D.A. Robinson (1,2) investigated the mechanics of the eyeball, the extraocular muscles and the supporting tissues of the orbit which manifests itself during the saccadic and smooth pursuit movements. The development of the suction contact lens made practical a closer investigation of the mechanics of these movements for it provided a simple method of applying known forces and loads to the eye while measuring subsequent rotations without fear of lens slippage.

Saccadic Movements

Robinson (1) conducted experiments to record normal isometric and isotonic saccades of human subjects. For isometric saccades a stiff beam, clamped at the base, ball-shaped at the tip was inserted into a cylindrical socket cemented to the contact lens so as to restrain the eye from moving. A coil of wire mounted on the beam served as a strain guage to measure the force applied to the beam tip by extra-ocular muscles. The subject's head was restrained on a bite bar, the isometric beam applied to the left eye and the subject made normal saccadic movement with his right eye, while tension in the beam was recorded.

For recording isotonic movements, a loop of surgical thread was tied to the socket of the contact lens and
attached through an isotonic system to a set of weights.

The time course of eye ball movement on application of steps of forces (e.g. 5 gms, 10 gms, 15 gms, etc) was recorded. During the experiment, the subject fixated his unencumbered eye in the primary position.

The moment of inertia of the eye ball was effectively increased about 100 times by attaching it through the ball and socket joint on the contact lens to a sled carried by knife edges on a double pendulum suspension system in such a way that the sled followed movements of the eye without offering either frictional or spring-like reacting forces. With the load attached to the left-eye, the subject made a series of saccades with right eye vision in 5° steps. The high inertia saccades of left eye were recorded.

Experiments were conducted on live cats to record isometric tension in the lateral rectus muscle. The cats were anaesthetized and their heads fixed as a stereotaxic holder. A portion of the molar bone and its frontal process was removed to facilitate the approach to the lateral rectus whose insertion was secured by surgical thread and attached to a strain guage mounted on a manipulator. Isometric muscle tension was recorded while muscle length, stimulus amplitude and stimulus rate were varied. (The results of experiment on cat were utilised to predict isometric tension in human extra-ocular muscles and also
to form a picture of mechanics of human orbit. The data obtained from these experiments were fitted to form a mechanical analogue (Fig. 4.1) of the human orbit (for details refer to (1)). The globe is a mass of moment of inertia 'm' on which three forces may act. \( F_a \), the force applied externally on the contact lens during isotonic experiments, \( F_m \) the force exerted by the net added tension of the horizontal recti and \( F_p \), the net restraining force of all passive tissues in the orbit. During isometric experiments, a spring of stiffness \( K_i \) is placed in parallel with the passive structure of the eye to represent the isometric beam. The moment of inertia can be artificially increased by attaching the mass \( M \) to the mass \( m \) of the eyeball during high inertia saccades. Fig. (4.1) shows the contractile component (CC) and series elastic component (SEC) of the muscle. All passive elements of the orbit and muscles (PE) are grouped into a slow (S) and a fast (F) viscoelastic element. Experimental procedures with viscoelastic beam (IB), high inertia load (HIL) and isotonic forces (Fa) are indicated.

Net added muscle force \( F_m \) is a reflection of net added active state tension \( F_o \) and equals it in the steady state. Assuming a linear approximation of the force-velocity relation and stress-strain curve (Refer Chapter IX on Muscle Models) the analogue of the muscle is a dashpot of viscous damping coefficient \( R_m \) gm./sec./deg. and a spring of stiffness \( K_e \) gm./deg. as shown in Fig. 4.1.
Fig 4.1 Analogue of the mechanical elements of the orbit from the study of saccadic movements (1)

Fig 4.2 A composite graph to show various types of eye movement responses \([T + OT]\) to a 10°/sec ramp. F: net isometric muscle tension in horizontal recti (1)
The equations of motion are:

\[
\frac{m \ddot{\theta}}{dt^2} + F_p = F_m + F_a
\]

\[ \text{-- } (4.1) \]

The equation for the way in which net added muscle tension \( F_m \) is related to the net active state tension \( F_a \) is

\[
\frac{R_m}{K_e} \frac{dF_m}{dt} + F_m = F_o - R_m \frac{d\theta}{dt}
\]

\[ \text{-- } (4.2) \]

