CHAPTER 4

AGENT-BASED ARTIFICIAL STOCK MARKETS TO
MODEL INVESTOR BEHAVIOR

4.1 Introduction

A novel bottom-up approach to studying and understanding stock markets comes from the area of computational finance as artificial financial markets or, more specifically, as artificial stock markets (ASM). Agent-based ASM can be mathematical or computational models, and are usually comprised of a number of heterogeneous and boundedly rational agents, which interact through some trading mechanism, while possibly learning and evolving. These models are built for the purpose of studying agents’ behavior, price discovery mechanisms, the influence of market microstructure, the reproduction of the stylized facts of real-world financial time-series (e.g. fat tails of return distributions and volatility clustering). A critical survey of agent based ASM to model financial markets in Chapter 3, led to the choice of computational models of LeBaron [70] to model the BSE. Analogous methodology to agent-based modeling comes from the physical sciences, namely the Microscopic Simulation. This methodology is a tool for studying complex systems by simulating many interacting microscopic elements, and has been also applied to financial markets by Levy et al. [71]. They believe that Microscopic Simulation models could be used to extend existing analytical models in finance by inspecting the role of their assumptions, or to build new models that could be as realistic as desired, e.g. models that incorporate various technical and fundamental strategies observed in experiments and real markets; dynamic models with heterogeneous investors that can learn and change strategies [75].

Complexity behaviour can emerge from very simple behavior, with the addition of some heterogeneity, interaction, and/or learning. The famous spatial proximity model by Schelling [100] is an example of an early agent-based work where unexpected aggregate pattern of segregation appears on the macro level even though it was not coded as such in the micro-level behavior of agents. Sugarscape model
is another example of a simple local behavior that leads to interesting macro patterns. The idea of complexity which emerges from nonlinear interactions between heterogeneous components forms the foundation of Complex Adaptive Systems (CAS), which is closely related to the agent-based modeling approach. The agent-based approach can lead to the development of homeomorphic models [44], that not only reproduce the stylized facts of real-world markets, but also achieve them through processes that are grounded on reasonable (psychologically plausible) assumptions, and resemble actual human behavior and realistic market mechanisms. Agent-based models can easily accommodate complex learning behavior, asymmetric information, heterogeneous preferences, and ad hoc heuristics [16]. Computational models can easily implement mathematical models and also, with further complex algorithms model the intricate interactions between agents, capture complex behaviors and interactions, trading strategies, as well as realistic market mechanisms that are employed in actual markets. For that reason, computational models are suited for studying behavioral finance topics and the role of individual investor biases. Agent-based ASM are potentially able to fully bridge the gap between individual investor behavior and aggregate market phenomena, by allowing the modeler to specify the behavior of market participants, to implement various market mechanisms, and to analyze the resulting asset prices [71, 75].

Developing models of financial markets that are able to produce realistic outputs which are qualitatively similar to the empirical data is important, but it constitutes only one possible goal of these simulations. Different models can be focusing more on other aspects of agent-based modeling, such as realistic representations of agents’ behavior, their interactions, or realistic market mechanisms [75]. Incorporating realistic elements of agents’ behavior can be achieved by implementing those behaviors that have been observed and documented in behavioral finance literature. In the rest of this chapter, brief overview of a few examples of such studies are given, focusing on the representation of behaviors, market mechanisms, and obtained market dynamics. Even though most agent-based models incorporate some behavioral aspects into the agents’ implementation, the last two examples given in this overview are interesting because they have explicitly accounted for a number of
behavioral finance topics. Overviews of artificial financial markets from different perspectives can also be found in [8, 70, 71].

4.2 Kim and Markowitz - Portfolio Insurers Model

This model is considered one of the first modern agent-based models of the financial market [75]. The main focus of the study was exploring the link between portfolio insurance strategies and market volatility. In this model the market consists of two types of investors: Rebalancers and Portfolio Insurers. There are two asset classes in the market: a risky stock and cash with zero interest. At the beginning of the simulation all investors are endowed with the same value of the portfolio, allocated half in stock and half in cash. The pricing mechanism is based on the order book. Buy and sell orders are stored in the order book and executed in the case of a match (or kept until the end of the trading day). The strategy of Rebalancers is to keep the same proportion of their wealth in stocks (50%) and in cash (50%). The strategy of Portfolio insurers is based on the so-called Constant Proportion Portfolio Insurance of Black and Jones, according to which the proportions of wealth in stock is kept in a constant proportion to the cushion (the value of the portfolio above some minimal level of wealth called floor). Timing in the model is discrete with trading happening at random time points and with each agent reviewing his portfolio at random intervals. Investors do not interact directly amongst themselves. However, they can perceive all orders places by other investors and use that information to form their prediction of the price. If their allocation according to a predicted price does not match their portfolio strategy, they will act by issuing a buy or sell order. Other exogenous influences on investors are modeled as randomly occurring withdrawals or deposits of random amounts of cash.

