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Effect of alpha Q-value on incomplete fusion

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Effect of $\alpha$-$Q$ value on incomplete fusion

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The probability of incomplete fusion in $^{13}\text{C}$+$^{159}\text{Tb}$ interactions has been measured in the energy range $\approx$4–7 MeV/nucleon. The variation of the incomplete fusion fraction has been studied in terms of projectile energy and type. Present results are compared with the existing $^{13}\text{C}$+$^{159}\text{Tb}$ data, where a strong projectile structure effect on the incomplete fusion fraction has been observed. It has been found that the probability of incomplete fusion is higher in the case of $^{12}\text{C}$ than for a one-neutron rich $^{13}\text{C}$ projectile. For better insight into the projectile structure effect, a systematic study is presented on the incomplete fusion measured in $^{12,13}\text{C}$+$^{16}\text{O}$+$^{199}\text{Tb}$ and $^{12,13}\text{C}$+$^{16}\text{O}$+$^{181}\text{Ta}$ systems by Singh et al. [Phys. Rev. C 80, 014601 (2009)] and by Babu et al. [J. Phys. G 29, 1011 (2003)]. The present analysis indicates a strong dependence of incomplete fusion probability on the $\alpha$-$Q$ value of the projectile at these low energies.

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I. INTRODUCTION

Much interest has been shown in recent years in the study of incomplete fusion (ICF) reaction dynamics in heavy-ion interactions at low incident energies, i.e., from slightly above barrier energies to well above them [1–10]. The observation of ICF events in heavy-ion (HI) induced reactions dates back to the 1970s, when Britt and Quinton first observed the fast $\alpha$ particles in massive transfer reactions at $E_{\text{lab}} \gtrsim 10.5$ MeV/nucleon [11]. Since then ICF has been extensively investigated and established as one of the competing modes of reaction at $E_{\text{lab}} \approx 7–10$ MeV/nucleon [2,8–10,12–17]. Although, complete fusion (CF) has been considered to be the sole contributor to the total fusion cross section at these energies [18,19], recent studies demonstrate substantial ICF contributions at energies $<10$ MeV/nucleon [2–4,8–10].

A variety of theoretical models have been proposed to understand ICF reaction dynamics [1,12–14,20–22]. The most widely used and accepted descriptions of ICF are based on the breakup fusion (BUF) [13] and sum-rule models [21,22]. According to the BUF model, CF and ICF events can be disentangled on the basis of the degree of linear momentum transfer (LMT) from the projectile to the target nucleus. In the case of CF, entire nucleonic degrees of freedom of projectile and target nucleus blend to form an equilibrated compound nucleus (CN) with predetermined physical properties, e.g., charge, mass, recoil velocity, etc. However, the ICF events originate from the fractional LMT followed by projectile breakup. It may be pointed out that the additional breakup degrees of freedom may give rise to several reaction processes, such as (a) the noncapture breakup (NCBU), when none of the breakup fragments are captured, (b) sequential complete fusion (SCF), the successive capture of all the projectile fragments by the target nucleus, and (c) incomplete fusion (ICF), when one of the breakup fragments is captured. Experimentally, it is not possible to distinguish normal and sequential CF events because of the identical residues in the exit channel. On the other hand, the sum-rule model takes driven input angular momenta ($\ell$ values) into account to describe CF and ICF processes. The $\ell$ values from $\ell = 0$ to $\ell_{\text{crit}}$ lead to the CF events; however, for $\ell \geq \ell_{\text{crit}}$, ICF events are expected to set in. In the latter case, the $P^\ell$ values higher than $\ell_{\text{crit}}$ the absence of potential pocket forbids fusion until a part of the projectile is released ($P^\ell_{\text{spectator}}$) to provide sustainable input angular momenta [21–23]. After such an emission, the remnant ($P^\ell_{\text{participant}}$) is supposed to carry input angular momenta less and/or equal to its own critical limit ($\ell_{\text{eff}} \leq \ell_{\text{crit}}$) for fusion to occur.

The LMTs in CF and/or ICF events obtained from the analysis of recoil-momentum distributions measured at different energies have been explained fairly well by the BUF model, and suggest the onset of ICF even at slightly above barrier energies [10]. Other existing models and theories explain the ICF data obtained at energies $\gtrsim 7$ MeV/nucleon to some extent, but completely fail at lower energies [12,14,20]. Contrary to the experimental observations, the sum-rule model predicts negligibly small ICF cross sections at $\approx$4–7 MeV/nucleon [10,24]. Existence of ICF at low incident energies and/or below the values of $\ell_{\text{crit}}$ (for CF) has been claimed by different groups [25–27]. In addition to this, the unclear or ambiguous dependences of ICF on various entrance channel parameters, viz., projectile type and energy, driving angular momentum ($\ell$) into the system, binding energy and/or $\alpha$-$Q$ value ($Q_{\alpha}$), mass asymmetry ($\mu_A = A_T/(A_T + A_P)$), deformation of interacting partners, etc., are also required to

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be explored. Several contradicting dependences of the fraction of incomplete fusion ($F_{ICF}$), which is a measure of relative strength of ICF to the total fusion, have been discussed in recent reports [3, 4, 24–30]. In Refs. [4, 25] it has been found that the $F_{ICF}$ is independent of the target charge ($Z_T$) and thus from $Z_PZ_T$. However, in Ref. [26] of the same group the fusion suppression is predicted to be almost proportional to the charge $Z_T$ of target nucleus. In a recent paper by Gomes et al. [27] a trend of systematic behavior for the $F_{ICF}$ as a function of the $Z_T$ is discussed. Morgenstern et al. [28] correlated the ICF fraction with entrance channel mass asymmetry ($\mu_\lambda$). Recently, Singh et al. [29] supplemented Morgenstern’s mass-asymmetry systematics by introducing the importance of projectile structure. Apart from this, one of our recent works [30] reported the dependence of $F_{ICF}$ on the target mass or $Z_P Z_T$ of interacting partners for a wide range of projectile-target combinations. Furthermore, Geoffroy et al. [15] suggested the origin of ICF events from undamped noncentral interactions. The noncentral nature of ICF events has also been emphasized by Trautmann et al. [16], Inamura et al. [17], and Zolowonski et al. [31]. In an outstanding review, Gerschel [32] presented several dependences of ICF. In the case of rare-earth targets, the ICF has been found to be originated from relatively high $\ell$ values [33], but the results obtained by Tricoire et al. [34] with semimagic targets suggest the contribution of ICF events from even smaller than 0.5$\ell_{cut}$ [35, 36]. Almost similar conclusions have been drawn by Tseruya et al. [37] and Oeschler et al. [38], who observed both CF and ICF below and above the value of $\ell_{cut}$.

The ambiguous dependence of ICF on various entrance channel parameters needs serious attention. To investigate ICF reaction dynamics in detail, we have undertaken a program to study ICF fractions in terms of various entrance channel observables. In this work, the ICF fraction ($F_{ICF}$) has been deduced from the analysis of experimental excitation functions (EFs) of individual reaction residues produced in the $^{13}$C + $^{159}$Tb system at energies $\approx$4–7 MeV/nucleon. The present results are compared with the existing $^{12}$C + $^{159}$Tb data [30]. This reveals the first sign of an $\alpha$-$Q$ value effect on the ICF fraction.

II. EXPERIMENTS

The experiments were performed using the 15UD-Pelletron accelerator of the Inter-University Accelerator Center (IUAC), New Delhi, India, employing an activation technique. The experimental setup and procedures are the same as in Ref. [30]. Here, a short account of experimental conditions are given, and we refer the reader to the recent paper [30] for further details. Natural $^{159}$Tb targets of thickness $\approx$1.2 mg/cm$^2$ and Al foils of thicknesses $\approx$1.5–2.5 mg/cm$^2$ were prepared by a rolling technique. Each target was backed by an Al foil of appropriate thickness (hereafter called the target-catcher foil assembly) to stop heavy recoiling products produced in the reactions. To cover a wide energy range in the limited beam time, a stacked-foil energy degradation procedure was used. Five stacks, with three target-catcher foil assemblies in each, were bombarded by a $^{13}$C beam at energies $E_{lab}$ $\approx$58, 60, 70, 73, 85, and 88 MeV with beam intensities $\approx$3–4 pnA. The target-catcher foil assemblies were taken out from the scattering chamber for off-line activity measurements. The activities produced in the individual target-catcher foil assemblies were counted with a precalibrated HPGe detector coupled to an in-line CAMAC data-acquisition system [39]. The HPGe detector was calibrated for energy and efficiency using standard $\gamma$ sources of known strength. The evaporation residues were identified by their characteristic $\gamma$ lines, and verified by their decay-curve analysis. Figure 1 shows a part of the $\gamma$ spectra and the decay curve of $^{168}$Lu ($t_{1/2} = 5.5$ min) residues (inset of Fig. 1) populated via $^{13}$C + $^{159}$Tb interactions at $E_{lab} \approx$ 87.6 $\pm$ 0.38 MeV. Some of the $\gamma$ lines are marked with the corresponding evaporation residues. The energy-dependent production cross section of evaporation residues ($\sigma_{ER}$) have been determined [24]. The overall error in the measured $\sigma_{ER}$ is estimated to be $\approx$15%. A detailed discussion on error analysis is given elsewhere [30].

