CHAPTER 3

AGENT-BASED ARTIFICIAL STOCK MARKETS (ASM)

3.1 Introduction

Market models enable study of dynamics in any market. The various approaches used to study stock markets are: theoretical analytical models, empirical studies, experiments, market microstructure, nonlinear economic dynamics, micro simulation and agent-based computational economics. In general, market models within these approaches are referred to as ASM. This chapter elaborates on agents and a number of popular ASM.

3.2 ASM

3.2.1 Characteristics of ASM

ASM are models of financial markets used to study and understand market dynamics. They incorporate a representation of market participants and a well defined price formation mechanism. The one important aspect in ASM environments is that prices should emerge internally as a result of trading interactions of the market participants represented. According to LeBaron [70], models in the realm of agent-based computational finance view financial markets as inter-acting groups of learning, boundedly-rational agents, in which dynamic heterogeneity is critical. ASM can enable study the stock market as a complex adaptive system since they are rich in dynamics, and in emergent properties. In classical theoretical models, homogeneous behavior is assumed, whereas, traders are heterogeneous in behavior. Traders are interacting in order to achieve their objectives, communicating and trading with each other. They can perceive the changes in their environment and act upon it and hence are adaptive in nature. Also a variety of dynamics can emerge as a result of interaction between them. Prices emerge as a result of interaction among market participants. Chen and Yeh [21] make a more strict distinction between ASM and conventional models pointing out that ASM are composed of many heterogeneous interacting adaptive traders, and prices emerge as a result of interaction among market participants. In LeBaron [62],
emergent properties of ASM due to the interaction between heterogeneous traders are explained.

### 3.2.2 Agents

Russell and Norvig [97] define an agent very broadly as anything that can be viewed as perceiving its environment through sensors and acting upon that environment through effectors. Agent-based modeling is characterized by the existence of many agents who interact with each other with little or no central direction. In the literature on agent-based computational economics (Tesfatsion [116]), “agent” refers broadly to a bundle of data and behavioral methods representing an entity constituting part of a computationally constructed world. Examples of possible agents include individuals (e.g., consumers, producers), social groupings (e.g., families, firms), institutions (e.g., markets, regulatory systems), and physical entities (e.g., weather, and geographical regions). Thus, agents can range from active data gathering decision makers with sophisticated learning capabilities to passive world features with no cognitive function. Moreover, agents can be composed of other agents, permitting hierarchical constructions. A mathematical formalization of agents is given in Wooldridge [122]. The agent’s environment can be characterized as a set of environment states $S = \{s_1, s_2, \ldots, s_n\}$ that the agent can influence only partially. The influence of the agent is effected through a set $A = \{a_1, a_2, \ldots, a_n\}$ of actions that the agent can perform. The agents observe the environment states only partially. Therefore, the agent’s actions will depend only on a set $P$ of percepts, which consists of a subset of the environment states and quantities that can be derived from the environment states. It is part of the agent’s design to determine which percepts it can derive from the available states. This mapping can be represented as a function: $S \rightarrow P$. The agent’s decision-making mechanism now maps (sequences of) percepts to the actions of the agent. Let $f$ denote this mapping. Then $f (P; \Theta): P \rightarrow A$ where $\Theta$ denotes a set of parameters with which the mapping $f$ can be parameterized. An agent’s function can now be specified by defining its set $P$ of percepts, set $A$ of actions and the mapping $f (P; \Theta)$ from the percepts to the actions as shown in Figure 3.1[97].
3.2.3 Agent-based Environments

Agents are also defined as specific computer programs. Wooldridge and Jennings [124] define an agent as “a computer system that is situated in an environment, and that is capable of autonomous action in this environment in order to meet its design objectives. Tesfatsionm[116] tries to associate agents’ autonomy to humans’ autonomy. She claims that “autonomy, for humans, means a capacity for self-governance”. What differentiates an intelligent agent from a simple agent is its flexibility. According to Wooldridge and Jennings [124], a flexible agent has the following properties:

(a) **Responsive.** Agents should perceive their environment and should be able to respond in a timely fashion to changes that occur in their environment;

(b) **Proactive.** They should be able to exhibit opportunistic, goal-directed behavior and take the initiative where appropriate; and

(c) **Social.** They should be able to interact with other agents in order to achieve their goals.

Agents are situated and interact in an environment. Russell and Norvig [97] list six dimensions along which an environment can be categorized.

(a) **Fully observable vs. partially observable.** An environment is fully observable if the complete state of the environment is observable. A complete, up to date information can be obtained about the state of the environment. An environment is partially observable if the observer cannot get complete insight into the environment due to the state of the environment or because of his own limitations and capabilities. Partially observable environments have some degree of uncertainty.
(b) **Deterministic vs. stochastic.** The environment here is deterministic, whereas, in a stochastic environment, a variety of effects can occur each with some probability. The probability that a certain effect will take place might be known, but it may lead to actions failing to have the desired result.

(c) **Episodic vs. sequential.** In an episodic environment, decisions are taken periodically. The state of the current episode does not depend on actions in the previous episodes. An episodic environment can be viewed as an environment with isolated decision problems that do not affect each other. In sequential environments, the current decision generally affects the state of the environment, and as such, all future decisions. The order of decision problems in sequential environments is sacrosanct, because in case the order is varied, the characteristics and outcomes are prone to changes.

(d) **Static vs. dynamic.** The state of a static environment stays the same while decisions are being made. In a dynamic environment there are many processes that operate concurrently to modify the environment in ways that are beyond control. A dynamic environment might thus change while decisions are being made, entailing uncertainty.

(e) **Discrete vs. continuous.** The discrete/continuous distinction can be applied to the state of the environment, to the way time is handled, and to the percepts and actions of the agent.

(f) **Single agent vs. multi-agent.** Based on the number of agents, an agent-based environment can be classified as single agent or multi-agent. Most problems require or involve multiple agents, which will need to interact with one another, to achieve their individual objectives or to manage the dependencies arising from the common environment.

### 3.2.4 Nature of Stock Markets

Stock markets are complex environments: they are partially observable; stochastic; sequential; dynamic; continuous with many participants or decision makers. Computational agent-based techniques are most suitable to model such a complex environment prevailing in the stock markets, as against the conventional financial
models, which are constrained by mathematical modeling limitations and consequent complexity [21].

3.2.5 Features of a Stock Market

The returns achieved from investing in shares in a stock market accrue partly from changes in the share price as capital gains and partly from dividends paid. Returns on shares are volatile and are consequently expected to be higher than for a safer investment such as risk-free bonds. A key property of returns on shares is their apparent randomness which follows from the concept of an efficient market based on the Random Walk Hypothesis (RWH). This hypothesis states that it is impossible to predict future price changes based on historic data. Therefore a realistic model of a stock market should show autocorrelation of price return close to zero; a distribution of returns which is non-Gaussian having a kurtosis significantly above three with “fat tails” and persistence in asset return volatility. However in practice the Gaussian is often used to represent the stock market returns since it is amenable to easier analytical tractability, notwithstanding the fact that in actuality, the returns are non-Gaussian.

3.2.6 Agent-based Computational Finance

Agent-based computational finance focuses on agent-based modeling of stock markets. In agent-based financial markets, dynamic heterogeneity is represented by a distribution of agents with a changing set of strategies. Stock markets are suitable for agent-based modeling due to the uncertainty, dynamic and volatile nature of these markets.

3.2.7 Agent-based Modeling of Stock Markets

Agent-based modeling is a bottom-up systems approach to forecast and understand the behavior of non-linear systems [65]. The emergent properties of an agent-based model are the result of “bottom-up” processes, rather than a “top-down” direction. [63,65]. Interaction between agents is a key feature of the agent-based systems. As an alternative to regarding stock prices as stochastic processes, in ABM, prices arise from simulating the interactions of autonomous entities with different profit-making strategies. The collective behavior of such groups of individuals is not determined
by a single mechanism, but by the interaction of individual behaviors distributed across the group, and it is only by the individual behaviors that the group behavior can emerge (Figure 3.2).

