Chapter 1

Introduction

1.1 Inventory Control

Inventory is the stock of items or a resource of facilities used by an organization. Inventories are maintained in agriculture, industry and retail establishments. The objective of inventory control is often to balance conflicting goal of making available the required item at a time of need and minimizing the related costs. Scientific inventory control is generally understood to be the use of mathematical models to obtain optimal rules for operating inventory systems. The use of mathematical methods in inventory analysis seems to have been supplied by the simultaneous growth of the manufacturing industries and the various branches of engineering especially industrial engineering. The real need for analysis was first recognized in industries that had a combination of production scheduling problems and inventory problems.

The first quantitative analysis in Inventory studies goes back to the work of Harris [104] who derived the simple lot size formula. Since Wilson successfully popularized it, the same is also known as Wilson’s formula. The first full length book to deal with inventory problems was that of R. H. Raymond [191]. It explains how various extensions of the simple lot size model can be
used in practice. The most important impulse, however, came after the II World War. The economists Jacobs Marschak, Kenneth Arrow, Samuel Karlin [15] were some of the firsts to provide a rigorous mathematical analysis of a simple type of inventory model.

A collection of classical models by Arrow, Karlin and Scarf [16] provided much of the impetus for later work in this area. The relationship between the inventory management issues and classical economic thinking was presented in the monograph by Whitin [235]. The practical applications of inventory theory were provided by Wagner [231], Hadley and Whitin [103] and Peterson and Silver [176]. Numerous survey articles of stochastic inventory theory exist. The work by Veinott [229] is still one of the most comprehensive survey, despite its age. Aggarwal [2], Nahmias [154], Wagner [232, 233], Silver [212] and Porteus [181] provided other pertinent surveys. Girlich [96] provided a survey on dynamic inventory problems and implementable problems. The above reference citations are intended to be representative rather than encyclopedic. This is a necessity rather than a convenience since the number of articles on the subject is enormous. Dozens of books have also been published in this field. To mention a few, Naddor [151], Love [144], Odanaka [165], Bartmann and Beckmann [35] and Buzacott and Shantikumar [49] provided extensive treatment of the subject.

1.2 System Characteristics of Inventory Management

A simple inventory system represents a physical stock of goods kept in shelf for a smooth and efficient business transaction. While there are many good reasons to hold inventory / production, the demand variation is probably the most important one. In fact, inventory serves more or less as a simple insur-
ance against uncertainty. Inventory management systems can be classified according to the following:

1. **Demand process:** One of the most important aspect of an inventory model is the customer’s demand process.
   
   (a) **Deterministic:** The rate of demand is known with certainty and to be constant over time.
   
   (b) **Stochastic:** The demand has uncertainty and it may be stationary or non-stationary.

2. **Lead time:** The duration between the time at which the order is placed and the time at which it is received.
   
   (a) **Deterministic:** Lead time is known with certainty.
   
   (b) **Stochastic:** Lead time is a random variable.

3. **Review policy:** The inventory literature is concerned on the following types of review policies:
   
   (a) **Continuous review:** The level of inventory is monitored continuously, and we know the inventory level exactly at any point in time. As soon as an items are released from the stock or are added to stock the inventory level is updated automatically.

   The continuous review system has two cases according to its reviewing pattern. First kind depicts a system in which the review of system takes place at any time, and consequently an order is placed whenever the inventory level crosses a pre-specified level. But in the second, every transactions such as demand etc., occurred in the system are noticed and transactions are reported so
as to update the inventory level. So, it is also termed as transaction reporting system. Throughout this thesis, we call this simply a continuous review inventory system. Because of the advent of information technology and the use of electronic data processing machines in inventory management, the continuous review system has now received more attention.

(b) Periodic review: The level of inventory is monitored at prefixed time points, and hence the inventory level at any point in time is not known exactly.

The periodic review inventory system is considered as a multi-stage decision making system, for which dynamic programming technique provides suitable tools to obtain optimal decision rule.