4.3 Brock and Hommes - Adaptive Belief Systems

This paper of Brock and Hommes [13] investigates market dynamics in a simple present discounted value asset pricing model with heterogeneous beliefs. The authors investigate possible bifurcation routes to complicated asset price dynamics, by using a mixture of bifurcation theory and numerical methods. A few simple belief types are considered in the experiments. Investors can choose between two asset types: one risky asset and one risk free asset. The risky asset is paying a dividend
that is exogenously given as a stochastic process. Price fluctuations in the market are driven by an evolutionary dynamics between different expectation schemes. Individual Investors are agents who choose from a finite set of predictors of future prices of a risky asset and revise their beliefs in each period. Predictor selection (i.e. strategy selection) is based upon a fitness or performance measure such as past realized profits. Intensity of choice is a parameter measuring how fast agents switch between different predictions strategies. If intensity of choice is infinite, the entire mass of traders uses the strategy that has highest fitness. If intensity of choice is zero, the mass of traders distributes itself evenly across the set of available strategies. In terms of their goal all investors can be characterized as myopic mean variance maximizers with a homogeneous degree of risk aversion. Simple belief types that are characterized in experiments are: trend chasers, contrarians, upward biased type, downward biased type, fundamentalists (who believe prices revert to their fundamental value) and rational agents with perfect foresight (who not only know past prices and dividends, but also the market equilibrium equation and fractions of other types in the market).

4.4 Levy, Levy, Solomon - Microscopic Simulation

Levy, Levy, Solomon model (LLS) is a prominent model of the financial market based on the microscopic simulation approach which has roots in physics [71]. It is a numerical model developed in the framework of expected utility maximization. The LLS model maps micro level modeling of Investor behavior to the marco level emergence of the share price. Even a change in a behavior creates ripples in the market and thus affects share price significantly. This model comes under agent based modeling where each agent depicts different behaviors, and they interact among each other for price formation. In the LLS model, the market consists of two types of investors: Rational Informed Investors (RII) and Efficient Market Believers (EMB). There are two asset classes in the market: a risky stock that pays a dividend following a multiplicative random walk and has a finite number of outstanding shares; and a risk-less bond that pays a sure interest and has an infinite supply. At the beginning of the simulation all investors are endowed with the same amount of wealth that is comprised of cash and a number of shares. The trade facilitators and the sell side of financial services are not explicitly modeled in the LLS model.
Instead, the pricing mechanism based on the temporary market equilibrium is used to determine the price in such a way that the total demand for the risky asset equals the total number of outstanding shares [70]. The goal of all the investors in the LLS model is the maximization of the expected utility of the next period wealth. The risk attitude of the investors is risk aversion, and is captured by the parameter of the utility function. In the LLS model, a myopic power utility function with DARA (Decreasing Absolute Risk Aversion) and CRRA (Constant Relative Risk Aversion) properties is used. Due to this myopia property of this utility function, it can be assumed that investors maximize their one-period-ahead expected utility, regardless of their actual investment horizon [71]. An additional temporal characteristic of EMB investors is their memory length of past return realizations which are used in the prediction of future returns. Even though both RII and EMB investors have the same goal of expected utility maximization, their strategies are different because of the differences in information that they possess. RII investors know the properties of the dividend process, and can estimate the fundamental price of the risky asset as a discounted stream of future dividends. That fundamental price is used in their prediction of the next period return. EMB investors, however, do not know the dividend process, and must use ex post distribution of returns to estimate ex ante distribution. EMB investors use a rolling window of a fixed size, and in the original model are called unbiased if, in the absence of any additional information, assume that returns come from a discrete uniform distribution. In each period, investors perceive information about the new price and new dividends, which they can use to update their wealth status. Both types of investors are expected utility maximizers, which are considered as the cornerstone of rationality. Nonetheless, with EMB investors some noise is added to the optimal proportion to account for other factors that could cause such departures from optimal behavior [71]. In the LLS model, investors do not interact in the sense of the exchange information or strategies [75]. If there is some volume (the change in portfolio holdings) of an individual investor, we can say that some trading occurred because the shares have exchanged hands. However, in this model we are not concerned with how exactly that happened. Actions, such as market orders, are not modeled explicitly, since this is a model based on the temporary market equilibrium.
4.5 Lux and Marchesi - Stochastic Interaction and Scaling Laws

The model of Lux and Marchesi [76] is an agent-based model of the financial market that follows the tradition of earlier attempts to capture herding behavior by means of stochastic modeling. An example of this is the ant recruiting model of Kirman [56], which has also been proposed as an analogy for herding behavior of investors in the financial markets [75]. The market consists of two types of investors, fundamentalists and non-fundamentalists, and two types of investments, a risky stock and a risk-free asset. In addition, the risky asset pays out a stochastic dividend at the beginning of each period. The market mechanism is based on the price adjustment process which determines the market price based on the differences between the supply and the demand. According to their type of strategy, agents can be either fundamentalists or chartists (which are further divided into optimists (buyers) and pessimists (sellers)). In addition, there is a probabilistic switching between these groups of agents. The probabilities of switching between two types of chartists are based on the majority opinion and current price trend, while the probabilities of switching between chartists and fundamentalists are based on the observed differences in profits.

4.6. Takahashi and Terano - Investment Systems Based on Behavioral Finance

Although various behavioral aspects of agents, including investor biases, have been studied in earlier literature, the model of Takahashi and Terano [111] is one of the first agent-based models that explicitly studied a number of investor biases proposed in the behavioral finance literature [75]. The market consists of two types of investors, fundamentalists and non-fundamentalists, and two types of investments, a risky stock and a risk-free asset. In addition, the risky asset pays out a stochastic dividend at the beginning of each period. Market mechanism is based on the temporary market equilibrium, i.e. the traded price of the stock is derived so that the demand meets the supply. Fundamentalist investors use a strategy based on the fundamental value of the stock, which they estimate using a dividend discount model. Non-fundamentalist investors are trend predictors, and since they use past information to predict future prices, they could also be classified as technical traders or chartists. Both fundamental and non-fundamental investors predict next period
price and dividend. Non-fundamental investors have additional temporal parameters, in the sense of short-term, medium, or long-term trend predictors. Two types of biases are studied within the model, overconfidence of investors and loss-aversion. There is no social interaction between various investor groups.

