CHAPTER 4

RIPPLET TRANSFORM BASED SHRINKAGE FUNCTION FOR DENOISING MAGNETIC RESONANCE IMAGES

4.1 INTRODUCTION TO MAGNETIC RESONANCE IMAGE DE-NOISING

Magnetic Resonance Imaging is a standout technique for carrying out conducive research of the different internal organs. The texture and finesse of MR images is improved greatly by De-noising techniques. Earlier filtering techniques were based on averaging spatial patterns. One such filter is Gaussian filter, which was based on averaging dissimilar patterns. However, it suffered the disadvantage of blurred edges. Anisotropic Diffusion Filters (ADFS) were able to mitigate the above drawback by successfully removing noise withstanding the respecting edges by averaging pixels in the orthogonal direction of the local gradient. Such a filtering results in unnatural images due to edge enhancement effect. Wavelet based filters have also been effective for MR image De-noising. In 2007, Guleryuz proposed 2-D hard Thresholding technique that uses the sparseness of the image. Guleryuz technique (2007) is based on utilizing the sparseness of the image. i.e., the ability of the image to be represented by a smaller number of base functions. In this method an over complete set of 8x8 DCT blocks were used. The denoised estimate at block j is obtained by applying the hard thresholding rule.

\[ c_j = H(y_j) \hat{c}_j = T(c_j, \lambda) \hat{x}_j = H^{-1}(\hat{c}_j) \]  

(4.1)
where H represents 2-D DCT, T represents the hard thresholding technique with \( \lambda \) being the threshold value. A higher weightage to estimates with more null values after thresholding led to Gibbs effects in this technique.

Not very long ago, a simple but efficient filter called Non local means filter (NLM) had been introduced. Due to its redundant nature NLM algorithm has proven to be one of the successful image restoration technique in the state-of-the-art-methods. However the computation time proves to be a major setback. Sizable amount of research is being conducted in 3-D MRI de-noising. However this thesis plays within the confines of 2-D MRI de-noising. The proposed work can be continued with 3-D MR imaging in future.

4.2 NLM FILTER

In the classical NLM algorithm (2005) if \( x_i \) represents a noise free MR image, \( n_i \) represents the noise and \( y_i \), its noisy observation then \( y_i \) is defined as \( y_i = x_i + n_i \) at each pixel \( i \).

If \( N_i \) denotes a \( m \times m \) square neighborhood centered on the \( i^{th} \) pixel, then \( p(N_i) \) represents a matrix whose elements are gray level values of \( y_i \) at pixels in \( N_i \). Let \( S_i \) represent a search window centered on the \( i^{th} \) pixel. The restored intensity \( c \) is defined in NLM algorithm as the weighted average of all pixels in \( c \) and is given as

\[
\hat{x}_{NLM}(y_i) = \sum_{j \in S_i} w(i, j) y(j)
\]

(4.2)

where \( Z(i) = \sum_{j \in S_i} e^{-\|p(N_i) - p(N_j)\|^2 / h^2} \) represents a normalizing term, \( w(i, j) \) represents the family of weights that are represented by the similarities between the pixels \( i \) and \( j \) satisfying the two conditions \( 0 \leq w(i, j) \leq 1 \) and
\[ \sum_{j \in S_i} w(i, j) = 1. \quad (4.3) \]

\( h \) represents the smoothing kernel width parameter. However the estimation of an NLM filter is biased and the restoration process tends to keep the noise. The NLM filter is based on the weighed averaging process. This averaging process involves the control of the target pixel for the restoration process. The target pixel computes the weights based on its own value. Hence if the target pixel is highly corrupted then the entire process is biased and tends to retain higher noise levels. Computational time is another important drawback of NLM algorithm. A large amount of time is spent on search process thus making the process a time consuming one. In order to combat these issues a new Ripplet based shrinkage process is being proposed.

The matlab implementation of Jose Vincente Manjon and Dr. Kroon’s NLM filter version is used in this work for comparison. Due to better higher dimensionality properties instead of DCT based hard thresholding technique, a Ripplet based hard thresholding technique is used for comparison.