The excitation functions (EFs) of residues $^{166, 168}$Lu ($\alpha x n$; $x = 3–5$), $^{167}$Yb ($\alpha 4n$), $^{166, 165, 163}$Tm ($\alpha x n$; $x = 2, 3, 5$) and $^{162, 161, 160}$Ho ($2 \alpha x n$; $x = 2–4$) produced in $^{13}$C + $^{159}$Tb interactions in the energy range $\approx$1.01$V_b$ to 1.68$V_b$ ($V_b \approx$ 52 MeV) have been measured and are analyzed within the framework of the statistical model code PACE4 [40]. Detailed definition and listing of input parameters of this code are presented elsewhere [29, 30, 40–42]. The code PACE4 takes formation and decay of CF events into account according to the Hauser-Feshbach theory of CN decay, therefore, any deviation in the experimental EFs from the PACE4 calculations may be attributed to the onset of ICF. In this code, the level density parameter ($\alpha = A/K$) is an important input parameter which may be varied to reproduce the experimental EFs.

A. Analysis and interpretation of results

The experimentally measured and theoretically calculated EFs of all $\alpha x + px n$ channels ($\Sigma_{\alpha x + px n}$, i.e., the sum of the cross sections of $^{169, 168, 167}$Lu and $^{167}$Yb residues) are

![Graph](image-url)
FIG. 2. (Color online) (a) Sum of cross sections for $xn$ and $pxn$ channels to calibrate the parameters of PACE4 code for the $^{13}$C $+$ $^{159}$Tb system, which shows production of these channels via the CF process. (b) Comparison of cross section of $\alpha$-emitting channels with PACE4 code, which shows enhancement over theoretical predictions with same set of parameters as used to reproduce $xn$ and $pxn$ channels (for details see the text).

FIG. 3. (Color online) (a) Comparison of total, complete, and incomplete fusion cross sections for $^{13}$C $+$ $^{159}$Tb system. (b) Comparison of $F_{\text{ICF}}$ for $^{12}$C $+$ $^{159}$Tb and $^{13}$C $+$ $^{159}$Tb systems (for details see the text).

The $^{166,165,163}$Tm and $^{162,161,160}$Ho residues are likely to be populated via CF and/or ICF in the following ways:

(i) CF: the projectile $^{13}$C completely fuses with the target nucleus $^{159}$Tb to form an exited system $^{172}$Lu*, which eventually decays via light nuclear particles and/or one or two $\alpha$ clusters together with the neutrons and/or protons to produce Tm and Ho isotopes.

(ii) ICF: the projectile may break up into its constituent $\alpha$ clusters (i.e., $^{13}$C $\rightarrow ^{8}$Be $+$ $^{4}$He $+$ $n$). One of the fragments fuses with the target nucleus to form a reduced CN, and the remnant behaves as a spectator. The reduced CN may also decay via neutron and/or proton emission to reach the aforementioned isotopes.

As shown in Fig. 2(a), the enhancement in the production cross sections for $\alpha$-emitting channels over the PACE4 calculations increases with the incident energy, which directly correlates the incident energy and ICF fraction. To confirm this aspect, the data presented in Fig. 2 have been analyzed using a well-established data reduction procedure [29,30]. The fraction of ICF in $\alpha$-emitting channels has been accounted as $\sum \sigma_{\text{ICF}} = \sum \sigma_{\text{expt}} - \sum \sigma_{\text{th}}$, and is plotted as a function of energy in Fig. 3(a). The lines and curves are the outcome of best-fitting procedure. To show how the ICF contributes to the total fusion cross section, the sum of all CF channels ($\sum \sigma_{\text{CF}}$) is also plotted in Fig. 3(a) along with total fusion (i.e.,

compared in Fig. 2(a) with corresponding PACE4 calculations. It is not out of place to mention that the evaporation residue $^{167}$Yb ($p4n$) is found to be strongly fed from its higher charge isobar (precursor hereafter) $^{167}$Lu ($5n$) through $\beta^+$ emission. As such, the independent production cross section of $^{167}$Yb has been deduced using the prescription given in Ref. [43]. Lines and symbols are self-explanatory. As can be seen from this figure, the PACE4 calculations reproduce, fairly well, the experimental data with a value of level density parameter $a = A/8 \text{ MeV}^{-1}$. This confirms the population of $^{169}$, $^{168}$, $^{167}$Lu ($xn; x = 3$–$5$), and $^{167}$Yb ($p4n$) residues via CF of $^{13}$C with $^{159}$Tb. As such, the value of $a = A/8 \text{ MeV}^{-1}$ can be used as the default parameter for further analysis. To figure out if the $\alpha$-emitting channels are populated via CF, the experimental EFs of all $\alpha$-emitting channels ($\sum \sigma_{\alpha xn} + 2 \sum \sigma_{\alpha pxn}$, i.e., the sum of the cross sections of $^{166,165,163}$Tm and $^{162,161,160}$Ho residues) are compared with the predictions of PACE4 in Fig. 2(b). The calculations are performed using the same set of input parameters used to reproduce the $xn$ and $pxn$ channels. As can be seen from Fig. 2(b), the experimental EFs are significantly enhanced as compared to the PACE4 predictions, which points towards the observation of ICF contributions at these energies.
ICF strength functions for $^{13}$C 

out how the projectile structure affects the ICF strength, the $^{13}$C induced reactions. The strikingly different ICF fractions $\sigma$ value at a constant $\alpha$ is a well-known $\sigma$-cluster nucleus with $Q_\alpha \approx -7.37$ MeV. However, $^{13}$C has a larger $Q_\alpha$ value ($\approx -10.64$ MeV) than $^{12}$C. The higher $Q_\alpha$ value for $^{13}$C translates into the smaller breakup probability into constituent $\alpha$ clusters, resulting in a smaller ICF fraction than for $^{12}$C induced reactions [44].

To validate the above-mentioned $Q_\alpha$-value systematics, the probability of ICF ($\%F_{\text{ICF}}$) has been deduced for $^{12}$C, $^{13}$C, and $^{16}$O induced reactions on the two sets of targets $^{159}$Tb [29,30] and $^{181}$Ta [45,46] at a constant relative velocity $v_{\text{rel}} = 0.053$ [29,30,45,46], and plotted with $Q_\alpha$ values in Fig. 4. The values of $F_{\text{ICF}}$ for all six projectile-target combinations are found to follow the same trend as observed for $^{12}$C,$^{13}$C + $^{159}$Tb systems presented in Fig. 3(b). The probability of ICF is found to be less for larger $Q_\alpha$-value projectiles. For example, the value of $F_{\text{ICF}}$ for the $^{16}$O ($Q_\alpha \approx -7.16$ MeV) + $^{159}$Tb system [29] is found to be $\approx 19\%$ which is reduced to only $\approx 3\%$ for the $^{13}$C ($Q_\alpha \approx -10.64$ MeV) + $^{159}$Tb system [46]. The same systematics was followed for the $^{181}$Ta target. Hence, from the data presented in Fig. 4, it can be inferred that the $Q_\alpha$ value is an important entrance channel parameter which essentially dictates the probability of ICF.