![Figure 3.2: Market Dynamics – Micro to Macro level](image)

This indeed is the mechanism prevailing in stock markets and hence the aptness of agent based models for analysis. A large number of agent-based stock market models have been proposed by researchers [8,31,36,51,61,63,67,75,90].

### 3.2.8 Weaknesses of Agent-based Modeling

Agent based modeling has its limitations as well, which needs to be appreciated prior to embracing the methodology [8]. Most models consider small number of generic assets and agents. The dynamics can change drastically when these are enhanced to large numbers. The large number of parameters increases the computational complexity. Any method would be wholly acceptable only after due validation and calibration to real data is done. Since the stock market has a wealth of trading data, this can be addressed. The calibration issue has been discussed by Palin [90].

### 3.2.9 A Definition of ASM

ASM can be defined as market models that have at least a well-defined price formation mechanism for at least one asset for which prices emerge internally through the interaction of market participants. In agent-based ASM, market participants are represented as agents. It is a representation of a market participant
(or a group of market participants using similar strategies), having a form that varies from a simple equation to complex software components endowed with human-like artificial-intelligence based adaptive behavior [70].

3.2.10 An overview of ASM

ASM are models of financial markets used to study and understand market dynamics. Agent Based ASM can be seen as any market model in which prices are formed endogenously as a result of participants’ interaction and in which the representation of participants varies from simple equations of forecast functions to intricate software components endowed with human-like artificial-intelligence based adaptive behavior [62,113,114]. There are various ASM in existence that are created using different strategies and customized for specific requirements. Trading sessions may be call market sessions or continuous sessions. Call market (Discrete) sessions occur at predefined intervals of time whereas trading happens continuously in continuous sessions [28]. Some ASM from the literature are presented here. The criteria for selecting the ASM that are studied here is, that they should incorporate an endogenous price formation mechanism, and a representation of the behavior of traders. More precisely, market models are selected that have at least a well-defined price formation mechanism for at least one asset to the extent that prices emerge internally through the interaction of represented market participants. It is admitted that the ASM chosen is selective and incomplete, but it covers the most widely discussed approaches. The ASM can depict either discrete or continuous trading environment. The discrete/continuous distinction is applied to the state of the environment, to the way time is handled [97].

3.3 A Survey of Call Market (Discrete) ASM

In this section, five ASM models have been discussed, namely: Santa Fe Artificial Stock Market (SF-ASM), Genoa Artificial Stock Market (GASM), Agent Based Model for Investment (ABMI), Business School (BS) and LeBaron’s Model (BM), all being call market or discrete time sessions. Their features, design, pros and cons based on a few important parameters are analyzed. The Santa Fe ASM is studied first, which is considered as a very revolutionary model and has been accepted as a pioneering effort implemented over a decade ago. Other models are explored which
have come up subsequently and then a fair comparison of the five models are made based on a few selected parameters.

3.3.1 Santa Fe ASM

The Santa Fe Artificial Stock Market (SF-ASM) [67,115] consists of a central computational market and a number of artificially-intelligent agents. The agents choose between investing in a stock and leaving their money in the bank, which pays a fixed interest rate. Agents make their investment decisions by attempting to forecast the future return on the stock using Genetic Algorithm (GA) to generate and evolve predictive rules. The artificial market shows two distinct regimes of behavior namely, the rational expectations behavior and the complex realistic market behavior. The parameter settings and the initial conditions control the strategy. One of the parameters that can be used is the exploration rate, which governs how rapidly the agents explore new hypothesis with their GAs. At low exploration rates, the market settles into rational expectations equilibrium and at high exploration rates it falls into the realistic regime. In the rational expectations equilibrium theory, the agents select their optimum behavior by assuming that the agents have complete information, are perfectly rational, have common expectations and they know that everyone else have the same properties. Because of these assumptions there is neither any dynamics, nor learning nor evolution and everything is decided ab-initio.

3.3.1.1 The Agents

The agents classify the available information; notice patterns in the information and generalize internal models from the noticed patterns and act on the basis of these models. However, the agents have to evaluate and adapt after seeing how well they work. In actuality, the agents have a number of different ways of predicting the future and they continually compare and evaluate them. The ones which work well gain more weight and are used more often. The market and the agent are co-evolving in the environment, each action affecting the behavior of each other.

3.3.1.2 Market Structure

The basic structure of the market is N agents, ranging from 50 to 100, interacting with the central market. The interaction between the agents is not direct but only via
the market. A single stock exists with price \( p(t) \) per share at time \( t \). The stock pays a dividend of \( d(t+1) \) per share at the end of time period \( t \). The dividend time series \( d(t) \) is a stochastic process independent of the market and the agents’ actions. The dividend \( d(t) \) is given by the simple random process

\[
d(t+1) = [p d(t) + \alpha n(t)]
\]  

(3.1)

where \( p \) and \( \alpha \) are parameters and \( n(t) \) is a Gaussian random variable, chosen independently at each time \( t \) from a normal distribution with mean 0 and variance \( \sigma \). There is also a fixed-rate asset, the bank, which pays a constant rate or return \( r \) per period. The agents have to decide how much money they want to put into the stock and how much money they want to leave in the bank. At any time \( t \), each agent \( i \) holds some number of shares, of stock \( h_i(t) \) and has some amount of cash \( M_i(t) \) in the bank. Its total wealth is then given by

\[
w_i(t) = [M_i(t) + h_i(t) p(t)]
\]  

(3.2)

At the end of the period, one time step later, this portfolio becomes worth

\[
w_i(t+1) = [(1+r) M_i(t) + h_i(t) p(t+1) + h_i(t) d(t+1)]
\]  

(3.3)

where the three terms are the money in the bank with interest, the new value of the stock, and the dividend pay-out. The trading process is managed by a specialist inside the market. The specialist also has the job of setting the \( p(t+1) \). If there are more bids than offers, then the price is raised, so the bids drop and the offers increase, until they match closely.

### 3.3.1.3 World Bits

The information that is available to the agents at any given time in the market consists of the price, dividend, total number of bids, and total number of offers at each past time step and also includes a predictor of the future dividend and a random “sunspot” variable around which the agents might coordinate their actions. However, this information, known as the world, is condensed into a string of 80 bits and some recent price and dividend information, called as the world bits, each of which is either true or false.
3.3.1.4 Structure of Agents

The agents decide whether to invest in the stock or the bank. The forecasting agents are considered that use a number of predictors each of which attempts to predict the future return (price plus dividend). By seeing how well their predictors work, the agents can estimate their accuracy (prediction variance) and update or replace poor ones.

3.3.1.5 Constant Absolute Risk Aversion (CARA)

Because they know the variance of their overall predictions, the agents can also perform a risk aversion analysis- Constant Absolute Risk Aversion (CARA). When the mean and variance of the return is known for each asset, an optimal division of funds between two possible assets is made based on an exponential utility function. If agent i’s estimate of the mean return is \( E_i \[ p(t+1) + d(t+1) \] \) with variance \( \nu \), then under CARA, the optimum number of shares to hold is given by

\[
h_i^{\text{desired}}(t) = \frac{E_i \[ p(t+1) + d(t+1) \]}{\nu} \lambda
\]

where \( \lambda \) is the degree of relative risk aversion. The agents’ predictors actually consist of two parts, a condition part and the forecast part. The condition part determines when each particular predictor is activated, as explained below. Only activated predictors produce forecasts, using their forecast part which is a linear rule

\[
E_i \[ p(t+1) + d(t+1) \] = a_{ij} \[ p(t) + d(t) \] + b_{ij}
\]

where \( E_i \) means the expected (predicted) value for i’s jth predictor and \( a_{ij} \) and \( b_{ij} \) are the coefficients that constitute the forecast part of this predictor.

3.3.1.6 Classifier Systems

The condition part of the predictor is implemented with a classifier system, in which the condition part is represented by a ternary string of the symbols \{0,1,#\}, one for each of the world bits that the agent can observe. 0 means false, 1 means true and # means either true or false.