4.7 Hoffmann et al. – SimStock Exchange Model

Hoffmann [50] is a study that combines a number of behavioral phenomena within an agent-based simulation of the financial market, focussing on the social aspects of investor behavior and the consequences of two distinct network topologies. Asset classes available in the market are one risky stock and cash. The market mechanism is based on the order book. Using the rules of this mechanism, the limit orders submitted by agents are mutually crossed and executed. The market price is calculated as the average of the bid and ask prices, weighted by the number of asked and offered shares. This model also depicts the news arrival process, as a normally distributed noise around the current price. The investors are characterized by their level of confidence, which determines how much their private information (price expectation) is weighted compared to the expectations of their neighboring investors. Strategy used by investors is based on the comparison between the current market price and the expected market price of the stock: when the expected price is higher than the current price, it is attractive to invest in stock, and when the expected price is lower, it seems attractive to divest. The strategy also determines the proportion of cash to invest or the proportion of stock to divest. The perception of risk depends on investors’ level of confidence. Investors who have high confidence perceive lower risk, whole those investors who have lower levels of confidence perceive high risk and apply risk reducing strategies, which can be a simplifying strategy (heuristic) or a clarifying strategy (collecting more information). Time perspective of investors is myopic. They base their decisions only on the currently available information and expectations for the next period. Agents perceive information about the current market price, the news about the stock, and the price expectation of investors in their social network, which are all finally translated into their own market price expectation. The model pays special attention to the social interaction between agents [75]. In different experiments the investors are connected
in two different types of social networks which are used for the dissemination of market price expectations. Investors who exhibit social type of simplifying risk reducing strategy copy the behavior of other investors in the social network, while those who exhibit social type of clarifying risk reducing strategy ask other investors for more information. The investors act by sending limit orders, which consist of the number of shares that they want to buy or sell, and the limit price which is set to their expected price.

4.8 Discussion

The findings on agent based ASMs discussed above are compiled below [75]:-

(a) **Kim and Markowitz - Portfolio Insurers Model.** The main result of the simulations is that portfolio insurance strategy can have a destabilizing effect on the market.

(b) **Brock and Hommes - Adaptive Belief Systems.** In this model numerical evidence of chaotic attractors is shown when the intensity of choice to switch prediction strategies is high. The paper shows how an increase in the intensity of choice to switch predictors can lead to market instability and the emergence of complicated dynamics for asset prices and returns. This includes irregular switching between phases where prices are close to the fundamental value, phases of optimism where traders extrapolate upward trends, and phases of pessimism where traders are causing a sharp decline in asset prices.

(c) **Levy, Levy, Solomon - Microscopic Simulation.** One of the main results of the simulations is that investors who use past information create cyclic bubbles and crashes which can be related to the size of their memory window. This happens in the case when investors are homogeneous with respect to their memory lengths. When investors are heterogeneous in memory lengths, the market dynamics becomes more realistic in the sense that it does not display such prominent and semi-predictable bubbles.

(d) **Lux and Marchesi - Stochastic Interaction and Scaling Laws.** The model is able to generate the properties of the market prices: unsystematic deviations
of the market price from the fundamental price, heavy tails of return distributions, and volatility clustering.

(e) Takahashi and Terano - Investment Systems Based on Behavioral Finance. When the market consists of the same number of fundamental and technical traders, the market price agrees with the fundamental price. However, when there is a large fraction of technical traders in the market, the market price deviates largely from the fundamentals and fundamentalists are eventually eliminated from the market. There are also deviations from the fundamental price in the case of over confident investors and when non-fundamentalists act asymmetrically towards losses.

(f) Hoffmann et al. - SimStockExchange Model. The results of the simulations indicate that the structure of the social network of investors influences the dynamics of the prices. When investors were forming a Barabasi and Albert scale-free network, there was no indication of volatility clustering in the market, but when they were forming a torus network, such evidence was found. The authors speculate that networks of investors may behave more like torus networks with respect to information diffusion, and that information may sometimes take longer to travel to distant parts of the networks, allowing the old shocks to influence the presence for a considerable period of time.

4.9 Extensions of LLS Model by Lovric M.

Since agent-based models can easily accommodate complex learning behavior, asymmetric information, heterogeneous preferences, and ad hoc heuristics [16], such simulations could be particularly suitable to test and generate various behavioral hypotheses [75]. This complementarity of behavioral finance research and the agent-based methodology has been recognized and examples of agent-based papers that pursue the idea of explicit accounting for behavioral theories in financial market simulations are Takahashi and Terano [111] and Hoffmann et al. [49,50]. In Takahashi and Terano [111] the focus is on overconfidence and loss aversion, while Hoffmann et al.[49, 50] focus on social dimensions of investor behavior. Lovric M. [75] extended the LLS model by including various behavioral issues into the
character of EMB investors. The following discussion explains the basics of the
LLS model in which the investor behavior has been incorporated one by one.