Further, as shown in Fig. 4, the value of $F_{\text{ICF}}$ is found to be $\approx 3\%$ and $\approx 7\%$ for $^{13}$C + $^{159}$Tb and $^{13}$C + $^{181}$Ta, $\approx 9\%$ and $\approx 11\%$ for $^{12}$C + $^{159}$Tb and $^{12}$C + $^{181}$Ta, and $\approx 19\%$ and $\approx 15\%$ for $^{16}$O + $^{159}$Tb and $^{16}$O + $^{181}$Ta, respectively. The value of $F_{\text{ICF}}$ for $^{16}$O + $^{181}$Ta is expected to go up; as indicated in Ref. [45], all the $\alpha$ channels could not be measured for this system. The value of $F_{\text{ICF}}$ for the given projectile-target combinations supports Morgenstern’s mass-asymmetry systematics [28] along with the projectile structure supplement given by Singh et al. [29].

III. CONCLUSION

In summary, the probability of low energy ICF has been measured in the $^{13}$C + $^{159}$Tb system from the analysis of differential EFs within the framework of statistical model code PACE4 . To the best of our knowledge, the first sign of an $\alpha$-$Q$-value effect on the ICF fraction has been observed for strongly bound projectiles. The fraction of ICF has been found to decrease for projectiles having large negative $\alpha$-$Q$ values. If confirmed for other projectile-target combinations, this may provide an important input to understanding the complex ICF dynamics at low incident energies. More experiments are planned to cover this aspect thoroughly.

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FIG. 4. (Color online) Comparison of $F_{\text{ICF}}$ on the basis of $Q_\alpha$ value at a constant $v_{\text{rel}} = 0.053$ (for details see the text).

$\sigma_{\text{TF}} = \Sigma \sigma_{\text{CTF}} + \Sigma \sigma_{\text{ICF}}$ as a function a energy. The onset of ICF is clearly evident at energies as low as $\approx 63$ MeV (i.e., 21% above the barrier) and increases almost linearly for higher energies.

For better insight into the onset and influence of ICF in terms of various entrance channel parameters, the percentage fraction of ICF ($F_{\text{ICF}}$) has been deduced from the analysis of data presented in Fig. 3(a). The $F_{\text{ICF}}$ is a measure of the relative strength of ICF to the total fusion, defined as $F_{\text{ICF}}(\%) = (\Sigma \sigma_{\text{ICF}}/\sigma_{\text{TF}}) \times 100$. The mapping of ICF strength with incident energy is termed the ICF strength function [29]. To figure out how the projectile structure affects the ICF strength, the ICF strength functions for $^{13}$C + $^{159}$Tb and $^{12}$C + $^{159}$Tb (from Ref. [30]) systems are plotted in Fig. 3(b). The energy axis is normalized to correct for the different Coulomb barriers of the two systems. As shown in this figure, the probability of ICF (% $F_{\text{ICF}}$) for the $^{13}$C projectile is noticeably smaller than that for the $^{12}$C projectile in the entire energy range. In case of $^{12}$C, the onset of ICF is at a relatively lower energy (i.e., 1.1 $V_b$) than for $^{13}$C induced reactions. The strikingly different ICF fractions for $^{13}$C and $^{12}$C induced reactions point mainly towards the projectile structure effect. It may be pointed out that $^{12}$C is a well-known $\alpha$-cluster nucleus with $Q_\alpha \approx -7.37$ MeV.
EFFECT OF $\alpha$-$Q$ VALUE ON INCOMPLETE FUSION

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Effect of entrance-channel parameters on incomplete fusion reactions

Effect of entrance-channel parameters on incomplete fusion reactions

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In order to disentangle the contribution of complete and incomplete fusion components and to study their
dependence on various entrance-channel observables, the measurement and analysis of forward recoil ranges,
which is the direct measure of linear momentum transfer from projectile to the target nuclei, has been done
in the interaction of $^{12}$C beam with $^{159}$Tb target nucleus at three distinct above-barrier energies $\approx$74, 80, and
87 MeV. The recoil-catcher technique followed by off-line $\gamma$-ray spectroscopy has been used. The complete and
incomplete fusion events have been tagged by full and partial linear momentum transfer components, respectively.
The observed incomplete fusion events have been explained on the basis of the breakup fusion model where these
events may be attributed to the fusion of $^{8}$Be and/or $^{4}$He from $^{12}$C projectile to the target nucleus. Analysis of the
data indicates that the incomplete fusion has significant contribution at the studied energies and its contribution
has been found to increase with the beam energy. An attempt has also been made to understand the projectile
structure effect on the underlying reaction dynamics.

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I. INTRODUCTION

The interest to study the incomplete fusion (ICF) reactions at energies $\approx$4–7 MeV/nucleon has increased recently due to
the observation of these reactions at such low energies, where complete fusion (CF) is expected to be the sole contributor
to the total fusion cross section [1–10]. In the case of CF, the projectiles for $\ell < \ell_{\text{crit}}$ merge with the target nucleus, with the
dominance of the nuclear force field, leading to the formation of a completely fused excited composite system. However, in
the case of ICF, for $\ell > \ell_{\text{crit}}$, the projectile may break up into clusters, and one of the clusters may fuse with the target
nucleus, forming the reduced excited composite system with relatively less mass, charge, and excitation energy compared
to the completely fused composite system. The remnants flows in the forward direction with almost beam velocity. Due to
this partial fusion of the projectile, the fractional momentum transfer takes place in the ICF processes. Each of these
processes (CF and ICF) leads to the characteristic velocity distribution of the reaction residues. As such, the distribution
of measured yields of the residues as a function of velocity and/or the ranges in a stopping medium may give an insight
into the reaction mechanism involved. Though the differences in the velocity and ranges of CF and ICF reaction products are
not so significant, by using very thin catcher foils ($\approx \mu g/cm^2$), it is possible to separate the CF and ICF residues. The breakup
of heavy-ion projectiles may also be understood on the basis of disappearance of the fusion pocket in the one-dimensional
effective potential energy curve, as the angular momentum ($\ell$) increases beyond the critical limit ($\ell_{\text{crit}}$) of complete fusion. In

order to provide sustainable input angular momentum and/or to restore the fusion pocket in the potential energy curve, the
projectile may break up into clusters and a part fuses with the target nucleus, while the other may escape and carries away the
excess angular momentum. As such, there is a deficit in the linear momentum of composite system, compared to the total
linear momentum [11–16]. It may be pointed out that there are conflicting reports on the dependence of ICF on the angular momenta. The $\gamma$-multiplicity measurements done by Inamura et al. [17], Wilczynski et al. [18], Gerschel et al. [19], and Trautmann et al. [20] showed that ICF involves $\ell \geq \ell_{\text{crit}}$. However, studies [21] on spherical targets showed involvement of $\ell$ in ICF lower than $\ell_{\text{crit}}$ for CF. This suggests that ICF competes with CF even at $\ell \leq \ell_{\text{crit}}$, contrary to the hypothesis of the SUMRULE model [22,23] of ICF. Parker et al. [4] observed forward peaked $\alpha$ particles in reactions of $^{12}$C on $^{51}$V at $E \approx 6$ MeV/nucleon. Morgenstern et al. [24] measured the velocity spectra of evaporation residues (ERs) in reactions of $^{12}$C, $^{20}$Ne beams of energies $\approx$10–25 MeV/nucleon with $^{40,44,48}$Ca, $^{58,60,62}$Ni targets, where the deviation in velocity spectra from the mean velocity of complete fusion has been observed, indicating the incomplete momentum (mass) transfer from projectile to the target nucleus. Tseruya et al. [21] found the evidence for ICF from time-of-flight (TOF) measurements of ERs in the reactions of $\approx$5.5–10 MeV/nucleon $^{12}$C with $^{120}$Sn, $^{160}$Gd, and $^{197}$Au. In one of our recent letters [25] it has also been shown that the ICF reactions may originate during the peripheral interactions.