3.3.1.7 Genetic Algorithm (GA)

Some of the agents’ predictors may give good predictions when they are activated, while others may not. A Genetic Algorithm (GA) is used to adjust and evolve a
better set of predictors. The GA eliminates some of the worst predictors, those that have the highest variance, and generates some new ones to replace them. To generate new predictors, cloning, crossover and mutation are carried out.

3.3.1.8 Reported Results

The agents are given the initial beliefs in the rational expectations result by setting the initial conditions for $a_{ij}$ and $b_{ij}$ to calculated rational expectations values which is a local attractor, resulting in a very stable market with very little trading, and homogeneous agent behavior. Two regimes of behavior are seen viz, the rational expectations regime and the complex regime. Varying the parameter K indicated how often the GA is run which controlled how often the new predictors were evolved. K=250 gave fast exploration (complex regime) and K=1000 slow exploration (rational expectations regime).

3.3.1.9 Discussion

SF-ASM was a trendsetter, one of the most complex artificial markets of the time. This market allows agents to explore a fairly wide range of possible forecasting rules. They have flexibility in using and ignoring different pieces of information. The interactions that cause trend following rules to persist are endogenous; they are not forced to be in the market. On the other hand, the market is relatively difficult to track in terms of a computer study. This makes it harder to make strong theoretical conclusions about the reflections of this market on real markets. Further, impact of wealth and improved prediction are not considered or explored in SF-ASM.

3.3.2 Genoa Artificial Stock Market (GASM)

Genoa Artificial Stock Market (GASM) is characterized with heterogeneous agents which exhibit random buy or sell patterns and interact with each other [81]. The orders thus placed are processed by a module called the Market Maker, which decides the price of the asset. Orders which find their limit price are satisfied.

3.3.2.1 Micro-structure of the GASM

(a) Traders. Each trader is modeled as an autonomous agent with certain amount of cash and stocks at the beginning. It is up to the trader to decide whether to sell his stock or use his cash to buy more stock. Their decisions depend on
their current state i.e., the cash he possesses at hand and the stocks he owes. The system has mainly three state variables (degrees of freedom), namely: The amount of cash in the system, the number of stock in the system and the price of each stock.

(b) Market Maker. The purpose of the market maker is to fix the price of the stock. It does so from the demand and supply curves. Demand curve gives the price per stock against the demand for the stock (ordered quantity). Similarly supply curve gives the price per stock against the ordered quantity. The price formation process is given by the intersection point of these two curves. Only orders compatible with prices can be executed. The market maker can also add cash into the system or add assets to the system. The size of the buy order or sell order may vary. If the size of sell orders is larger than the size of the buy orders then the market maker adds cash to the system and subtracts assets from it and vice versa for the reverse. The market maker may thus be thought of as having unlimited cash and assets capable of satisfying any order. All orders that do not satisfy the clearing price are discarded.

(c) Functioning of the Market. At the beginning of the simulation, the current price is set and each trader is given an amount of cash and an amount of stocks. The trader issues buy and sell orders with the same probability and are totally independent from each other. In this model, each trader is marked with a tendency to be optimist or pessimist. At each time step, random links are added among traders sharing the same tendency, with a probability, hence clusters of traders sharing the same opinion gradually take shape. A link between two traders belonging to different clusters, results in merging of clusters into a bigger one. At each simulation step clusters of both optimist and pessimist traders are randomly chosen with a probability. All traders belonging to a chosen cluster receive a message to buy (if they are optimists) or to sell (if they are pessimists) as far as they can. After the aggregate orders are placed, all links of traders belonging to the chosen clusters are broken, and these traders change their tendency. As optimists have bought almost all the stocks they could, their tendency switches to pessimist as they don’t have any more cash for buying, but only stocks to sell (vice-versa for pessimists).
3.3.2.2 Discussion

The market’s main features include the following:

(a) The portfolio and cash of every single trader, order and transaction are tracked.

(b) It follows the realistic price formation mechanism.

(c) With a mechanism called aggregation of traders, the GASM is able to reproduce the phenomenon of “fat-tails” seen in real markets.

This aggregate behavior reflects in a simplified way mechanisms of opinion formation actually in place in real markets. However, in this model, there is no learning for the agents and hence there is no evolution in their behavior.

3.3.3 Agent Based Model for Investment (ABMI)

This ASM illustrates how simple agent-based systems can be used for modeling and studying stock markets [31]. There are a few types of investors and a market maker, all represented as agents. The role of the market maker is to adjust prices as a function of the order imbalance. The study shows in what sense the market mechanism matters. Risk-averse behavior of the market maker, for example, introduces trends in prices. This is caused by the fact that if the market maker acquires a position he wants to get rid of it. Structure in price series creates opportunity for technical traders. In the model there is a point at which the market is efficient (i.e. everyone breaks even). The authors analyze under which conditions the market will converge to this point.

3.3.3.1 Market-Making Model

The ABMI model is based on an order-driven market. Traders place orders in this case, market orders and the market maker provides liquidity by buying and selling. The agents do not know at what price the orders will be filled, so the transactions automatically take place out of equilibrium. A single asset and a single representative market maker is assumed. Market orders only are allowed and actions
are synchronized, so that trades take place at \( t, t + 1, \) and so on, using the following price-formation rule:

\[
p(t + 1) - p(t) = \frac{1}{\lambda} \left[ \sum_{i=1}^{N} \omega_i(t) - \beta X(t) \right]
\]

where

- \( \omega_i = \) market order of agent \( i \)
- \( p = \) log price
- \( \omega_i^{i(t+1)} = \left[ x_i^{i(t+1)} - x_i^{i(t)} \right] \)
- \( \lambda = \) liquidity
- \( X = \) market-maker position
- \( \beta = \) market-maker risk aversion
- \( x_i = \) position of agent \( i \)

The change in the logarithm of the price is proportional to the sum of the net order imbalance. The first term is the sum of the orders that are placed by the agents at time \( t \), and the second term is the order placed by the market maker, which is always a fraction \( \beta \) of the market maker’s current position. The variable \( X \), the market maker’s total position, is the total amount that supply and demand are out of balance in the market. The constant of proportionality \( 1/\lambda \), can be thought of as liquidity, and it determines the amount that an order of a given size will move the price. Any particular agent behavior can be studied by specifying a set of function \( x_i^{(t)} \) and iterating the resulting equations.

### 3.3.3.2 Behavioral Models

The behavioral model involves four classes of investors: Market makers as seen above, fundamental (or value) investors, technical traders (or chartists), and liquidity demanders (or noise traders). Value investors take a position based on the perceived fundamental value of an asset. The more underpriced the asset, the larger the position that value investors take in the asset. The asset’s perceived fundamental value will change over time according to a logarithmic random walk. Thus, it is assumed that there is some positive number that the value investor perceives as being associated with the asset, which changes randomly with time.
3.3.3.3 Discussion

The agent based model replicates some of the properties of real prices. This model is simple, but only certain basic points are illustrated. The market mechanism operation is illustrated: Using an order-based market with market makers as liquidity providers can result in patterns in prices that sustain trend followers. When a large number of different strategies are introduced and the market let to select them, the dynamics of the strategies interacting with each other become prominent.

3.3.4 Business School (BS)

The Business School [20,21] is an agent-based model of a so-called “school” (actually strategies) which is used to forecast future values and then evolves over time as a function of their performance. Investors update from time to time the forecast function selected from the school if it does not predict satisfactorily.

3.3.4.1 Traders

All traders share the same Constant Absolute Risk Aversion (CARA) utility function. Traders can accumulate their wealth by making investments. There are two assets available for traders to invest in. One is the risk less interest-bearing asset (money), and the other is the risky asset (stock). Stocks pay dividends following a stochastic process not known to traders. The goal of each trader is to myopically maximize the one-period expected utility function. The key point in relation to this ASM is the formation of expectation, which is modeled by genetic programming. The call market sessions are implemented; traders simultaneously submit orders that are centrally matched at a price at equilibrium.