4.9.1 Overconfident Investors

The LLS model has been discussed above briefly. The market consists of two types
of investors: Rational Informed Investors (RII) and Efficient Market Believers
(EMB). Although both RII and EMB investors have the same goal of expected
utility maximization, their strategies are different because of the differences in
information that they possess. RII investors are presumed to know the properties of
the dividend process, and can estimate the fundamental price of the risky asset as a
discounted stream of future dividends. The fundamental price is used in their
prediction of the next period return. EMB investors, however, do not know the
dividend process, and must use *ex-post* distribution of returns to estimate *ex-ante*
distribution. EMB investors use a rolling window of a fixed size, and in the absence
of any additional information, in the original model assume that returns come from a
discrete uniform distribution. In each period investor perceive information about the
new price and new dividends, which they can use to update their wealth status. Both
types of investors are rational expected utility maximizers. With EMB investors
some noise is added to the optimal proportion to account for any departure from
optimal behavior. LLS model is based on the temporary market equilibrium. All
agents will have information about the current market price, which will allow them
to update their wealth status and decide on their investment strategies. They will also
receive news, such as dividends on risky assets and interest on risk-less assets.
However, some agents will also have information about the determinants of the
fundamental price, such as the dividend generating process. Depending on their
strategies, agents will translate their price expectations or predictions into orders or
demand functions, which will be cleared using a specific market mechanism, and
finally a new market price will be formed. According to Levy et al. [71], when
empirical support is taken into account, the choice of utility function would be
Decreasing Absolute Risk Aversion (DARA) and Constant Relative Risk Aversion
(CRRA).
Hence,

\[ U(W) = \frac{W^{1-\alpha}}{1-\alpha} \]  \hspace{1cm} (4.1)

Where

\begin{align*}
W & : \text{Wealth of an investor} \\
U(W) & : \text{is the utility at a given wealth W} \\
\alpha & : \text{Risk aversion factor.}
\end{align*}

4.9.1.1 Behavior of RII

In the LLS model, the RII invest based on dividend process of the given entity. They estimate the fundamental value as a discounted stream of future dividends, according to the Gordon model [42]:

\[ P_{t+1}^f = \frac{D_t(1+z)(1+g)}{k-g} \]  \hspace{1cm} (4.2)

where

\begin{align*}
P_{t+1}^f & : \text{the fundamental price of a stock} \\
D_t & : \text{the dividend} \\
k & : \text{the discount factor of the expected rate of return demanded by the market for the stock} \\
g & : \text{the expected growth rate of the dividend} \\
z & : \text{a random variable.}
\end{align*}

The RII would take a constant growth rate that the company has been maintaining in the past 10 years and these investors assume that the price will converge to the fundamental value in the next period, i.e.

\[ P_{t+1} = P_{t+1}^f \]  \hspace{1cm} (4.3)

In each period, the RII investor \( i \) chooses the proportion of wealth to invest in stocks and bonds so that the expected utility of wealth is maximized in the next period, given by the equation from Levy et al [71]:

\[ EU \left( \bar{W}_{t+1}^i \right) = EU(W_h^i \left[ (1 - x)(1 + r_f) + x \bar{R}_{t+1}^i \right]) \]  \hspace{1cm} (4.4)

Where:

\begin{align*}
EU & : \text{Expected utility}
\end{align*}
\( W^i_t \) represents wealth of investor \( i \) in period \( t \) given that the price in period \( t \) is some price \( P_h \)

\( x \) is the proportion of wealth invested in shares

\( r_f \) is the risk free interest rate

\( W^i_h \) consists of the previous period wealth \( W^i_{t-1} \), interest and dividend accumulated from the last period, and capital gains or losses incurred on the difference between \( P_h \) and \( P_{t-1} \).

The returns of each year are calculated using,

\[
R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}
\]  

(4.5)

Where

\( D_t \) is the dividend in the given time period,

\( P_t \) is the price at time step \( t \).

The dividend \( D_t \) is a random variable with a constant growth rate. Based on the optimal proportion \( x \), which maximizes the expected utility of next period wealth, fundamental investors determine the number of stocks demanded by multiplying their wealth with this optimal proportion. The rest of their wealth is invested in the risk-less asset.

4.9.1.2 Overconfidence as a Judgmental Bias

Overconfidence as a judgmental bias has been proposed as an explanation for the observed high levels of trading in financial markets [75], as well one of the causes for poor investor performance: some investors trade too much, which might be a manifestation of their over confidence [86]. In order to study the effects of overconfidence, one could measure investor overconfidence by means of psychometric tests (survey responses) and relate those measures to trading records of the same persons. Such studies have recently become available [38] and they shed important light on a number of behavioral phenomena, including overconfidence. On another note, they also show challenges in conducting empirical research on behavioral finance topics, which stem from the data collection efforts needed to close the gap between the micro-level investor behavior and the macro-level effects.
of those behaviors. Such micro-macro mapping is one of the main advantages of agent-based simulations of financial markets. A type of overconfidence that has been prevalently considered in theoretical and computational studies is the miscalibration, which means overestimating the precision of own information, e.g. by setting too narrow confidence intervals in the assessment of the value of a financial asset. In the experimental market of Biais et al. [6], miscalibration was found to have an effect of reducing trading performance. One of the first papers to explicitly study a number of behavioral biases in an agent-based model of the financial market [111], also models overconfidence as miscalibration, more specifically as underestimated variance of stock returns. Lovric M. makes a similar implementation of overconfidence in the sense of miscalibration, on top of LLS model [75].

4.9.1.3 Modeling Overconfidence in the Behavior of EMB

The EMB investors believe that the price accurately reflects the fundamental value. However, since they do not know the dividend process, they use ex-post distribution of stock returns to estimate the ex-ante distribution. In the original LLS model, the EMB investor $i$ uses a rolling window of size $m_i$, and is said to be unbiased if, in absence of additional information, the same probability is assigned to each of the past $m_i$ return observations. Hence, the original, unbiased EMB’s assume that returns come from a discrete uniform distribution:

$$\Pr^i(R_{t+1} = R_{t-j}) = \frac{1}{m^i} \text{ for } j=1,\ldots,m^i. \quad (4.6)$$

The expected utility of EMB investor $i$ is given by

$$\text{EU}(\hat{W}_{t+1}^i) = \frac{(w_{1}^i)^{1-\alpha}}{1-\alpha} \sum_{j=1}^{m^i} \Pr^i(\hat{R}_{t+1} = R_{t-j})[(1-x)(1+r_f) + x R_{t-j}]^{1-\alpha} \quad (4.7)$$

Where

- $R_t$ is the returns at time stamp $t$.

