(k)} + \delta_g B_{31}^{(k)}) S_1^{(k)} + (\delta_s + \delta_\alpha + \delta_l A_{33}^{(k)} + \delta_g B_{33}^{(k)}) S_3^{(k)}, \]

\[ F_{4k} = A_{31}^{(k)} S_1^{(k)} + (A_{33}^{(k)} - 1) S_3^{(k)}, \]

\[ F_{5k} = B_{31}^{(k)} S_1^{(k)} + (B_{33}^{(k)} - 1) S_3^{(k)}, \quad (k = 1, 2, 3, 4), \]

\[ F_{15} = F_{45} = F_{55} = 0, \quad F_{25} = -\rho f v_0, \quad F_{35} = \cos \theta_0, \]

\[ g_1 = g_4 = g_5 = 0, \quad g_2 = -F_{25}, \quad g_3 = F_{35}. \]

The superscript \(^{(')}\) on matrices \( A \) and \( B \) means the matrices are evaluated for slowness vector \( p = (p_1^{(k)}, 0, p_3^{(k)}) = (p_1, 0, q_k) \).

### 2.3.4 Energy partition

Reflection (refraction) coefficient is defined as the energy share of a reflected (refracted) wave in the energy of incident wave. Distribution of energy between reflected and different refracted waves is considered across a surface element of unit area at the
plane interface $x_3 = 0$. The average energy flux of a wave across the interface, denoted by $\langle P^* \rangle$, is the time average of scalar product of its surface traction and particle velocity. With $x_3$-axis as the normal to the interface in the fluid medium, the average energy fluxes of incident wave and reflected wave are $\langle P^*_I \rangle = -0.5\omega^2\rho_f v_o \cos \theta_0$ and $\langle P^*_R \rangle = 0.5\omega^2|f_5|^2\rho_f v_o \cos \theta_0$, respectively. The value of the ratio $\langle P^*_R \rangle/\langle P^*_I \rangle$ defines the reflection coefficient of the reflected wave as $E_R = |f_5|^2$. The argument of complex amplitude $f_5$ defines the phase shift (denoted as $\Phi_R$) of the reflected wave. Energy-amplitude relation for the reflected wave is given by

$$f_5 = \sqrt{E_R}e^{i\Phi_R}. \quad (2.28)$$

The variation of phase angle (or shift) with propagation direction implies the spatial dispersion of reflected waves. The dispersive phase behaviour of reflection coefficient is important for tracing of ray-paths.

The waves refracted across the interface to dissipative porous solid propagate as inhomogeneous waves accompanied with interaction energy [Borcherdt, 1977]. It is a net energy flux across the interface which comes from the interaction of the velocity field of one wave with the stress field of another wave. This local energy flow does not result in any net radiation but is associated with net energy dissipation. It is significant for calculation of energy transmission coefficients and ensures the conservation of energy across the interface. In the present problem, an energy matrix

$$E_{ij} = \langle P^*_{ij} \rangle/\langle P^*_I \rangle, \ (i, j = 1, 2, 3, 4), \quad (2.29)$$

is defined to explain the partition of energy refracted to dissipative porous medium. In porous medium, the average energy flux, corresponding to the interaction between stress of wave ‘$i$’ and the velocity of wave ‘$j$’ is defined as

$$\langle P^*_{ij} \rangle = 0.5 \Re \left[ (\delta_s + \delta_\alpha) \tau^{(d)}_{31} \ddot{u}_1 + (\delta_s + \delta_\alpha) \tau^{(d)}_{33} \ddot{u}_3 + \delta_l \tau^{(l)}_{33} \ddot{v}_3 + \delta_g \tau^{(g)}_{33} \ddot{w}_3 \right]. \quad (2.30)$$

Solving the system of equations (2.27), by Gauss elimination method, provides the values for excitation factors ($f_k$), which are required to calculate the matrix (2.29) for $x_3 = 0$. This energy matrix explains the energy partition at the boundary $x_3 = 0$ of the porous solid with overlying water. The sum of all the non-diagonal entries of this
energy matrix calculates the share of interaction energy \( E_{int} \) in the medium. The diagonal entries \( E_{11}, E_{22}, E_{33} \) and \( E_{44} \) of this matrix denote the refraction coefficients (or, energy shares) of refracted \( P_I, P_{II}, P_{III} \) and \( SV \) waves (in the incident energy). The conservation of energy incident at the interface is verified through the relation

\[
E_{11} + E_{22} + E_{33} + E_{44} + E_{int} + E_R = 1. \tag{2.31}
\]