3.3.4.2 Evolution

The population of forecast functions in the school is evaluated and evolves over time using a GA-based technique. In an evolution phase badly performing strategies are eliminated and give place to new strategies. At BS at every trading period in the experiments there is a probability for each trader to go back to learn. This probability depends on the relative net change in wealth (compared to all traders)
and on the growth-rate of wealth. Learning means choosing a forecast function (randomly) from the set of functions that would have performed better for the latest given number of periods.

3.3.4.3 Discussion

In the BS, the fundamental value depends on the current price and the dividend paid which result in a random Independent and Identically Distributed (IID) return series in a world with technical traders. The basic framework is the standard asset pricing model. The dynamics of the market are determined by the interactions of many heterogeneous agents. Each of them, based on his forecast of the future, maximizes his expected utility. This models an individual as a collection of decision rules. These decision rules are continuously under review and revision; new decision rules are tried and tested against experience, and rules that produce desirable outcomes supplant those that do not. This ASM has successfully demonstrated the emergence of macro-phenomena of financial markets, endogenously generated from interactions among evolving decentralized system of autonomous adaptive agents without exogenously imposing any conditions.

3.3.5 LeBaron’s Model (BM)

The design proposed by LeBaron [63,65] is reviewed in the following paragraphs. The investment decisions of agents are based upon an information set. A Walrasian auction is adopted to determine the price. The model contains two assets for investment – cash and equity. Cash pays a constant guaranteed rate of return \( r_f \) (risk free). The equity pays a dividend at each time step. This is random and the log-dividend follows a random walk. The equity is available in a fixed supply of one share for the population. The sum of holding of all agents will be always maintained at unity. The equity price arises through the interactions of the agents.

3.3.5.1 Agents

The model contains a number of agents with a certain wealth and at each time step it decides how much of its wealth should be allocated to equity and how much to cash. The agents are of Constant Relative Risk Aversion (CRRA) of logarithmic
form and at time t makes these decisions in an attempt to maximise its lifetime utility. The optimal amount of wealth to consume at a single time step is taken as a constant proportion of wealth. The agent restricts itself over the next single time step. In order to maximise the utility, the expected log-return is maximised. Since the equity returns distribution is not known in advance (these arise from the interaction of the agents), the agents maximise a sample expectation taken from historic returns. Because the distribution of returns may change over time, agents look at the last few periods, to maximize the wealth. Allocation to equities is done using one of a pool of rules. A rule recommends the proportion of savings an agent should allocate to equities, taking information about the current state of the market. The rules are implemented as simple Feed Forward Neural Network (FFNN) with a single hidden unit giving an output.

3.3.5.2 The Information Set

The information set consists of six items reflecting various fundamental and technical trading strategies. The first three technical trading inputs are the returns on equity in the previous three time-steps. The fourth is a measure of how the current price differs from the rational-expectations price. The last two inputs measure the ratio between the current prices and exponentially weighted moving averages of the price.

3.3.5.3 Trading and Price-Setting

For a given share price, each agent can determine how much of its wealth is to be invested in shares and arrives at a demand function for shares. A Walrasian auction is then used to find the price.

3.3.5.4 Adaptation and Evolution

The model contains three forms of adaptation and evolution. At each time step a proportion of the agents adapt by selecting a randomly chosen rule after comparing with the current rule. Agents evolve at each time step, wherein agents with the least wealth is removed and replaced by a new agent. The rules are also evolved by being replaced if it has not been used for 10 time steps.
3.3.5.5 Discussion

This ASM demonstrates some of the empirical features generated in an agent-based computational stock market with market participants adapting and evolving over time. Investors view differing lengths of past information as being relevant to their investment decision making process. The interaction of these memory lengths in determining market prices creates a kind of market ecology in which it is difficult for the more stable longer horizon agents to take over the market. The market generates some features that are similar to those from actual data, viz, magnifying the volatility from the dividend process, inducing persistence in volatility and volume, and generating fat-tailed return distributions. One of the goals of this market has been to streamline some of the complexities of the Santa Fe Artificial Market.

3.3.6 Comparative Study of Discrete ASM

The design and mechanisms of these ASM is analysed based on structure and trading.

3.3.6.1 Structure

The various architectural elements of the markets they model is given below.

(a) Assets traded. In general to reduce computational complexity, ASM trade only few assets, usually two types of assets viz one risk-free and one risky stock. Exception being GASM where two risky assets can be traded. Risk free assets might represent PPF or Govt bonds paying a constant interest rate. Dividends paid by risky assets are represented in SF-ASM, BS and BM, wherein additional dynamics are observed. Dividends are generally modeled as stochastic processes. In the BS the fundamental value depends on the current price and the dividend paid. In the SF-ASM and BM, the dividend paid by the risky stock is compared to the interest rate of the risk-free stock to get its real value.

(b) Orders Generated. Trading is either by market or by limit orders. In GASM and SF-ASM it is limit orders.
(c) **Market Participants.** Individual investors are simulated in all ASM. Brokers are not modeled. The behavior of market makers is modeled in SF-ASM and ABMI.

(d) **Execution System.** In all models traders simultaneously submit orders that are centrally matched at a price at equilibrium - the execution system being “single-price auction”.

### 3.3.6.2 Trading

(a) **Investor Objectives.** The main investment objectives are to maximize profits by fundamental or technical strategy as follows:-

- **ABMI** - Arbitrage opportunities.
- **SF-ASM, BS and BM** - Utility function.
- **GASM** - Portfolio optimization.

(b) **Time Horizon.** The majority of the objectives is long-term as they remain fixed during the full length of the experiments. In BM however, Long and Short horizons are considered.

(c) **Attitude to Risk.** Investors’ attitude to risk is modeled by some ASM by introducing risk averse traders:-

- **SF-ASM and BS** - CARA.
- **BM** - CRRA of logarithmic form.

(d) **Investment strategy.** Investors are classified as informed or fundamentalist traders:-

- Fundamentalists or informed traders are in ABMI and GASM. At SF-ASM and BM, investors compare the dividend paid to the interest rate of the risk free asset.
- Technical trading strategies are considered in all ASM. While in ABMI it is trend followers, in GASM the technical trading strategies include mean-variance trading.
- Both fundamental and technical strategy is used by traders at SF-ASM and BM for forecasting future values, applying moving average functions.
- In BS, forecast functions generated are combinations based on past prices and dividends.

(e) Learning.
- In SF-ASM, BS and BM, traders can switch strategies if they are not successful enough.
- At SF-ASM and BM, each trader has its own set of strategies, from which they choose the most suitable one every trading round.
- Neural networks and evolutionary algorithms are two commonly used techniques to implement learning. SF-ASM, BS and BM apply this technique. Selection, mutation and crossover are applied to adapt the set of strategies to the changing conditions. In the BS model, investors try to find the best trading strategies by genetic programming. Fitness function in GA is maximum return, wealth, utility, or the minimum forecast error.
- At SF-ASM and BM, agents who learn are selected centrally with some probability every given trading period.
- In BS, at every trading period in the experiments there is a probability for each trader to learn.
- At SF-ASM, the distance of the forecast value from the real outcome indicates the fitness.

(f) Various Timing Issues. All traders simultaneously make a trading decision whereby it usually results in placing a new order. In all ASM except BM, the time-horizon of the investment objectives hold during the whole experiment. In BM, long and short memory traders are modeled. Forecast horizon of the investment strategies is one period ahead. Asynchronous behavior is modeled by selecting only a fraction of traders at any time.
(g) Execution of Order. The brokers are not represented in any of these ASM. At ABMI the market maker trades based on his position. All orders are market orders, and price is centrally set according to an automated mechanism. The market maker is represented by a simple equation. At SF-ASM and BM, the market price is defined by an auctioneer.

(h) Equilibrium price.

- At SF-ASM, equilibrium is determined at the price at which trading volume is maximized.
- In GASM a new market price is often at the intersection of demand and supply curves.
- At BS price is based on the excess demand/ supply discounted with some adjustment value.
- At BM, Walrasian auction is done.

3.3.7 Findings of Call Based Trading Session ASM

ASM mainly focus on the analysis of price or return dynamics [8,59].

(a) Patterns Observed. Many ASM find evidence for stylized facts.