### 4.3 WAVELET TRANSFORM

Traditional de-noising techniques deployed wavelet methods for appropriate recovery of a noisy image. Wavelet transform provides an appropriate basis for separating the image from noise. Wavelet transform has the major advantage of high-energy compaction. De-noising the noisy image includes the decomposition of the image into appropriate sub bands. Each sub-band carries approximate and detail coefficients corresponding to low frequency and high frequency components. The de-noised image can be extracted by applying the inverse wavelet transform. This wavelet oriented de-noising technique includes the following steps:
- Apply Forward wavelet transform
- Apply linear or non-linear shrinkage method
- Take Inverse wavelet transform

However wavelet Transform is at its best for point singularities, however it fails when it comes to line singularities. These disadvantages led to the wide range of use of Curvelet transform. Wavelet Transform has made a high impact in the field of signal processing, image processing and several other applications. The main drawback of wavelet is that it lacks sparse representation along $c^2$ curve. However the usage of wavelet transform has reduced to a great extent due to poor directionality.

4.4 CURVELET TRANSFORM

Curvelet transform discards the inherent limits such as multidimensional singularities, which are a major drawback in earlier wavelet implementation. In case of decimated wavelet transform there is a substantial loss of important details along the edges. Curvelet reconstruction appreciably reduces the artifacts along the edges when compared to its counterpart wavelet transform. Curved edges can be represented efficiently at fine scales, as they appear straight at such scales.

4.5 RIPPLET TRANSFORM

Ripplet transform is a higher dimensional generalization of Curvelet transform, which provides a comprehensive representation of 2D signals. Ripplet transform as a generalization of Curvelet has almost all the properties of Curvelet except parabolic scaling.
4.5.1 Continuous Ripplet Transform

For a 2D integrable function \( f(\vec{x}) \), the continuous Ripplet function is defined as the inner product of \( f(\vec{x}) \) and Ripplets

\[
R(a, \vec{b}, \theta) = \left< f, \rho_{ab\theta} \right> = \int f(\vec{x}) \rho_{ab\theta}(\vec{x}) \, d\vec{x} \tag{4.4}
\]

where \( R(a, \vec{b}, \theta) \) represents the Ripplet coefficients and \( \rho_{ab\theta}(\vec{x}) \) represents the Ripplet element function.

4.5.2 Discrete Ripplet Transform

Discretization of Continuous Ripplet Transform involves the discretization of the parameters of Ripplets. The scale parameter ‘\( a \)’ is sampled at dyadic intervals. The position parameter ‘\( \vec{b} \)’ and rotation parameter ‘\( \theta \)’ are sampled at equal intervals. If \( a, \vec{b}, \theta \) represent the continuous parameters, then there corresponding digital parameters are given by

\[
a_j = 2^{-j}, \quad \vec{b}_k = [c.2^{-j}k_1, 2^{-id}k_2]^T \quad \text{and} \quad \theta_l = \frac{2\pi}{c} . 2\left\lfloor j(1-l/d) \right\rfloor , \]

where \( \vec{k} = [k_1, k_2]^T \), \((.)^T\) denotes the transpose of a vector and \( j, k_1, k_2, l \in \mathbb{Z} \). The degree of Ripplets can take value from \( \mathbb{R} \). With \( d = n/m, n,m \neq 0 \in \mathbb{Z} \). The frequency response of Ripplet Transform in the frequency domain is given by:

\[
\hat{\rho}_j(r, \omega) = \frac{1}{\sqrt{c}} a^{m+n} W(2^{-j}, r)V \left( \frac{1}{c} \left\lfloor \frac{m-n}{n} \right\rfloor \omega^{-l} \right). \tag{4.5}
\]

where \( W \) and \( V \) satisfy admissibility conditions as given by:
\[
\sum_{j=0}^{+\infty} |W(2^{-j} \cdot r)|^2 = 1 \quad (4.6)
\]

\[
\sum_{l=-\infty}^{+\infty} \left| \frac{1}{c} \cdot 2^{-|j-l/d|} \omega - l \right|^2 = 1 \text{ given } c, d \text{ and } j. \quad (4.7)
\]

The wedge corresponding to the Ripplet function in frequency domain is given by

\[
H_{j,l}(r,\theta) = \{2^j \leq |r| \leq 2^{2j}, \left| \theta - \frac{\pi}{c} \cdot 2^{-|j-l/d|} \cdot l \right| \leq \frac{\pi}{2} 2^{-j} \} \quad (4.8)
\]

The discrete case gives insight about the parameters ‘c’ and ‘d’. The parameter ‘c’ controls the number of directions in the high-pass bands. ‘d’ controls how the number of directions changes across bands. For a particular value of ‘c’, ‘d’ helps to control the resolution in directions at each high-pass band. For a particular value of ‘d’, ‘c’ controls the number of directions at all high-pass bands.