However, for continuous review systems, this technique is not so much adaptive, since it yields complicated expressions. The continuous review inventory models have been analyzed by a strategy which has two parts. In the first part, the ordering policy of a given type is chosen and the stationary behaviour of the inventory level is analyzed without specifying the cost structure of the problem. The performance measures like reorder rate, shortage rate, mean inventory level and mean backorder level are computed. In the second part, a cost structure is imposed on the system, and the relevant total expected cost per unit time for operating the whole inventory system is minimized. The first approach to solving optimal inventory problem is more flexible and implementable to real situations. It also provides more information regarding various operating characteristics of the system, and enables one to test the sensitivity of the total cost with regard to different parameters.
4. Excess demand:

(a) Backlog: The demands that arrive when the inventory system is out of stock are back logged. This means that these demands would be satisfied as soon as the replenishment is received.

(b) Lost Sales: The demands that arrive when the stock level is zero are completely lost. This means that the demands that occurred when the stock level is zero are not satisfied even after the replenishment is received.

5. Inventory life: Obsolescence and/or perishability of items may occur, and goods could not be stocked for an infinite amount of time.

(a) Obsolescence: Obsolescence refers to items that lose their value through time because of rapid changes of technology or the introduction of a new product by a competitor. Fashionable goods must be sharply reduced in price or otherwise disposed off after the season is over. For example, spare parts of military aircrafts depend on model and they become obsolete when a new model is introduced.

(b) Perishability: Examples of such perishable inventories include blood, fresh produce, food, pharmaceutical, photographic films, chemicals and assembly components in aerospace and semi-conductors. While dealing with perishable systems, the consequent loss must be explicitly taken into account, as its exclusion from the model may yield inaccurate performance measures. Analysis of systems stocking perishable items is far more difficult than their counterparts having infinite lifetime and a primary reason is that the stock depletion rate is a function of the on-hand inventory level.
A number of Economic Order Quantity models (EOQ) and their variations have been proposed for exponentially decaying items. Ghare and Scharder [95] were the first to address the inventory problem with deteriorating items. Under the assumptions of constant demand and constant deterioration rate and with the help of some mathematical approximations, they developed a simple EOQ model for such systems. For continuously deteriorating inventory with deterministic demand, several EOQ-type formulations using various functions for the rate of deterioration have been proposed. For example, the deteriorating rate is assumed to have a Weibull law (Covert and Philip [63], Philip [177]), a gamma law (Taddikamalla [219]) and an arbitrary distribution (Shah [201], Shine [207]). Shah and Jaiswal [202] and Aggarwal [3] have presented stochastic demand models for items that deteriorate continuously in time. In all the above models the replenishment is assumed to be instantaneous. Misra [150] and Heng et al. [105] have considered deteriorating items in a production system with finite production rate. Nahmias and Wang [155] derived a heuristic lot size reorder policy for an exponential decay problem with constant lead times.

For continuous review systems when the ordering policy or replenishment is not a decision variable, Nahmias [156], Graves [99], Kaspi and Perry [120, 121], Perry and Posner [173] have obtained the performance measures by applying the results from queueing theory with reneging customers. Here the queue is identified with inventory, the service pattern with demand, time to impatience with lifetime of fresh stock and arrivals of customers to the replenishment of inventory. But this method has some serious shortcoming. In the inventory context the timing and the amount of replenishment are controllable, while in the queueing context occurrences of arrivals are not controllable.
The first paper that dealt with the determination of optimal policy for continuous review systems is due to Weiss [234], who proved that \((0, S)\) policy is optimal for simple model with Poisson demand, fixed lifetime and instantaneous supply. A few Markovian \((s, S)\) inventory systems with exponential lifetimes and zero lead times have been studied by Kumarasamy and Sankarasubramanian [132], Kalpakam and Arivarignan [111] and Liu [141]. The analysis of perishable inventory systems with zero lead times is relatively straightforward, but if the lead time is positive, this becomes difficult as the decay applies only to items on hand and not to those on-order. The first paper that dealt analytically a model with positive lead times is that of Schmidt and Nahmias [199] in which it is assumed that a \((S - 1, S)\) lost sales model with fixed life and lead times under Poisson demand. Ravichandran [189] dealt with an \((s, S)\) system where the batch of \((S - s)\) items replenished will perish together and the lifetime has an Erlangian distribution. Pal [167] and Kalpakam and Sapna [116] analyzed Markovian models with Poisson demand and exponential life and lead times; the former dealt with and \((S - 1, S)\) model with full backlogging and the latter with a lost sales \((s, S)\) system. Arivarignan [12] considered a perishable \((-r, S)\) inventory system with renewal demand and exponentially distributed life time of each item. Pal and Ghosh [168] considered a perishable \((-s, S)\) inventory system with Poisson demand, exponentially distributed life time of each item and instantaneous supply.