- Fat tails and volatility clusters are observed in SF-ASM, BM and GASM.
- In SF-ASM and BM, a correlation between the trading volume and volatility is found.
- A reversion to mean is reported in the study on the GASM market.
- Trend followers in the ABMI, mean-reversion traders at the GASM market, and fast learning technical traders at SF-ASM and BM can systematically dominate the market, meaning that the market is not efficient in these cases.

(b) Investment Objective. The investment objective parameter analyses the goal of the ASM.
• In Santa Fe, the agents form demands based on a utility factor, CARA.
• In BM it is CRRA.
• In ABMI, agents have the simple objective of maximizing the profit.
• GASM aims at maximizing liquidity.

c) Execution Systems. The execution systems applied is the single price auction system.

d) Market Participants. Also based on market participants, ABMI and GASM models can have any number of individual investors; Santa Fe accommodates 50-100 individual investors.

3.3.8 A Comparison of Important Parameters.

Table 3.1 gives a comparison of important parameters of selected discrete ASM studied above.

Table 3.1: Comparison: Important parameters of Discrete ASM

<table>
<thead>
<tr>
<th>ASM PARAMETERS</th>
<th>Santa Fe (SF-ASM)</th>
<th>Agent Based Model for Investment (ABMI)</th>
<th>Genoa Artificial Stock Market (GASM)</th>
<th>Business School (BS)</th>
<th>LeBaron’s Model (BM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traded Assets</td>
<td>Experiments conducted with both risky and risk free assets.</td>
<td>Experiments conducted with only risky assets.</td>
<td>Experiments conducted with both risky and risk free assets.</td>
<td>Experiments conducted with both risky and risk free assets.</td>
<td>Experiments conducted with both risky and risk free assets.</td>
</tr>
<tr>
<td>Fundamental Value</td>
<td>Price, Dividend &amp; Risk free Interest rate</td>
<td>Log random walk</td>
<td>-</td>
<td>Price, Dividend</td>
<td>Price, Dividend &amp; Risk free Interest rate</td>
</tr>
<tr>
<td>Dividend Auto Regressive</td>
<td>-</td>
<td>-</td>
<td>IID(stochastic)</td>
<td>Auto Regressive</td>
<td></td>
</tr>
<tr>
<td>Types of orders</td>
<td>Limit Orders</td>
<td>Market Orders</td>
<td>Limit Orders</td>
<td>Market Orders</td>
<td>Limit Orders</td>
</tr>
<tr>
<td>Trading Sessions</td>
<td>Discrete Time (Call Based Trading Session)</td>
<td>Discrete Time (Call Based Trading Session)</td>
<td>Discrete Time (Call Based Trading Session)</td>
<td>Discrete Time (Call Based Trading Session)</td>
<td>Discrete Time (Call Based Trading Session)</td>
</tr>
<tr>
<td>Investment Objective</td>
<td>Maximize CARA utility</td>
<td>Arbitrage/ Maximize profit</td>
<td>Optimize by maximizing utility/ liquidity</td>
<td>Maximise CARA</td>
<td>CRRA of logarithmic form</td>
</tr>
<tr>
<td>Market Participants</td>
<td>50 – 100 individual investors 1 Market Maker</td>
<td>N individual investors 1 Market Maker</td>
<td>N individual investors No Market maker</td>
<td>500 N individual investors 1 Market Maker</td>
<td></td>
</tr>
<tr>
<td>Evolution</td>
<td>Evolving(NN &amp; GA)</td>
<td>No evolution</td>
<td>No evolution</td>
<td>Evolving(NN &amp; GA)</td>
<td>Evolving(NN &amp; GA)</td>
</tr>
</tbody>
</table>
3.4 A Survey of Agent Based Continuous Session ASM

Continuous trading sessions and quote driven mechanisms are more towards how real financial markets actually work. These markets try replicating actual markets in continuous order matching and asynchronous decision making. In this section, four continuous quote-driven ASM Models have been analyzed. As stated before, the discrete/continuous distinction is applied to the state of the environment, to the way time is handled, and to the percepts and actions of the agent [97]. The Continuous session ASM under this study are: Continuous Time Asynchronous Model (CTAM), Electronic Market Maker (EMM) Model, Continuous Extended Glosten Milgrom Model (CEGM) and KapSyn. On markets with continuous trading sessions, an order can be placed at any time and transactions are made whenever possible. Consequently, traders do not need to make decisions simultaneously.

3.4.1 Continuous Time Asynchronous Model (CTAM)

In the Continuous Time Asynchronous Model (CTAM), the agents are characterized to ‘sleep’ and ‘wake up’ at predefined times or due to previous decisions made. No central mechanism controls agents and thus each investor may play a different action at the same point in time [104].

3.4.1.1 The Market

(a) The market place. The trading sessions are continuous and quote driven (hence the presence of a market maker). Multiple investors and one market maker, who ensure liquidity of stocks and execution of orders, are present. One stock which doesn’t pay dividends is traded and investors place orders for that single stock. The stock has an underlying fundamental value, computed exogenously.

(b) The Fundamental Value (FV). The fundamental value at the time $t$ is $V_t$. It is constant most of the time and changes occasionally (i.e.) it follows a jump process. The jump process is modeled as a random process,

$$V_{t+1} = V_t + \omega(0, \sigma)$$

(3.7)
where $\omega (0, \sigma)$ represents a sample from a normal distribution with mean zero and variance $\sigma$.

(c) **The investors.** The investors are classified as informed and uninformed investors depending on the information regarding the fundamental value. Informed traders are further classified into perfectly informed and noisily informed. The perfectly informed knows the correct fundamental value and the noisily informed know the fundamental value with some noise. Finally, uninformed traders do not know what the underlying fundamental value is, and they trade randomly. Based on their observations, the traders buy when their observed fundamental value is higher than the ask quote of the market maker and sell when their value is lower than the market maker bid quote. Uninformed traders buy and sell with an equal probability.

(d) **The Market Maker.** The fundamental value is unknown to the market maker and he tries to capture it by responding to the behavior of the traders. The market maker keeps a probability density estimate over a whole range of possible values for the stock, which is updated after the arrival of an order. The market maker knows some factors such as the fraction of informed and uninformed traders, the probability with which the uniformed traders trade, the initial fundamental value, an occurrence of a jump process. The market makers carry out the following tasks:

- Receive and execute orders;
- Update the probability density estimate based on the received orders;
- Calculate the expected value of the stock based on the updated probability values;
- Adjust the bid/ask quotes according to changes in the expected value.

The market maker sets the bid and asks prices such that they reflect the fundamental value of the stock.

### 3.4.1.2 Market Price Discovery

(a) **Perfectly Informed Traders.** As the perfectly informed traders know the exact fundamental value, when the market maker releases the bid quote, all
these traders place orders resulting in a condition called overshoots. When
the market maker encounters such a condition he comes to know that the
fundamental value is a value well above his bid quote. The market makers
thus are always in line with the fundamental value except during the jump
periods. On the contrary, the fact that a sell order is submitted helps the
market maker understand that the current bid-ask spread being probably
above the actual fundamental value. After this, the market maker’s bid-ask
spread is set around the actual fundamental value so the probabilities within
the area between the bid and ask price are increased while the others are
decreased, creating a high peak around the actual fundamental value, until
another trade arrives.

(b) Noisily Informed Traders. In this case, for every update of the market
maker, the probabilities for a whole range of values are taken into account
because of the additional noise in the traders’ decisions. The case is similar
to that of the perfectly informed traders except for the observation of the
fundamental value with some noise. Because the market maker has to rely on
noisy information, the expected value most likely does not exactly follow the
fundamental value, but the fundamental value with additional noise that is
received by the noisily informed trader.

(c) Combination of both Perfectly and Noisily Informed Traders. Here the
market maker takes more time to decide upon the fundamental value because
the market now consists of both the groups. Large jumps are encountered
often which at times is difficult to track.

3.4.1.3 Discussion

- This model is continuous and asynchronous and works in line with the real
  financial markets.

