CHAPTER - 5

OPTIMIZATION PRODUCTION SCHEDULING MODEL FOR OIL REFINERY

5.1 Introduction

Aden Refinery Company (ARC) is receiving its crude oil through a pipeline, which is linked to a docking station to oil vessels (e.g. Masealah or Bab-Al Mandab or Qana which belong to ARC. Their Capacities are 13,800, 6,000 and 3,000 tons, respectively) or any other tankers unloaded.

The unloading schedule of these tankers is defined at the corporate level and cannot be changed easily (Annul Bulletin et al. [4]). Thus, for a given scheduling horizon, the number, type, arriving and departure times of the oil tankers are known a priori (Brown, Graves and Ronen [19]).

This Chapter is focused on the production scheduling optimization of operation modes concerning crude oil vessel unloading, storage, blending and feed to Crude Distillation Units (CDUS).

The model proposes the strategic operation for the system in accordance with the given conditions and the optimal operational cost calculated.

The strategic operation, if it is feasible, must follow the proposed suitable unloading days for vessels and the proposed flow rates among vessels and tanks, among storage and charging tanks and among tanks and plants, for keeping the optimal production
scheduling given by the model results. Moreover, this model can be used as a viable tool, not only for supporting the shipment planning, also, for discovering system infeasibilities and for strategic decisions concerning investments in storage and pumping systems. (Jia, Ierapetritou and Kelly [62], Karuppiah, Furman and Grossmann [66] and Robertson, Palazoglu and Romagnoli [123]).

The purpose is to satisfy the demand while the inventory volumes and the pumping system capacities are not violated. The shipment plan requires being realizable in terms of production. A cross checking between the shipment plan and the corresponding required feeding to plants according to the production schedule should be done to be sure that the feeding can be done in due time. Moreover, the model will help to do this cross checking and answer whether a shipment plan definitely can match in terms of production or not .(Brown, Graves and Ronen et al.[19], Lukac, Hunjet and Nerali[86], Taher and Abduljabbar [143] and Yuzgec, Palazoglu and Romagnoli [162]).

5.2 Problem Definition

The system configuration for this production scheduling optimization model corresponds to a multistage system consisting of vessels, storage tanks, charging tanks, and CDUs as is illustrated in figure (5.1).
For a given scheduling horizon, crude oil vessels arrive to the refinery docking station which only allows one vessel for unloading at a time. In accordance with the planning level, each vessel will have a reasonable date for leaving that should be at most the date when the next vessel arrives. The day one vessel arrives could start to unload depending of the optimal results recommended by the model. In accordance with all the current conditions of the system analyzed. The day one vessel finishes unloading should be up to one day before its maximum departure date fixed at the planning level, the reason is because the maximum departure date allowed for the preceding vessel is the same planned for arriving date of the next vessel so that potentially this next vessel could start unloading on the arriving date (Saharidis and Ierapetritou [129], Saharidis and Minoux and Dallery [130], Wenkai and Hui [156] and Yuzgec, Palazoglu and Romagnoli [162]).

The crude oil is unloaded into storage tanks at the docking station. Then, the crude oil is transferred from storage tanks to charging tanks. Each crude oil inside the charging tank must carry out to be within a range of blended crude oil composition determined at
the planning level for the scheduling horizon. The fulfillment of this blended composition range is made per tank having in account the material balance of the remaining volume and the different flows coming in or coming out of the tank with their different compositions per time interval. Then, blended crude oil from charging tanks are charged into the CDUs and whenever that it is optimally required feed switches are done from one kind of blended crude oil to another for each CDU. Finally, it is important to mention that when a charging tank is feeding a CDU, it must not be fed by any storage tank, or vice versa (Kallrath [65], Lee, Pinto, Grossmann and Park [77], Magalhaes and Shah [88] and Pan, Li and Qian [108]).

5.3 MILP Model Formulation

Given the configuration of this multistage system and the arrival time of the vessels, the equipment capacity limitations and the key component concentration ranges for crude oil. The problem will focus to determine the following operating variables to minimum costs (Fagundez, Xavier and Faco [34]).

1. Waiting time for each vessel in the sea after arriving.
2. Unloading duration time for each vessel.
3. Crude oil unloading rate from vessel s to storage tanks.
4. Crude oil transfer and blending rates from storage tank to charging tanks.
5. Inventory volumes of storage and charging tanks.
6. Crude distillation unit charging rates fulfilling the demand per each CDU.
7. Sequence of type of blending crude oil to be changed in each CDU in accordance with the optimal mode changeovers.
The following are the operating rules that have to be obeyed (Joly, Moro and Pinto [63]).

1) In the scheduling horizon each vessel for unloading should arrive and leave the docking station.
2) If a vessel dose not arrives at the docking station, it cannot unload the crude oil.
3) If a vessel leaves the docking station, it cannot continue unloading the crude oil.
4) The vessel can start unloading the same arriving date.
5) The vessel cannot unload on its maximum departure date. Except for the last vessel.
6) While the charging tank is charging one CDU, crude oil cannot be fed into the charging tank and vice versa.
7) Each charging tank can feed at most one CDU at one time interval.
8) Each CDU is charged by only one blended crude oil at one time interval.

