CHAPTER IV
A CLOSED ECONOMY MODEL OF LABOUR-AUGMENTING TECHNICAL CHANGE

The analysis in the previous chapter was constrained by the unrealistic assumption that the modern sector produces a single homogeneous output which can be transformed without cost into either a consumption good or a capital good. However, it is well recognised that consumption and capital goods are inherently different commodities. In this chapter, therefore, we consider the implications of labour augmenting technical change for employment and relative distribution of factor shares when the modern sector produces two goods—consumption goods and capital goods. The rate of labour augmenting technical change is assumed to be the same in the two industries. This is an assumption made in the interests of clarity, so that the issue is not blurred by the extraneous factor of disparities in rates of technological progress between the two industries.

The fundamental difference, between the two models, however, is that the present model is a closed economy one. The analysis in the previous chapter was concerned
with a small open economy where all prices were determined internationally. There was a single homogeneous global market so that a country's production pattern was independent of local demand. In the present model, however, the closed economy assumption implies that all prices are internally determined. Demand relationships play an important role and production is geared to demand. This stands in contrast with the previous model where demand relationships had no significant role to play. Further there is no fixed price relationship between consumption and capital goods in the present model. Thus, the present model has to take account of changing terms-of-trade since all prices would be changing.

Thus, the production function in the consumption goods industry is given by

\[ C(t) = F_\alpha(K_\alpha, L_\alpha e^{\lambda t}) \]

\[ = L_\alpha e^{\lambda t} f_\alpha(K_\alpha/L_\alpha e^{\lambda t}, 1) \]

or

\[ C(t) = L_\alpha e^{\lambda t} f_\alpha(\theta_\alpha) \]

(1)

where \( L_\alpha \) is the labour employed in the consumption goods industry, \( K_\alpha \) is the capital employed and \( \theta_\alpha = \frac{K_\alpha}{L_\alpha e^{\lambda t}} \)

Similarly, the production function in the investment
goods industry is given by

\[ I(t) = L_\infty e^{\mu t} f_\infty(\Theta_\infty) \]  

(2)

where \( \Theta_\infty = \frac{K_\infty}{L_\infty e^{\mu t}} \).

Perfect mobility of capital coupled with profit maximising behaviour implies that the value of marginal product of capital is equal in the two industries. Or,

\[ p_\infty f'_\infty(\Theta_\infty) = p_x f'_x(\Theta_x) = r_p \]

or

\[ r = f'_x(\Theta_x) \]  

(3)

where \( r_p \) is the price of investment goods and \( r \) the rate of interest.

Similarly, given perfect mobility of labour we have

\[ p_\infty e^{\mu t} \{ f_\infty(\Theta_\infty) - \Theta_\infty f'_\infty(\Theta_\infty) \} = p_x e^{\mu t} \{ f_x(\Theta_x) - \Theta_x f'_x(\Theta_x) \} = \tilde{w} \]  

(4)

where \( \tilde{w} \) is the fixed wage rate in the modern sector.

As noted earlier, the internal determination of prices entails a description of demand relationships. All kinds of demand relationships can be used. However, only the two simplest ones are considered here. It can be assumed that the expenditure fraction devoted to food is
constant. Thus, if δ is the fraction spent on food and (1-δ) the fraction spent on manufactures, then by the above assumption, the ratio of the value of food to the value of manufactures, \( \frac{A}{p_oC} \) equals δ where A is the agricultural output and the price of food is the numeraire. The agricultural output is assumed to remain constant despite a withdrawal of labour from agriculture (see Chapter II). Since the right-hand-side is assumed to remain constant, \( \frac{p_oC}{\delta} \) will also be constant, that is,

\[
\frac{p_oC}{\delta} = 1-\delta \quad A
\]

(5)

Alternatively, it can be assumed that the fraction of surplus over subsistence requirements devoted to food is constant. This assumption conforms to the Stone-Geary\(^1\) form of the utility and demand functions. Now the surplus is given by the difference between the consumer expenditure in terms of food and the subsistence requirements of the population.