For EMB investors, an investor specific noise is added to the optimal investment proportion $x^*$ to account for variation from the rational optimal behavior:

$$x^i = x^* + \varepsilon^i \quad (4.8)$$
\( \varepsilon \sim \mathcal{N}(0, 0.2) \) for each investor is drawn from a normal distribution with mean 0 and standard deviation of 0.2 \cite{71}. In the modified model, various behavioral characteristics of technical investors viz normal and overconfident \cite{75} are incorporated:

(a) Normal EMBs assume that returns come from a stable normal distribution, and in each period estimate the mean \( \hat{\mu} \) and standard deviation \( \hat{\sigma} \) using the rolling window of size \( m_i \).

\[
\Pr(\bar{R}_{t+1} = R_{t-1}) = \frac{pdf(R_{t-1} | \hat{\mu}, \hat{\sigma})}{\sum_{k=1}^{m_i} pdf(R_{t-k} | \hat{\mu}, \hat{\sigma})} \tag{4.9}
\]

\[
pdf(x | \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{4.10}
\]

(b) Overconfident EMB also estimate normal distribution from the sample, but they underestimate the variance of the distribution. The standard deviation is multiplied by a factor to model this departure in behavior:

\[
\sigma = oc \cdot \hat{\sigma}, \text{ where } oc \text{ is the overconfidence coefficient, } 0 < oc < 1. \tag{4.11}
\]

Here the expected utility of wealth is given by:

\[
\text{EU}(\bar{W}_{t+1}) = \left(\frac{\bar{W}_h}{1-\alpha}\right)^{1-\alpha} [(1 - x)(1 + r_t) + x\hat{\mu}]^{(1-\alpha)} \tag{4.12}
\]

The probabilities are calculated and normalized using the probability density function (pdf) of the peaked normal distribution:

\[
pdf(x | \mu = \hat{\mu}, \sigma = oc \cdot \hat{\sigma}) = \frac{1}{oc \cdot \hat{\sigma} \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2(oc \cdot \hat{\sigma})^2}} \tag{4.13}
\]

### 4.9.1.4 Price Mechanism

The pricing mechanism is based on the temporary market equilibrium, according to the classification by LeBaron \cite{70}. All categories of investors determine optimal proportion in the stock so as to maximize the expected utility of their wealth in the next period. Since expected utility is a function of the price, which is still unknown in the current period, investors therefore determine optimal proportions of wealth to invest in the risky asset \( x_{ih}(P_h) \), and generate respective demands for shares \( N_{ih}(P_h) \),
for various hypothetical prices $P_h$. The equilibrium price $P_t$ is set to that hypothetical price for which the total demand of all investors in the market equals the total number of outstanding shares, according to:

$$\sum_i N_h^i(P_t) = \sum_i x^i_h(P_t) \frac{W^i_h(P_t)}{P_t} = N. \quad (4.14)$$

Where

- $N$ is the total number of shares
- $P_t$ is the share price at timestep $t$
- $x^i_h$ being the proportion of wealth invested in risky assets at hypothetical price $P_h$.

### 4.9.2 Modeling Sentiments in EMBs

One of the key characteristics that govern investor behavior is the optimism or pessimism (sentiments) of the investors. The link between asset valuation and investor sentiment has been the subject of considerable debate in the finance, and has been studied in the context of mispricing (departures from the fundamentals) [12], the limits of arbitrage [27], as well as the under reaction and overreaction of stock prices [4]. Two methodological approaches can be found in the finance literature. One is concerned with finding adequate proxies for the aggregate investor sentiment, and using them in statistical analysis to explain the variation of stock prices and the occurrences of mispricing, such as bubbles and crashes. The other one is a bottom-up approach that aims at modeling individual investor optimism and pessimism by using the insights from psychological theories. For these theories, it is important to have a flexible framework that can be adapted to capture the complexity of human decision making behavior. In fuzzy decision theory, a wide range of connectives (aggregation operators) has been proposed and studied in order to model the flexibility of human decision making. A model of investor optimism is proposed based on fuzzy aggregation [75].

#### 4.9.2.1 New Agent Behavior: Optimists and Pessimists

The optimism of investors is explicitly related to the investors’ opinions on the future market returns. Whether these opinions are going to be translated into exclusive buying or selling behavior is something that is not imposed by the
definition [75]. The implications of different levels of optimism and pessimism have been tested by market simulations. A model of investor optimism has been proposed based on the generalized averaging operator. The advantage of this approach is that the influence of different levels of optimism can be studied by varying a single parameter. The effects of investor optimism are based on the LLS model as was done in the case of overconfidence.

A new EMB type, called the sentiment EMBs is described by using a fuzzy set connective to model both optimists and pessimists. Sentiment EMBs use generalized aggregation operator to estimate future returns, using the rolling window of size m. The prediction of the next period return for each investor i is given by

\[
\tilde{R}_{t+1} = \left( \frac{1}{m} \sum_{i=1}^{m} (R_{t-i})^{s} \right)^{1/s}
\]

(4.15)

The higher the parameter s, the higher the estimate of the return (closer to the maximum value from the sample), and vice versa. In such a way, the parameter s is used to capture the phenomena of investor optimism and pessimism. Several values of the parameter s have been considered which also coincide with the special cases of the generalized mean:

- \( s \to -\infty \), the minimum of the sample;
- \( s = -1 \), the harmonic mean;
- \( s \to 0 \), the geometric mean;
- \( s = 1 \), the arithmetic mean;
- \( s = 2 \), the quadratic mean;
- \( s \to \infty \), the maximum of the sample.