- The herd like behavior of the traders reacting to a situation mitigates the
  versatility of the system.

- Adapting the learning algorithm of the market maker to account for both
  perfectly informed and noisily informed traders is a challenge.
3.4.2 Electronic Market Maker (EMM) Model

EMM Model [85] presents an adaptive learning model for market-making under the reinforcement learning framework. Reinforcement learning is a learning technique in which agents aim to maximize the long-term accumulated rewards. No knowledge of the market environment, such as the order arrival or price process, is assumed. Instead, the agent learns from real-time market experience and develops explicit market-making strategies, achieving multiple objectives, including maximizing of profits and minimization of the bid-ask spread. Reinforcement learning can be considered as a model-free approximation of dynamic programming. Bid and ask prices are dynamically determined to maximize some long term objectives, such as expected profits or expected utility of profits. The knowledge of the underlying processes is not assumed, but learned from experience.

3.4.2.1 The Model

In the basic model, the market-maker quotes a single price. The optimum strategies can be determined analytically and it can be shown that the reinforcement algorithms successfully converge to these strategies. The basic model is then extended to allow the market-maker to quote bid and ask prices. While the market-maker controls only the direction of the price in the basic model, it has to consider both the direction of the price as well as the size of the bid-ask spread in the extended model.

3.4.2.2 Discussion

- Since reinforcement learning assumes no knowledge of the underlying market environment, this model can be applied to market situations for which no explicit model is available.
- The main limitation of these models is that, specific properties of the underlying processes (price process and order arrival process) have to be assumed in order to obtain a closed-form characterization of strategies.
- The environment state is only partially observable and reward signals may not be available at each time step.
• The simulation results show initial success in bringing learning techniques to building market making algorithms.
• This ASM aims to model the market-making problem in a reinforcement learning framework, explicitly develop market-making strategies, and discuss their performance.

3.4.3 Continuous Extended Glosten Milgrom Model (CEGM)

This model is an extension of the Glosten Milgrom information based model [39], proposed to show the influence of informational asymmetry on the bid-ask spread, based on the learning market maker from Das [25,26]. The market maker tries to discover the fundamental value of a stock by means of Bayesian learning. He determines the bid and ask quotes based on his expectation of the real value, the order flow, taking into account his prior knowledge regarding the ratio of informed and uninformed traders. A nonparametric density estimation technique is used to maintain a probability distribution over a range of expected true values. The market-maker uses these probability estimates to set bid and ask prices. Discrete time simulation is applied in this extended model, as well as a probabilistic representation of the order flow.

3.4.3.1 The Framework

Trading sessions are continuous and the execution system is quote-driven. There is one stock traded. One market maker and multiple investors are represented. The stock does not pay dividends.

(a) The Fundamental Value of Stock. It is assumed that the stock has an underlying fundamental value, which is generated exogenously to the market. The underlying fundamental value of the stock follows a jump process, being constant most of the time.

(b) Trading. Trading is organized in trading rounds as a sequence of bilateral trading opportunities. Each trading opportunity involves a single potential investor who is selected at random from an unchanging pool of potential traders. The selected trader can buy at the offer, sell at the bid, or choose not to trade.
(c) **The Market Maker.** The market maker is responsible for the liquidity of the stock and the execution of orders at the current bid or ask price. He sets bid and ask prices as a function of the order flow and the market information he possesses.

(d) **Probability Density Estimate (PDE).** All orders are assumed to be market orders of one unit. The market maker does not know the fundamental value, but, in order to ensure an efficient market, he tries to capture it by maintaining a probability density estimate (PDE) over a range of expected true values. The probability estimates are initialized according to the normal distribution. The initial bid and ask prices are calculated from this initial PDE and a priori expectations of the market maker.

(e) **Trading Rounds.** After initialization, the trading rounds start. A round consists of the following steps:

- The probability is evaluated for a jump in the fundamental value, and the jump is carried out if it is the case.
- An investor is selected randomly from the pool to place an order.
- A Buy/Sell/No order is sent by selected trader to the market maker.
- The market maker processes the order and carries out the transaction.
- The market maker updates his PDE.
- The market maker updates the bid and ask prices.

In the EGM model a jump in the fundamental value occurs with some probability (0.001 in the experiments) at every trading period, i.e. at every discrete point in time. The jump process is modeled as a random process

**3.4.3.2 The Investors’ Behavior**

Investors are differentiated based on the information they receive regarding the fundamental value. There are two types of investors:

(a) Informed traders and

(b) Uninformed traders.
The informed traders are further classified as perfectly informed or noisily informed. Perfectly informed traders observe the correct fundamental value, while noisily informed investors observe a distorted fundamental value. Finally, uninformed traders do not know what the underlying fundamental value is, and they trade randomly. Informed traders decide whether to trade or not, based on their observation of the fundamental value. An informed trader will buy if the fundamental value that he observes is higher than the market maker’s ask price. He will sell if the fundamental value that he observes is below the bid price. He will place no order if the observed fundamental value is within the bid-ask spread. Uninformed traders place buy and sell orders with equal probability. They can also decide not to place orders with probability

3.4.3.3 The Behavior of the Market Maker

After an investor has been selected, and has placed an order, it is the market maker’s task to carry out the rest of the actions within a trading round. On his turn, the market maker needs to carry out the following tasks:

- Receive and execute orders;
- Update the probability density estimate (PDE) based on the received orders;
- Adjust the bid and ask quotes according to the changes in the PDE.

The market maker executes sell orders at the current bid price and buy orders at the current ask price. The private information regarding the fundamental value is revealed implicitly by the type of the orders submitted by the (informed) traders. Information regarding the fundamental value of the stock diffuses from the informed traders to the market maker in this way. A series of sell orders might indicate that the fundamental value is lower than the current bid price, and a series of buy orders might indicate that the fundamental value is higher than the current ask price. However, the market maker will have to take into account the noise incorporated by the orders of the noisily informed traders, and the noise implied by the orders submitted by the uninformed traders. The market maker aims to set bid and ask prices to capture the underlying fundamental value of the stock. The fundamental
value is known only by the (perfectly) informed investors, and is not known by the market maker. The main task of the market maker is to learn this value. As mentioned above, he tries to do this by maintaining a set of possible true values with probability estimates attached to each of them. The range of possible values, the corresponding probabilities, and the learning process of the market maker is based on a set of current and a-priori known information.

3.4.3.4 Discussion

- This model consist of “informed” trading agents who decide to trade based on received signals of the true or fundamental value of the stock, and “uninformed” trading agents who trade for exogenous reasons and are modeled as buying and selling stock randomly.
- The combination of traders and market-making agents replicate properties observed in real financial markets.
- The bid-ask spread increases in response to uncertainty about the true value of a stock.
- Average spreads tend to be higher in more volatile markets; and
- Market-makers with lower average spreads perform better in competitive environments.
- The time series data generated showed volatility clustering and fat-tail return distributions.

3.4.4 KapSyn

The KapSyn ASM [72] presents an approach to calculating transaction costs that is based on the capital market’s microstructure. KapSyn can imitate the microstructure of any stock exchange, especially with respect to its price-finding procedures. Since trading has increasingly been concentrating on cutting costs, several alternatives have evolved for placing orders, and transaction costs have become increasingly important as a means of evaluating them.
3.4.4.1 The KapSyn framework

The model replicates the microstructure of various stock exchanges, especially with respect to their price-finding procedures. The stock markets focused on are the XETRA and the NASDAQ. Investors are represented at individual level. The model’s basic structure is presented in [72]. The model applies a Markov process with continuous time and discrete state space to model the capital market’s dynamics. It uses a stochastic utility approach to evaluate the trader’s utility. Like real stock exchanges, traders are able to place orders or revise their orders. The modeling of intra-day market simulates trading processes of a realistic environment. At any point during the trading session the market participants (‘agents’) can submit bids and asks. Whenever two orders match in volume and price, the quote is fixed. Under certain conditions, an auction may take place. Every trader believes that, at the end of the planning period, the price of a security will equal the individual estimate. Rational expectations at the level of the individuals are applied. This assumption leads to the basic rule of trading: traders aim to buy securities whose share price is above their individual intrinsic value estimates, and they aim to sell those securities whose share price is below their individual estimates and invest the proceeds elsewhere. Transaction costs and the opportunity costs of alternative investments also have to be taken into account. KapSyn also represents brokers. More than one stock is traded on the KapSyn. Investors have actions at their disposal associated to each stock. An action can be placing a buy order, placing a sell order or changing the expected value. With each action a reaction time is associated. The reaction time depends on the expected utility that the action can generate and is lower for higher utilities. At each simulation round an action is selected for execution. Selection depends on the reaction time associated to the actions. Actions with smaller reaction times have a higher probability to be selected. The continuous time is implemented by using a discrete state space. Transition from one state into another depends on time related parameters. The agents’ decision process regarding which action to take next is independent of each other and takes place simultaneously in parallel. A decision process contains the following steps.