Thus, food demand,

\[
A - N\theta = \delta(G - N\theta)
\]

---

where \( N \) is the total population, \( \beta \) the subsistence requirement and \( G \) the consumption expenditure in terms of food. And manufacturing demand, in terms of food, is given by

\[
p_C = 1 - \delta (G - N\beta)
\]

Therefore,

\[
\frac{p_C}{\delta} = \frac{1 - \delta}{A - N\beta}
\]

and

\[
p_C = \left( \frac{1 - \delta}{\delta} \right) (A - N\beta)
\]

We find that as long as the population \( N \) remains constant, the results are qualitatively the same under both assumptions. Therefore, either of the two assumptions can be invoked. Further, the price and income elasticities are both unity in the former case. However, in the Stone-Geary case, the income elasticity of demand for food is less than one and is greater than one for manufactures. The price elasticity is unity for both food as well as manufactures. Thus, the particular form of the demand function is irrelevant to us, as long as \( p_C \) is equal to a constant, which implies that the demand curve for consumer manufactures is a rectangular hyperbola and the expenditure on these is constant.
Now there exists a one-to-one relationship between \( \theta_o \) and \( \theta_i \), for the wage-rental ratio in the two industries is equal given the perfect mobility of factors. Thus,

\[
p_{o\theta} e^{\lambda e} [f_{\theta}(\theta_o) - \theta_o f'_{\theta}(\theta_o)] / p_{o\theta} f'_{\theta}(\theta_o) = \frac{p_{i\theta} e^{\lambda e} [f_{i\theta}(\theta_i) - \theta_i f'_{i\theta}(\theta_i)]}{p_{i\theta} f'_{i\theta}(\theta_i)} = \frac{\bar{\omega}}{r} \tag{6}
\]

Or

\[
\frac{f_{\theta}(\theta_o) - \theta_o f'_{\theta}(\theta_o)}{f'_{\theta}(\theta_o)} = \frac{f_{i\theta}(\theta_i) - \theta_i f'_{i\theta}(\theta_i)}{f'_{i\theta}(\theta_i)} = \frac{\bar{\omega} e^{-\lambda e}}{r} = \omega
\]

where \( \omega \) is the wage-rental ratio. \( \theta_o \) can therefore be expressed as a function of \( \theta_i \). Or \( \theta_o = \mu(\theta_i) \). \( \tag{7} \)

Also, equation (6) can be rewritten as

\[
MRS_{\theta}(\theta_o) = MRS_{\theta}(\theta_i) = \omega \tag{8}
\]

since the marginal rate of substitution is equal in the two industries. Differentiating equation (8) logarithmically, we get,

\[
\frac{\delta \ln(MRS_{\theta})}{\delta \ln \theta_o} . \frac{d\theta_o}{\theta_o} = \frac{\delta \ln(MRS_{\theta})}{\delta \ln \theta_i} . \frac{d\theta_i}{\theta_i}
\]
\[
\frac{1}{\sigma_c} \frac{d\theta_c}{\theta_c} = \frac{1}{\sigma_x} \frac{d\theta_x}{\theta_x}
\]

Therefore, \(\frac{d\theta_c}{\theta_c} = \frac{\sigma_c}{\sigma_x} \frac{d\theta_x}{\theta_x}\) \hspace{1cm} (9)

where \(\sigma_c\) and \(\sigma_x\) are the elasticities of factor substitution in the consumption goods and capital goods industries respectively.