On the other hand these are the following operating constrains that must be met (Moro and Pinto [100]).

1) Equipment capacity, limitations: tank capacity and pumping rate.
2) Quality limitations of each blended crude oil: range of component concentration in each blended crude oil.
3) Demand per interval of time (day) of each CDU.

The model minimizes the operation cost for the total system shown. The model is formulated using general notation and MILP formulation showing the possibility of using the model for a general problem of this matter. In order to develop the multi-period MILP model, continuous and binary variables are associated with the system network. The following are the assumptions for the proposed model;
1) Only one vessel docking station for crude oil unloading is considered.

2) The time applied for the changeover are neglected and also, the transient flows generated during either start up or shut down when a changeover is done.

3) Perfect blending is assumed for each charging tank while, it is being fed by different crude oils and additional blending time inside the tank is not required before it charges the CDU.

4) The composition of the crude oil is decided by the amount of key components presented in the crude oil or in the blended crude oil. In general, Sulfur is at least one of the key components for differentiating between crude oils.

A Uniformed discretization is chosen in the given scheduling horizon for the proposed scheduling model. The selection of the length of each discretized time span involves a trade-off between accurate operation and computational effort (Lee, Pinto, Grossmann and Park et al. [77], Lundgren, Lundgren and Purson [87] and Shah [134]).

5.4 Nomenclature

5.4.1 Sets

VE : \{V = 1, 2,...,V\} / Crude oil vessel or tankers).

ST : \{I = 1,2 ,...,I \} / Storage tanks).

CT : \{j,y =1,2 ,...,J,y \} / Charging tanks).

COMP : \{K = 1,2 ,...,K\} / Crude oil components).

CDU : \{L = 1,2 ,...,L\} / Crude distillation units).

SCH : \{T =1,2,...,T\} / Time intervals along the scheduling Horizon}
5.4.2 Parameters

\( V_{S_i}^{\text{max}} \) : Storage tank maximum capacity.

\( V_{S_i}^{\text{min}} \) : Storage tank minimum capacity.

\( V_{B_i}^{\text{min}} \) : Charging tank minimum capacity.

\( V_{B_i}^{\text{max}} \) : Charging tank maximum capacity.

\( C_{U_v} \) : Unloading cost of vessel V per unit time interval.

\( C_{SEA_v} \) : Sea waiting cost of vessel V per unit time interval.

\( C_{SINV_i} \) : Inventory cost of storage tank i per unit time interval.

\( C_{BINV_j} \) : Inventory cost of charging tank j per unit time interval.

\( C_{CHANG} \) : Changeover cost of CDUL.

\( T_{ARR_v} \) : Crude oil vessel arrival to the docking station.

\( T_{LEA_v} \) : Crude oil vessel maximum departure date from the docking station.

\( P_{UMPCAP_v} \) : Maximum pumping system capacity or total flow capacity from each storage tank i to charging tanks.

\( E_{V_v, k} \) : Concentration of component k in the crude oil vessel V

\( E_{S_i, k} \) : Concentration of component k in the crude oil of Storage tank i.

\( E_{S_i, k}^{\text{min}} \) : Minimum concentration of component k in the blended crude oil of storage tank i.

\( E_{S_i, k}^{\text{max}} \) : Maximum concentration of component k in the blended crude oil of storage tank i.

\( E_{B_j, k}^{\text{min}} \) : Minimum concentration of component k in the blended crude oil of charging tank j.
\( \text{EB}_{j,k}^{\text{max}} \): Maximum concentration of component \( k \) in the blended crude oil of charging tank \( j \).

\( \text{DMCDU}_{t,l} \): Demand of each CDU\(_{L} \) per time interval.

\( \text{TOTDMCDU}_{L} \): Total demand of each CDU\(_{L} \) along the scheduling horizon.

\( \text{DMBCO}_{t,j} \): Demand of each blended crude oil \( j \) along the scheduling horizon.

\( \text{FVS}_{v,i,t}^{\text{max}} \): Maximum crude oil rate from vessel \( V \) to one storage tank \( i \).

\( \text{FVS}_{v,i,t}^{\text{min}} \): Minimum crude oil rate from vessel \( V \) to one storage tank \( i \).

This variable is not mandatory to have a value. It could be either 0 or a positive value depending of the minimum flow restriction for the vessel pumping system that normally is assisted working in series with the refinery pumping system to storage tanks.

\( \text{FSB}_{i,j,t}^{\text{max}} \): Maximum crude oil rate from storage tank \( i \) to one charging tank \( j \).

\( \text{FSB}_{i,j,t}^{\text{min}} \): Minimum crude oil rate from storage tank \( i \) to one charging tank \( j \).

By default, this parameter has a value of 0 as the minimum value. A small optimal flow rate if it is optimally required could be managed with centrifugal pumps and process control instrumentation.