- Each agent determines a set of alternative actions for each stock.
Then, each agent selects one action for each stock.
Finally, one action is selected for each agent.

All the selections depend on the utility and execution time of the actions. The greater the expected benefit gained from the action of an agent, the shorter will be the reaction time and the higher the selection probability. After all agents have made their decision with respect to the next action to be carried out, the system executes the action with the shortest reaction time. After an action that is observable by all market participants, i.e. a buy or a sell action, the decision process determining the next action is repeated on the basis of the new market conditions. Therefore, in this case, only one action is executed in a simulation round.

3.4.4.2 Discussion

- Continuous time is implemented by using a discrete state space.
- KapSyn is the only ASM in which intra-day data is generated.
- Brokers and market makers are represented.
- More than one stock is traded on the KapSyn

3.4.5 Comparative Study of Continuous ASM

EGM, EMM and KapSyn are artificial markets that represent continuous quote-driven trading orders wherein market makers set bid and ask quotes. The difference between continuous auction markets, and continuous quote-driven markets is that in the second one a market maker is responsible for executing orders, traders can trade with each other only with the help of this market maker, and the market maker has to take position [8,57].

3.4.5.1 Structure

The various architectural elements of the markets they model are given below:

(a) Assets.
- ASM trade only two types of assets viz one risk-free and one risky stock. Risk free assets might represent PPF or Government bonds paying a constant interest rate.
- Dividends paid by risky assets are represented in SF-ASM. Dividends are
generally modeled as stochastic processes. In the SF-ASM the dividend paid by the risky stock is compared to the interest rate of the risk-free stock to get its real value.

- In Kapsyn multiple (risky) assets can be traded. A well-determined fundamental value exists in most of the ASM. This value typically follows a random IID (Independent and Identically Distributed) process. In KapSyn there is no unique fundamental value.

(b) Orders Generated.

- Trading by limit orders or both limit and market orders can be placed.
- Both limit and market orders are placed in CTAM, EGM and EMM.
- In EGM and EMM, investors’ give market orders, and the market maker determines bid and ask quotes with limit prices.
- In the KapSyn market, limit orders are generated, but traders might also decide to accept quoted orders.
- Unexecuted orders at CTAM and KapSyn can be canceled.

3.4.5.2 Market Participants

- Individual investors are simulated in all ASM. Traders place orders as the result of some decision that is mainly utility maximization in every trading round. In this way only investors are represented by these traders.
- The behavior of market makers is modeled in SF-ASM, EGM, EMM and Kapsyn.
- In EGM and EMM, investors are not modeled individually; instead, just the orders placed are modeled arriving according to some distribution.
- In CTAM and KapSyn, individual investors with their own decision-making behavior are simulated.
- Brokers are modeled only in KapSyn.
3.4.5.3 Trading Sessions

- At the KapSyn stock market continuous time is implemented by using a discrete state space.
- At CTAM, discrete-event simulation is applied to model continuous trading.

3.4.5.4 Execution System

- Price formation is based on automated central execution systems.
- In KapSyn model, price formation depends on the microstructure of the stock exchange modeled.

3.4.5.5 Formation of Price and Traders’ behavior

(a) Investor Objectives. The objectives of the investors are to maximize profits and depend on the strategy they use: fundamental or technical:-

- EGM and EMM: through arbitrage opportunities.
- KapSyn: maximize a utility function.
- CTAM: maximize wealth.

(b) Risk. Investors’ risk profile is modeled by some ASM by introducing risk averse traders.

- CTAM -strive to minimize risk.
- KapSyn - risk is measured as the deviation of the actual portfolio from the desired portfolio.

(c) Investment Methodology. Investment methodology differs and investors are classified as:-

- Fundamentalist traders - CTAM, EMM and EGM.
- Technical traders- CTAM and Kapsyn.
- Random traders -CTAM, EGM and EMM.

(d) Asset. The weight of the stocks depends on the utility function applied.

- In CTAM shares to trade is based on forecast and a fraction of the
wealth.

- The traders decide to sell/buy if the expected price is below/above the current market price.

- Stochastic functions are applied to determine trading.

- Buy and sell orders are placed with equal probability in EGM and for random traders in CTAM.

- At KapSyn the probability of being selected is related to the expected utility.

- In CTAM, for the limit orders, the limit price defined by the traders is the forecasted price of the stock.

(e) Adaptation. Adaptations vary from simple value adjustments to intricate evolution of strategies. An agent monitors the new market prices; perceived fundamental values; bid and ask quotes when applicable; and the performances.

- In SF-ASM traders can switch strategies if they are not successful. Neural networks and evolutionary algorithms are two commonly used techniques to implement learning. Selection, mutation and crossover are applied to adapt the set of strategies to the changing conditions. At SF-ASM agents who learn are selected centrally with some probability every given trading period. Fitness function in GA is maximum return, wealth, utility, or the minimum forecast error. At SF-ASM, the distance of the forecast value from the real outcome indicates the fitness. Most often traders use one single strategy during the full length of a simulation.

- In KapSyn technical traders correct the expected value of a stock upward or downward.

(f) Time Horizon. The time-horizon of the investment objectives hold during the whole experiment. All traders simultaneously make a trading decision whereby it usually results in placing a new order. Asynchronous behavior is
modeled by selecting only a fraction of traders at any time. In every simulation round usually one order is generated thereby depicting the asynchronous behavior of traders. At KapSyn, the next event to be carried out depends on the reaction time of all events prepared by investors. In CTAM the timing for placing an order can be predefined by some event. Timing of orders in CTAM and KapSyn is determined by the investors and not by the system centrally, thereby representing autonomy of traders.

(g) Equilibrium Price. In KapSyn(XETRA), equilibrium is determined at the price at which trading volume is maximized. The behavior of the market makers is adaptive: EGM - Bayesian learning and EMM - Reinforcement Learning. The market makers at KapSyn modify their quotes with respect to the limit order book. In EMM, the electronic market maker reacts when the order imbalance reaches a predefined threshold. In EGM, EMM and KapSyn the market microstructure approach is used. Execution systems at CTAM represent a continuous automated order execution mechanism.

3.4.6 Findings of Continuous ASM

ASM mainly focus on the analysis of price or return dynamics and that it will not be possible to earn above-average profits.

3.4.6.1 Patterns in Price Series

Many ASM find evidence for stylized facts. Bubbles and crashes occur within CTAM.

3.4.6.2 Autonomy

Representations of traders in ASM illustrate passive agents, while agents on the market are autonomous. Autonomous behavior can only be accomplished if the agents themselves decide when they want to trade.

- Traders are selected globally by the system to trade in the next trading cycle. In CTAM traders react to some events or wait for a while before they analyze the market conditions.
• In KapSyn a reaction time is associated to each individually planned action as a function of their utility.
• All investors are goal-directed, except for cases in which decisions are represented by random order flows.
• Interactive behavior occurs since the agents need to trade with each other.
• Learning is Bayesian learning in EGM, and AI based techniques in EMM.

3.4.6.3 Continuous and Asynchronous Behavior

In KapSyn and CTAM continuous order matching and asynchronous decision-making is done. Whereas in other models, continuity and asynchronous behavior is depicted stochastically, selecting one trader whose decision is carried out, and automatically matching new orders with pending ones if possible. Here the agents are no longer autonomous regarding their actions.