Full employment of factors in both industries implies that

\[
K = K_c + K_x \hspace{1cm} (10)
\]

and \(L = L_c + L_x\) \hspace{1cm} (11)

From the marginal productivity relationship \([\text{equation (4)}]\) we have

\[
p_{o_c}e^{\lambda t} = \frac{\bar{w}}{f_c(\theta_c) - \theta_c f'_c(\theta_c)} \hspace{1cm} (4')
\]

And substituting for consumption goods output in equation \(5\) we get
Equating (4') and (5') we get

$$\frac{L_0}{s_{L0}(\theta_0)} = \frac{1-\delta}{\delta} \frac{A}{\overline{w}}$$

(12)

where $s_{L0}$ is the relative share of labour in the consumer goods industry and is given by

$$\frac{f_0(\theta_0) - \theta_0 f'_0(\theta_0)}{f_0(\theta_0)}$$

Equation (12) can be rewritten as

$$L_0 = \frac{1-\delta}{\delta} \frac{A}{\overline{w}} s_{L0}(\mu(\theta_1))$$

(12)

Now $\dot{K} = I(t)$

(13)

that is, net capital formation is identically equal to the output of new capital. Equation (13) assumes that capital is ever lasting.

Also, since capitalists' are assumed to save a proportion "s" of their income and s is assumed to be a
Equation (14) gives us the demand for investment. Therefore, equating the demand for investment with the supply of investment goods we have

\[ L_x e^{\lambda x} f_x(\theta_x) = s(r).r.K \]

Or substituting for \( K \) from the full employment equation (10) we get

\[ L_x e^{\lambda x} f_x(\theta_x) = s(r).r.[K_\alpha + K_x] \]

or

\[ L_x e^{\lambda x} f_x(\theta_x) = s(r).r.e^{\lambda x}[L_\alpha \theta_\alpha + L_x \theta_x] \]  

(15)

since \( \theta_x = K_x \) and \( \theta_\alpha = K_\alpha \)

Solving for \( L_x \), the labour employed in the investment goods industry, from equation (15) we get

\[ L_x = \frac{s(r).r.L_\alpha \theta_\alpha}{f_x(\theta_x) - s(r).r.\theta_x} \]  

(16)
Now \( K = e^{\lambda t} [L_0 \theta_0 + L_\lambda \theta_\lambda] \)

given the full employment of capital.

Substituting for \( L_\lambda \) we have

\[
K = e^{\lambda t} \left[ L_0 \theta_0 + \frac{s(r).r.L_0 \theta_0}{f_x(\theta_\lambda) - s(r).r.\theta_\lambda} \right]
\]

or \( K = e^{\lambda t} L_0 \theta_0 \left[ \frac{s(r).r + 1}{f_x(\theta_\lambda) - s(r).r.\theta_\lambda} \right] \)

And substituting for \( L_0 \theta_0 \) from equation (12) we get

\[
K = e^{\lambda t} \left[ 1 - \delta \frac{A s_{\lambda} \mu(\theta_\lambda) \mu(\theta_\lambda)}{\delta \bar{w}} \right] \left[ \frac{s(r).r + 1}{f_x(\theta_\lambda) - s(r).r.\theta_\lambda} \right]
\]

(17)

Differentiating equation (17) logarithmically and equating it with \( s(r).r \) we have
\[
\frac{\theta_x}{\sigma_x} = \frac{s(r).r - \lambda}{\sigma_x (1 + (1-s_L)(1-\sigma_o)) + (1-s_L) \left[ \frac{f_x(\theta_x) - \theta_x.r}{f_x(\theta_x) - s(r)r\theta_x} \right] \left[ s - (s + rs') \right] \sigma_x}
\]

(18)

(see Appendix B)

where \( \sigma_o \) and \( \sigma_x \) are the elasticities of substitution in the consumer and investment goods industries respectively, \( s_L \) is the share of labour in the investment goods industry and \( s' \) is the derivative of the savings proportion, \( s \), with respect to the rate of interest. \( \frac{\sigma_o}{\sigma_x} \) describes the behaviour of \( \theta_o \) relative to \( \theta_x \).

\( \theta_x \). And \( (1 + (1-s_L)(1-\sigma_o)) \) is the rate of growth of employment in the consumer goods sector.