The breakup fusion model [26,27], hot spot model [28], SUMRULE model [22,23], promptly emitted particles [29],
and exciton model [30,31] etc., are some of the models generally used to describe such reactions. These models
are found to fit the experimental data at projectile energies, $E_{\text{lab}} \geq 10$ MeV/nucleon to a large extent. However, the onset
of ICF from just near to well above the Coulomb barrier energies observed recently triggered a resurgence of interest in the study of ICF dynamics at low energies [32–37]. However, some of the most debated and outstanding issues related to low-energy ICF have been, (i) the energy dependence of ICF processes, (ii) the localization of the ℓ window, (iii) the usefulness of ICF to populate high-spin states in final reaction products, and (iv) the effect of entrance-channel parameters on the onset and strength of ICF. In recent years, high-quality experimental data on cross sections [9], spin distributions (SDs) of residues [25], and linear momentum distributions [16] of reaction products have been obtained at the Inter-University Accelerator Center (IUAC), New Delhi. These studies concluded that the ICF contributes at low energies, but is limited only to a few projectile-target combinations. The measurement of forward recoil range distributions (FRRDs) can be used as one of the irrefutable methods to distinguish the different ICF components, where the same residue may be formed by more than one fusion channel. In the present work, in order to facilitate the experimental disentanglement of these competing processes (CF and ICF), the FRRDs of reaction residues populated in $^{12}$C + $^{159}$Tb interactions at three beam energies ≈74, 80, and 87 MeV have been measured. In the present work, an attempt has also been made to have quantitative information of ICF reactions. The present work is in continuation of our recent investigation on the same system $^{12}$C + $^{159}$Tb, where the measurement and analysis of excitation functions have been used to investigate the role of breakup processes [35,36].

The present paper is organized as follows. A brief description of the experimental setup is given in Sec. II, while Sec. III deals with the details of the measurements and analysis of RRDs, and finally the conclusion is presented in Sec. IV.

II. EXPERIMENTAL DETAILS

The experiments have been performed using $^{12}$C ion beam delivered from the 15UD-Pelletron accelerator at the Inter-University Accelerator Center (IUAC), New Delhi, India. Although the experimental methodology is similar to that in our earlier work [16,33], for quick reference a brief description is given here. In the present work, three different stacks, each consisting of a $^{159}$Tb target (abundance = 100%) followed by a series of thin Al-catcher foils (different in each irradiation depending on the energy of the beam), to trap the recoiling residues, have been irradiated separately by the $^{12}$C beams of ≈74, 80, and 87 MeV energy. The targets were prepared by the rolling method, while, the thin Al-catcher foils were made by vacuum evaporation technique. The thickness of each sample and catcher foil has been measured by the α-transmission method. The thickness of the target was ≈190 µg/cm², however, the thicknesses of Al catchers ranged from ≈15–50 µg/cm². The samples were pasted on rectangular Al holders (size ≈2.5 x 2.1 cm²) having concentric holes of 10 mm diameter. The irradiations have been performed in the general purpose scattering chamber (GPSC) having an invacuum transfer facility (ITF). A stack of the thin Al-catcher foils (sufficient to stop the compound nucleus (CN) formed via full linear momentum transfer) was placed just after the target, so that the heavy recoiling residues populated via CF and/or ICF could be trapped at their respective ranges in Al-catcher foil thicknesses. The irradiations have been carried out for ≈12 h, with a beam current ≈4pnA. The total charge collected in the Faraday cup has been used to obtain the beam flux during the irradiations. The activities produced in each Al-catcher foil have been recorded separately using a precalibrated high-resolution HPGe spectrometer of 100 c.c. active volume coupled to the CAMAC-based CANDLE [38] software. The resolution of the γ-ray spectrometer was ≈2 keV, for 1.33 MeV γ-ray of $^{152}$Eu source. The geometry-dependent efficiency of the HPGe detector for various γ-ray energies at different source-detector separations was determined using the standard $^{155}$Eu source. The identification of populated reaction products have been made on the basis of their characteristic γ-ray energies, and has been further confirmed by measuring their half-lives as well. A list of identified reaction residues populated in $^{12}$C + $^{159}$Tb interactions are tabulated in Table I, along with their spectroscopic properties [39,40]. The present technique of measuring cross sections has been found to work well in the mass region of interest with well established level schemes where the γ lines are well separated for different residues.

III. ANALYSIS AND INTERPRETATION OF RESULTS

As has been mentioned earlier, the measurement of the projected ranges of the reaction products gives the degree of linear momentum transfer (ρLMT) from the projectile to the target nucleus and thus is an irrefutable method to disentangle the CF and/or ICF reactions. The velocity distribution of a given type of reaction products is symmetric about $v_γ$, having a width that depends upon the evaporation process and, in particular, on the particles evaporated from CN. The mean

<table>
<thead>
<tr>
<th>Residue</th>
<th>$T_{1/2}$ (min)</th>
<th>$J^\pi$</th>
<th>$E_γ$ (keV)</th>
<th>$I^\gamma$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{168}$Lu$^{+2+}$ (3n)</td>
<td>5.5</td>
<td>3$^+$</td>
<td>198.86</td>
<td>180.0$^*$</td>
</tr>
<tr>
<td>$^{167}$Lu (4n)</td>
<td>51.5</td>
<td>7/2$^+$</td>
<td>213.21</td>
<td>3.5</td>
</tr>
<tr>
<td>$^{165}$Lu (6n)</td>
<td>10.74</td>
<td>1/2$^+$</td>
<td>120.58</td>
<td>25</td>
</tr>
<tr>
<td>$^{167}$Tb (3p3n)</td>
<td>17.5</td>
<td>5/2$^-$</td>
<td>176.2</td>
<td>20.4</td>
</tr>
<tr>
<td>$^{165}$Tm (α2n)</td>
<td>30.06</td>
<td>1/2$^+$</td>
<td>242.85</td>
<td>35</td>
</tr>
<tr>
<td>$^{163}$Tm(α4n)</td>
<td>1.81</td>
<td>1/2$^+$</td>
<td>190.07</td>
<td>1.28</td>
</tr>
<tr>
<td>$^{161}$Ho(2α2n)</td>
<td>2.48</td>
<td>7/2$^-$</td>
<td>103.03</td>
<td>3.6</td>
</tr>
<tr>
<td>$^{160}$Ho(2α3n)</td>
<td>25.6</td>
<td>5$^-$</td>
<td>645.25</td>
<td>16.20</td>
</tr>
<tr>
<td>$^{160}$Ho$^{+6+}$ (2α3n)</td>
<td>5.02</td>
<td>2$^-$</td>
<td>645.25</td>
<td>16.20</td>
</tr>
</tbody>
</table>

These intensities are relative.
where, $P_{\text{frac}}$ is the linear momentum of the fused fraction of projectile. As already mentioned, the projectile and $P_{\text{proj}}$ is the entire linear momentum of the projectile. As already mentioned, $\rho_{\text{LMT}}$ is proportional to the fused mass of the projectile (i.e., maximum LMT may give maximum recoil velocity to the reaction products). This is a promising way of investigating the full momentum transfer in the case of the complete-fusion process, and relatively small momentum transfer in a partial momentum transfer reaction (ICF). Since, in the CF process, the maximum $\rho_{\text{LMT}}$ is transferred from the projectile to the target nucleus, therefore, for a given entrance channel the CN has predetermined mass, energy, and momenta. While, in the case of ICF, a partial $\rho_{\text{LMT}}$ results due to the formation of an incompletely fused composite system in the excited state. For an incompletely fused composite system, the mass, energy, and momenta of CN may not have unique values. This may be because of the fluctuations in the fused mass from the projectile to target nucleus and various interaction trajectories. Thus, the experimentally measured forward recoil ranges of final reaction products in the stopping medium gives information about the $\rho_{\text{LMT}}$ involved.

As already mentioned, the identification of the trapped recoiling reaction products in the catcher foils was made by their characteristic $\gamma$ radiations as well as by measuring their half-lives. The production cross sections ($\sigma_{\text{ER}}$) for identified reaction products were computed using the standard formulation given in Ref. [32]. In order to obtain the normalized yields as a function of cumulative depth in the Al stopping medium, the cross section of the reaction products in each catcher foil was divided by its thickness. The resulting normalized yields have been plotted against cumulative catcher foil thicknesses. The identified reaction products were computed using the standard formula-(1).