The two spring-dashpot elements (Voigt elements) offer a passive force \( F_p \) related to eye movement \( \theta \) by

\[
R_1 R_2 \frac{d^2 \theta}{dt^2} + (R_1 K_2 + R_2 K_1) \frac{d\theta}{dt} + K_1 K_2 \theta = (K_1 + K_2) F_p + (R_1 + R_2) \frac{dF_p}{dt}
\]

\[ \text{-- } (4.3) \]

Equations (4.1), (4.2) and (4.3) when combined result in a 4th order linear differential equation which describes eye movements \( \theta \) in response to forces \( F_a \) externally applied or neural commands via active state tension \( F_o \).

The parameters \( R_1, K_1, R_2 \) and \( K_2 \) are found by iterative approximations that yield the best match with experimental data.
## Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net series elastic stiffness</td>
<td>$K_e$</td>
<td>3.6 g/deg</td>
</tr>
<tr>
<td>Net muscle force velocity slope</td>
<td>$R_m$</td>
<td>0.072 g sec/deg</td>
</tr>
<tr>
<td>Muscle Time Constt.</td>
<td>$T_m$</td>
<td>0.02 sec.</td>
</tr>
<tr>
<td>Past Passive</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spring Stiffness</td>
<td>$K_1$</td>
<td>2.06 g/deg</td>
</tr>
<tr>
<td>Viscoelastic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Viscosity</td>
<td>$R_1$</td>
<td>0.025 g sec/deg</td>
</tr>
<tr>
<td>Time constant</td>
<td>$T_1$</td>
<td>0.012 sec.</td>
</tr>
<tr>
<td>Slow Passive</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spring stiffness</td>
<td>$K_2$</td>
<td>6.36 g/deg</td>
</tr>
<tr>
<td>Viscoelastic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Viscosity</td>
<td>$R_2$</td>
<td>1.81 g sec/deg</td>
</tr>
<tr>
<td>Time constant</td>
<td>$T_2$</td>
<td>0.285 sec.</td>
</tr>
<tr>
<td>Combined passive spring stiffness</td>
<td>$\frac{K_1}{K_1} \cdot \frac{K_2}{K_2}$</td>
<td>1.5 g/deg</td>
</tr>
<tr>
<td>Isometric beam stiffness</td>
<td>$K_i$</td>
<td>15.0 g/deg</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eye + Lens</td>
<td>$m$</td>
<td>$0.677 \times 10^{-4}$</td>
</tr>
<tr>
<td>With isometric beam</td>
<td>$m + m_i$</td>
<td>$2.16 \times 10^{-4}$</td>
</tr>
<tr>
<td>With sled load</td>
<td>$m + M$</td>
<td>$28.9 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

(units: g sec$^2$/deg)
The four types of saccadic eye movements viz. normal, isometric, isotonic and high inertia saccades were simulated for this model and agreement was found between theoretical predictions and experimental results.

Robinson analysed the experimental data to argue that the eye is impulsively driven in a saccade by a brief burst of force much larger than that needed to hold the eye in its new position. The force applied tonically to deviate the eye against the restraining orbital tissue was 1.5 g/deg. The excess active state tension briefly applied during a saccade was estimated at 25 gm, independently of the saccade magnitude. The mass of the eye ball was found to play negligible role in determining the time course of a saccade.

The frequency at which the mechanical response of the eye and its attachment is 3 db below that at zero frequency was found to be 1.1 c/s. However, when the data on saccadic durations is analysed, the analysis indicates a range of apparent bandwidths from 19.1 c/s for the 5° saccade to as low as 5.7 c/s for the 40° saccade. The reason for rapid execution of saccade (irrespective of poor frequency response of the system as such) is due to the excess force briefly applied in the beginning of the saccade. This was named as 'Pre-Emphasis'. The neural discharge pattern is centrally shaped to cause the pre-emphasis due to which the sluggish system of the orbit appears to have a bandwidth between 6 and 20 c/s.
4.2 Smooth Pursuit Movements

D.A. Robinson (2) measured eye movements and net isometric tension of the horizontal recti while subjects followed horizontally moving targets with one eye. Techniques similar to those described in (3.1) were used. Ramp velocities varied from 5 to 20°/sec. The typical average responses (Fig. 4.2) for a target velocity of 10°/sec were:

Response 1 - occurs most frequently (59%). Movement begins after 125 msec (± 20 S.D.) latency. Under a rate of change rise of muscle force of 21.7 g/sec (± 7.1 S.D.) the eye accelerates at a mean velocity of 60 deg/sec to reach a velocity of 6.1 deg/sec (± 2.2 S.D.) and a displacement of 0.38 deg (± 0.14 S.D.) in 112 msec before the interruption of a saccade. The saccade occurs 237 ms (± 30 SD.) after the stimulus commences and has an amplitude of 1.24° (± 0.24 S.D.) so that an error of 0.7° remains. The eye leaves the saccade at a smooth velocity of 12.2 deg/sec (± 1.6 S.D.) which it maintains for about the next 200 msec thereby reducing the error to almost zero at which time the globe velocity slacks off to 10°/sec. to match target velocity. The steady state rate of change of net muscle force to maintain this velocity is 12 g/deg.

Response 2 - occurring 31 p.c. of the time shows no velocity overshoot. There is, however, a secondary correcting saccade after the first.
Response 3: The saccade comes too late to interfere with the completion of the S.P. movement. This is a rare case.

Response 4: Occurs 10 p.c. of the time. It shows almost no early smooth component. It's saccade occurs quite late and is accompanied by large abrupt change in S.P. velocity.

The variability in amount of velocity overshoot, the occurrence of saccadic component and amount of abrupt velocity change accompanying the saccade poses a rather difficult situation for analytical description of S.P. system.

A step-ramp target movement in which the magnitude in degrees is 0.15 to 0.2 times the ramp velocity in °/Sec (so that the target recrosses its initial position after 150-200 milli-seconds) the eye will respond with a smooth pursuit movement devoid of saccades. Robinson measured the time course of globe rotation and muscle force for a step-ramp input of 1.5° step and 10°/sec ramp in opposing directions. He observed an increase in latency (21 p.c). Also there was the characteristic excess rate of change of muscle force during the first 150 m.sec. of the movement reaching a peak of 20.8 g/sec before dropping to a steady rate of 12 g/sec. The existence of the excess rate of change of muscle force reveals a characteristics similar to the pre-emphasis of the saccadic system. The globe changes its velocity under the impetus of a large burst of excess rate of change of force. However, it was pointed out that the same central mechanism cannot be responsible for pre-emphasis in saccadic as well as
smooth pursuit movements. This is inferred from examination of the two forms of pre-emphasis. Saccadic pre-emphasis compresses the 10° saccade to 45 m/sec, while the 10 deg/sec velocity change is only compressed to about 130 milli secs. by S.P. pre-emphasis.

The static and dynamic relations between the net muscle active state tension and globe rotation was investigated with the help of a mechanical analogue as in the case of saccadic movements. With minor modifications in the analogue of Fig. (1), it was possible to obtain a match between theoretical predictions and experimental results. The modified analogue is shown in Fig (4.3).

Important modification is the inclusion of one more Voigt element characterized by $K_2 = 2.86$ g/deg, $R_2 = 1.49$ g/sec deg ($T_2 = 500$ m/sec). Effective long term orbital spring stiffness was revised to 1.2 g/deg. from the previous value of 1.5 g/deg. $K_e$ was increased from 2.06 to 3.0 g/deg and $K_e$ was reduced from 3.6 to 3.0 g/deg. These modifications improved the agreement with experimental data for saccadic movements beyond 350 msecs.

With this model, it was possible to construct by trial a time course (Fig.4.4) of net active state tension $F_0$, which, when applied to the model produced curves of eye rotation (M) and isometric tension (F) which agree fairly well with the experimentally determined average curves.
Fig 4.3 Analogue of the mechanical elements of the orbit from the study of smooth pursuit as well as saccadic movements (2).