The system attains steady state growth if the numerator equals zero, that is, \( s(r).r \) equals \( \lambda \). In other words, steady state growth is attained when the rate of capital accumulation equals the rate of labour augmenting technical progress. Thus, capital and augmented labour
will grow at the same rate, \( \lambda \), as long as there is no migration. Now, \( s(r)r \) can be rewritten as \( s(f'(\theta_x))f'(\theta_x) \). In order to obtain the conditions for the existence of equilibrium, we have to consider the limit of the function \( s(r)r - \lambda \).

It is assumed by Inada\(^2\) that

\[
f'(\infty) = 0 \quad \iff \quad \lim_{\theta_x \to \infty} \sigma_x \leq 1
\]

and

\[
f'(0) = \infty \quad \iff \quad \lim_{\theta_x \to 0} \sigma_x \geq 1
\]

This is equivalent to the fact that if the elasticity of substitution, \( \sigma_x \), is less than one, there will be a maximum rate of interest which \( r \) will not exceed. And if the elasticity of substitution, \( \sigma_x \), is greater than one, there will be a minimum rate of interest and \( r \) will never fall below this minimum. As can be seen from figure 4.1, the necessary and sufficient condition for the existence of steady state is that \( s_{x_{\text{max}}} \geq \lambda \geq s_{x_{\text{min}}} \).

In the case where \( \lim_{\theta_x \to 0} \sigma_x \geq 1 \) and \( \lim_{\theta_x \to \infty} \sigma_x \leq 1 \),

\( r_{\text{max}} \) is infinity and \( r_{\text{min}} \) is zero and the existence of steady state is guaranteed. This, however, is a sufficient

Fig 3.1(a)

Fig 3.1(b)
condition. The equilibrium attained by \( \theta_x \) will be unique because \( s(r).r \) is continuously diminishing in \( \theta_x \) and there is only one point at which \( s(r).r \) could equal 0.

Stability of the steady state solution depends on whether or not the denominator in equation (18) happens to be positive at the equilibrium point. The sufficient condition for the steady state solution to be locally stable is that the denominator be positive at the point of equilibrium.

Now consider the equilibrium in equation (18). Under the plausible assumption that \( \sigma_0 \leq 2 \) the expression

\[
1 + (1-s_{LO})(1-\sigma_0) \geq 1 + (1-s_{LO})(1-2).
\]

i.e., \( \geq 1 - (1-s_{LO}) \)

i.e., \( \geq s_{LO} \geq 0 \).

Thus, the denominator will be positive since the share of labour in the consumer goods industry will be positive.

As regards \( 1-s_{LO} \)

\[
\begin{bmatrix}
\left[ f(x(\theta_x) - r.\theta_x) \right] / \left[ f(x(\theta_x) - s(r).r.\theta_x) \right] & \left[ s - (rs'+s) \right] / \sigma_x
\end{bmatrix}
\]

\]
It can be shown that for very high and very low $\theta_x$, this term reduces to zero and the denominator is almost certainly positive (except in the unlikely event of an unrealistically high elasticity of substitution in the consumer goods industry).

Thus, $\theta_x \to 0$, we have the two possibilities

(i) $\lim \sigma_x < 1$

and (ii) $\lim \sigma_x \geq 1$

When $\lim \sigma_x < 1$, the share of labour in the investment goods industry, $s_{Lx}$, would tend to zero and $r$ would tend to a finite maximum. Therefore, $(1-s_{Lx}) \to 1$.

Now $f_x(\theta_x) - r.\theta_x$ can be rewritten as:

$$f_x(\theta_x) - s(r).r.\theta_x$$

$$\frac{1 - r.\theta_x/f_x(\theta_x)}{1 - s(r).[r.\theta_x/f(\theta_x)]}$$

$$= \frac{s_{Lx}}{1 - s(1-s_{Lx})}$$
Growth Rates

Fig 4.2
Therefore, 

\[ \frac{\frac{f_x(\theta_x) - x.\theta_x}{f_x(\theta_x) - s(r).x.\theta_x}}{(1-sLx)} = \frac{slx}{1-s(1-sLx)} \]

tends to zero as \( sLx \) tends to zero. Thus the denominator would be positive.