As a representative case, to show CF and ICF contributions from straggling. The identified reaction products are populated via $4n$ channel at $\approx$74, 80, and 87 MeV beam energies. The forward recoil range distribution (FRRD) for identified reaction products is shown in Figs. 1–3, at three different beam energies $\approx$74, 80, and 87 MeV. In the recoil range measurements, the cross sections for the production of a given residue as a function of the range are affected by relative errors, which depend essentially only on the counting statistics and the uncertainty in the catcher thicknesses and in presently studied cases, are less than or, at most, around 15%. The size of the circles in Figs. 1–3, includes the uncertainty in the yield values. The measured FRRDs clearly indicate the different momentum transfer components, depending on the fused mass of the projectile with the target nucleus. In case of the $4n$ channel (Fig. 1), the measured RRDs show only a single peak at all the three bombarding energies, indicating only one linear momentum transfer component (a characteristic of the CF process) involved in the production of $^{167}$Lu residues. A close observation of the range distribution of $^{167}$Lu residues (Fig. 1) reveals that the FRRD peak shifts toward relatively higher cumulative catcher thickness with increase in beam energy. Further, it may be pointed out that, the neutron emission from the forward recoiling residues may change their energy and momentum of the final residue, depending on the direction of emission. This is reflected in the width (FWHM) of the experimentally measured recoill range distributions. The widths may also arise because of the contributions from straggling. The identified reaction products and their experimentally measured most probable ranges, $R_p$ in Al, in units of $\mu$g/cm$^2$, for all the CF residues along with the theoretically estimated (using the code SRIM [41]) mean ranges $R_p^{\text{theo}}$ in Al, in units of $\mu$g/cm$^2$, are given in Table II. The most probable recoil ranges ($R_p$) have been calculated, assuming that in the case of CF, the incoming ion completely fuses with the target nucleus and transfers its total linear momentum to the fused system, which recoils for the conservation of

\[ v_C = v_{CN} = \sqrt{\frac{2M_P E}{M_{P+T}-T}} \]

where, $M_P$ is the projectile mass, $M_{P+T}$ is the total mass of the composite system (projectile+target), and $E$ is the projectile energy. Thus, the degree of linear momentum transfer may be given as

\[ \rho_{\text{LMT}} = \frac{P_{\text{frac}}}{P_{\text{proj}}} \]

FIG. 1. (Color online) Measured FRRDs for $^{167}$Lu residues populated via $4n$ channel at $\approx$74, 80, and 87 MeV beam energies.
linear momentum. An attempt has also been made to check the consistency in the FWHM of the observed FRRDs. The normalized FWHM ($\text{FWHM}/R_{\text{p}}^{\text{exp}}$) has been deduced for the observed distributions and tabulated in Table III. The normalized FWHM has been found to be consistent for the CF and ICF residues individually. As can be seen from Table III that for $\alpha$-emitting channels, the average peak resolution for CF is $\approx 0.28$, while for ICF-$\alpha$ and ICF-$2\alpha$ the average peak resolution increases to $\approx 0.69$ and 1.78, respectively, as expected. On the basis of the previous description, it is clear that the population of reaction products $^{167}\text{Lu}$ produced via $4\, n$ channel is associated with the entire LMT from projectile to the target nucleus, and may be represented as

$$^{12}\text{C} + ^{159}\text{Tb} \Rightarrow ^{171}\text{Lu}^* \Rightarrow ^{167}\text{Lu} + 4n$$

In a similar way, the FRRDs for the residues $^{168}\text{Lu}$ ($3n$), $^{165}\text{Lu}$ ($6n$), and $^{167}\text{Yb}$ ($p3n$) are found to have a single peak associated with complete linear momentum transfer from projectile to the composite nucleus, indicating the production of these residues via the CF process only.

Further, in case of $\alpha$-emitting channels, the residues $^{165}\text{Tm}$, $^{163}\text{Tm}$, $^{161}\text{Ho}$, $^{160}\text{Ho}^g$, and $^{160}\text{Ho}^m$ are expected to be populated, respectively via $\alpha2n$, $\alpha4n$, $2\alpha2n$, and $2\alpha3n$ channels. The observed FRRDs were resolved into two Gaussian peaks, for $\alpha xn$ channels, using the ORIGIN software. As a representative case, the FRRDs for the residues, $^{163}\text{Tm}$ ($\alpha4n$), have been plotted at three different energies in Figs. 2(a)–2(c).

As can be seen from Fig. 1, the FRRDs may be fitted using the range energy relation along with the reaction products produced in the interaction of $^{12}\text{C}$ with $^{159}\text{Tb}$ at $\approx 74$ MeV.

**TABLE II.** Experimentally measured most probable ranges $R_{\text{p}}^{\text{exp}}$ deduced from RRD curves, and theoretically calculated forward mean ranges $R_{\text{p}}^{\text{the}}$ in Al in units of $\mu\text{g/cm}^2$ for CF and ICF components using the range energy relation along with the reaction products in the interaction of $^{12}\text{C}$ with $^{159}\text{Tb}$ at $\approx 74$ MeV.

<table>
<thead>
<tr>
<th>Residues</th>
<th>$R_{\text{p}}^{\text{CF}}$</th>
<th>$R_{\text{p}}^{\text{ICF-4He}}$</th>
<th>$R_{\text{p}}^{\text{ICF-3He}}$</th>
<th>$R_{\text{p}}^{\text{ICF-He}}$</th>
<th>$R_{\text{p}}^{\text{the}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{168}\text{Lu}$</td>
<td>315 ± 43</td>
<td>321</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{167}\text{Lu}$</td>
<td>312 ± 48</td>
<td>321</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{165}\text{Lu}$</td>
<td>314 ± 52</td>
<td>321</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{167}\text{Yb}$</td>
<td>330 ± 28</td>
<td>321</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{165}\text{Tm}$</td>
<td>340 ± 32</td>
<td>321</td>
<td>163 ± 23</td>
<td>150</td>
<td>-</td>
</tr>
<tr>
<td>$^{163}\text{Tm}$</td>
<td>333 ± 61</td>
<td>321</td>
<td>158 ± 19</td>
<td>150</td>
<td>-</td>
</tr>
<tr>
<td>$^{161}\text{Ho}$</td>
<td>334 ± 53</td>
<td>321</td>
<td>150 ± 21</td>
<td>150</td>
<td>22 ± 8</td>
</tr>
<tr>
<td>$^{160}\text{Ho}^g$</td>
<td>348 ± 32</td>
<td>321</td>
<td>141 ± 28</td>
<td>150</td>
<td>23 ± 9</td>
</tr>
<tr>
<td>$^{160}\text{Ho}^m$</td>
<td>337 ± 61</td>
<td>321</td>
<td>145 ± 26</td>
<td>150</td>
<td>25 ± 11</td>
</tr>
</tbody>
</table>
with two Gaussian peaks, one at $\approx 333 \pm 32$, $351 \pm 60$, and $396 \pm 65 \mu g/cm^2$ in AI for three beam energies, indicating the complete momentum transfer events, however, another peak at lower cumulative depth at $\approx 155 \pm 23$, $162 \pm 40$, and $168 \pm 45 \mu g/cm^2$ corresponds to the fusion of $^8$Be (if $^{12}$C is assumed to break up into $^8$Be + $^\alpha$ and $^8$Be fuses) with $^{159}$Tb target nucleus. Similarly, the FRRDs for other $\alpha n$ channels have been resolved into two Gaussian peaks, indicating the presence of more than one linear momentum transfer components. It is observed that the complete as well as the incomplete momentum transfer peaks in the range spectra are centered at the expected position shown by the arrow. It may be observed from Fig. 2 that the mean range $R^{\text{exp}}$ shifts towards higher cumulative catcher thickness as the beam energy increases, as expected. It may be inferred that the residues $^{163}$Tm populated through $\alpha 4n$ channel may be populated via two ways

(i) Fusion of $^{12}$C

$$^{12}$C + $^{159}$Tb $\Rightarrow$ $^{171}$Lu$^*$ $\Rightarrow$ $^{163}$Tm + $\alpha 4n$ and/or $2p6n$$

or

(ii) Fusion of $^8$Be ($\alpha$ as spectator)

$$^{12}$C($^8$Be + $\alpha$) $\Rightarrow$ $^8$Be + $^{159}$Tb

$\Rightarrow$ $^{163}$Tm$^*$ + $\alpha$ (as spectator)

$\Rightarrow$ $^{163}$Ho$^*$ + $^8$Be (as spectator)