In addition, the review articles of Nahmias [157] and Raafat [183] point out the need for analysis of random lifetime perishable systems under realistic assumptions like stochastic demands, non-instantaneous lead times and situations corresponding to lost sales.
1.3 Operating Policy

One of the major operating policies in the study of stochastic inventory systems is the well established $(s, S)$ policy. Inventory systems based on $(s, S)$ policy have been studied quite extensively during the last three decades. This policy has a simple operating rule: when the inventory position is less than or equal to a prefixed level $s$, an order is placed so as to raise to a level $S$ as it ordered items are received immediately. The probabilistic approach for such systems was proposed by Arrow, Karlin and Scarf [16], who provided a systematic treatment of inventory models based on renewal theory. The celebrated $(s, S)$ policy has been given importance because of its simplicity, weakest set of assumptions and catching power of the essence of all control problems (Wagner [231]).

For continuous review systems, which may be considered as a limiting case of periodic review models, one can argue heuristically that the $(s, S)$ policy is optimal. The rigorous proof was already available (Beckmann [36], Hadley and Whitin [103] and Sahin [197]). In continuous review systems, the prime managerial level of interest is to determine the optimal values of $s$ and $S$, that minimize the long-run expected cost rate. The management practitioners expect a closed form solution for this problem, which is some times very difficult to obtain. The computational complexity of cost function is high, because it is a function of two variables $s$ and $S$.

The book by Hadley and Whitin [103] provides an excellent account of applications. A computational approach for finding optimal $(s, S)$ inventory policy is given by Veinott and Wagner [228]. An excellent review of Veinott [229] summarizes the status of the mathematical theory of inventory until early sixties. The cost analysis of the different inventory systems along with several other characteristics is given by Naddor [151]. Gross and Harris
[100] developed a continuous review \((s, S)\) inventory model with state dependent lead time. Sivazlian [216] considered a continuous review inventory system with arbitrary distribution for inter-arrival time points of demand occurrences where each demand requires exactly one item from the inventory. The same result for the case with arbitrarily distributed demand quantity have been obtained by Richards [192]. An indepth study of \((s, S)\) inventory policy with arbitrary distribution for lead time and for inter demand arrivals was made by Srinivasan [217].

An \((s, S)\) inventory system for which the demand for items depend upon an external environment was studied by Feldmann [86]. Ramaswami [185] obtained algorithms for an \((s, S)\) inventory model where the demand is according to a versatile Markovian point process. A survey on dynamic inventory problems and implementable models was given by Girlich [96]. An efficient algorithm for computing optimal policies was designed by Federgruen and Zipkin [84]. Bookbinder and Cakanyildirim [44] considered a continuous review inventory system with random lead times and orders with \((Q, R)\) policy. Kalpakam and Sapna [116] considered a continuous review \((s, S)\) inventory system with stochastic lead times and performed optimal cost analysis. An inventory system with random life times was analyzed by Liu [141]. A continuous review production inventory system was studied elaborately by Sharafali [206].