When \( \lim_{\theta_x \to 0} \sigma_x \geq 1 \), \( sLx \) tends to one and \( (1-sLx) \) tends to zero. Therefore, \( sLx.(1-sLx) \) would tend to zero and hence the denominator is positive.

Now consider the case where \( \theta_x \to \infty \). We have

(i) \( \lim \sigma_x < 1 \), \( sLx \) tends to one,

and (ii) \( \lim \sigma_x \geq 1 \), \( sLx \) tends to one.

Therefore, in both cases the denominator will be positive. As can be seen from figure 4.2, the curve will have the right kind of slope and will intersect the axis only once. So long as the curve is continuous, there will exist a unique and stable steady state solution. We find that the stability of the steady state solution depends crucially on the rate of growth of employment in the consumer goods industry.
Fig 4.3
It is possible, though unlikely, that the curve is not continuous. If the denominator equals zero at some intermediate point, the whole function may approach positive or negative infinity [see figure 4.3]. Thus, there will be singularity at the point at which the function approaches positive or negative infinity. The function approaches positive infinity when $\theta_x$ is very low and starts again from negative infinity (point A). Again, when $\theta_x$ is very high, the function approaches negative infinity and starts from positive infinity (point B). If the equilibrium is trapped between the two singularities, the equilibrium so attained is unstable but the value of $\theta_x$ will be limited by the dotted line and hence there will be limited stability.

Let us now consider the implications of steady state growth for employment and relative distribution of factor shares in the modern sector.

From the full employment equation we have

$$L = L_0 + L_x$$

Or substituting for $L_0$ and $L_x$ we get,
\[ L = \left[ 1 - \delta \right] \frac{A}{\delta} \frac{s_L a[\mu(\theta_x)]}{\bar{w}} + \frac{s(r) \sigma(r) \mu(\theta_x)}{f_x(\theta_x) - s(r) r \theta_x} \left[ 1 - \delta \frac{A}{\delta} \frac{s_L a[\mu(\theta_x)]}{\bar{w}} \right] \]

(19)

And \( K = e^{\lambda x} \left( L_0 \theta_x + L_x \theta_x \right) \) \hspace{1cm} (20)

Over time both \( \dot{L} \) and \( \dot{K} \) would depend on \( \dot{\theta_x} \). And \( \frac{\dot{L}}{L} \) \( \frac{\dot{K}}{K} \) in the case of a steady state solution, \( L \) would equal \( L \) zero, that is, employment would be constant. \( K \) will be constant at the rate \( \lambda \)--the rate of labour augmenting technical progress. Output of each industry would be growing at the rate \( \lambda \) because the effective factor inputs would be growing at the rate \( \lambda \). Prices would be falling at the rate \( \lambda \). Further, the wage bill is constant in terms of food.

Now if \( D = \bar{W}_L \) is the relative distribution of \( \frac{rK}{r} \)
income in the modern sector,

\[ \frac{\dot{D}}{D} = \frac{\dot{L}}{L} - \frac{\dot{r}}{r} - \frac{\dot{K}}{K} \]

since the real wage \( \bar{w} \) is assumed to remain constant. \( \dot{r} \) is the rate of growth of interest rate.

In steady state, \( \frac{\dot{D}}{D} = -\lambda \) since \( \frac{\dot{L}}{L} = \frac{\dot{r}}{r} = 0. \)

Thus, over time the relative distribution of income in the modern sector would be shifting against labour and in favour of capital. However, the real income of the employed would be increasing since the price of consumption goods, \( p_\sigma \), would be declining at the rate \( \lambda \).