\( \text{FBC}_{j,L,t}^{\text{max}} \): Maximum crude oil rate from charging tank \( j \) to one CDU\(_{L} \).
In general, this value corresponds to the maximum \((\text{CDU}_L)\) feed capacity or it could be adjusted to a lower value.

\[ \text{FBC}_{j,L,t}^{\min} \] : Minimum crude oil rate from charging tank \(j\) to one \(\text{CDU}_L\).

This value could be adjusted to the same value \(\text{DMCDU}_{t,l}\), depending on the problem conditions.

### 5.4.3 Variables:

there are many variables as follows:

- **Binary Variables**
  - \(X_{F_{v,t}}\): Variable to denote if vessel \(V\) starts unloading at time \(t\).
  - \(X_{L_{v,t}}\): Variable to denote if vessel \(V\) finishes unloading at time \(t\).
  - \(X_{W_{v,t}}\): Variable to denote if vessel \(V\) is unloading its crude oil at time \(t\).
  - \(F_{i,j,t}\): Variable to denote if the crude oil blended in storage tank \(i\) is feeding charging tanks at time \(t\); otherwise storage tank \(i\) could be being fed by vessel \(V\).
  - \(D_{j,L,t}\): Variable to denote if the crude oil blended in charging tank \(j\) charges \(\text{CDU}_L\) at time \(t\); otherwise charging tank \(j\) could be being fed by storage tanks.
  - \(Z_{j,y,L,t}\): Variable to denote switch of the blended crude oil fed to \(\text{CDU}_L\) from the charging tank \(j\) to the charging \(y\).

- **Integer Variables**
  - \(T_{F_v}\): Vessel \(V\) unloading initiation time.
  - \(T_{L_v}\): Vessel \(V\) unloading completion time.
• **Continuous Variables**

\[ V_{v,t} \]: Volume of crude oil in vessel V at time t.

\[ V_{S_{i,t}} \]: Volume of crude oil in storage tank i at time t.

\[ V_{B_{j,t}} \]: Volume of crude oil in charging tank j at time t.

\[ F_{V_{S_{v,t}}} \]: Volumetric flow rate from vessel V to storage tank i at time t.

\[ F_{S_{B_{i,j,t}}} \]: Volumetric flow rate from storage tank i to charging tank j at time t.

\[ F_{B_{C_{j,l,t}}} \]: Volumetric flow rate from charging tank j to CDU L at time t.

\[ F_{K_{V_{S_{v,t}}}^{k_i}} \]: Volumetric flow rate of component k from vessel V to storage tank I at time t.

\[ F_{K_{S_{B_{i,j,k,t}}}^{k_i}} \]: Volumetric flow rate of component k from storage tank i to charging tank j at time t.

\[ F_{K_{B_{C_{j,l,k,t}}}^{k_i}} \]: Volumetric flow rate of component k from storage tank j to CDU L at time t.

\[ V_{K_{S_{i,k,t}}} \]: Volume of component k in storage tank i at time t.

\[ V_{K_{B_{j,k,t}}} \]: Volume of component k in charging tank j at time t.

\[ E_{S_{i,k,t}} \]: Concentration of component k in the blended crude oil of storage tank i at time.

\[ E_{B_{j,k,t}} \]: Concentration of component k in the blended crude oil of charging tank j at time t.

**Cost** : Total optimal operational cost.
5.4.4 Conditions

- **Initial Conditions**

\( VV_{V,TARR_v} \) : Initial volume of crude oil vessel at time equal \( TARR_v \).

\( VS_{i,1} \) : Initial volume of storage tank \( i \) at time equal 1 in the startup of the scheduling horizon.

\( VB_{j,1} \) : Initial volume of charging tank \( j \) at time equal 1 in the startup of the scheduling horizon.

\( ES_{i,k,1} \) : Concentration of component \( k \) in the blended crude oil of storage tank \( i \) at time \( t \) equal 1 in the startup of the scheduling horizon.

\( EB_{j,k,1} \) : Concentration of component \( k \) in the blended crude oil of charging tank \( j \) at time \( t \) equal 1 in the startup of the scheduling horizon.

\( VKS_{i,k,1} \) : Initial volume of component \( k \) in storage tank \( i \) at time \( t \) equal 1 in the startup of the scheduling horizon.

\( VKB_{j,k,1} \) : Initial volume of component \( k \) in charging tank \( j \) at time \( t \) equal 1 in the startup of the scheduling horizon.

5.5 Mathematical Formulation of Model

The model focuses on minimizing the following operation cost of the system for the operations of crude oil vessel unloading, storage, blending and feeding to crude distillation units in an oil refinery. Then, this is the main objective equation that represents the total operation cost of the system:
\[
\text{Cost} = \sum_{V=1}^{V} \left[ (TL_{V} - TF_{V} + 1)CU_{V} \right] + \sum_{V=1}^{V} \left[ (TF_{V} - TARR_{V} )CSEA_{V} \right] + \\
\sum_{i=1}^{I} \sum_{t=1}^{T} \left[ (VS_{i,t} + VS_{i,t+1})CSINV \right] / 2 + \\
\sum_{j=1}^{J} \sum_{t=1}^{T} \left[ (VB_{j,t} + VB_{j,t+1})CBINV \right] / 2 + \\
\sum_{j=1}^{J} \sum_{y=1}^{Y} \sum_{l=1}^{L} \sum_{t=1}^{T} \left[ CHANG_{L,Z} \right]_{j,y,l,t} \]
\]

\[5.1\]