The discrete Ripplet transform of an M x N image \( f(n_1, n_2) \) will be in the form of

\[
R_{j,k,l} = \sum_{n_1}^{M-1} \sum_{n_2}^{N-1} f(n_1, n_2) \rho_{j,k,l}(n_1, n_2) \quad (4.9)
\]

where \( R_{j,k,l} \) are the Ripplet coefficients. The image is reconstructed through the inverse transform

\[
\tilde{f}(n_1, n_2) = \sum_{j} \sum_{k} \sum_{l} R_{j,k,l} \rho_{j,k,l}(n_1, n_2) \quad (4.10)
\]
Comparison with other transforms such as Discrete Wavelet transform, Discrete Curvelet Transform and Discrete Cosine transform (DCT) shows that Ripplet transform outperforms Curvelets with respect to PSNR, Wavelets in avoiding the ringing artifacts and DCT in avoiding the blocking artifacts. The standard Ripplet Transform type-I uses hard shrinkage technique for all image-processing applications. However the hard shrinkage technique is not continuous at the threshold resulting in oscillations in the recovered signal. We introduce a new Ripplet based shrinkage technique, which is continuous at the threshold, thus eliminating oscillations in the recovered signal.

This chapter focuses on introducing of a new Ripplet based shrinkage technique used to suppress noise from Magnetic Resonance Images. The propitious properties of Ripplet transform such as anisotropy, high Directionality, good localization and high-energy compaction make the proposed method efficient and feature preserving when compared to other transforms. Ripplet transform provides efficient representation of edges in images with a higher potential for image processing applications such as image restoration, compression and de-noising. The proposed method implies a new non-linear Ripplet based shrinkage technique to extract the spatial and frequency information from Magnetic Resonance Images (MRI) corrupted by noise. The choice of this new shrinkage technique is due to its simplicity, versatility and its efficiency in removing noise from homogenous regions and those regions with singularities, when compared to the existing filtering techniques. Experiments were conducted on several diffusion weighed Images and anatomical images. The results show that the proposed de-noising technique shows competitive performance compared to the current state-of-art methods. Qualitative validation was performed based on several quality metrics and profound improvement over existing methods was obtained. Higher values of Peak signal to noise ratio (PSNR), Correlation Coefficient,
Mean structural similarity index (MSSIM) and lower values of Root Mean Square Error (RMSE) and computational time were obtained for the proposed Ripplet based shrinkage technique, when compared to the existing ones.

4.6 s-trim SHRINKAGE

The main argument of this work is to obtain an appropriate shrinkage technique to solve the basic pursuit of de-noising problem and to show how the proposed shrinkage can be implemented in order to enhance the de-noising effect. Consider a model \( y_i = x_i + n_i \). With \( n_i \) representing a Gaussian zero mean white noise. Here \( x_i \in \mathbb{R}^N \), \( y_i \in \mathbb{R}^N \) and \( n_i \in \mathbb{R}^N \).

The commonly used approach is to minimize the function

\[
f(x_i) = \frac{1}{2} \left\| x_i - y_i \right\|_2^2 + \lambda P_r(x_i) \tag{4.11}
\]

where \( x_i \) represents the original signal, \( y_i \) its noisy observation and \( \lambda \), the threshold. This expression emerges from Bayesian concept while implementing maximum a posteriori probability (MAP) estimation. The first term is known as log-likelihood, which gives the relationship between the original signal \( x_i \) and its noisy version \( y_i \). The term \( P_r(x_i) \) stands for prior posed on \( x_i \). Several such expressions as given by equation (4.11) have been used in the literature. Several popular methods are used in signal processing one of which considers the sparsity of the unknown signal with respect to its transformed representation given as \( P_r(x_i) = \| Tx_i \|_1 \). Donoho and Johnstone conducted extensive research in de-noising using sparse signal. Simoncelli and others related these ideas to the MAP formulation as presented above and used the prior \( P_r(x_i) = \| W x_i \|_p \) where \( W \) represents the wavelet transform.
matrix $W \in \mathbb{R}^{N \times N}$ and $0 \leq p \leq 1$. This leads to the solution known as shrinkage. The stepwise process of De-noising involves

$$y \rightarrow \text{RippletTransform} \rightarrow \text{shrinkage} \rightarrow \text{Inv RippletTransform} \rightarrow \hat{x}$$