The existing computational schemes for finding the optimal decision values of decision variables, are classified into two categories (i) exact method and (ii) heuristic.

Works in the first category include Inglehart [107], Veinott [228], Wagner [231], Johnson [108], Archibald [11], Sahin [197], Federgruen [83] and Zipkin [240]. Studies of heuristic methods can be found in Wagner [231], Silver [213],
Nadoor [151], Erharat and Wagner [75], Porteus [180], Freeland and Porteus [89], Tijms [227] and Sahin and Sinha [196].

Because of the complicated form of the cost function, the computation of optimal values for \( s \) and \( S \) has been found to be prohibitively expensive. Success stories of applications of \((s, S)\) inventory models can be found in Nahmias [153]. Recently Feng and Xiao [87] gave a new algorithm to tackle optimal \((s, S)\) policy problems. They introduced an auxiliary function that is built around the cost function and a dummy cost factor.

When the demands that occurred during stockout period are assumed to be lost, this introduces a new complexity in the modelling processes. But this aspect was not considered in most of the earlier inventory models due to the complexity of the situation. Since 1960, most of the inventory models are studied by assuming full backlogging. Archibald [11] analyzed a lost sales continuous review \((s, S)\) system and derived the long run expected cost rate for the same. A generalized study of lost sales \((s, S)\) inventory system was done by Srinivasan [217]. In 1985, Buckmann and Love [48], studied a simple \((s, S)\) inventory system with lost sales, Poisson demand and Erlangian lead times. A general system having Markov renewal demands and exponential lead time was analyzed by Kalpakam and Arivarignan [112]. Recently a modified \((s, S)\) inventory system with lost sales was studied by Kalpakam and Sapna [115]. They considered a system having renewal demands and exponential lead times, with a policy involving placement of an emergency order at the time of stock-out, if available. But all the models cited above deal with lost sales and restricted reorder policy that the number of pending orders at any instant be unity \((S - s > s)\) to preserve the tractability of the analysis. Kalpakam and Arivarignan [114] introduced multiple reorder level policy. Elango and Arivarignan [73] considered an inventory model
in which they introduced an ordering policy with contiguous collection of reorder levels. The demand points form a renewal process and the lead time is distributed as negative exponential. Perumal et al., [175] considered a Markovian inventory system with a set of reorder levels but the lead time is assumed to be distributed as exponential with parameter depending upon the level at which it was initiated.

1.4 Discrete Time Inventory Systems

In discrete time inventory systems, the time is measured in discrete units with epochs numbered 0, 1, 2, \ldots. All events are assumed to occur only at these epochs and the system state is monitored (reviewed) continuously at each and every epoch. Though many inventory systems are conveniently characterized by fixed length intervals at which events occur and decisions are made, few articles in the literature deal with discrete-time inventory models.

In 1989, the first paper in discrete time inventory model was considered by Bar-Lev and Perry [34]. In this paper, they assumed that the number of items which arrive into a system as well as the number of demands for those items are discrete random variables having a common support \{0, 1, \ldots\}. Each item is classified into exactly one of \(N\) age categories. New items arriving into the system are placed into the first age category. The items of age \(j\) (\(j = 1, 2, \ldots, N - 1\)), which have not been removed by a demand during a period, are transferred into age category \(j + 1\) at the beginning of the next period. The items of age \(N\), which are not removed by demand are lost. If the demand is larger than the total number of items in the system, then the excess demand is registered as unsatisfied demand. Otherwise the demand is satisfied according to a FIFO issuing policy.

Later Lian and Liu [139] developed a discrete time inventory model with
geometric inter-demand times and constant life time. They assumed that the demands arrive in batches and the batch-size is random. They also assumed that the lead time is zero and demands are backlogged. They used matrix-analytical methods to get steady state solution for the inventory and obtained a closed-form solution for the average cost function.