The above equation is subjected to following constrains:

**5.5.1 Vessel Arrival and Departure Operation Rules**

Each vessel must arrive to the docking station for unloading only once through the scheduling horizon:

\[
\sum_{t=1}^{T} X_{F_{V}} = 1, \quad \forall V \in VE
\]

\[5.2\]

Each vessel leaves the docking station only once through the scheduling horizon:

\[
\sum_{t=1}^{T} X_{L_{V}} = 1, \quad \forall V \in VE
\]

\[5.3\]

The unloading initiation time is denoted by the following equation:

\[
TF_{V} = \sum_{t=1}^{T} tX_{F_{V}} = 1, \quad \forall V \in VE
\]

\[5.4\]

The unloading completion time is denoted by the following equation:

\[
TL_{V} = \sum_{t=1}^{T} tX_{L_{V}}, \quad \forall V \in VE
\]

\[5.5\]
Each vessel must start unloading either after or on the arrival time established at the planning level:

\[ TF_V \geq TARR_V , \quad \forall V \in VE \]  
(5.6)

Each vessel must finish unloading up to one interval of time before the maximum departure time established at the planning level:

\[ TL_V < TLEA_V , \quad \forall V \in VE , \forall V \neq V \]  
(5.7)

Except for the last vessel:

\[ TL_V \leq TLEA_V , \quad \forall V = V \]  
(5.8)

Minimum duration of the vessel unloading is two time intervals:

\[ TL_V - TF_V \geq 1, \quad \forall V \in VE \]  
(5.9)

The preceding vessel must finish unloading one time interval before the next vessel in the sea arrives and starts to unload:

\[ TF_{V+1} \geq TL_V + 1, \quad \forall V \in VE \]  
(5.10)

Unloading of vessel V only will be possible between times TFV and TLV:

\[ XW_{VJ} \leq \sum_{t=1}^{T} XF_{VJ} , \quad \forall t \leq m , \forall V \in VE , \forall m \in \{ m = TARR_V , \ldots TLEA_V \} \]  
(5.11)

\[ XW_{VJ} \leq \sum_{t=1}^{T} XL_{VJ} , \quad \forall t > m , \forall V \in VE , \forall m \in \{ m = TARR_V , \ldots TLEA_V \} \]  
(5.12)

5.5.2 Material Balance Equations for the Vessel

The crude oil in vessel V at time \( t + 1 \) must be equal to; the
crude oil in vessel $V$ at time $t$ taking away the crude oil transfer from vessel $V$ to storage tank $i$ at time $t$:

$$VV_{V,t} = VV_{V,TARR_V}, \quad \forall t = TARR_V$$

(5.13)

$$VV_{V,t+1} = VV_{V,t} - \sum_{i=1}^{I} FVS_{V,i,t}, \quad \forall V \in VE, \forall t \in SCH$$

(5.14)

The crude oil of each vessel $V$ has different composition. Then, if there is one storage tank assigned to each vessel, the solution must guarantee that the crude oil from each vessel $V$ is transferred to the corresponded storage tank $i$ which will handle the same vessel crude oil composition. Then, for this general case, $i$ is equal to $V$ to identify the respective storage tank for the vessel:

$$\sum_{i=1}^{I} \sum_{t=1}^{T} FVS_{V,i,t} = \sum_{i=1}^{I} \sum_{t=1}^{T} FVS_{V,i,t}, \quad \forall V \in VE, \forall i = V$$

(5.15)

If there are more storage tanks than vessels, the problem could consider managing crude oil blending composition ranges per storage tank. Then, each vessel could optimally unload to several tanks meeting with the pre-established blending range for each storage tank as it was explained above. Whether this is the case, equation (5.15) is not useful and must not be used for this situation. However, instead, the following equation should be used to control the total flow from each vessel to storage tanks within a reasonable margin:

$$\sum_{i=1}^{I} FVS_{V,i,t} \leq PUMP\text{CAP}_V, \quad \forall V \in VE, \forall t \in SCH$$

(5.16)
The volume of the total crude oil transferred from vessel V to storage tanks during the schedule horizon, must be equal to the initial crude oil volume of vessel V:

\[ \sum_{i=1}^{I} \sum_{t=1}^{T} FVS_{V,i,t} = VV_{V,TARR_v}, \quad \forall V \in VE \]  

Equation (5.17)

Equation (5.18) and (5.19) are the operation constraints on crude oil transfer rate from vessel V to storage tank i at time t:

\[ FVS_{V,i,t} \min .XW_{V,t} \leq FVS_{V,i,t} \leq FVS_{V,i,t} \max .XW_{V,t}, \]

\[ \forall V \in VE, \forall i \in ST, \forall t \in SCH \]  

Equation (5.18)

\[ FVS_{V,i,t} \min .(1-F_{i,j,t}) \leq FVS_{V,i,t} \leq FVS_{V,i,t} \max .(1-F_{i,j,t}), \]

\[ \forall V \in VE, \forall i \in ST. \forall J \in CT, \forall t \in SCH \]  

Equation (5.19)

Equation (5.19) will be applicable only if storage tanks are managing crude oil blending ranges. The term \((1-F_{i,j,t})\) denotes that if there is any oil transfer from vessel to storage tank i, there is no oil transfer from storage tank i to any charging tank j at time t.