What would be the course of the economy if the steady state solution did not exist? This contingency would arise, if, in the limit, either \( \lambda \) exceeds \( sr_{\text{max}} \) or \( \lambda \) is less than \( sr_{\text{min}} \) (see Figure 4.4).

\[ \frac{\dot{\sigma}_x}{\theta_x} \] would be negative when \( \lim \sigma_x < 1 \), \( \lambda \) is greater than \( sr_{\text{max}} \) and would be positive when \( \lim \sigma_x \geq 1 \), \( \lambda \) is less than \( sr_{\text{min}} \).
Fig 4.4(a)

Fig 4.4(b)
Let us therefore consider the implications for employment growth and relative distribution of factor shares when a steady state solution does not exist.

Now \( L = L_0 + L_x \)

Substituting for \( L_0 \) and \( L_x \) we get

\[
L = \left[ \frac{1-\delta}{\delta} \right] \left[ \frac{A \sigma L\sigma[\mu(\theta_x)]}{\bar{w}} \right] \left[ \frac{s(r).r.\mu(\theta_x)}{f_x(\theta_x) - s(r).r.\theta_x} + 1 \right]
\]

(19)

Differentiating equation (19) logarithimically we get

\[
\frac{\dot{L}}{L} = \frac{\dot{\theta}_x}{\theta_x} \left[ \sigma_0[(1-sL_0)(1-\sigma_0)] + (1-sL_x) \right] \left[ \frac{f_x(\theta_x) - r.\theta_x}{f_x(\theta_x) - s(r)r\theta_x} \right]
\]

\[
* \left[ \frac{1}{1 + s(r).r[\mu(\theta_x) - \theta_x]} \right] s\left[ 1 + 1/sL_x \right]
\]

\[
* \left[ \frac{[\mu'(\theta_x)-1]/\theta_x + s(r).r[\mu(\theta_x)-\theta_x\mu'(\theta_x)]/\theta_x}{f_x(\theta_x)} \right] - (1-sL_x)[\mu(\theta_x)-\theta_x]/\theta_x = (rs'+s)/\sigma_x
\]
\[
* \left[ 1 + \frac{[\mu(\theta_x) - \theta_x]}{\theta_x} \right] \]

(21)

[see Appendix B]

\[ \frac{\partial}{\partial \theta_x} \text{ will be negative when } \lim_{\theta_x \to \theta_{x_{\text{max}}}} \sigma_x < 1, \lambda \text{ exceeds} \]

\[ \text{lim}_{\theta_x} \]

Now, as noted earlier, the share of labour in the investment goods industry, \( s_{Lx} \), would tend to one as \( \theta_x \) tends to infinity.

And given \((1-s_{Lx}) f_x(\theta_x) - r.\theta_x = (1-s_{Lx}) \frac{s_{Lx}}{f_x(\theta_x) - s(r).r.\theta_x} \]

\[ \frac{s_{Lx}}{1 - s(1-s_{Lx})} \]

the second term in equation (21) would reduce to zero.

Thus, as long as the elasticity of substitution in the consumption goods industry, \( \sigma_c \), is lesser than one \( \sigma_c < 1 \) and \( \lambda \) and \( \theta_x \) hence employment will decline over time when \( \theta_x \) is
negative. If, however, $\sigma_c$ is greater than one, employment would be growing in the consumption goods industry. The peculiar feature of this model is that employment remains constant in the investment goods industry despite the fact that the share of labour in this industry approaches one.

Again, $\theta_x$ will be positive when $\lim \sigma_x \geq 1$, $\lambda$ is less than $s_{\text{min}}$. In this case, $s_{Lx}$, the share of labour in the investment goods industry would tend to zero as $\theta_x$ tends to infinity. Thus, the second term is again reduced to zero and $L$ would be growing only in the consumption goods industry, as long as $\sigma_c$ is less than one. If $\sigma_c$ is greater than one, employment in the consumption goods industry actually declines. Thus, the impact of labour saving technical change on employment would depend entirely on the relative ease of substitution between factors in the consumption goods industry.