In 2001, Abboud [1] analyzed a discrete time Markov model for the production inventory systems with machine breakdowns. He assumed that the demand and production rates are constant and the production rate is greater than the demand rate. The failure times and the repair times are independently distributed as geometric and backorder of demands is assumed.

More recently, Lian, Liu and Neuts [140] discussed a discrete time model for common life time inventory systems. In this paper, the demands arrive in batches according to a discrete phase-type renewal process and the lifetime of items has a discrete PH-distribution. They assumed that the lead time is zero and unmet demands are back ordered.

1.5 Queueing Systems

A queueing model is usually characterized in terms of customers requiring service, service facilities providing service and queues containing customers waiting for service. Queueing problems come up in a variety of situations in the real world and have stimulated an enormous literature which, though in part quite mathematical and abstract, is not of a purely academic nature. In fact, there has been a considerable interaction between the developments at the various degrees of abstraction in the field. Thus, though the more theoretical-oriented part of the literature tends to deal with models and problems too simplified to be of any great direct practical applicability, the notions and techniques that are studied are also important for the practical
worker in the field. Conversely, the call for solutions to particular problems has of course stimulated not only the theory of queueing but also that of probability as a whole, fields like Markov processes, renewal theory and random walk owing their present state and importance to a large extend to the impact from queueing theory. Queueing problems present a great challenge to the probabilists and a \textit{memento mori} to probability theory as a whole. The development of abstract probability theory may be of great beauty, but seldom sheds much light on how to come up with the numbers the practical worker asks for.

Queueing situations from daily life are almost too obvious, but we shall list a few anyway: customers queueing up before the $m$ cashiers in a supermarket; telephone callers waiting for one of the lines of an exchange to become available; aircraft circling over the airport before a runway becomes free; and so on. Of more recent date than these classical examples are a number of problems connected with computer organizations or networks in teletraffic theory or data transmission: in a time-sharing computer, we may think of the jobs as customers who are served by the central processor unit and possibly input/output facilities.

The pioneering work on the theory of queues was done by Erlang \cite{Erlang1, Erlang2} of the Copenhagen Telephone Company during 1909 to 1920 (for a study of Erlang’s work from the modern point of view, see Brockmeyer et al. \cite{Brockmeyer})). Independently, some valuable work was done by Fry \cite{Fry}. Mention must also be made of major contribution to the theory by Palm \cite{Palm}, Khintchine \cite{Khintchine} and Pollaczek \cite{Pollaczek, Pollaczek2}. A systematic treatment of the theory from the point of view of stochastic processes is due to Kendall \cite{Kendall1, Kendall2}, and this has greatly influenced subsequent work in this field.

A good summary of the history of queueing theory up to 1961 can be
found in Saaty [194]. This book has a bibliography of 910 items. Neuts book [160] lists 315 bibliographic items, most subsequent to Saaty’s list. Gross and Harris book [101] lists 267 items, most of them are subsequent to Neuts.

In 1953, Kendall [124] introduced the A/B/C : X/Y/Z type queueing notation. Cox introduced the “Supplementary variables technique” [64] to analyze queues. Brill developed the level crossing method (1975). Wolff named and popularized the PASTA principle [236], although the principle was known earlier (see Cooper [62]).

The best known textbooks in queueing theory are those by Gross and Harris [101], Kleinrock [127, 128] and Cooper [62]. Other major works in queueing include the voluminous book by Cohen [60], the three volumes by Takagi [222, 223, 224] and Bocharov et al. [46].

1.6 Queueing Systems with Vacations

Queueing systems with server vacation have a new branch of queueing systems, namely, “Vacation Queueing Systems”. A vacation period is a duration in which the server will not be available to the system. In vacation queueing systems, the vacation durations correspond to the length of times the server or system spends on secondary jobs. The utilization of the server is idle time in a vacation queueing system corresponds to performing other tasks like server taking rest in queueing systems, attending preventive maintenance jobs in production systems, testing and maintenance jobs in computer and communication systems and serving secondary jobs in some other queueing systems (Doshi [69]).