### 2.4 Numerical example

A reservoir rock (sandstone) saturated with a liquid and gas is chosen for the numerical model of porous medium [Garg and Nayfeh, 1986]. The solid grains of the rock with bulk modulus \( K_s = 36 \text{ GPa} \), rigidity modulus \( \mu_s = 9 \text{ GPa} \), and density \( \rho_s = 2650 \text{ kg/m}^3 \) form a porous frame of porosity \( f = 0.3 \). The connected pore space is filled with the bubbles of gas of bulk modulus \( K_g = 0.0037 \text{ GPa} \) and density \( \rho_g = 100 \text{ kg/m}^3 \) mixed in a liquid of bulk modulus \( K_l = 2.3 \text{ GPa} \) and density \( \rho_l = 980 \text{ kg/m}^3 \). Both the pore-fluids are viscous and the values chosen for dissipation coefficients are \( d_l = 1 \text{ MPa s/m}^2 \) and \( d_g = 0.04 \text{ MPa s/m}^2 \). The same liquid with viscosity \( \nu_\alpha = 10^{-12} \text{ GPa/ sec} \) and Reynolds number \( Re_\alpha = 100 \) is assumed in the bound pores. The value of ratio \( K_{cap}/K_l \) is used to calculate \( \gamma = (1 - \sigma)K_{cap} \). Low-frequency propagation regime is ensured with \( \omega \leq 2\pi \times 5 \text{ kHz} \). The vanishing of Henry’s constant \( \alpha_h \) (or, \( \eta_0 = 0 \)) in some cases, is used to consider the immiscibility of gas and liquid in connected pore space.

#### 2.4.1 Velocity and attenuation

The numerical values of various parameters given above are used to calculate the complex velocities of the four attenuated (i.e., \( P_I, P_{II}, P_{III}, SV \)) waves in the porous medium. Phase velocities \( (c_j, j = 1, 2, 3, 4) \) and attenuation coefficients \( (\zeta_j, j = 1, 2, 3, 4) \) are computed for inhomogeneous propagation of these waves. Effect of various parameters on \( c_j \) and \( \zeta_j \) are exhibited in figure 2.2 to figure 2.6.

**Effect of frequency**

The variations of phase velocities and attenuation coefficients of the \( P_I, P_{II}, P_{III} \) and \( SV \) waves with \( d \in [0, 1] \) are exhibited in figure 2.2 for three different frequencies,
\( \omega/2\pi = 0.1, 1, 5 \text{kHz} \). Values chosen for other parameters are \( \alpha = 0.2, \) \( \sigma = 0.4, \) \( K_{cap} = 0.001 K_l \) and Henry’s constant \( \alpha_h = 4/Gpa \). It is quite evident that with the increase of inhomogeneity of an attenuated wave of a given frequency, its phase velocity decreases but attenuation increases. The velocities of all the waves may reduce upto one-half with the change in propagation from homogeneous (i.e. \( d = 0 \)) to nearly-evanescent (i.e. \( d \approx 1 \)). However, at low-frequency, the velocity decrease of two slower longitudinal (i.e. \( P_{II}, P_{III} \)) waves is less as compared to that for two faster (i.e. \( P_I, SV \)) waves. The increase of frequency may not have much effect on the velocities \( (c_1, c_4) \) of two faster waves. But, the two slower longitudinal waves propagate faster at higher frequency. For each of the four waves in porous medium, an increase of frequency (and/or inhomogeneity) increases its attenuation. However, the effect of inhomogeneity on attenuation may be mild at low frequencies. The general observation is that slower a wave higher is its attenuation. Then, homogeneous waves may travel faster but cannot represent the wave motion with larger attenuation.