Now $K = e^{\lambda t}(L_0\theta_0 + L_x\theta_x)$

Substituting for $L_0$ and $L_x$ we get

$$K = e^{\lambda t} \left[ 1 - \frac{1}{\delta} \frac{A s_{Lc}[\mu(\theta_x)]}{\bar{W}} \right] \left[ \frac{s(r) \cdot r}{f_x(\theta_x) - s(r) \cdot r \cdot \theta_x} + 1 \right]$$
Differentiating logarithmically we get,

\[ \dot{K} = \dot{\theta}_x \psi_0 \left( (1-s_\theta)(1-\psi_0) + 1 \right) + (1-s_\theta) \]

\[ \dot{K} \]

\[ \theta_x \]

\[ \left( \frac{f_x(\theta_x) - x, \theta_x}{f_x(\theta_x) - s(r), x, \theta_x} \right) \left[ \frac{s - (rs' + s)}{\sigma_x} \right] \]

(22)

[see Appendix B]

The analysis in this case is very straightforward. When \( \dot{\theta}_x \) is positive capital accumulates at a rate faster than the rate of labour augmenting technical change and when \( \dot{\theta}_x \) is negative, capital grows at a rate which is less than \( \lambda \).

Now \[ \frac{\dot{D}}{D} = \frac{\dot{L}}{L} - \frac{\dot{r}}{r} - \frac{\dot{K}}{K} \]

Substituting for \( \frac{\dot{L}}{L} \), \( \frac{\dot{K}}{K} \) and \( \frac{\dot{r}}{r} \) we have
\[
\frac{D}{\theta_x} = -\lambda + \frac{\dot{\theta}_x}{\theta_x} \left[ -\sigma_0/\sigma_x + (1-s_{\text{Lx}}) \right] \left[ \frac{f_x(\theta_x) - r.\theta_x}{f_x(\theta_x) - s(r).r.\theta_x} \right] \\
\times \left\{ \frac{[r\sigma^t + s]}{\sigma_x} \left[ 1 - \frac{1}{1 + s(r).r[\mu(\theta_x) - \theta_x]} \right] \right. \\
\left\{ 1 + \frac{[\mu(\theta_x) - \theta_x]}{\theta_x} \right. \\
\times \left\{ \frac{1}{s_{\text{Lx}}} \left[ 1 + \frac{[\mu'(\theta_x) - 1]}{\theta_x} - (1-s_{\text{Lx}})[\mu(\theta_x) - \theta_x] \right] \right. \\
\left. + \frac{s(r).r[\mu(\theta_x) - \theta_x\mu'(\theta_x)]}{\theta_x} \right] - 1 \right\} + \frac{s_{\text{Lx}}}{\sigma_x} \\
\text{[see Appendix B]} \quad (23)
\]
Again, given (i) \( \lim \sigma_x < 1 \), \( s_{Lx} \) tends to one and (ii) \( \lim \sigma_x > 1 \), \( s_{Lx} \) tends to zero.

\[
(1 - s_{Lx}) \frac{f_x(\theta_x) - r_1 \theta_x}{f_x(\theta_x) - s(r_1) r_1 \theta_x}
\]

tends to zero in both cases.

\[
\dot{D} \text{ can be rewritten as } -\lambda + \frac{\dot{\theta}_x}{\theta_x} \left[ \frac{-\sigma_x}{\sigma_x} \right] + \frac{s_{Lx}}{\sigma_x} \frac{\dot{\theta}_x}{\theta_x}
\]

or

\[
\dot{D} = -\lambda + \left[ \frac{s_{Lx} - \sigma_x}{\sigma_x} \right] \frac{\dot{\theta}_x}{\theta_x}
\]

(24)