There is a natural interest in the study of queueing systems with server vacations or interruptions. Systems of this nature have been studied extensively over the last four decades. Many variations of the single server queue-
Queueing systems with vacations have been investigated in detail and expressions for important performance measures have been obtained. The techniques and results developed have been fruitfully used in a variety of applications. These applications include the analysis of processor schedules in computer and switching systems, the effects of administrative and maintenance work on the basic functions of such systems, the analysis of polling systems frequently used in data communication networks, the analysis of manufacturing systems with machine breakdowns, the analysis of many single server queues with priorities, etc.

For example, in a polling type situation a single server may provide service at a number of service locations moving from one to other in some pre-defined sequence. One way of such a system would be to assume that from the service station’s point of view, the server goes for a vacation (i.e., moves to the other stations) after completing service to all the customers at its present location. Service at the service station being considered resumed only after the server returns to this station (see Takagi [220]).

Two surveys summarizing many of the developments in the analysis of single server queues with vacations are due to Tegham [225] and Doshi [70]. For the single server queues with vacations and a Poisson arrival process, an extensive set of results in terms of Laplace transform and moments can be found in Takagi [222, 221]. An excellent summary of applications to polling models is provided in Takagi [220].

Many important new results have been obtained for single server queues with vacations. Some of these are fundamentally new while others show the results obtained earlier in greater generality. Together, these new results have provided considerable additional insight, resulting in expressions for quantities not available earlier and ability to analyze a whole new set of
applications. The vacation aspect of a queueing model can be classified as follows.

**Single Vacation:** In single vacation policy, the server goes on only one vacation after the busy period ends. Once it comes back from this vacation, it does not go for another vacation even if the system is still empty at that time. Immediately after the departure of a customer, if the system is not empty, then the server will serve another customer according to the FCFS discipline. Otherwise, the server will withdraw from the system for a vacation independent of the arrival and service process. Upon terminating this vacation period, the server returns to the system and begins to serve those customers, if any, that have arrived during that vacation. If, upon returning, the server finds no customer, he stays in the system waiting for the first one to arrive. This queueing models with single vacation have been studied by Levy and Yechiali [136] and Shanthikumar [204].

**Multiple Vacation:** In multiple vacation queueing models, if the server finds the system empty at the end of a vacation, it will immediately take another vacation, the duration of which has the same distribution as that of the first vacation period. He continues in this manner until he finds at least one waiting customer upon returning from a vacation. This model has been studied by Cooper [61], Levy and Yehicali [136] and Shanthikumar [204] and has numerous applications in computer and data communication systems.

**Bernoulli Vacation:** In the Bernoulli vacation model after each service completion the server takes a vacation with probability $p$ and starts a new service (if a customer is present) with probability $(1 - p)$. If the system is idle, after a service completion or a vacation completion, the server always takes a vacation. The decision about taking a vacation after each service completion or vacation completion are independent. If $p = 0$, this reduces
to the multiple vacation model. In general, if the server finds a customer upon returning from a vacation, it always starts the service of the first arrival. Also the vacations are independent and identically distributed (see Keilson and Servi [122] and Ramaswamy and Servi [184]).

1.7 Scope of the Thesis

This thesis provides a comprehensive analysis of some continuous and discrete time inventory systems hitherto not studied in the literature. More specifically, expression for the various operating characteristics of the systems in the long run are derived. In addition, adequate emphasis has been laid on the cost optimization. The cost functions which are derived in this work are non-linear and complex form and hence it is an arduous task to determine the optimal values of the decision variables analytically. Hence we have carried out the detailed cost analysis with the help of numerical examples, which provide valuable insight into the operation of the system.

The thesis is divided into eight chapters. Chapter 1 is an introduction to some of the literature and ideas surrounding the inventory and queues.

Chapter 2 sets up the basic notations and established some results which are used throughout the thesis. It contains a preliminary consideration of the Discrete and Continuous time Markov Chains, Matrix-Geometric Method, Phase-type distribution, Markov arrival processes and some Measures of System Performance.

