CHAPTER V
A MODEL OF LABOUR AUGMENTING TECHNICAL CHANGE WITH AN IMPERFECTLY ELASTIC LABOUR SUPPLY

The analysis so far was based on the assumption that the supply of labour to the modern sector is perfectly elastic. It would be interesting to study the implications of labour augmenting technical progress in a model where the supply of labour is imperfectly elastic. The presentation in this chapter follows the papers by Johansen¹ and Taylor². The imperfect elasticity of labour supply can be explained in terms of the Harris-Todaro³ model. The urban-rural wage-differential acts as an inducement to migrate. And the number of job seekers invariably exceeds the number of jobs, resulting in unemployment. Therefore, urban unemployment is an imminent possibility in this model. Also this model is a fixed coefficients model and hence the capital-augmented labour ratio remains unchanged. The possibility of substitution of cheaper augmented labour for capital—the factor which

1. Ibid. Johansen.
acted in the interests of labour in the earlier models—does not operate in this model. The main contradiction, in the present model, is between the labour displacing effects of a declining capital-labour ratio and the favourable effects of higher profits and accelerated accumulation for employment.

Modern sector output, Q, is assumed to be determined by the available capital stock, K, through a fixed output-capital ratio "a",

\[ Q = a.K \]  \quad (1)

Employment, L, can be related to the capital stock by a coefficient "b", since "a" is assumed to remain constant,

\[ L = b.K \]  \quad (2)

Continuous labour-augmenting technical progress would imply that the labour-capital ratio declines from time 0 at a rate \( \lambda \),

\[ b(t) = b_0 e^{-\lambda t} \]  \quad (3)

As noted earlier, unemployment in the modern sector is possible in this model. If the number of people looking for jobs in the modern sector (the economically active
population) is denoted by "E", then the Harris-Todaro migration theory points to a differential equation such as,

\[ \dot{E} = h(w - w_s) \]

for the rate of growth of E. w is the current industrial wage and w_s is the subsistence wage set by the level of living in the countryside. The size of E can vary as people move back and forth between the subsistence and modern sectors. The function \( h(w - w_s) \) is non-linear, but in some cases the linear approximation

\[ \dot{E} = \alpha(w - w_s) \]

\[ \alpha = h'(0) \] (4)

can be applied when the difference between the modern sector wage w, and the subsistence wage w_s is not too large.

Now, \[ L = g(w - w_s) \] (5)

Here \( L \) measures the momentary nonsubsistence supply of labour and equation (5) gives the "supply price"
of labour reflecting the presumption that an increase in employment as a share of the economically active population bids up the wage. It represents Ricardo's expression: "Labour is dear when it is scarce and cheap when it is plentiful."

Capital accumulation is assumed to be determined by profits,

\[
dK = s \cdot Y_k \quad \text{(6)}
\]

where \( Y_k \) is the profit income and "s" is the savings rate from it. Total profits are given by

\[
Y_k = Q - wL
\]

Now differentiating equation (2) we get

\[
L' = K' + b' = K' - \lambda
\]

since \( \lambda \) is the rate of labour augmenting technical change.

Now substituting for \( K' \) from equation (6) we get

\[
K' = s(Q - wL)/K
\]

since \( Y_k = Q - wL \). And substituting for \( Q \) we have

4.Ibid.Ricardo, Chapter V.
\[ K' = s(a.K - w.L)/K \]

\[ = s(a - bw) \quad \text{since } L = b.K \]

Therefore, \[ L' = s(a - bw) - \lambda \] (7)

This equation determines the rate of growth of labour demand.

Labour supply growth is given by

\[ L' = E' + \gamma w' \]

\[ = g(w - w_m) + \gamma w' \] (8)

where \( \gamma \) is the elasticity of the function \( g(w - w_m) \) and is assumed to be constant.

Equating (8) with (7) we get the market clearing growth rate of the wage \( w \),

\[ w' = \frac{1}{\gamma} \left[ \beta - (\lambda + s.b).w \right] \] (9)

where \( \beta = sa + 2w_m - \lambda \)

Now equation (9) conforms to the Bernoulli form of differential equations. One particular solution is \( w = 0 \); the rest of the solution can be obtained by the transformation \( w = 1/z \) which gives the linear equation
\[
\frac{dz}{dt} = \frac{1}{\gamma} (-\beta z + \omega + \beta \lambda)
\]  \hspace{1cm} (10)

Solving equation (10) and taking the inverse transformation we get

\[
w(t) = \left[ \frac{z^* e^{-\beta t/\gamma} + w_w(t)^{-1}}{1} \right]^{-1}
\]  \hspace{1cm} (11)

where \( z^* \) is the constant of integration and \( w_w(t) \) is the wage "warranted" by the rate of technical progress and other parameters,

\[
w_w(t) = w_a + \beta \left[ s(a - b w_a) - \chi + \gamma \lambda (\lambda - sa) \right] \quad \frac{\beta (\omega + \beta \lambda)}{\beta (\omega + \beta \lambda)}
\]  \hspace{1cm} (12)

The constant \( \beta \) is almost certainly positive. Therefore, \( w(t) \) will approach \( w_w(t) \) as \( t \to \infty \). The derivative of \( w_w \) with respect to the labour-capital ratio, "b", is negative. Thus the warranted wage as well as the actual wage in the long run will rise since the labour-capital ratio, "b", falls with technical progress. When \( \beta \) is negative, that is, when the savings rate and the
output-capital ratios are very low and the rate of technical change is very high, the wage tends to zero.

Consider what happens in the short and medium run. In this case it is possible for the warranted wage \( w_w \) to be lesser than \( w_a \). This depends on the sign of the numerator. Here the term \( Y.\lambda(\lambda - sa) \) will be negative for the same reason as \( \beta \) is positive. Hence the warranted wage will be lesser than \( w_a \) if \( sa - bw_a \) - \( \lambda \) is also negative. This condition implies that labour demand is declining in the short run and is pulling wages down with it. Continuing technical progress will finally reduce the labour-capital ratio and generate profits high enough to call forth positive employment growth.

One of the major differences between this analysis and the analysis in the previous chapters is that the elasticity of substitution in this model is assumed to equal to zero. There is no a-priori reason for such an assumption to hold. An elasticity of substitution greater than zero might alter the above arguments substantially. As noted in the previous chapter, the elasticity of substitution plays a crucial role. It is quite possible that results quite contrary to those obtained in this model might hold if the elasticity of substitution is greater than zero.