5.5.3 Material Balance Equations for the Storage Tank

The crude oil in storage tank i at time \( t + 1 \) must be equal to; the crude oil in storage tank i at time t plus the crude oil transferred from vessel V to storage tank i at time t taking away the crude oil transferred to charging tanks j at time t:

\[ VS_{i,t+1} = VS_{i,t} , \quad \forall t = 1 \]  

Equation (5.20)

\[ VS_{i,t+1} = VS_{i,t} + \sum_{V=1}^{V} FVS_{V,i,t} - \sum_{j=1}^{J} FSB_{i,j,t} , \]

\[ \forall i \in ST, \forall t \in SCH \]  

Equation (5.21)
Volume capacity limitation for storage tank $i$:

$$ VS_{i}^{\min} \leq VS_{i,t} \leq VS_{i}^{\max}, \quad \forall i \in ST, \forall t \in SCH $$

(5.22)

Equations (5.23) and (5.24) indicate the operating constraints on crude oil transfer rate from storage tank $i$ to charging tank $j$ at time $t$:

$$ 0 \leq FSB_{i,j,t} \leq FSB_{i,j,t}^{\max}(1-D_{j,l,t}), $$

$$ \forall i \in ST, \forall j \in CT, \forall l \in CDU, \forall t \in SCH $$

(5.23)

The term $(1 - D_{j,l,t})$ denotes that if charging tank $j$ is charging any CDU$_L$; there is no oil transfer from any storage tank $i$ to charging tanks $j$:

$$ 0 \leq FSB_{i,j,t} \leq FSB_{i,j,t}^{\max}F_{i,j,t}, $$

$$ \forall i \in ST, \forall j \in CT, \forall t \in SCH $$

(5.24)

Equation (5.24) will be applicable only if storage tanks are managing crude oil blending ranges. The term $F_{i,j,t}$ denotes that if there is oil transfer from storage tank $i$ to charging tanks, it must not have oil transfer from a vessel $V$ to storage tank $i$ at time $t$.

There is no minimum set rate different from 0, because the problem considers the use of centrifugal pumps commonly used for this kind of service. Therefore, the optimal flow $FSB_{i,j,t}$ could be controlled by process control instruments in any optimal range including the optimal lower flows that would be required for one or more time intervals along the scheduling horizon, if the transfer between the storage tank and one charging tank is not null at any interval of time $t$. 

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However, the total maximum flow allowed by the capacity of the pump or the pumping system from one storage tank $i$ to several charging tanks $j$ at time $t$ is controlled by the following constraint:

$$
\sum_{j=1}^{J} FSB_{i,j,t} \leq PUMPCAP_i , \quad \forall i \in ST , \forall t \in SCH
$$

(5.25)

Generally speaking, the flow rate from one storage tank to one charging tank $FSB_{i,j,t}$, almost, always is a little smaller than the total flow rate from the same tank to several charging tanks ($\sum FSB_{i,j,t}$), because the flow restrictions are bigger when the same pumping system taking from one storage tank $i$ is discharging to another charging tank $j$ circuit, when is discharging to several charging tanks. Furthermore, $FSB_{i,j,t}^{max}$ depends on the system characteristics from each storage tank $i$ to each charging tank $j$. However, for the cases analyzed in this Chapter a symmetrical system is considered. Therefore, $FSB_{i,j,t}^{max}$ will have the same value for each circuit from one storage tank $i$ to one charging tank $j$.

### 5.5.4 Material Balance Equations for the Charging Tank

The crude oil blended in the charging tank $j$ at time $t+1$ must be equal to; the crude oil in the charging tank $j$ at time $t$ plus the crude oil transferred from the storage tanks taking away the crude oil transferred to the CDU$_L$ at time $t$:

$$
VB_{j,t+1} = VB_{j,1} + \sum_{i=1}^{I} FSB_{i,j,t} - \sum_{i=1}^{I} FSC_{j,L,t} , \quad \forall j \in CT , \forall t \in SCH
$$

(5.27)
Volume capacity limitation for charging tank j:

\[ VB_{j}^{\text{min}} \leq VB_{j,t} \leq VB_{j}^{\text{max}}, \quad \forall j \in CT, \forall t \in SCH \]  

(5.28)