The inability of orthogonal wavelets in representing singularities led to the wide scale usage of redundant transforms such as Curvelet, Contourlets. Recently, Ripplets have emerged to be better than state-of-art methods.

It has been shown that shrinkage is the paramount technique for de-noising. In the current research Ripplets have surfaced to be the prime redundant technique for image de-noising. The proposed technique weaves in both the techniques in this work. However the most important aim of this thesis is to choose an appropriate shrinkage function. The classical shrinkage techniques such as hard and soft shrinkage suffer from discontinuity and linearity resulting in Gibbs phenomenon and blurring effect. An optimum shrinkage technique requires being continuous and non-linear. Hence a new Ripplet based shrinkage technique is introduced in this thesis.

Shrinkage is an appealing De-noising technique, which is designed with the aim of mitigating the disadvantages of NLM filter and other state of art methods. This thesis introduces a new shrinkage technique called $s$-trim. The $s$-trim filter is continuous and non-linear. The non-linearity of the $s$-trim technique gives equal weightage to the homogenous regions and those containing edges. In case of the $s$-trim filtering technique, the insignificant empirical coefficients below threshold are killed and the remaining, significant coefficients are retained based on the equation.
\[
P = \begin{cases} 
\frac{2}{(1 + \exp(-\eta y))} - 1 |y| - \lambda 
& \text{for } |y| > \lambda \\
0 
& \text{for } |y| \leq \lambda 
\end{cases}
\] (4.12)

\(\lambda \in [0, \infty]\) represents the threshold value and \(\eta\) represents the scaling parameter, which determines the slope of the distribution curve. Whereas in classical methods \(\lambda\) is the only parameter, in the proposed \textit{s-trim} method there are two parameters \(\lambda\) and \(\eta\) thus improving the versatility of the proposed method. While \(\lambda\) distinguishes between the coefficients with null values and those that undergo a non-linear shrinkage, the \(\eta\) value decides on the orientation of the coefficients in the Ripplet domain. Figure 4.1 gives the distribution of \textit{s-trim} shrinkage method. A set of 1-D samples for different values of \(\eta\) namely, \(\eta = 0.1, \eta = 0.2, \eta = 0.3\) are taken to which the proposed shrinkage function is applied. The same threshold value is maintained for all the three curves.

It can be found that the curves corresponding to \(\eta = 0.2\) and \(\eta = 0.3\) do not null down the coefficients below threshold rather certain bumps are found in the region below the threshold value which may lead to Gibbs phenomenon.

The curve corresponding to \(\eta = 0.1\) has flatter response in the region below threshold and also establishes an optimum shrinkage of coefficients in the region above threshold. Hence \(\eta = 0.1\) is chosen for the proposed shrinkage technique. However for \(\eta\) less than 0.1 the curve above the threshold value was found to be with a very low slope and hence lower
PSNR. Hence, $\eta = 0.1$ proves an optimum value for the current de-noising problem.

![Figure 4.1 Distribution curve of the proposed function for different values of $\eta$](image1)

![Figure 4.2 Distribution curve of the sigmoid function](image2)
4.7 THEORETICAL FRUITION OF THE PROPOSED METHOD

The basic idea behind the development of the proposed model was the requirement of an optimum technique that is simple, non linear, real valued and differentiable with a high convergence rate. A sigmoid function is one such function with an ‘S’ shaped curve as shown in the Figure 4.2. A sigmoid function is a bounded, real, differentiable function that is defined for all input values and has a positive derivative at each point. The sigmoid function can be defined by the formula

\[ S(t) = \frac{1}{1 + \exp^{-t}} \]  

(4.13)

Another important factor about the sigmoid function is that its value varies between 0 and 1. The s-trim shrinkage is derived from the sigmoid function, which finds wide range of applications in neural networks. The modification of the basic sigmoid function results in s-trim. In case of s-trim the term $|y|^{-\lambda}$ decides on the noisy and signal coefficients, while the term $\eta$ helps in determining the slope of the characteristic curve. From the discussion in section 4.6 it can be understood that $\eta = 0.1$ is the optimum value for different kinds of images. The exponential function in the denominator aids to achieve high noise rejection capability.