Further, in case of $\alpha 2n$ channels, the measured FRRDs have been found to be resolved into three Gaussian peaks. The measured FRRDs for $\alpha 2n$ channel have been plotted in Figs. 3(a)–3(c) at three beam energies. In this figure the observation of three peaks may be understood assuming the breakup of $^{12}$C into possible $\alpha$ clusters. The peaks at $\approx 334 \pm 53$, $378 \pm 47$, and $396 \pm 67 \mu g/cm^2$ depths for three beam energies are attributed to the complete momentum transfer (i.e., fusion of $^{12}$C with the target nucleus). However, the peaks at $\approx 150 \pm 21$, $193 \pm 37$, and $207 \pm 29 \mu g/cm^2$ for three beam energies belongs to the partial linear momentum transfer ($\frac{1}{2}\sigma_{\text{LM}}^\text{ICF}$) (i.e., the fusion of $^8$Be). Another peak at the lowest cumulative depth corresponds to the fusion of the $\alpha$ particle with the target nucleus, involving $\frac{1}{2}\sigma_{\text{LM}}^\text{CF}$. As such, it can be inferred that the residues $^{161}$Ho produced through $\alpha 2n$ channel have the contribution from both the processes, namely, CF as well as ICF, which may be represented as

(i) Fusion of $^{12}$C

$$^{12}$C + $^{159}$Tb $\Rightarrow$ $^{171}$Lu$^*$ $\Rightarrow$ $^{161}$Ho + $\alpha 2n$ and/or $4p6n$,

(ii) Fusion of $^8$Be ($\alpha$ as spectator)

$$^{12}$C($^8$Be + $\alpha$) $\Rightarrow$ $^8$Be + $^{159}$Tb

$\Rightarrow$ $^{167}$Tm$^*$ + $\alpha$ (as spectator)

$\Rightarrow$ $^{167}$Ho$^*$ + $^8$Be (+ $^{167}$Tm$^*$ + $\alpha 2n$, or

(iii) Fusion of $^8$Be as spectator)

$$^{12}$C($^8$Be + $^4$He) $\Rightarrow$ $^4$He + $^{159}$Tb

$\Rightarrow$ $^{163}$Ho$^*$ + $^8$Be (as spectator)

$\Rightarrow$ $^{163}$Ho$^*$ $\Rightarrow$ $^{161}$Ho + $2n$.

The above description is based on the breakup fusion model, where it is assumed that the incident $^{12}$C ion breaks into fragments (e.g., $^8$Be + $\alpha$ or $^8$Be + $^4$He) as it enters in the nuclear field of the target nucleus. The fragments so produced are assumed to move nearly with the same velocity as that of incident ion. One of the fragments ($^8$Be or $^\alpha$) fuses with the target nucleus forming an incompletely fused composite system, which recoils in the forward direction to conserve the input linear momentum. In order to compare the range-integrated yields of CF and ICF reactions, the statistical model calculations have been done using the code PACE4 [42], which is the upgraded version of code PACE2 [43]. The code PACE2 [43] is a modified version of JULIAN, the Hillman-Eyal evaporation code using a Monte Carlo code coupling angular momentum. The code PACE4 [42] has several new features, including a user friendly Windows interface, where explanation for each parameter is displayed. Further, a database for binding energies is also included in this program. The code PACE4 [42] gives almost similar results except that it is quite user friendly and simple. This code is based on the statistical approach of CN de-excitation by Monte Carlo procedure. In code PACE4 [42] the angular momentum projections are calculated at each step of de-excitation. The angular momentum conservation is explicitly taken into account, and the CF cross sections are calculated using the BASS formula [44]. The partial cross section ($\sigma_\ell$) for the formation of compound nucleus at a particular angular momentum $\lambda$, and specific bombarding energy, $E$ is given by

$$\sigma_\ell = \frac{\lambda^2}{4\pi}(2\ell + 1)T_\ell,$$

where $\lambda$ is reduced wavelength. Transmission coefficients $T_\ell$ may be given by the expression

$$T_\ell = \left[1 + \exp\left(\frac{\ell - \ell_{\text{max}}}{\Delta}\right)\right]^{-1},$$

TABLE III. Comparison of normalized FWHM of the distributions.
TABLE IV. Experimentally measured forward recoil range integrated cross section \( \sigma_{\text{exp}}^{\text{RRD}} \) deduced from RRD curves, and theoretically calculated cross-section \( \sigma_{\text{Pace}}^{\text{RRD}} \) at \( \approx 74 \), 80, and 87 MeV.

<table>
<thead>
<tr>
<th>Residues</th>
<th>Energy (E) ( \approx ) 74 MeV</th>
<th>Energy (E) ( \approx ) 80 MeV</th>
<th>Energy (E) ( \approx ) 87 MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma_{\text{exp}}^{\text{RRD}} )</td>
<td>( \sigma_{\text{Pace}}^{\text{RRD}} )</td>
<td>( \sigma_{\text{exp}}^{\text{RRD}} )</td>
</tr>
<tr>
<td>(^{168})Lu (3n)</td>
<td>3.20</td>
<td>3.18</td>
<td>1.10</td>
</tr>
<tr>
<td>(^{167})Lu (4n)</td>
<td>297</td>
<td>314</td>
<td>96</td>
</tr>
<tr>
<td>(^{165})Lu (6n)</td>
<td>1.4</td>
<td>0.61</td>
<td>120</td>
</tr>
<tr>
<td>(^{167})Yb (p3n)</td>
<td>33</td>
<td>29</td>
<td>13.3</td>
</tr>
<tr>
<td>(^{165})Tm (p2n)</td>
<td>6.79</td>
<td>0.83</td>
<td>11.15</td>
</tr>
<tr>
<td>(^{167})Tm (2n4p)</td>
<td>165.32</td>
<td>10.98</td>
<td>260.9</td>
</tr>
<tr>
<td>(^{161})Ho (2α2n)</td>
<td>7.27</td>
<td>0.20</td>
<td>5.94</td>
</tr>
</tbody>
</table>

where \( \Delta \) is the diffuseness parameter, while \( \ell_{\text{max}} \) is the maximum amount of \( \ell \) determined by total fusion cross section,

\[
\sigma_F = \sum_{\ell=0}^{\infty} \sigma_{\ell} \tag{5}
\]

The optical model potentials of Becchetti and Greenlees [45] have been used for calculating the transmission coefficients for neutron and proton, and for \( \alpha \)-particle emission. In the description of \( \gamma \)-ray competitions, emission of E1, E2, M1, and M2 \( \gamma \) rays are included and the \( \gamma \)-ray strength functions for different transitions are taken from tables of Endt [46].

Further, the relative contributions of complete and incomplete fusion in the production of a particular reaction product may be computed by fitting the experimentally measured RRDs with a Gaussian distribution using the ORIGIN software. The Gaussian yield curves of evaporation residues obtained from RRD are given by

\[
Y = Y_0 + \frac{A}{\omega_A \sqrt{2\pi}} e^{-(R-R_p)^2/2\pi \omega_A^2}, \tag{6}
\]

where \( R_p \) is the most probable mean range, \( \omega_A \) is the width parameter (FWHM) of the distribution, and \( A \) is the area under the peak. Further, the normalized yield \( Y \) may be estimated by the \( \chi^2 \) square fit of the experimentally determined range distributions and may be represented as follows:

\[
\chi^2 = \frac{1}{(m-p-1)} \{Y(A) - Y_0(A)\}^2 \tag{7}
\]

The value of the \( \chi^2 \) square was minimized in this analysis using a nonlinear least-square fit routine, keeping the width parameter \( \omega_A \) and most probable mean range \( R_p \) in the FRRD as a free parameter. Moreover, as indicated in Figs. 2 and 3, the residues involving \( \alpha \)-emitting channels show more than one RRD component. In such cases, the experimentally measured normalized yields have been fitted using the multipeak option in a similar way as mentioned above. The contribution of different fusion components have been obtained by dividing the area under the peak of the corresponding fusion component by the total area associated with the experimental data. It has been observed that the contribution of CF satisfactorily matches with that predicted by PACE4 code with physically reasonable parameters [35,36], which were optimized to reproduce the evaporation residues populated in case of complete fusion reactions such as \( \alpha n \) and \( pxn \) channels. However, the contribution of ICF reactions (given in Table IV) could not be reproduced by calculations using the same set of parameters since PACE code does not take ICF into account.

A. Dependence on projectile energy

In order to study the energy dependence of CF (full LMT) and ICF (partial LMT) components, percentage relative contributions of the CF and ICF components are deduced using the relation,

\[
F_{\text{ICF}} = \frac{\Sigma \sigma_{\text{ICF}}}{\Sigma \sigma_{\text{CF}} + \Sigma \sigma_{\text{ICF}}} \times 10^2, \tag{8}
\]

where \( \Sigma \sigma_{\text{CF}} \) and \( \Sigma \sigma_{\text{ICF}} \) are the complete and incomplete fusion cross sections at the studied energies. The percentage ICF contributions of different fusion components have been obtained by dividing the area under the ICF peak of the corresponding fusion component by the total area associated with the experimental data. The values of \( F_{\text{ICF}} \) deduced from FRRDs data are also compared with the \( F_{\text{ICF}} \) obtained from the excitation function measurements [35], as a function of beam energy.

