This thesis entitled “Integer Optimization in Sample Surveys” is submitted to the Aligarh Muslim University, Aligarh, India, to supplicate the degree of Doctor of Philosophy in Statistics. It consists of the research work carried out by me in the Department of Statistics and Operations Research, Aligarh Muslim University, Aligarh, India.

The problem of deriving statistical information on several population characteristics, based on sample data, can be formulated as an optimization problem in which we wish to minimize the cost of the survey, which is a function of sample size, size of the sampling unit, the sampling scheme and the scope of the survey, subject to the restrictions that the loss in precision for the various characteristics arising out of making decisions on the basis of sample results are within a certain prescribed limit. Or alternatively, we may minimize the loss in precision, subject to the restriction that the cost of the survey is within the given budget. Thus we are interested in finding the optimal sample size or the optimal sampling scheme which will enable us to obtain estimates of the population characteristics with prescribed properties.

In stratified sampling the population is first divided into mutually exclusive and exhaustive groups called strata. An important problem in stratified sampling is the determination of sample sizes (allocations) for different strata. They may be chosen to minimize the sampling variance of the estimator for a fixed cost or to minimize the total cost of the survey for a desired precision.

The solution of the above problem for univariate cost i.e., when a single characteristic is studied on each and every population unit, exits in sampling literature. However,
the multivariate case is more complicated and few attempts have been made to crack the problem so far.

The fundamental problem of optimization is to arrive at the best possible decision in any given set of circumstances. However, sometimes what is best for one person is worst for another and more often we are not at all sure what is meant by best. The first step, therefore in mathematical optimization is to choose some quality, typically a function of several variables, to be maximized or minimized, subject possibly to one or more constraints. The next step is to choose a mathematical programming to solve optimization problem; such methods are usually called optimization techniques or algorithms.

The work in this thesis is spread over in seven chapters, starting with an introductory chapter.

Chapter I provides an introduction to the historical background of survey sampling, Non-Response in sample surveys, Optimization problem arise in Stratified Sampling, Mathematical programming techniques & their applications to various fields including sampling.

Chapter II: This chapter is based on my research paper “Optimum sample sizes in case of stratified sampling for non-respondents: an integer solution” published in International Journal of Operations Research and Optimization. Here the problem is to determine the sample sizes for the fixed total sample size to various strata in presence of non-response and then to find the optimum sub sample sizes among the non-respondents in stratified sampling. The integer solution is found by using branch and bound method. A computer program in C++ for calculating sample sizes and sub-sample sizes are included in this chapter.
Chapter III: This chapter is based on my research paper “A new cutting plane method for finding the integer solution of allocation problem in Stratified Sampling” published in *International Journal of Operations Research and Optimization*. Here we have developed a cut using the concept of greatest integer that reduces the feasible region of the original problem. We then search the integer point inside this feasible region. A numerical example is presented to illustrate the computational procedure. A computer program in \texttt{C++} for calculating sample sizes and value of the objective function are also contained in this chapter.

Chapter IV: This chapter is based on my research paper “A new cut applied to the formulated allocation problem of Stratified Sampling using Confidence interval where pre specified levels of precision are desired” published in *International Journal of Computer Engineering*. In this chapter, we have used a New Cut for reducing the feasible region of the stratified sampling problem formulated as a Non Linear Integer Programming problem (NLIPP) using confidence interval and solved the reduced problem for the integer solution by Lingo Software.

Chapter V: This chapter is based on my research paper “Goal Programming and Lexicographic goal programming approaches in bi-objective stratified sampling: An Integer Solution” accepted in *International Journal of Scientific Computing*. In this chapter goal programming (GP) and Lexicographic goal programming (PGP) approaches are used for allocating the sample sizes in stratified random sampling problem. We have discussed the methods how to solve the allocation problems in stratified sampling & compared both the methods.

Chapter VI: This chapter is based on my research paper “Allocation of Sample Size in bi-objective Stratified Sampling using Lexicographic Goal Programming” communicated in *International Journal of Engineering, Science and Technology*. In
this chapter we have discussed allocation of sample size in stratified sampling using $D_1$-distances. The solution corresponding to the minimum distance is the best compromise solution. A numerical example is presented to illustrate the computational procedure.

Chapter VII: This chapter is based on my research paper “Allocation in Multivariate Stratified Sampling in Presence of Non-Response: A New Approach” communicated in *Communications in Statistics - Simulation and Computation*. In this chapter the proposed Lagrange Multiplier’s Technique (LMT) provide a minimum coefficients of variance among the non-respondent as compared to the Goal programming technique. The numerical example is presented to illustrate the computational details.

A comprehensive list of references, arranged in alphabetical order is also provided at the end of the thesis.