The expected utility calculations and market mechanism remain as defined earlier, pertaining to general EMBs in the LLS model.

### 4.9.3 A Model of Investor Confidence and Sentiment

Having modeled the effects of investor overconfidence and investor sentiment separately, a general model which allows a combined study of both investor optimism and investor overconfidence is seen. Each behavioral phenomenon is modeled by only a single parameter and influences either the mean (sentiment) or
the standard deviation (confidence) of the return distribution investors uses to predict future returns. This enabled the study of interaction of overconfidence and investor sentiment in the same model, since it was expected that overconfidence would have different consequences for optimistic and pessimistic investors. Optimism determines the mean of distribution, while confidence determines the peak of the distribution. The prediction of the next period mean return $\mu_{t+1}$ is calculated as follows.

$$\mu_{t+1} = \left( \frac{1}{m} \sum_{j=1}^{m} (R_{t-j})^{\frac{1}{2}} \right)^{\frac{1}{2}}$$ (4.16)

Based on the level of optimism (or pessimism), a general EMB investor $i$ centers the prediction of the next period return around a value which can range from the minimum to the maximum value in the rolling window $R_{t-1}, \ldots, R_{t-m}$. The predicted deviation of the next period return $\mu_{t+1}$ is calculated from the standard deviation $\sigma$ of the sample of past returns $R_{t-1}, \ldots, R_{t-m}$ and the level of confidence $c$:

$$\sigma_{t+1} = c \times \sigma$$ (4.17)

In this general model, the coefficient of confidence $c \in [0, \infty)$ is used instead of the coefficient of over confidence $\omega$, as both the cases of investor overconfidence ($c \in [0, 1)$ and investor under confidence ($c > 1$) have been addressed. Overconfident EMBs are too confident about their predictions; they underestimate the standard deviation of the distribution, making it more peaked around the generalized mean of the sample. Underconfident EMBs are less certain about future returns; they overestimate the standard deviation of returns making the distribution broader around the generalized mean. In each period of the simulation, EMB investor $i$ predicts next period return by the following discrete probability distribution that incorporates the effects of investor sentiment and confidence:

$$\Pr^i(\tilde{R}_{t+1} = R_{t-1}) = \frac{\text{pdf}(R_{t-1} | \tilde{\mu}_{t+1}, \sigma_{t+1})}{\sum_{k=1}^{m} \text{pdf}(R_{t-k} | \tilde{\mu}_{t+1}, \sigma_{t+1})}$$ (4.18)

Where pdf is the probability density function of a normal distribution. Since probability mass function assigns probabilities only to the returns of the rolling window, for pronounced levels of optimism or pessimism (i.e. high offset of the generalized mean from the arithmetic mean) the resulting discrete probability
distribution will be skewed. The study of investor overconfidence presented earlier, can be considered a special case of the general model where overconfidence is varied but sentiment is set to the special case of $s = 1$, or the arithmetic mean. Similarly, the study of investor sentiment can be viewed as a special case of the general model where optimism level is varied, but confidence is set to the special case of $c = 0$, or the full overconfidence, where only one value of return is given as a prediction. In the case of extreme underconfidence, $c \to \infty$, the distribution becomes uniform, which represents the so-called unbiased EMBs of the original LLS [71]. These original EMBs are not influenced by the level of sentiment because the uniform distribution does not depend on the mean.

4.9.4 Recency and Primacy Effects

Recency and primacy effects, which concern investors who assign more importance to either more recent or older return observations, have been modeled. The effects of investor’s confidence and sentiment seen earlier represented two psychological phenomena which influenced shaping of the probability distribution that investors use to predict future returns. For both of these phenomena, the important aspect is the order of observed past returns based on their magnitude. Sentiment, in the case of optimism, gives more weight to higher returns observed in the memory window, while in the case of pessimism sentiment gives more weight to the lower returns. Confidence determines the weight based on the departures from the mean value which is determined by the sentiment. In this model timing did not play a role, in the sense that it did not matter whether a particular return observation occurred at the beginning or at the end of the memory window.

In the following model, timing aspects of observed returns have been explicitly taken into account. In psychological literature two inverse effects have been observed in the way people give salience to received stimuli or observations depending on their serial position [84]. These cognitive biases are known as recency and primacy effects. While primacy refers to the tendency to give more weight to the first received piece of information (the oldest one), recency describes the tendency to give more weight to the last received piece of information (the most recent one). Recency effects have been studied in the financial literature in relation with the
overreaction hypothesis [11]. When processing information, people tend to overweigh recent information compared with their prior belief. Thus, traders who are not sure of the intrinsic value of a stock will be too optimistic about its value when the firm is winning and too pessimistic when it is losing [87]. The empirical finding that a portfolio composed of past losers eventually beats a portfolio composed of past winners is considered an evidence for such overreaction. Recency and primacy in our model refer exclusively to the importance given to observation based on their time stamp. It is not relevant whether a particular observation was winning or losing, because those effects have already been captured with our experiments on investor’s optimism and pessimism. In such a way, we are disentangling between the effects of timing and the effects of the magnitude of return observations.