![FIG. 4. (Color online) The percentage incomplete fusion fraction \( F_{\text{ICF}} \) deduced from the analysis of forward recoil range distributions as a function of projectile energy. Data shown by black stars are obtained from Yadav’s analysis of EFs [35].](image-url)
TABLE V. Comparison of range-integrated cross-section with the cross-sections obtained from EFs measurement for $3n$, $4n$, $6n$ and $p3n$-channels.

<table>
<thead>
<tr>
<th>Residues</th>
<th>$\approx 74$ MeV</th>
<th>$\approx 80$ MeV</th>
<th>$\approx 87$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{165}$Lu ($3n$)</td>
<td>3.2 (3.4)</td>
<td>1.1 (0.76)</td>
<td>-</td>
</tr>
<tr>
<td>$^{167}$Lu ($4n$)</td>
<td>297 (320)</td>
<td>96 (90)</td>
<td>12 (10)</td>
</tr>
<tr>
<td>$^{165}$Lu ($6n$)</td>
<td>1.4 (1.05)</td>
<td>120 (124)</td>
<td>510 (550)</td>
</tr>
<tr>
<td>$^{167}$Yb ($p3n$)</td>
<td>33 (32.2)</td>
<td>13.3 (11.6)</td>
<td>2.8 (3.6)</td>
</tr>
</tbody>
</table>

energy ($E_{lab}$) in Fig. 4. As can be seen from this figure, the ICF fraction increases rapidly with energy at lower energies, however, at relatively higher energies the $F_{ICF}$ seems to increase with slow rate. Nevertheless, it may also be observed from Fig. 4, that both the measurements of FRRDs and the EFs give nearly same $F_{ICF}$, which strengthens the present measurements and indicates the self-consistency of the data. In Table V comparison of range-integrated cross sections with the cross sections obtained from EFs measurement for $xn$ and $p3n$ channels has also been done, which matches reasonably. It may not be out of place to mention that similar observations of ICF contributions increasing with energy and mass asymmetry have been obtained by Morgenstern et al. [24]. However, their work involved measuring the velocity spectra employing the time-of-flight method in lighter systems and also at relatively higher energies $\approx 10-25$ MeV/nucleon.

B. Comment on mass-asymmetry and projectile structure effect

In order to have better understanding about the dependence of underlying reaction dynamics on mass asymmetry and/or projectile structure, the presently deduced $F_{ICF}$ values have been compared with the $F_{ICF}$ obtained in the $^{16}$O induced reactions on same target $^{159}$Tb [14]. The Fig. 5, shows the comparison of $F_{ICF}$ for both the systems. It is evident from this figure that the $^{16}$O as projectile has higher ICF contribution than for the $^{12}$C, at the same normalized projectile energies. But according to Morgenstern’s mass-asymmetry systematics the more asymmetric system would have more ICF probability. The mass asymmetry of interacting partners is defined as $\mu = A_T/(A_T + A_P)$. So $^{12}$C + $^{159}$Tb ($\mu = 0.9298$) should have more ICF than $^{16}$O + $^{159}$Tb ($\mu = 0.9086$). However, the binding energy aspect ($E_{\text{_binding}}^{^{16}$O} > E_{\text{_binding}}^{^{12}$C}$) is also unable to explain the present picture. One of the possible explainations may be the excess of $\alpha$ cluster in $^{16}$O versus $^{12}$C. In addition to this the $\alpha$-Q value for $^{16}$O is less than the $^{12}$C (i.e., $^{16}$O requires less energy to break up into $\alpha$ clusters than $^{12}$C and thus gives large ICF contributions). As such, the $\alpha$-Q value of the projectile seems to be a reasonable parameter to explain the presently observed large $F_{ICF}$ values for $^{16}$O as projectile than for the $^{12}$C.

C. SUMRULE calculations: sharp cutoff in $\ell$ distribution

In the SUMRULE model [22,23], which is based on the partial statistical equilibrium and on the idea of generalized concept of critical angular momentum, the transfer of mass may occur only if the angular momentum of relative motion of the captured fragment $P^p$ ($P^p$: participant, $P^s$: spectator) with respect to the target nucleus is smaller than the critical angular momentum for this incompletely fused system (i.e., $\ell_{\text{eff}} \leq \ell_{\text{crit}}^{P^p+T}$. The limiting angular momentum in the reference frame of the entrance channel, $\ell_{\text{limit}}$, is related to the critical angular momentum $\ell_{\text{crit}}^{P^p+T}$ of fused part as

$$\ell_{\text{limit}} = \frac{A_P A_T}{(A_P A_P + A_P A_T) \ell_{\text{crit}}^{P^p+T}}. \quad (9)$$

However, for a mass-asymmetric projectile-target combination, the limiting angular momentum may be rewritten as

$$\ell_{\text{limit}} \sim \frac{A_P}{A_{P^s}} \ell_{\text{crit}}^{P^p+T}. \quad (10)$$

By assuming the smooth cutoff in the $\ell$ space the transmission coefficient for each individual reaction channel is given as

$$T_\ell(i) = \left[1 + \exp \left(\frac{\ell - \ell_{\text{limit}}(i)}{\Delta}\right)\right]^{-1}, \quad (11)$$

where $\Delta$ gives the diffuseness in the $\ell$ distribution. For small $\ell$ values the transmission coefficients $T_\ell$ are almost unity for all channels. The different reaction channels open up one after the other with increasing angular momentum and depending upon their corresponding limiting angular momenta $\ell_{\text{limit}}(i)$, hence the reaction probabilities for a given partial wave $\ell$

$$N_\ell \sum_i T_\ell(i) \times \exp \left[\frac{Q_{gg}(i) - Q_{c}(i)}{T}\right] = 1, \quad (12)$$

where $N_\ell$ is the $\ell$-dependent normalization factor common for all reaction channels. Thus, absolute cross sections for the individual reaction channels are defined as

$$\sigma(i) = \pi \lambda^2 \sum_{\ell=0}^{\ell_{\text{max}}} (2\ell + 1) \times \frac{T_\ell(i) p(i)}{\sum_j T_\ell(j) p(j)}, \quad (13)$$

where $\lambda^2 = h^2/(2\mu E)$ is the reduced wavelength for the entrance channel and $p(i)$ is the reaction probability for a
given channel $i$, which is proportional to $\sim \exp[(Q_{\text{ext}}(i) - Q_{\text{int}}(i))/T]$, $T$ is an effective temperature, and $Q_{\text{ext}}(i)$ is the change of the Coulomb interaction energy due to the transfer of charge. The $\ell_{\text{max}}$ is defined as the largest $\ell$ for which the colliding system penetrates into the region where the total nucleus-nucleus potential is attractive and/or the distance of closest approach is smaller than the sum of the half-density radii, however, the critical angular momenta $\ell_{\text{crit}}$, which determine the magnitude of the transmission coefficients $T_{\ell}$, for individual reaction channels were calculated from a simplified formula as

$$
\ell_{\text{crit}}^2 = \frac{\mu m(C_1 + C_2)^3}{\hbar^2} \left[ 4\pi g \frac{C_1 C_2}{C_1 + C_2} - \frac{Z_1 Z_2 e^2}{(C_1 + C_2)^2} \right],
$$

where $\mu m$ is the reduced mass of the interacting partners, $g$ is the surface tension coefficient, $Z_1$, $Z_2$ and $C_1$, $C_2$ are the atomic numbers and half-density radii of projectile and target nuclei, respectively. With this model one may calculate absolute cross sections for CF, ICF channels, and other binary reactions, which presumably proceed via the formation of a dinuclear system. The model contains three free parameters: the effective temperature $T$, the effective Coulomb interaction radius $R_c$, and the diffuseness in the $\ell$ distributions, $\Delta$. Wilczynski et al. [22], to fit the experimental data in the $^{14}$N + $^{159}$Tb reaction at $E_{\text{lab}} = 140$ MeV, used $T = 3.5$ MeV, $R_c/(A_p^{1/3} + A_T^{1/3}) = 1.5$, and $\Delta = 1.7\hbar$. In order to obtain the magnitude of the ICF-reaction cross section in the present work, the same parameters have been retained. Using these parameters, the SUMRULE model calculations highly underestimate the measured cross sections for residues of interest. As a typical example the experimentally measured cross sections for the ($\alpha$+n) and ($\alpha$+2n) channels are $\approx 64.0 \pm 9.6$ mb and $5.0 \pm 0.7$ mb, however, the theoretically calculated SUMRULE values are 1.32 mb and 0.02 mb at 86 MeV beam energy. These substantial discrepancies indicate the need for refinement in the assumptions of the SUMRULE model. Similar deviations have also been found by Parker et al. [4] in their study on the $^{12}$C + $^{51}$V system up to 100 MeV.