5.5.5 Material Balance Equations for Component k in the Storage Tank

The volume of component k in storage tank i at time t +1 is equal to; the volume of component k in storage tank i at time t plus the volume of component k in crude oil transferred from vessel V to the storage tank i taking away the volume of component k in the blended crude oil i transferred to charging tanks at time t (These equations must be considered only if storage tanks have blending functions):

\[ VKS_{i,k,t+1} = VKS_{i,k,t} + \sum_{v=1}^{V} FKVS_{v,j,k} - \sum_{j=1}^{J} FKBS_{i,j,k}, \quad \forall i \in CT, \forall k \in COMP, \forall t \in SCH \]  

(5.29)

The following is the operating constraint on volumetric flow rate of component k from vessel V to storage tank i:

\[ FVKS_{v,i,k} = FVS_{v,i} \cdot EV_{v,k}, \quad \forall v \in VE, \forall i \in ST, \forall k \in COMP, \forall t \in SCH \]  

(5.30)

The following are the operating constraints on volumetric flow rate of component k from storage tank i to charging tank j:

\[ FSB_{i,j}^{\text{min}} \leq FKS_{i,j,k} \leq FSB_{i,j}^{\text{max}}, \quad \forall i \in ST, \forall j \in CT, \forall k \in COMP, \forall t \in SCH \]  

(5.31)
The limitations in the volume capacity for component \( k \) in storage tank \( i \) at time \( t \) are given by:

\[
V_{S_{i,t}} \leq V_{KS_{i,t,k}} \leq V_{S_{i,t}} \quad \forall i \in ST, \forall k \in COMP, \forall t \in SCH
\]

\[
V_{S_{i,t}} \leq V_{ES_{i,t,k}} \leq V_{S_{i,t}} \quad \forall i \in ST, \forall k \in COMP, \forall t \in SCH
\]

(5.33)

**5.5.6 Material Balance Equations for Component \( k \) in the**

**Charging Tank**

The volume component \( k \) in charging tank \( j \) at time \( t+1 \) is equal to; the volume of component \( k \) in charging tank \( j \) at time \( t \) plus the volume of component \( k \) in crude oil transferred from storage tanks to the charging tank \( j \) taking away the volume of component \( k \) in the blended crude oil \( j \) transferred to CDU\(_L\) at time \( t \):

\[
V_{KB_{j,k,t+1}} = V_{KB_{j,k,t}} + \sum_{i=1}^{L} F_{KS_{i,j,k,t}} - \sum_{i=1}^{L} F_{KBC_{j,i,k,t}},
\]

\[
\forall j \in CT, \forall k \in COMP, \forall t \in SCH
\]

(5.34)

The following is the operating constraint on volumetric flow rate of component \( k \) from storage tank \( i \) to charging tank \( j \):

\[
F_{KS_{i,j,k,t}} = F_{SB_{i,j,k}} \cdot ES_{i,k},
\]

\[
\forall i \in ST, \forall j \in CT, \forall k \in COMP, \forall t \in SCH
\]

(5.35)

Constraint (5.35) is not needed when storage tanks are operating as blending tanks.

The following are the operating constraints on volumetric flow rate of component \( k \) from charging tank \( j \) to CDU\(_L\):

\[
F_{BC_{j,l,j}} \leq F_{KBC_{j,l,k,j}} \leq F_{BC_{j,l,j}} \quad \forall j \in CT, \forall k \in COMP, \forall t \in SCH
\]

(5.36)
\( \forall j \in CT, \forall l \in CDU, \forall k \in COMP, \forall t \in SCH \) (5.36)

The limitations in the volume capacity for component \( k \) in charging tank \( j \) at time \( t \) are given by:

\[
V_{B_{j,k,t}} \leq V_{K_{B_{j,k,t}}} \leq V_{B_{j,k,t}}^{\text{max}},
\]

\( \forall j \in CT, \forall k \in COMP, \forall t \in SCH \) (5.37)

### 5.5.7 Operating Rules for Crude Oil Charging to Crude Distillation Units

As it was stated above each \( CDU_L \) only can be charged by one charging tank \( j \) at time \( t \):

\[
\sum_{j=1}^{J} D_{i,l,t} = 1, \quad \forall l \in CDU, \forall t \in SCH
\] (5.38)

On the other hand, each charging tank \( j \) can charge at most one \( CDU_L \) at time \( t \):

\[
\sum_{l=1}^{L} D_{j,l,t} \leq 1, \quad \forall j \in CT, \forall t \in SCH
\] (5.39)

Equation (5.39) is useful when the system analyzed considers more than one \( CDU \).

If the \( CDU_L \) is charged by crude oil blended \( j \) at time \( t \) and after is charged by crude oil blended \( y \) at time \( t+1 \) then, changeover cost must be involved. The following is the condition that confirms that changeover cost shall be charged:

\[
Z_{j,y,l,t} \geq D_{j,l,t} + D_{y,l,t+1} - 1,
\]

\( \forall j,y (j \neq y) \in CT, \forall l \in CDU, \forall t \in SCH \) (5.40)
5.5.8 Problem Solving Direction

Only one of the following groups of constraints must be added to the model depending of the production planning demand target for the crude distillation units along the scheduling horizon.