4.9.4.1 EMB Investors with Recency Effects

Here additional experiments in the LLS model are conducted in order to study recency effects among the EMB investors. The recency effect refers to the tendency of EMB investors to give more weight to more recent return observations compared to those farther in the past. This is modelled by assigning exponentially decaying probability mass towards the older return values in the rolling window of size $m$. In Figure 4.1 probability mass functions for two different levels of recency effect represented by parameter $\mu \in [0, 1]$ is depicted [75]. The case of $\mu = 1$ represents full recency effect where only the most recent observation is taken into account (this is equivalent to the memory length of size $m = 1$). The case of $\mu \to 0$ represents no presence of recency effect, where each observation is given the same probability mass (this is the case of a uniform distribution, which is studied in the original model of LLS [71]. The calculation of the probability mass function is carried out as follows. The most recent return observation is initially given weight of $\mu$ and the weights of older observations are iteratively reduced by factor $(1 - \mu)$. These initial weights are then normalized to give the probability mass function. The EMB investors aggregate returns from their memory window by calculating a weighted average of returns, and only this value is used for prediction.
The portfolio choice is then based on the comparison between that aggregated value and the risk-free return (similarly to the study of investor optimism).

\[ W(R_{t-1}) = \mu \]  
\[ W(R_{t-j}) = W(R_{t-j+1})(1 - \mu), j = 2..m^l \]  
\[ Pr^i(R_{t+1} = R_{t-j}) = \frac{W(R_{t-l})}{\sum_{k=1}^{m^l} W(R_{t-k})} \]

In the case of full recency effect ($\mu = 1$), much smaller departures from the fundamental value than in the original model of uniform EMBs was observed. However, the market price is still deviating from the fundamental value. For other values of recency ($\mu =0.7$ and $\mu =0.4$) it was seen that the departures from the fundamentals were increasing. In the case of low recency effect ($\mu =0.1$), more prominent departures from the fundamental value is seen. For no recency effects ($\mu \to 0$), the case of aggregation becomes the case of neutral sentiment (arithmetic mean) and full overconfidence and the case of no aggregation becomes the original market model of uniform EMB investors.

### 4.9.4.2 EMB Investors with Primacy Effects

In the case of EMB investors exhibiting primacy effects, the tendency of investors is to give more importance to the return observations that they encountered first, i.e. the oldest observations in their memory window. This is modelled by assigning exponentially decaying probability mass from the oldest return observation towards the most recent return observation in the rolling window of size $m^l$. Probability mass
functions for four different levels of primacy effect represented by parameter $\eta \in [0, 1]$ are shown in Figure 4.2.

$$W(R_{t-m!}) = \eta$$  \hspace{1cm} (4.22)

$$W(R_{t-j}) = W(R_{t-j-1})(1 - \eta), j = 1..(m! - 1)$$ \hspace{1cm} (4.23)

$$Pr^i(R_{t+1} = R_{t-j}) = \frac{w(R_{t-j})}{\sum_k^{m!} w(R_{t-k})}$$ \hspace{1cm} (4.24)

In the case of full primacy effect ($\eta = 1$) only the oldest observation is taken into account, whereas in the case of no primacy effect ($\eta \to 0$) each observation is given the same probability mass. The latter is the case of uniform distribution, which is studied in the model of LLS. Hence, both in the case of recency and primacy effects, as those effects diminish, the resulting behavior becomes the original behavior of the EMB investor. The probability mass function which captures the primacy effect is calculated as follows. First, the oldest return observation is given weight of $\eta$, and then the weights of more recent observations are iteratively reduced by factor $(1 - \eta)$. Finally, all the weights are normalized. EMB investors calculate a weighted average of returns and use it for their prediction, while in the second variation (without aggregation) the investors use the entire probability mass function (pmf) for their prediction. In the case of full primacy effect ($\eta = 1$), much smaller departures from the fundamental value occur, than in the original model of uniform EMBs. As the primacy effect becomes lower, the market dynamics show more excess volatility and become similar to the behavior of the original model with uniform EMB investors. An important observation is that with market dynamics, both primacy and recency effects are reducing the excess volatility, when compared to the original behavior of uniform EMB investors. In cases of full primacy and full recency effects, the investment choice in each period is based on the comparison between only one return observation and the risk-less return. When a high return on the stock is observed, it entices investors into buying the stock. However, as opposed to the original model where such a high return is influencing investors as long as it is in their memory window (creating a bubble in such a way), in this case the high return is not relevant in the next time step because only the new return observation is taken into account.
Although, the effects of primacy and recency seem to be similar, there are still some differences in their impact on market dynamics. By comparing the market dynamics of full recency effect and full primacy effects, it has been observed that extrapolating the most recent return observation (recency effect) causes somewhat higher excess volatility than extrapolating the return observation with a larger time lag (primacy effect). Hence, timing differences in strategies do matter, but their effects are not as prominent as the effects of magnitude captured by the investor sentiment (optimism or pessimism). When compared to the original experiment with EMB investors who use uniform distribution to predict future returns, this time-series looks more realistic, since large movements are not followed by calm periods as in the original model [75]. In the original model, more realistic price dynamics was achieved only after adding heterogeneity in memory lengths, while in the model of Lovric M., investors are homogeneous both with respect to their memory lengths and the type and amount of behavioral bias they exhibit. This illustrates that variations in behavior can have impact and adds to the realism of the market dynamics [75].

### 4.9.5 Self-Attribution Bias and Loss Aversion

It has been seen that the market can also influence the confidence of investors: the over confidence of successful investors can be reinforced through self-attribution bias, i.e. a belief that their trading success should be attributed mostly to their own abilities [86]. In Lovric et al. [74] emerging overconfidence due to self-attribution
bias has been studied. That study demonstrates the advantages of agent-based approach, because it can easily model the dynamics of investor attitudes based on some feedback from the market.