**Effect of bound liquid film**

The variations of phase velocities and attenuation coefficients of the \( P_I, P_{II}, P_{III} \) and \( SV \) waves with \( d \in [0,1] \) are exhibited in figure 2.3 for three different values of \( \alpha = 0.05, 0.5, 0.95 \). Values chosen for other parameters are unchanged except a fixed \( \omega = 2\pi kHz \). The velocities of all the four waves appear to be decreasing with \( \alpha \). This implies that waves propagate faster when bound pores are less and connected pores are occupying much of the pore space. In other words, an absence of interconnections between saturated pores may slow down the wave propagation. Such a slow down is observed mainly on slower longitudinal waves. On the other hand, bounding of pores may increase the attenuation of these slower waves but may not affect the attenuation of two faster waves.

**Effect of gas share in pores**

The variations of phase velocities and attenuation coefficients of the four (i.e., \( P_I, P_{II}, P_{III}, SV \)) waves with \( \sigma \in (0,1) \) are presented in figure 2.4, for three different values \( d = 0, 0.3, 0.5 \) of inhomogeneity parameter. Except a fixed \( \alpha = 0.3 \), other parameters are unchanged. Increasing of gas share seems to have a little effect on the velocities
Figure 2.2: Phase velocities \( c_j \), \( j = 1, 2, 3, 4 \) and attenuation coefficients \( \zeta_j \), \( j = 1, 2, 3, 4 \) of \( P_I, P_{II}, P_{III}, S \) waves respectively; variations with inhomogeneity parameter \( d \) and frequency \( \omega \); \( \alpha = 0.2, \sigma = 0.4, K_{cap} = 0.001 K_t, \alpha_h = 4 \text{ Gpa} \).
Figure 2.3: The same as the figure 2.2 but variations with inhomogeneity parameter \((d)\) and bound liquid film \((\alpha)\); \(\omega = 2\pi kHz\), \(\sigma = 0.4\), \(K_{cap} = 0.001K_t\), \(\alpha_h = 4/Gpa\).
Figure 2.4: Phase velocities ($c_j$, $j = 1, 2, 3, 4$) and attenuation coefficients ($\zeta_j$, $j = 1, 2, 3, 4$) of $P_I$, $P_{II}$, $P_{III}$, $S$ waves respectively; variations with inhomogeneity parameter ($d$) and frequency ($\omega$); $\alpha = 0.2$, $\sigma = 0.4$, $K_{cap} = 0.001 K_I$, $\alpha_h = 4/Gpa$. 
Figure 2.5: The same as the figure 2.4 but variations with gas saturation ($\sigma$) and Henry’s constant ($\alpha_h$); $\omega = 10\pi kHz$, $\alpha = 0.1$, $K_{cap} = 0.001K_I$, $d = 0.1$, $c_1$, $c_2$, $c_3$, $c_4$, $\zeta_1$, $\zeta_2$, $\zeta_3$, $\zeta_4$. 
Figure 2.6: The same as the figure 2.4 but variations with gas saturation ($\sigma$) and capillary pressure ($K_{cap}$); $\omega = 2\pi kHz$, $\alpha = 0.1$, $\alpha_h = 4/Gpa$, $d = 0.1$. 
Figure 2.7: Energy shares of reflected wave, refracted ($P_I, SV, P_{II}, P_{III}$) waves, and interaction among refracted waves; variations with incident direction ($\theta_0$) and frequency ($\omega$); $\alpha = 0.2$, $\sigma = 0.4$, $K_{cap} = 0.001K_l$, $\alpha_h = 4/G_{pa}$. 

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and almost no effect on the attenuation of two faster waves. However, effect of $\sigma$ is quite significant on the velocities and attenuation of slower longitudinal waves. From this figure, it is observed that increase in the attenuation of main waves is attributed more to their inhomogeneous propagation than the presence or increase of gas bubbles in interstitial liquid.

**Effect of miscibility of pore-fluids**

For three different values of Henry’s constant $\alpha_h = 0, 1, 5$ per GPa, the variations of phase velocities and attenuation coefficients of the $P_I, P_{II}, P_{III}$ and $SV$ waves with $\sigma \in (0,1)$ are as shown in figure 2.5. Except $\omega = 10\pi kHz$ and $\alpha = 0.1$, the values of other parameters are unchanged. A non-zero Henry’s constant (i.e. $\alpha_h$) represents the instantaneous diffusivity of gas in the liquid phase in connected pores. Then, a gas soluble in the liquid may reduce the phase velocities and increase the attenuation of three longitudinal waves. But such a change is observed only when gas is in smaller proportion (i.e. $\sigma < 0.5$). It does not have any effect on the propagation and attenuation of shear wave.