1. If it is requested a fixed demand of blended crude oils from charging tanks along the scheduling horizon to charging all CDUs and a feed rate variation range to feed the crude distillation units is allowed. The following constraints are applicable:

\[
FBC_{j,l,t} \min D_{j,l,t} \leq FBC_{j,l,t} \leq FBC_{j,l,t} \max D_{j,l,t},
\]

\[
\forall j \in CT, \forall l \in CDU, \forall t \in SCH
\]

\[
\sum_{l=1}^{L} \sum_{t=1}^{T} FBC_{j,l,t} = D_{MCDO_{j}}, \quad \forall j \in CT
\]

(5.41)

(5.42)

2. If it is requested a charging rate demand per time interval for each CDU and it is allowed a feed rate variation up to either the maximum plant capacity per time interval or any other maximum allowed feed per time interval, the following constraints are applicable:

\[
DMCDU_{l,t} D_{j,l,t} \leq FBC_{j,l,t} \leq FBC_{j,l,t} \max D_{j,l,t},
\]

\[
\forall j \in CT, \forall l \in CDU, \forall t \in SCH
\]

(5.43)

The application of this equation is particularly interesting, if there is a short charging variation range allowed between the feed demands from plants and their maximum charging feed previously fixed.

Now, by solving cases using this option II will allow an easier understanding of the model not only for solving scheduling problems, also, for review the impact of future development in the system.
Particularly, a case with the CDU shutting down one day $n$ in the middle of the schedule horizon was analyzed using the additional constraint:

$$FBC_{j,L,n} = 0, \quad \forall t = n, \forall n \in SCH$$  \hspace{1cm} (5.44)

3. If it is requested a fixed feed rate demand per time interval for each CDU the following constraint inapplicable:

$$FBC_{j,l,t} = D_{MCDU_{t,l}} \cdot D_{j,l,t},$$

$$\forall j \in CT, \forall l \in CDU, \forall t \in SCH$$ \hspace{1cm} (5.45)

4. If it is requested a fixed total demand per CDU along the scheduling horizon and a charging rate variation range to feed the crude distillation units is allowed. The following constraints are applicable:

$$\min_{j,l,t} D_{j,l,t} \leq FBC_{j,l,t} \leq \max_{j,l,t} D_{j,l,t},$$

$$\forall j \in CT, \forall l \in CDU, \forall t \in SCH$$ \hspace{1cm} (5.46)

$$\sum_{j=1}^{J} \sum_{l=1}^{T} FBC_{j,l,t} = TOTDMCDU_{l,t}, \forall l \in CDU$$ \hspace{1cm} (5.47)

The following explanation applies to all groups presented. The term $D_{j,l,t}$ denotes that if there is any oil transfer from storage tank $i$ to charging tank $j$, there is no oil transfer from charging tank $j$ to CDU$_L$.

It could have other options using for instance a combination of the above groups, but this situation hardly could get feasible solutions unless some data needed for the constraints are relaxed. Therefore, new combination easily could come out with similar solutions to the solutions given by one of the options presented above.
5.6 Numerical Example

Consider the following problem, there are two crude vessels will arrive at day 1 and 5. The unloading for both vessels should be completed by day 8. Vessel 1 and 2 contain 1,000,000 bbl. of crude oil A and B, respectively. There is one CDU which has to process 1,000,000 bbl. of mixed crude oil x and y, respectively. The weight fractions of sulfur which determine the quality of crude oil are 0.01 for crude oil A and 0.06 for crude oil B. Two crudes are mixed to make two types of mixers: crude oil mixes x and y. The sulfur concentration of x should be in the range of 0.015 and 0.025, while that of y is between 0.045 and 0.055. The initial volumes of the storage tanks for crude oil A and B are respectively 250,000 and 750,000 bbl. While the initial volumes of the charging tanks for crude mix x and y are all 500,000 bbl. The costs involved in this problem are inventory cost, vessel harboring cost, Vessel Sea waiting cost and CDU changeover cost for crude oil mix charging mode change. Table (5.1) shows the system information for numerical example.

Table (5.1): System information for numerical example

<table>
<thead>
<tr>
<th>Scheduling Horizon ( of unit times: days )</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of vessel arrivals</td>
<td>2</td>
</tr>
<tr>
<td>Vessel number</td>
<td>Arrival time</td>
</tr>
<tr>
<td>Vessel 1</td>
<td>1</td>
</tr>
<tr>
<td>Vessel 2</td>
<td>5</td>
</tr>
<tr>
<td>Number of storage tanks</td>
<td>2</td>
</tr>
<tr>
<td>Storage tanks</td>
<td>capacity</td>
</tr>
</tbody>
</table>