Modeling investor sentiment using a generalized average operator allowed modeling optimism and pessimism using just one parameter. The updating mechanism for investor sentiment depends on the market performance. Investors who increase their wealth subsequently increase their optimism, while those who lose their wealth decrease their optimism (i.e. increase their pessimism). However, loss aversion as a robust finding of human psychology and decision making, suggests that people can react differently towards loses and gains (“Losses loom larger than gains.”) [52]. In the model, this asymmetry is incorporated to study its impact on the dynamics of investor sentiment. Hence, loss aversion in the model operates through a different mechanism than in the Prospect Theory, where it is incorporated in the shape of the value function. Self-attribute bias and loss aversion has been incorporated in the model by describing new agent behavior for updating investor sentiment and investor confidence.

4.9.5.1 Updating Investor Sentiment

A change in investor sentiment is modelled based on the market performance of investors. The investors look at their return on investment in the last period (the relative change in their own wealth), and based on that change they update their index of optimism. If the relative return is higher than one (their wealth has increased), they increase their optimism (increase parameter $s$), and if the relative return is lower than one (their wealth has decreased), they decrease their optimism (decrease parameter $s$). In order to make this update, the parameter $s \in < -\infty, \infty >$ is first mapped into $S \in < 0, 1 >$, through a logistic function $L$:

$$L(s_t^i) = \frac{1}{1 + e^{-(s_t^i - 1)}} = s_t^i$$

(4.25)

The index of optimism is mapped using the logistic function in order to simplify the update equations. Optimism can now easily be increased (decreased) by multiplying with a value that is higher (lower) than one. In addition, the logistic function is translated horizontally so that the neutral sentiment, i.e. the border between
optimism and pessimism \((S = 1)\) corresponds to the middle point of the transformed interval \((S = 0.5)\). This has been done for the convenience in analyzing the graphs of the dynamics of investor sentiment, because the upper half of the graph represents investor optimism and lower half of the graph represents investor pessimism. After the transformation, the index \(S\) is modified based on the recent performance. If the wealth has increased, the optimism of EMB investors is increased by a factor \(\bar{l}\)

\[
\text{IF } \left( \frac{w^i_t}{w^i_{t-1}} > 1 \right) \text{ then } s^i_{t+1} = s^i_t \cdot \bar{l}
\]

and if the wealth has decreased, the optimism is decreased by a factor \(\overline{l}\):

\[
\text{IF } \left( \frac{w^i_t}{w^i_{t-1}} < 1 \right) \text{ then } s^i_{t+1} = s^i_t \cdot \overline{l}
\]

where \(\bar{l} > 1\) and \(0 < \overline{l} < 1\). Finally, the index \(S\) is mapped back into the origin al interval \((-\infty, \infty)\), using an inverse logistic function

\[
s^i_{t+1} = L^{-1}(s^i_{t+1}) = \ln \left( \frac{s^i_{t+1}}{1-s^i_{t+1}} \right) + 1
\]

Two types of updates have been studied, a symmetric update where investors are equally sensitive to losses and gains:

\[
1 - \overline{l} = \bar{l} - 1, \text{ e.g. } \bar{l} = 1.01 \text{ and } \overline{l} = 0.99
\]

and asymmetric updates in which investors are more sensitive to losses than gains:

\[
1 - \overline{l} = \lambda(\bar{l} - 1), \lambda > 1, \text{ e.g. } \lambda = 2, \bar{l} = 1.01 \text{ and } \overline{l} = 0.98
\]

This can be seen as a way of modelling loss aversion, since investors are more influenced by negative returns than positive returns. By increasing pessimism, loss aversion decreases the mean of expected returns.

4.10 Summary

In this chapter, an overview of a number of well-known agent-based ASM modeling behavioral aspects of investors is given. Agent-based ASM are bottom-up models of financial markets which allow us to study their dynamics and emerging properties. They do so by focusing on the behaviors of individual market participants and well-defined market mechanism which are able to translate those behaviors into artificially generated asset prices. One of the aims of these models is to reproduce realistic asset prices which contain the stylized facts of real-world time-
series, such as leptokurtic returns, volatility clustering, bubbles and crashes etc. Since agent-based modeling constitutes a bottom-up approach, deciding on the elements of agents’ behaviors to implement and which market mechanism to use is of uttermost importance. Most agent-based models use very stylized market mechanisms, as well as highly stylized representations of investor behaviors (e.g. fundamental, technical, zero-intelligence agents). However, complexity can easily be introduced into the models by, for example, allowing the agents to learn, co-evolve with the market, or (stochastically) switch between different strategies. Some of the newest developments in the field of agent-based ASM are to look for inspiration into the behavioral finance literature, especially for various behavioral biases of investors.

In the latter part of this chapter, the work of Lovric M. [75] is discussed, wherein various behavioral biases are modeled within the LLS model. The incremental approach has been adopted, wherein an existing computational model is firstly replicated and then new behavior is introduced into the model one by one. By comparing the results of the original model with the results of the incremented model, the implications of the newly introduced (biased) behaviors of investors have been studied. Investor sentiment and investor overconfidence was incorporated in the modified LLS model [71]. The overconfidence in the model refered to the peak of the return distribution around the mean of return observations, while sentiment in the model determined how that mean is chosen (ranging from the minimum observation to the maximum observation in the sample of past returns). It has been shown that changes in the formation of expectations by EMB investors can have a marked impact on the price dynamics. The modified LLS model proposed by Lovric M. [75] is intuitively appealing and appears to be a reasonable choice to study behavioral bias of investors in the BSE.