**Effect of capillary pressure**

Capillary pressure among immiscible fluids is represented through an equivalent bulk modulus $K_{cap}$. The variations of phase velocities and attenuation coefficients of the attenuated ($P_I, P_{II}, P_{III}, SV$) waves with $\sigma \in (0,1)$ are exhibited in figure 2.6, for three different values of capillary pressure defined by $K_{cap}/K_l = 0.0001, 0.01, 0.1$. The values of other parameters are unchanged except $\omega = 2\pi kHz$ and $\alpha_h = 0$. It is clear that velocities of longitudinal waves increase with the increase of capillary pressure. On the other hand these waves, in general, attenuate more when $K_{cap}$ is small. The exception is attenuation of $P_{II}$ wave (i.e. $\zeta_2$), which attains maximum at different gas saturations (i.e. $\sigma$ values) depending upon the capillary pressure. The shear waves are not at all sensitive to the presence of capillary pressure or interfacial tension between two different fluid phases in pores.
Figure 2.8: The same as the figure 2.7 but variations with incident direction ($\theta_0$) and bound liquid film ($\alpha$); $\omega = 2\pi kHz$, $\sigma = 0.4$, $K_{cap} = 0.001K_t$, $\alpha_h = 4/Gpa$. 

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Figure 2.9: The same as the figure 2.7 but variations with incident direction ($\theta_0$) and gas saturation ($\sigma$); $\omega = 2\pi kHz$, $\alpha = 0.2$, $K_{cap} = 0.001K_1$, $\alpha_h = 4/Gpa$. 

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Figure 2.10: The same as the figure 2.7 but variations with incident direction ($\theta_0$) and capillary pressure ($K_{cap}$); $\omega = 10\pi kHz$, $\alpha = 0.2$, $\sigma = 0.4$, $\alpha_h = 4/Gpa$. 
2.4.2 Reflection and Refraction

Acoustic wave with velocity $v_0 = 1463 m/s$ propagates through water of density $1000 kg/m^3$ incident at the porous bottom making an angle $\theta_0$ with normal to plane interface. The energy of this incident wave is shared among the reflected acoustic wave and four refracted ($P_I$, $P_{II}$, $P_{III}$, $SV$) waves. Due to the inhomogeneous propagation of refracted waves, a part of the refracted energy share is identified as the interaction energy. The variations in the energy partition with incident direction are presented in figure 2.7 to figure 2.10 and are discussed as follows:

Effect of frequency

The variations of energy shares of reflected wave, four refracted ($P_I$, $P_{II}$, $P_{III}$, $SV$) waves and interaction with $\theta_0 \in (0, 90^0)$ are exhibited in figure 2.7, for three different frequencies $\omega/2\pi = 0.1, 1, 5 kHz$. Values chosen for other parameters are $\alpha = 0.2, \sigma = 0.4, K_{cap} = 0.001 K_t, \alpha_h = 4/Gpa$. It is noted that the energy shares of two slower longitudinal ($P_{II}, P_{III}$) waves in porous medium are negligible. The effect of frequency is observed mainly for the incidence beyond a particular angle, which is the critical angle for refracted inhomogeneous $P_I$ wave. For post critical incidence, the energy shares of reflected wave, refracted $P_I$, $SV$ waves and interaction energy increase initially with the increase of frequency. Then, for increase of frequency beyond a level, these energies start decreasing. Negative sign of interaction energy implies the travel of energy towards the interface.

Effect of bound liquid film

The variations of energy shares with incident direction are shown through the plots in figure 2.8, for three different values of $\alpha = 0.05, 0.5, 0.95$. On the major energy shares, the effect of $\alpha$ is observed clearly for the post-critical incidence. Reflected energy increases with the increase of $\alpha$. This implies that a stronger reflection is the indication of a larger amount of liquid in a porous rock to be in bound pores. A near total reflection is observed for incidence after $60^0$ when almost all the pores are bound. This implies that porous aggregate is behaving like an elastic solid with the presence of very little dissipation due to the viscous fluid in bounded pores. However, the refracted waves are stronger when most of the saturated pores are connected and pore-fluids are
flowing through the porous rock.