Chapter 3 considers a mathematical modelling of an inventory system with a single server, operating under stochastic environment is carried out in this work. It is assumed that the demand times form a Poisson process. The operating policy is \((s, S)\) policy, that is, the maximum inventory level is \(S\) and whenever the inventory level drops to a prefixed level \(s(s < S)\),
an order for $Q(= S - s)$ units is placed. The ordered items are received after a random time which is assumed to have exponential distribution. The server goes for a vacation of an exponentially distributed duration whenever the inventory level reaches zero. If the server finds an empty stock when he returns to the system, he immediately takes another vacation. The demands that occur during stock out period or during the server vacation period either enter a pool of size $N$ or leave the system. The demands in the pool are selected one by one, while the stock is above the level $s$, with interval time between any two successive selections distributed as exponential with parameter depending on the number of customers in the pool. The joint probability distribution of the inventory level and the number of customers in the orbit is obtained in the steady state case. Various system performance measures in the steady state are derived and the long-run total expected cost rate is calculated.

In chapter 4, we have incorporated all the assumptions that are made in chapter 3 and further assumed that the life time of each item is assumed to have exponential distribution. The joint probability distribution of the inventory level and the number of customers in the pool is obtained in the steady state case. Various system performance measures in the steady state are derived and the long-run total expected cost rate is calculated.

In chapter 5, we consider a discrete-time $(s, S)$ inventory model in which demands arrive according to a discrete Markovian Arrival Process. The inventory is replenished according to an $(s, S)$ policy and the lead time for replenishment is assumed to follow a discrete phase type distribution. The demands that occur during stock out periods either enter a pool which has a finite capacity $N(< \infty)$ or leave the system with predefined probability. Any demand that arrives when the pool is full and the inventory level is zero,
is also assumed to be lost. The demands in the pool are selected one by one, if the on-hand inventory level is above \( s \), and the interval time between any two successive selections is assumed to be distributed as discrete phase-type distribution. The joint probability distribution of the number of customers in the pool and the inventory level is obtained in the steady state case. The measures of system performance in the steady state are derived and the total expected cost rate is also calculated. The results are illustrated numerically.

Chapter 6 considers a discrete-time perishable inventory model in which demands arrive according to a Bernoulli process. The inventory is replenished according to an \((s, S)\) policy and the replenishment lead time is assumed to follow a geometric distribution. The demands that occur during stock out periods enter a pool which has a finite capacity \( N(< \infty) \). Any demand that arrives when the pool is full and the inventory level is zero, is assumed to be lost. The demands in the pool are selected one by one, if the on-hand inventory level is above \( s \), and the interval time between any two successive selections is assumed to be distributed as geometric. The joint probability distribution of the number of customers in the pool and the inventory level is obtained in the steady state case. The measures of system performance in the steady state are derived and the total expected cost rate is also calculated. The results are illustrated numerically.

The chapter 7 considers a continuous review perishable inventory system with a finite number of homogeneous sources generating demands. The demand time points form a quasi-random process and demand is for single item. The maximum storage capacity is assumed to be \( S \). The life time of each item is assumed to have exponential distribution. The reorder policy is \((s, S)\) policy, that is, whenever the inventory level drops to a prefixed level \( s \), an order for \( Q(= S - s) \) items is placed. The ordered items are received
after a random time which is distributed as exponential. We assume that the demands that occur during the stock out periods either enter a pool or leave the system which is according to a Bernoulli trial. The demands in the pool are selected one by one only when the stock is above the level \( s \) with interval time between any two successive selections distributed as exponential. The joint probability distribution of the number of customers in the pool and the inventory level is obtained in the steady state case. Various system performance measures are derived to compute the total expected cost per unit time in the steady state. The optimal cost function and the optimal \((s, S)\) are studied numerically.

In the final chapter we present the summary of the thesis.