CHAPTER 7
SUMMARY AND CONCLUSION

7.1 Summary

The thesis proposed a cure model for recurrent event count data for the first time. This model turns out to be a zero inflated Poisson model. The parameter of interest is the cured proportion $p$. The estimation of parameters has been done and inference is carried out based on uncensored as well as censored data. The study has also been carried out using a covariate in both the discrete as well as a continuous setup for uncensored data.

In the case of uncensored data without covariates, seventeen tests were proposed for testing the cured proportion $p$. These include the likelihood ratio test, the Wald's test and its variants, and also score test and its variants. The thesis proposes an innovative approach for handling the censored data for the cure model for recurrent event count data. This leads to the development of the conditional inference for the cured proportion. Thirteen conditional tests are proposed for testing the cured proportion. Investigation is further carried out to compare the thirteen (out of seventeen) tests in the uncensored case by including covariates in the model. The asymptotic null distribution of these tests are derived by deriving the asymptotic distribution of the maximum likelihood estimators. The small sample performance of the various tests are compared using extensive simulations.

In the uncensored case without covariates, the variant of the Wald test when the specified value of the inflate parameter and the unrestricted maximum likelihood estimator of the mean parameter were used in the Fisher information matrix, emerges as the best. The same conclusion holds when the covariates are included in the model as well as in the case of censored data.

For several configurations of the simulations, the power functions of the score test and that of its variants exhibit fluctuations. The reason is that the test statistics become negative. At this point, the power function starts decreasing, or decreasing and increasing after a certain stage when the number of negative values of the test statistics is large. The inconsistency of the score test has received the attention of the
researchers in the past (Morgan, et al. (2007)). Our investigation demonstrates the inconsistency of the score test in small sample estimation of the power function at the boundary. The likelihood ratio test and the Wald's test also face this problem on the boundary of the parameter space for the cured proportion \( p \). From the results concerning type I error rates and the powers, it becomes clear that a minimum sample of size 100 is required. For these sample sizes, it is of interest to compare the coverage probability and length of the confidence interval for the cured proportion \( p \).

The focus of this thesis is on type I error rates and power of the tests. Censoring has no much impact on the power of the tests but the number of times the tests maintain type I error rate is less when compared to the uncensored case.

Czado and Min (2005) considered the problem of testing the significance of the inflate parameter \( p \) of the ZIGP distribution. They recommended the use of the Wald test compared to the score test when the sample size is 200 or more. For these sample sizes, the score test does not suffer from inconsistency. However, since a variant of the Wald test has emerged as the best in the uncensored case (both with and without covariates), we recommend the use of the variant of the Wald test in small sample cases. This variant is obtained when the specified value of the inflate parameter and the unrestricted maximum likelihood estimator of the mean parameter were used in the Fisher information matrix to obtain the standard error of the estimator.

A detailed investigation concerning the tests for cured proportion was carried. In the case of the score test and its variants, the performance of the score test is at par with one of its variants where the specified value of the inflate parameter and the unrestricted maximum likelihood estimator of the mean parameter were used in the Fisher information matrix. The overall conclusion that can be drawn regarding the tests for cured proportion in a recurrent event count data is that one of the variants of the Wald's test is better compared to the conventional Wald test. This variant is obtained when the specified value of the inflate parameter and the unrestricted maximum likelihood estimator of the mean parameter were used in the mean square error of the estimator.
7.2 Areas of future research

The cure model proposed in the thesis for testing the cured proportion for the recurrent event count data is the inflated Poisson distribution Inflated negative binomial and the inflated generalized Poisson are the competitors for the inflated Poisson distribution In the absence of real life data sets, we could not decide which of these three models is an appropriate one for recurrent event count data. Research in this direction is called for

In the present thesis, we have proposed the tests for cured proportion in the uncensored case when the baseline covariates are present We have not examined the performance of the tests when covariates are linked to the mean parameter in the censored case. The test can be extended to the case of censored data also by using the log link function $\log \lambda_i = t_i + X_i \beta$, $i = 1, 2, \ldots, n$. In the absence of a suitable real life data set, the performances of the thirteen tests could not be perceived in the present thesis and is a topic of future research

We have used conditional inference for the cured proportion for the censored observations This is a restrictive assumption and needs to be relaxed. A question that needs to be answered is how to handle the censored observations if we do not have any knowledge regarding the censoring times Work in this direction is needed

Throughout this thesis, the frequentist approach has been used for inference on the cured proportion The work by Bhattacharya, et al. (2008) can be extended for the Bayesian inference regarding cured proportion and needs to be perceived.