Fig 4.4 Computed and experimental time courses of isometric force F & eye movement M in response to 2°-10°/sec step-ramp target motion. F₀ denotes the net active state tension. F₀' denotes active state tension without preemphasis. M' is the eye movement in the absence of preemphasis.
The agreement between calculated and experimental results suggest that the transfer function of the orbital elements from the net active state tension to globe rotation is the same for saccadic and smooth pursuit movements. In Fig. 4.4 the maximum rate of change of $F_o$ is 30 g/sec which represents a 150 p.c. excess rate of rise of active state tension. It is possible to calculate on the basis of the isotonic eye movement experiments (Sec. 3.2) the time course of eye movement $M'$ in the absence of pre-emphasis, i.e. when the active state tension $F_o'$ rises at its steady state level of 12 g/sec from the start. This movement would be quite slow owing to the viscous nature of the supporting tissues and the steady state velocity would not be reached for about 500 m/sec. This clearly illustrates the importance of central-pre-emphasis in the tracking performance of the S.P. system.

The study of saccadic and smooth pursuit movement reveals a simple picture of the process in the orbit. All the passive elements of the orbit including passive elements of the muscles form a visco-elastic restraining medium for the globe. The spring stiffness of this medium is 1.2 g/deg. To displace the globe by 10° in this medium, the muscles must apply a differential force of 12 gms. To rotate the globe at 10°/sec in this medium, the muscles must apply a differential rate of change of force of 12 g/sec. These viscous elements of the medium represent the predominant impedance to sudden motions of the globe and far out-shadow the effect of the very small moment of inertia of the globe. These viscous elements
prevent the globe from coming to rest in a new position until 500 m sec after the application of a sudden constant force. In order to hasten the globe rotation against these viscous elements the CNS utilizes the large tension reserve of the extracocular muscles by causing a burst of excess force or rate of change of force to alter eye position in only 45 m. sec. in the 10° saccade and eye velocity in only 133 m.sec in the 10 deg/sec smooth pursuit response.

4.3 Distinction between the Saccadic and Smooth Pursuit Responses

As has already been pointed out, the typical response to pulse of target position, led Young (3) to suggest a sampled data model for the saccadic mode. In this theory, smooth pursuit movements are also considered to be due to a sampled data system in which the errors between eye and target velocity are discretely sampled and discretely corrected simultaneously with saccadic corrections. However, there are certain features of the smooth pursuit movements which raise doubts - presence in records of smooth pursuit movements without saccades, abrupt saccade-linked changes in smooth pursuit velocities and a variety of temporal relations between saccadic and smooth pursuit movements.

To investigate whether the smooth pursuit movements are also the result of a sampled data system, Robinson conducted an experiment in which two successive target ramps spaced 150, 100 and 75 m sec about were used to elicit the eye movement responses. The responses in each case consist of two
distinct smooth pursuit movements (S.P.M.) which although delayed in time by the latency, are temporally spaced from each other by 150, 100 and 75 m sec respectively. Thus the refractory period for the SP system is not larger than 75 m sec. The response time of the extra-ocular muscles and the globe with pre-emphasis is 133 m sec larger than the maximum refractory period of 75 m sec. A sampled data system whose intersampling interval is less than the response time of the sampling apparatus being controlled is indistinguishable from a continuous or non-discrete system. Thus, the above experiment suggests that the smooth pursuit system is a continuous system as opposed to a sampled data (S.D.) system. Even though the S.P.M. is regarded as a S.D. system with a sampling period of 75 m sec or smaller, its sampling rate would be 13/sec, whereas that of the saccadic mode is 5/sec, thus suggesting a different controlling mechanism for the two systems.

Robinson conducted experiments to show the complete temporal independence of saccadic and smooth pursuit movements by eliciting a variety of temporal relationships between S.P. and saccadic movements. Smooth pursuit movements were shown to occur before, during and after saccadic movements.

The abrupt velocity changes accompanying a saccade led him to suggest a non-linear interaction between the two types of movements in the final common path.
4.3.1 Variable Feedback Behaviour