4.8 ADAPTATION TO RICIAN NOISE

MR Images comprise of real and imaginary components with similar variance and Gaussian noise distribution. The magnitude MR images computed from the real and imaginary parts follow Rician distribution. The Rician noise built from Gaussian is given by:
\[ y_r(i) = x(i) + n_r(i), n_r(i) \sim N(0, \sigma), \]
\[ y_i(i) = n_i(i), n_i(i) \sim N(0, \sigma), \]

(4.14)

where, \( x(i) \) represents the original image. \( y_r(i) \) represents the imaginary component and \( y_i(i) \) represents the real component. \( \sigma \) represents the standard deviation of the Gaussian noise. Then the noisy image is computed as follows

\[ y(i) = \sqrt{y_r(i)^2 + y_i(i)^2} \]

(4.15)

4.9 RESULTS

Experiments were conducted on several test MR images. The results of some of the images are discussed below. In Figures 4.3-4.7 a 512 x 512 T_1-weighed image, a 512 x 512 T_2-weighed image and 512 x 512 3D TOF (Time of Flight) image were used for the experimental analysis with different Rician noise levels of 5\%, 10\% and 15\% respectively. Ripplet-I transform with \( c=1, d=3 \) has been implemented. The proposed \textit{s-trim} method was compared with the existing block-wise NLM filter and Rippled based hard shrinkage Technique. In case of block wise NLM filter several blocks are processed in parallel to reduce the computational time. It can be verified from the Figures (4.3-4.7) that the proposed technique has a higher noise rejection capability when compared to block-wise NLM filter and Ripplet based hard shrinkage technique.

![Figure 4.3 Original images](image-url)
Figure 4.4 Noisy images

Figure 4.5 Images De-noised via NLM filter

Figure 4.6 Images De-noised via Ripplet-hard

Figure 4.7 Images De-noised via s-trim shrinkage technique
To get better understanding of the efficiency and consistency of the proposed *s-trim* filtering technique another set of MR Images are also tested with the proposed and existing techniques. The second set of images also show similar results. The de-noised image using *s-trim* technique shows lesser noise speckles and higher resolution image when compared to its counterparts.

![Figure 4.8 Original images](image1)

![Figure 4.9 Noisy images](image2)

![Figure 4.10 Images De-noised via NLM filter](image3)
4.10 DISCUSSION

4.10.1 Comparison on the Cropped Image

In this experiment, a 512 x 512 real brain sagital image with a mass indicating Gliobastoma was used. A cropped patch indicating the mass with singularities and rich texture from the test image is compared in Figure 4.13 for a detailed analysis of the mass. Figure 4.13. F. shows that the proposed method achieves better noise reduction than other methods. An important feature of the proposed technique is that it also eliminates blocking and ringing artifacts.
4.10.2 Comparison with Respect to PSNR

For statistical parameter estimation analysis, the first set of images as indicated in Figures (4.3-4.7) are considered. Comparison of PSNR values from Figure 4.14 shows that the proposed \textit{s-trim} filter generates improved results when compared to Block-wise NLM and Ripplet \textit{hard} thresholding techniques. It can also be observed that \textit{s-trim} filtering technique achieves better results for all the test images. A specific advantage of the \textit{s-trim} technique is that it proves to be a better method as noise level increases.
When compared to the existing methods PSNR values are higher by a factor of 5 dB, 4 dB and 3 dB for 3-D TOF image, T₁-weighed image and T₂-weighed images respectively, at a Rician noise level of 15%. Though the performance of hard thresholding technique proves to be slightly better at low noise levels (5%), its fails to be an optimal technique due to Gibbs artifacts at higher noise levels.