104
<table>
<thead>
<tr>
<th></th>
<th>Tank 1</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>100</td>
<td>25</td>
<td>0.01</td>
</tr>
<tr>
<td>Tank 2</td>
<td></td>
<td>100</td>
<td>75</td>
<td>0.06</td>
</tr>
<tr>
<td>Number of charging Tanks</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Charging Tanks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tank 1</td>
<td>100</td>
<td>50</td>
<td>0.02(0.015-0.025)</td>
</tr>
<tr>
<td></td>
<td>Tank 2</td>
<td>100</td>
<td>50</td>
<td>0.05(0.045-0.055)</td>
</tr>
<tr>
<td>Number of CDUs</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Unit costs involved in vessel operation</td>
<td>Unloading cost =8, sea waiting cost =5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tank inventory unit cost</td>
<td>Storage Tank =0.08, charging tank = 0.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit changeover cost for charged oil switch</td>
<td>50(independent of sequence and CDU)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blended oils demand from Charging tanks to CDUs</td>
<td>Blended oil 1:100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum flow rate from vessel to one storage tank</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum flow from one storage tank to one charging tank</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum total flow rate from one storage tank to charging tanks at any time interval</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The given data for quantifying volumes and flow rates are given in barrels x 10,000 and barrels per time interval x 10,000 respectively. Changeover costs are given in US$ x 1,000; sea waiting costs and unloading costs are given in US$ x 1,000 per time interval day. Tank inventory unit cost is given in US$ x 0.1 per oil barrel. Therefore, optimal value results will be in US$ x 1,000.

We can represent the given data in flow network as it is represented in sketches below.
Table (5.2) points out the comparisons of the optimal unloading starting up date results from both models for numerical example vessel 1 starts to unload on day 3 for the existing model. That is one day more than the other model which considers this operation on day 2. Instead, for vessel 2 both coincide starting unloading on day 7.

Table (5.2): Optimal unloading starting date for vessels

<table>
<thead>
<tr>
<th>Vessels</th>
<th>Lee, Pinto and Park et al. [77] Model</th>
<th>X Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

The situation may be causing a big part of the difference between optimal values since the optimal operational schedule of the existing model should be paying more money in inventory total cost. On the other hand, the existing model results is paying one day more of sea waiting cost (US$ 5,000) than X model results as is indicated in table (5.2).
The optimization model table (5.3) involved 36 discrete variables, 192 single variables and 331 constraints.

The General Algebraic Modeling System (GAMS) (Brooke, Kendrick, Meerous and Raman [18]) and (IBM, OSL [57]), was used for setting up the optimization model. The number of variables and constraints was reduced by considering the data structure of binary variables.

The optimal results generated by the X model are shown and compared in advance with the optimal results of the existing model in table (5.3)

Table (5.3): Comparisons of optimal results with the existing model

<table>
<thead>
<tr>
<th>Items</th>
<th>Lee, Pinto and Park et al. [77], model</th>
<th>X model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal value (US$ x 1,000)</td>
<td>217,667</td>
<td>206,950</td>
</tr>
<tr>
<td>Equations and constraints</td>
<td>331</td>
<td>552</td>
</tr>
<tr>
<td>Single variables</td>
<td>192</td>
<td>337</td>
</tr>
<tr>
<td>Discrete variables</td>
<td>36</td>
<td>116</td>
</tr>
<tr>
<td>Iterations</td>
<td>1,695</td>
<td>4,393</td>
</tr>
<tr>
<td>Solving time (seconds)</td>
<td>17.1</td>
<td>5.21</td>
</tr>
</tbody>
</table>

As it can be observed in table (5.3), all the optimal objective values given by the X model as results are lower than the existing model values. In accordance with the mathematical explanation of X model, the equations and constraints handle by the X model really are more than the existing models ones; but the precision and the optimal value results of the X model are better than the existing model because they have achieved more money savings solving the same
operational scheduling problem. Furthermore, the computational advance has avoided that the machine time running of the model for each example has been much bigger proportional to the new number of equations and constraints.

In summary, the optimal results for numerical example are better for the X model indicating a total operational cost saving for the problem conditions of US$ 10,717 (4.92% cost reduction with respect to the existing model), during the 8 day scheduling horizon in accordance with the results showed in table (5.1).

5.7 Conclusions

This Chapter deeply described the problem definition and the mathematical formulation in an algebraic form of the optimization production scheduling model for managing and optimizing the operational cost in an oil refinery regarding the scheduling functions of unloading, storage, blending and feeding of crude distillation units.

The main objective function to minimize equation (5.1) was described including among other, the changeover cost, unloading cost and inventory cost. Moreover, all equations, constraints and variables required for the models were well described; explained in some cases their particularities. Also, the different direction for solving scheduling problems with this X model were indicated, pointing out the complementary different constraints that must be used in the model to solve each one of the different options.

This Chapter also, showed the test of the X model in detail, solving the same examples used by (Lee, Pinto, Grossmann and Park et al. [77]), when they presented their model; also, called in this Chapter “Existing model” for distinguishing from the X model. Both model results have been compared in detail, pointing out the
differences in the results. Although, the X model manages more equations and constraints, its optimal values with respect to minimize the total operational cost have been lower than the optimal values presented by the “Existing model”.

Furthermore, it has been demonstrated that the features of the X model design allow that it can be adapted to different demand conditions included the condition established by (Lee, Pinto, Grossmann and Park et al. [77]) for their example. Instead, the “Existing model” was developed to work in only one solved direction, Although, most of its results shown in this Chapter, apparently go in the same optimal path, it is less precise than the X model solving these kind of scheduling problems.

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