3.4.7 A Comparison of Important Parameters

Table 3.2 gives a comparison of important parameters in the selected ASM with continuous and asynchronous behavior.

Table 3.2 Comparison: Important Parameters of Continuous ASM

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>CTAM</th>
<th>EMM</th>
<th>CEGM</th>
<th>KAPSYN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traded assets</td>
<td>Risky and Risk free</td>
<td>Risky and Risk free</td>
<td>Only risky</td>
<td>Risky and Risk free</td>
</tr>
<tr>
<td></td>
<td>(N&lt;123)</td>
<td>(N&lt;123)</td>
<td>(N&lt;123)</td>
<td>(N&lt;123)</td>
</tr>
<tr>
<td>Trading Session</td>
<td>Continuous (Sleep &amp; Wake)</td>
<td>Continuous</td>
<td>Turn based and autonomous</td>
<td>Call and Continuous</td>
</tr>
<tr>
<td>Investment Objective</td>
<td>Own max number of instruments</td>
<td>Liquidity</td>
<td>Maximise utility</td>
<td></td>
</tr>
<tr>
<td>Investment Strategy</td>
<td>Based on knowledge of fundamental value</td>
<td>Fixed set of strategies</td>
<td>Based on knowledge of fundamental value</td>
<td>Individual expectation</td>
</tr>
</tbody>
</table>

3.5 Modular Continuous-Time Asynchronous Model of Katalin Boer Sorban

The work of [8] is an extension of a discrete-time agent-based financial market (as most models are) into a modular continuous-time asynchronous model, and it shows that a continuous-time framework entails much more intricate market dynamics. Although, continuous trading sessions are much more common in real markets [28, 45], these have been rarely focused on in early ASM, and have gained attention only recently. With respect to the behavior of market participants, in ASM attention is
rarely paid to the representation of financial traders involved in setting prices, such as market makers or brokers. Usually, the only market participants considered are investors, who are moreover modeled as being centrally selected and making trading decisions simultaneously. In contrast to this representation, in reality investors take decisions autonomously and asynchronously. The modeled strategies across the different AMS vary on a wide scale, given the fact that investment strategies are hardly observable, and the fact that arbitrary many possibilities exist to forecast future values. However, by far most ASM are one shot models with a predefined set of trading alternatives. In case other strategies / models are to be studied, an altogether new model is required to be built from scratch. It is not possible to easily plug in new alternatives in existing models. In an attempt to provide a better understanding of market dynamics, Boer has proposed a trading environment that addresses the above mentioned shortcomings. The market is defined from bottom-up, and continuously follows the trading behavior of market participants, including financial traders, portrayed as individual, autonomous agents. In order to accomplish this goal the ACE methodology has been applied. A modular representation of markets and trading behaviors has been given, so as to support multiple market structures and arbitrarily many trading strategies in a flexible way. The primary focus is on studying dynamics in relation with continuous trading sessions by using continuous time simulation.

3.6 Evaluation of Modeling Aspects of ASM

Even though the agent-based modeling approach allows for the implementation of arbitrary complex agent behaviors, most agent-based artificial markets developed focus on relatively simple agent behaviors. Artificial financial markets that are based on a small number of highly stylized behaviors have been labeled as few-type models [70]. Typically, strategies (or agents who employ them) can be divided into two groups: fundamental who trade based on a perceived fundamental value of an asset, and technical who trade based on the past prices, e.g. some form of trend extrapolation. In addition, many models have been rooted in the zero-intelligence framework in which agents basically trade randomly, possibly subject to a budgetary constraint. Sometimes a small number of such agents are included into a few-type
model in order to provide liquidity for other agents. Depending on their type, agents will have access to different types of information. Typically, all agents will have information about the current market price, which will allow them to update their wealth status and possibly tune their strategies (in case learning mechanisms are employed). They will also receive news, such as dividends on risky assets and interest on riskless assets. However, some agents will also have information about the determinants of the fundamental price, such as the dividend generating process. Depending on their strategies, agents will translate their price expectations or predictions into orders or demand functions, which will be cleared using a specific market mechanism, and finally a new market price will be formed. In agent-based financial markets, the behavioral aspects of individual agents are not determined only through fixed strategies, but also through learning (evaluating and updating strategies based on the past performance) and social interaction (exchanging/combining information and/or strategies) with other market participants, which can lead to herding type of behavior. Such aspects are sometimes conceptualized and modeled using the paradigm of GAs [63, 65, 66]. This idea is explored in the many-type models, where the pool of strategies is co-evolving with market conditions in order to see which ones will survive and which ones will fail. This has been implemented in LeBaron’s model [63]. Depending on their strategies, investors can use divergent information sources to assist their decision making and update their expectations. All investors are informed about the current market price which can be used to update the value of their portfolio and sometimes evaluate their strategies. Often there is an information asymmetry in the sense that some investors also know the fundamental price or the properties that allow them to estimate it (e.g. dividend process). The investment environment presented in Chapter 2 describes a number of asset classes existing in real markets. Asset classes modeled in agent-based markets are usually only a few, and they typically include risky assets (stocks), risk-less assets (bonds), and cash (Figure 3.3). In addition, many agent-based models generate and study time-series of only one risky asset. At first, this may seem as a very strong restriction, especially for models that aim to be realistic. However, that is not necessarily a strong restriction, if the focus is on modeling the
financial market as a whole, in which case the risky asset could be interpreted as a market index [59].

Figure 3.3: Investment environment in ASM

Four different types of pricing mechanisms described in the literature [8,32,70,75] are:

(a) Temporary market equilibrium is a market clearing mechanism where the price is determined so that the total demand of agents equals the total number of shares in the market. Given that the demand of each agent typically has to be determined for different hypothetical prices until the equilibrium price is found, this mechanism can be computationally costly.

(b) Price impact function formalizes a price adjustment process in which the new price is determined from the past price and net order (the difference between demand and supply) scaled by a fixed parameter. This captures the basic intuition that excess demand raises the price, while excess supply lowers the price. The main advantage of this pricing mechanism is that it is computationally very fast, while some of the disadvantages are that the price changes are very sensitive to the choice of the liquidity parameter, and the issue of who fills in the excess demand or supply in the market [70]. A variant of this mechanism is a log-linear price impact function which additionally ensures that the price remains positive [32].

(c) Order book is a pricing mechanism where actual order book is simulated, and buy and sell orders of agents are crossed using a certain well-defined procedure. This approach can be considered more realistic because it allows
a detailed analysis of trading mechanisms [70]. This particularly contrasts with the equilibrium-based mechanism where explicit trading is not modeled. (d) Matching is the type of mechanism where agents (randomly) meet, and, if it suits them, trade with each other. This mechanism may be appropriate for situations where no formal trading markets have been established [70].

3.7 Summary

The thrust of this chapter is a survey on agent-based ASM. The large scale of design and implementation approaches applied in the ASM studied here demonstrates that there are many methods to represent traders, to determine a forecasting strategy, to implement learning, to construct a portfolio and develop other decision strategies that lead the investors to place certain orders. In addition a variety of order execution and price setting mechanisms are represented. Several ASM have been considered and analyzed from the point of view of how they cover the important organizational and behavioral aspects of stock markets. A vast number of trading strategies in a broad range of market organizations is illustrated by the ASM. The model proposed by LeBaron [63] is inferred to be a suitable model to represent the BSE. This model is a “many-type model” with the following characteristics/ advantages over the other models surveyed:

- Produces realistic outputs of a stock market, which are qualitatively similar to the empirical data.
- Displays realistic representations of agents’ behavior, agents learning by evaluating and updating strategies based on the past performance.
- Demonstrates social interaction (exchanging/combining information and/or strategies with other market participants), conceptualized and modeled using the paradigm of GAs, where the pool of strategies is co-evolving with market conditions in order to see which ones will survive and which ones will fail.
- Incorporates realistic market mechanisms including price discovery.