![Graphs showing PSNR vs Rician Noise for different images](image)

(a). 3-D TOF image  
(b). T₁-weighed image  
(c). T₂-weighed image

**Figure 4.14** Comparison of different de-noising techniques with respect to PSNR
### 4.10.3 Comparison with Respect to RMSE

RMSE refers to the noise rejection capability of the de-noising method. RMSE value should be as low as possible. A low value of RMSE leads to an exact replica of the original image. It is inferred from Figure 4.15 that the RMSE values obtained for $s$-trim shrinkage are lesser when compared to the existing methods. It can be found that the proposed method is highly effective at high noise levels.

![Graphs showing RMSE comparison](image)

(a). 3-D TOF image  
(b). $T_1$-weighed image  
(c). $T_2$-weighed image

**Figure 4.15** Comparison of different de-noising techniques with respect to RMSE
4.10.4 Comparison with Respect to MSSIM

The SSIM is an image fidelity measure, capable of differentiating the structural and non-structural distortions in an image. While non-structural distortions such as Gamma distortions, luminance, contrast and spatial shift do not affect structure of the system, Structural distortions such as noise, blur and glossy compression tend to distort the image significantly. Human visual system tends to be highly sensitive to structural distortions, while non-structural distortions can be compensated. Under such condition retention of signal structure becomes an important image quality metrics.

![Graphs showing comparison of different de-noising techniques with respect to MSSIM](image)

Figure 4.16 Comparison of different de-noising techniques with respect to MSSIM
SSIM simulates this functionality. Figure 4.16 shows that \textit{s-trim} filtering technique proves to be successful in retaining the structural components of an image. When compared to other de-noising techniques the proposed method achieves higher values for MSSIM (Mean Structural Similarity Index). The retention capability is higher at higher noise levels. While the MSSIM index for NLM drops to 0.45 on an average and Ripplet \textit{hard} drops to 0.53 on an average at a noise level of 15\%, the proposed \textit{s-trim} technique shows its retention capability with MSSIM index of 0.74 at such high noise-levels. At low noise levels of 5\% \textit{s-trim} has an average MSSIM index of 0.93, Ripplet \textit{hard} and NLM achieve 0.89 and 0.83 respectively.

4.10.5 \textbf{Comparison with respect to Correlation Coefficient}

Correlation coefficient is a statistical measure of how well the De-noised image follows the trends in the original image. A high value of correlation coefficient indicates a stronger level of relationship between the original and de-noised images. Experiments conducted on the test images show that the de-noised image due to \textit{s-trim} shows stronger relationship with the original image when compared to the existing methods. Figure 4.17 shows that \textit{s-trim} maintain a high level of consistency at all the noise levels. The high value of correlation coefficient obtained from the experiments proves that \textit{s-trim} is insensitive to geometrical distortion, intensity inhomogeneity and data missing.
Figure 4.17 Comparison of different de-noising techniques with respect to Correlation Coefficient

4.10.6 Comparison on Running Time

The computational complexity in case of NLM and DCT based methods depends on the number of pixels in the image, the search window and the neighborhood patch. The running times also increase with the size of search window. However, Ripplet based techniques devoid the use of search window technique. The computational time of NLM technique has been
greatly reduced by using a block wise NLM in this comparison rather than the classical one. In spite of parallel processing of blocks Figure 4.18 shows that in NLM technique the computational time of the block wise NLM technique amounts to 64.9 sec on an average, while it is 4.34 sec and 4.12 sec for \textit{s-trim} and Ripplet-\textit{hard} respectively.

![Figure 4.18](image)

**Figure 4.18** Comparison of different de-noising techniques with respect to Computational Time

4.10.7 Comparison of Ripplet based \textit{s-trim} with other Transform Domains

In order to prove the superlative performance of Ripplet Transform when compared to other transforms a sample noisy MR image is de-noised using different transform domains such as Wavelet, Curvelet, Contourlet, Contourlet-SD and Ripplet. All the transform domains are implemented with \textit{s-trim} shrinkage technique as shown in Figure 4.19. It can be inferred from the figure that while Wavelet Transform and Curvelet Transform are effective in noise removal, both the transforms tend to introduce blurring effects. On the other hand de-noised images obtained by their counterparts namely
Contourlet and Contourlet-SD are devoid of any blurring. However they are lesser effective in noise removal. Ripplet Transform serves at its best in both noise removal and removing blurring effects.