**Effect of gas share in pores**

The variations of energy shares with incident direction are exhibited in figure 2.9 for three different values of $\sigma = 0.2, 0.5, 0.8$. Similar to the effect of $\omega$ in the figure 2.7 and $\alpha$ in the figure 2.8, the effect of $\sigma$ on energy partition is observed mainly for post-critical incidence. In this case, a larger gas share in pores may be responsible for stronger reflected waves. Whereas, refracted waves will get more energy share when connecting pores have a dominating presence of liquid as compared to gas. Larger interaction is observed when pores contain more liquid and less gas.

**Effect of capillary pressure**

The variations in energy partition among reflected and refracted waves with incident direction are presented in figure 2.10, for three different values of capillary pressure defined by $K_{cap}/K_l = 0.0001, 0.01, 0.1$. Similar to the last three figures, the effect of $K_{cap}$ on energy shares is observed only for post-critical incidence. A larger capillary pressure may results in a weaker reflected waves and hence a larger share of incident energy refracting to porous medium. The major difference is observed in the energy share of a slower ($P_{II}$) longitudinal wave, which is having a significant strength when $K_{cap}/K_l = 0.1$. This gain of strength may be contributed by the reflected acoustic wave and the refracted $P_I$ wave. The capillary pressure may not have a significant effect on the refracted $SV$ wave.

### 2.5 Concluding remarks

This chapter considers to study the inhomogeneous propagation of four attenuated waves in a porous solid containing a liquid film in bound pores and a connected pore space saturated with two miscible fluids. The porous medium is dissipative due to the presence of viscous fluids in bound as well as connected pores. The four attenuated waves in this dissipative medium are identified with complex velocities. The presence of bound liquid film modifies the rigidity ($\mu_p$) of the porous aggregate and hence affects the velocities of longitudinal waves as well as transverse wave. With pores saturated by the mixture of two fluids, particularly gas bubbles soluble in liquid, we have a
generalised as well as a more realistic model to study attenuation from the reservoirs rocks. The variable gas share in pores enables to represent the pore saturation from almost all liquid to all gas. Some interesting observations from the numerical example may be important and hence are explained as follows.

- The phase velocities of all the waves may reduce to half with the change in propagation from homogeneous to evanescent. Attenuation increases with frequency as well as with inhomogeneity. That means, the homogeneous waves may travel faster but cannot represent the wave motion with a larger attenuation. Moreover, a slower wave attenuates more in dissipative porous medium.

- The presence of a liquid film in bound pores may reduce the phase velocities of all the waves but increases the attenuation of only slower longitudinal waves.

- Presence of gas bubbles mixed in pore-liquid may decrease the velocities of three faster waves but increase of gas share beyond a level may reverse this effect. However, the share of gas bubbles affects the attenuation of only slower longitudinal (i.e., $P_{II}$, $P_{III}$) waves.

- Solubility of gas bubbles into pore-liquid may reduce the velocities of longitudinal waves when gas share is smaller. It certainly decreases/increase the velocity/attenuation of Biot’s slow (i.e., $P_{II}$) wave. Shear wave is unaffected.

- The presence of capillary pressure increases the velocities but decreases the attenuation of longitudinal waves. However, it does not affect the propagation and attenuation of shear wave.

- Effect of frequency, bound liquid film, presence of gas bubbles and capillary pressure on energy partition may be quite significant but only for post-critical incidence. However, the solubility of gas bubbles in pore-liquid may not affect the partition of incident acoustic energy at the ocean bottom.

- From the figure 2.8, the dotted curves show that there is almost total reflection for $\alpha = 0.95$, for incidence beyond $60^\circ$. This implies that when the pore-space is almost a bound liquid film then medium behaves as elastic. Or the medium behaves viscoelastic but with a little dissipation. The reason being the small
viscosity ($\nu_\alpha$) of the liquid in such pores. However, the main contribution for the attenuation of waves in dissipative porous medium comes from the viscoelasticity of the porous frame [Sharma, 2005, 2010].

- From the figure 2.10, the strength of Biot’s slow (i.e. $P_{II}$) wave refracted to the oceanic crust may be a direct indication for the presence of interfacial tension due to two-phase fluid in connected pores.