Experiments with variable visual feedback were conducted which offer additional evidence that the saccadic movements are sampled and the smooth pursuits are not. For the variable feedback experiments, a signal proportional to eye position $\theta_{e}$ is added to the target controlling stimulus $\theta_{s}$ so that the target position $\theta_{t}$ is given by

$$\theta_{t} = \theta_{s} + \alpha \theta_{e} \quad \ldots \quad (4.4)$$

where $\alpha$ is the proportion of eye movement electrically feedback to the target through a mirror drive system. The error $\theta_{r}$ between the target and eye position is

$$\theta_{r} = \theta_{t} - \theta_{e} = \theta_{s} - (1 - \alpha) \theta_{e} = \theta_{s} + K\theta_{e} \quad \ldots \quad (4.5)$$

where $k = (1 - \alpha) = \text{total feedback factor.}$

In normal vision, $\alpha = 0$ and $K = -1$. It was shown that under simulated open loop conditions ($\alpha = 1$ and $K = 0$), the saccadic system executes an increasing staircase pattern of eye movement in response to an initial $2^\circ$ stimulus $\theta_{s}$. However, under slight positive feedback ($K = +0.1$), the smooth pursuit system was stimulated when the initial retinal error was less than about 0.1 deg. Under this condition the saccadic system is not stimulated and the eye movement consists of a smooth growing time course with no detectable discrete velocity changes.

4.4 Effect of Negative Feedback:

Under normal vision, $\alpha = 0$ and $K = -1$. If, however, $\alpha = 1$, and $K = -2$, a negative feedback results. If the
target is initially $1^\circ$ to subject's left, and he attempts to fixate, his eye movement of $1^\circ$ causes the target to move right by $1^\circ$ to the spot from which the eye came. While the eye returns to the starting point in an attempt to correct for the error, the target again returns to its starting point. This is a condition of saccadic oscillation. Experimentally it is found that saccadic oscillations are only initiated and soon die out for $K = -2$. Between $K = -2$ and $K = -5$, the oscillations wax and wane which is attributed to the subject's attempt to adopt to the conditions of variable visual feedback. For values of $K$ more negative than $-5$, sustained saccadic oscillations occur at $2 - 2.5$ c/s.

Under the conditions of -ve feedback, there is no evidence of S.P. oscillations intermixed with the saccadic. By electrically processing the eye movement signal to remove large saccadic velocities, a condition of normal feedback ($K = -1$) for the saccadic system and variable feedback for the smooth pursuit system can be obtained. Thus smooth pursuit oscillations, if any, can be brought out by varying the amount of negative feedback.

It was found that the S.P. system did not break into spontaneous oscillations even for values of $K$ as low as $-8$ in the absence of a target stimulus. However, when the stimulus is a ramp ($10^\circ$/sec) the S.P. system breaks into sinusoidal oscillations which are superimposed on the steady state eye velocity.
For values of $K$ between $-2$ and $-6$, these oscillations wax and wane which is attributed to predictability of the system whereas for values of $K$ more negative than $-8$, sustained oscillations persist at $3.3$ c/s.

The important point to be noted is that the smooth pursuit system exhibits approximately sinusoidal oscillations under negative feedback which appear to be the product of a continuous system. Moreover, the frequency of oscillations of the S.P. system is considerably higher than that of the saccadic system. This shows complete temporal independence of saccadic and S.P. systems.

The important properties of the saccadic and S.P. systems as noted by Robinson are:

**TABLE II**

<table>
<thead>
<tr>
<th>Property</th>
<th>Saccadic system</th>
<th>Smooth Pursuit system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latency</td>
<td>200-250 m.sec.</td>
<td>125 m.sec.</td>
</tr>
<tr>
<td>Duration of movement</td>
<td>45 m sec (for 10°)</td>
<td>133 m.sec (for 10°/sec.)</td>
</tr>
<tr>
<td>percentage excess rate of force during pre-emphasis</td>
<td>258 p.c.</td>
<td>150 p.c.</td>
</tr>
<tr>
<td>Type of system</td>
<td>Sampled</td>
<td>Continuous</td>
</tr>
<tr>
<td>Need for target stimulus</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Barbituate interference</td>
<td>Insensitive</td>
<td>Sensitive</td>
</tr>
<tr>
<td>Frequency of oscillations during negative feedback</td>
<td>2 - 2.5 c/s</td>
<td>3.3 c/s</td>
</tr>
<tr>
<td>Amount of -ve f.b. $K$ to insure oscillations</td>
<td>-5</td>
<td>-8</td>
</tr>
</tbody>
</table>
References

(1) Robinson D.A.

The Mechanics of Human Saccadic Eye Movements


(2) Robinson D.A.

The Mechanics of Human Smooth Pursuit Eye Movements

Journal of Physiology 130 : 569 - 591, 1965

(3) Young, L.

A sampled data model for eye tracking movements.