Figure 4.19 (Continued)
In order to determine the efficacy of Ripplet Transform three MR Images were tested with \textit{s-trim} shrinkage in all the above mentioned transform domains. The quality of the resulting de-noised images were tested using two image quality metrics namely PSNR and RMSE. Table (4.1-4.3) shows the results. The results indicate the efficiency of Ripplet Transform over other transforms. While the PSNR values of all the three MR images de-noised using Ripplet transform are higher than other transform domains, the corresponding RMSE values are lower. Thus better quality images can be obtained using Ripplet Transform.

Table 4.1 Image Quality Metrics for MR Image-1

<table>
<thead>
<tr>
<th>Image</th>
<th>Parameters</th>
<th>Transform</th>
<th>Noise Variance in percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Img-1</td>
<td>PSNR</td>
<td>Wavelet</td>
<td>29.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Curve</td>
<td>28.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Contour</td>
<td>31.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cont-SD</td>
<td>31.79</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ripplet</td>
<td>32.45</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>Wave</td>
<td>7.79</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Curve</td>
<td>9.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Contour</td>
<td>6.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cont-SD</td>
<td>6.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ripplet</td>
<td>5.85</td>
</tr>
</tbody>
</table>
### Table 4.2 Image Quality Metrics for MR Image-2

<table>
<thead>
<tr>
<th>Image</th>
<th>Parameters</th>
<th>Transform</th>
<th>Noise Variance in percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Img-2</td>
<td>PSNR</td>
<td>Wavelet</td>
<td>28.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Curve</td>
<td>27.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Contour</td>
<td>30.61</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cont-SD</td>
<td>30.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ripplet</td>
<td>31.1</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>Wave</td>
<td>8.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Curve</td>
<td>10.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Contour</td>
<td>6.67</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cont-SD</td>
<td>6.95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ripplet</td>
<td>6.7</td>
</tr>
</tbody>
</table>

### Table 4.3 Image Quality Metrics for MR Image-3

<table>
<thead>
<tr>
<th>Image</th>
<th>Parameters</th>
<th>Transform</th>
<th>Noise Variance in percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Img-3</td>
<td>PSNR</td>
<td>Wavelet</td>
<td>27.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Curve</td>
<td>26.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Contour</td>
<td>29.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cont-SD</td>
<td>29.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ripplet</td>
<td>29.72</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>Wave</td>
<td>8.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Curve</td>
<td>9.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Contour</td>
<td>6.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cont-SD</td>
<td>6.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ripplet</td>
<td>6.23</td>
</tr>
</tbody>
</table>

Apart from statistical estimation of the quality of the de-noised image using $s$-$trim$ shrinkage the consistency of the $s$-$trim$ shrinkage...
technique was also determined by testing the proposed method with a set of MRI’s. Figure 4.20 indicates a set of nineteen MRI’s. For every MRI the original, its noisy version and the recovered image using \textit{s-trim} is indicated in Figure 4.20. An elaborate analysis of the results indicate that the proposed method is capable of removing noise and also extract the finest details of the original image aiding better inspection of the MRI by Medical professionals.

(a)

(b)

(c)

Figure 4.20 (Continued)
Figure 4.20 (Continued)
Figure 4.20 (Continued)
Figure 4.20 (Continued)
Figure 4.20 (a-s) A set of 19 MR Images with every set indicating the original, Noisy and de-noised image using *s-trim* respectively.
4.11 SIMULATION RESULTS

A set of 19 MR images is tested with the proposed \textit{s-trim} shrinkage technique as shown in Figure 4. 20. To every image 15\% noise level is added and tested with the proposed technique. It can be found from figure that recovered image almost resembles the actual image.

4.12 CONCLUSION

Thus the proposed \textit{s-trim} method proves to be a simple, efficient method in eliminating noise from Magnetic Resonance Images. The comparison on different quality metrics shows its functional capability of improved performance when compared to the existing methods. Implementing the proposed shrinkage technique using Ripplet Type-II method for different medical images can extend the research work. A lot of research is being carried out in restoring Optical Coherence Tomography (OCT) images corrupted by speckle noise. The Proposed method can be implemented to enhance such medical diagnosis. The De-noising process can also be enhanced using Graphics Process Unit (GPU) thereby accelerating the algorithm and making it applicable for clinical purposes.