Thus, the change in the relative distribution of income within the modern sector can be decomposed into two effects. The first effect can be attributed to the labour displacement effect that would have resulted from the labour augmenting technical change if the relative costs of capital and augmented labour had remained unchanged. The second effect is due to the fall in unit cost of augmented labour and the consequent substitution of labour for capital. Consider first the case where \( \frac{\dot{\theta}_x}{\theta_x} \) is
negative. As $\Theta_z$ tends to infinity, $s_{Lz}$ approaches one. If the elasticity of substitution in the consumption goods industry is less than one, the relative distribution of income moves against labour since both the labour displacing effect as well as the substitution effect are moving in the same direction. If, however, $\sigma_0$ is greater than or equal to one, nothing can be said categorically, because the ultimate result depends on whether or not the substitution effect is able to outweigh the labour displacing effect of technical change. Now, if $\frac{\dot{\Theta}_i}{\Theta_i}$ is positive, $\Theta_i \to \infty$, $\lim \sigma_i \geq 1$, $s_{Lz}$ tends to zero. In this case $\frac{\dot{D}}{D}$ would always be negative, because the share of labour in the investment goods industry falls to zero, thereby enhancing the share of capital and hence the returns to capital, irrespective of the value of elasticity of substitution in the consumer goods industry.

Now the price change in the consumer goods industry is given by

$$\frac{\dot{p}_0}{p_0} = \lambda - \frac{1-s_{Lz}}{\sigma_i} \frac{\dot{\Theta}_i}{\Theta_i}$$

(25)

[see Appendix B]
$s_{x}$ would always be less than one. Hence when \( \theta_{x} \)

is negative, the price of the consumer goods would be falling at a rate less than the rate of labour augmenting technical progress, \( \lambda \). And when \( \dot{\theta}_{x} \) is positive, prices

would be falling at a rate faster than the rate of labour augmenting technical change.

\[
\begin{align*}
\dot{p}_{x} &= -\lambda - (1-s_{x}) \frac{\dot{\sigma}_{x}}{\sigma_{x}} \frac{\dot{\theta}_{x}}{\theta_{x}}
\end{align*}
\]

Also \( \dot{p}_{x} = -\lambda - (1-s_{x}) \frac{\dot{\theta}_{x}}{\theta_{x}} \)

When \( \theta_{x} \) is negative, \( \lim \sigma < 1, \theta \rightarrow 0, s_{x} \)

tends to one, the numerator in the second expression is reduced to zero and the price of investment goods would be falling at a rate equal to the rate of labour augmenting technical change, which is the same as in the case of steady state growth. However, when \( \dot{\theta}_{x} \) is positive,

lim \( \sigma_{x} \geq 1, \theta_{x} \rightarrow 0, s_{x} \) tends to zero. And the price of investment goods would be falling at a rate faster than the rate of labour augmenting technical progress.
Thus, we find that although the relative distribution of income in the modern sector worsens when \( \theta_x \) is positive, the fall in the prices of both consumption and investment goods tends to mitigate a worsening in the real income of the employed. Even in the case where \( \theta_x \) is negative, prices tend to decline, though at a much lower rate, thus raising the real income of the employed.

From the above analysis we can infer that labour augmenting technical change in a two-commodity, closed economy model, results in a constancy of employment, given the existence, uniqueness and stability of steady state growth. The necessary and sufficient condition for the existence of steady state is that \( \lambda \), the rate of labour augmenting technical progress, lies between the maximum and minimum rates of capital accumulation. In steady state, capital accumulates at a rate equal to the rate of labour augmenting technical progress. And the relative distribution of income in the modern sector shifts steadily against labour and in favour of capital. The real income of the employed, however, increases
because of a fall in prices. The system does not attain steady state when either of the conditions is violated. In the case where the rate of labour augmenting technical progress exceeds the maximum rate of capital accumulation, $\theta_x$ is negative and when the rate of labour augmenting technical progress falls short of the minimum rate of accumulation, $\theta_x$ is positive. When $\theta_x$ is positive, capital accumulates at a rate faster than the rate of labour augmenting technical change, $\lambda$, and is less than $\lambda$ when $\theta_x$ is negative. Employment in both the cases depends on the elasticity of substitution in the consumption goods industry. The relative distribution of income in the modern sector worsens considerably when $\theta_x$ is positive and depends upon the relative strength of the substitution effect vis-a-vis the labour displacing effect of technical progress, when $\theta_x$ is negative.
In both cases the real income of the employed increases due to a fall in prices.