IV. CONCLUSION

The recoil range distributions of a large number of radionuclides viz; $^{168}$Lu (3$n$), $^{167}$Lu (4$n$), $^{165}$Lu (6$n$), $^{167}$Yb ($p3n$), $^{165}$Tm ($a2n$), $^{163}$Tm ($a4n$), $^{161}$Ho ($a2n$), $^{160}$Ho ($a2n$) and $^{160}$Ho ($a3n$) populated in $^{12}$C + $^{159}$Tb interactions at three above-barrier energies have been measured. The analysis of the measured FRRDs of reaction products presented strongly reveals a significant contribution from the partial LMT of the projectile associated with ICF in several $\alpha$-emitting channels. Different partial LMT components are attributed to the fusion of $^{8}$Be and $\alpha$ from the $^{12}$C projectile to the target nucleus. The percentage ICF contributions are found to have onset from $\approx 12\%$ above CB. It has been found, in general, that the residues are populated not only via CF but ICF is also found to play an important role in the production of different reaction products involving direct $\alpha$-cluster emission. The present results have also been compared with literature results, and it may be concluded that instead of Morgenstern’s mass-asymmetry

FIG. 6. (Color online) Fusion spin distributions calculated using the code CCFULL [47] for $^{12}$C + $^{159}$Tb system at $E_{\text{lab}} \approx 87$, 84 and 74 MeV, where $\ell$ is in units of $\hbar$. The value of $\ell_{\text{crit}}$ for fusion calculated using the formulation of Ref. [18].
systems, the projectile structure along with $\alpha - Q$ value is also important at the energy range of interest. On the other hand, the SUMRULE model calculations highly underestimate the ICF cross section, which may be due to the assumption that a substantial contribution to ICF comes from the collision trajectories with $\ell > \ell_{\text{crit}}$. However, in the energy range of the present study, $\ell_{\text{max}} < \ell_{\text{crit}}$, thus, significant cross sections for ICF at these beam energies indicate the contribution from collision trajectories with $\ell < \ell_{\text{crit}}$. The results obtained from the FRRDs give valuable information for establishing the CF and ICF yields at relatively low bombarding energies and also indicate that the $\ell$ values lower than $\ell_{\text{crit}}$ significantly contribute to the ICF reactions. More data on such reactions is needed to explore the above aspects, so that the assumptions of the SUMRULE model for energies near the barrier, where $\ell < \ell_{\text{crit}}$, may be improved upon to explain the experimental data.

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Large influence of in-complete fusion in $^{12}\text{C}+^{159}\text{Tb}$ at $E_{lab} \approx 4-7$ MeV/A

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Large influence of incomplete fusion in $^{12}$C + $^{159}$Tb at $E_{\text{lab}} \approx 4$–7 MeV/nucleon

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Low energy incomplete fusion has been studied in $^{12}$C+$^{159}$Tb system at energies $\approx 4$–7 MeV/nucleon. The excitation functions of individual reaction channels have been measured using off-line $\gamma$ spectroscopy, and analyzed in the framework of statistical model code PACE4 based on the concept of equilibrated compound nucleus decay. A significant fraction of incomplete fusion has been found in the production of residues involving $\alpha$ particle(s) in the exit channels. For better insights into the onset and strength of incomplete fusion, the incomplete fusion strength function has been deduced as a function of various entrance channel parameters. Large influence of incomplete fusion has been observed at slightly above barrier energies, and increases smoothly with incident projectile energy. Present results have been compared with the results obtained in the interactions of $^{12}$C with nearby targets to probe the dependence of incomplete fusion on entrance channel mass asymmetry. It has been found that the percentage fraction of incomplete fusion increases linearly with mass asymmetry of interacting partners in the studied mass region.

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I. INTRODUCTION

Considerable efforts are being employed to synthesize superheavy elements using heavy-ion induced complete fusion (CF) reactions [1–6]. In addition to the fission and quasifission [7–12], the existence of incomplete fusion (ICF) [13–27] at low incident energies (i.e., $\approx 4$–7 MeV/nucleon) may add complexity to the synthesis of superheavy elements. In general, at these energies, CF is supposed to be the sole contributor to the total fusion cross section [26,27]. However, a large fraction of ICF has been observed at energies as low as $\approx 4$–7 MeV/nucleon [13–21]. The onset of ICF at slightly above barrier energies triggered the resurgent interest to study ICF at these energies. In a qualitative way, CF and ICF processes can be disentangled on the basis of driving angular momenta ($\ell$ waves) [27–29]. The central and/or near-central interactions ($0 \leq \ell \leq \ell_{\text{crit}}$) form a completely fused composite (CFC) system after intimate contact and transient amalgamation of entire projectile and target nucleus. Eventually, the projectile’s kinetic energy and angular momenta are distributed among all the accessible internal degrees of freedom of the composite system, leading to the formation of a fully equilibrated compound nucleus (CN). However, at relatively higher $\ell$ values ($\geq \ell_{\text{crit}}$) imparted into the system due to noncentral interactions (or at sufficiently higher energies), the pocket in the entrance channel potential vanishes [28]. As a consequence, fusion of entire projectile is hindered and gives way to ICF. In this case, a part of projectile is emitted as a spectator ($P^*$) to release excess driving angular momenta. After such an emission, the remnant (participant: $P^*$) is supposed to carry input angular momenta less than or equal to its own critical limit ($\ell_{\text{eff}} \leq \ell_{\text{crit}}^{P^*+T}$) for fusion to occur [29]. The partial fusion of incomplete projectile results in an incompletely fused composite (IFC) system, and direct projectile-like fragments (PLFs) are dominantly ejected in the forward cone. The CN formed via CF is expected to have predetermined mass/charge, excitation energy and angular momenta. However, the IFC system is formed with relatively less mass/charge and excitation energy (due to fractional fusion of projectile), but at high angular momenta (imparted into the system in noncentral interactions) as compared to the CN originated from CF process [17].

The concept of partial fusion of projectile in heavy-ion interactions was set in after first experimental observation of ‘direct PLFs’ emitted in massive transfer reactions by Britt and Quinton [30]. Since then, several experimental/theoretical studies have been carried out to understand ICF dynamics. Some of the important studies are summarized in an outstanding review by Gerschel [31]. However, some of the widely used descriptions of ICF are discussed here for ready reference. Wilczynski et al. [29], established ICF as a natural extension of CF for higher $\ell$ values (above $\ell_{\text{crit}}$) associated with noncentral interactions. The noncentral nature of ICF has also been emphasized by Geoffroy et al. [32], Trautmann et al. [33], and Inamura et al. [34]. In the breakup fusion (BUF) model [35], of Udagawa and Tamura, the projectile is assumed to break up into constituent $\alpha$ clusters (e.g., $^{12}$C $\rightarrow ^{9}$Be $+ \alpha$) within the nuclear field of the target nucleus. The concept of the BUF model modifies the picture of fusion of two nuclei as (i) sequential CF where all projectile fragments fuse with target nucleus, and (ii) ICF where only a part of projectile fuses with target nucleus and the remnant behaves like a spectator.
The sequential CF is experimentally indistinguishable from direct CF. The total fusion cross section may be defined as the sum of CF and ICF cross sections, i.e., \( \sigma_{\text{TF}} = \sigma_{\text{CF}} + \Sigma\sigma_{\text{ICF}} \). Further, Morgenstern et al. [36] correlated the probability of ICF with the entrance channel mass asymmetry. In Ref. [36], the probability of ICF is found to be higher for more mass-asymmetric systems. It may be pointed out that the existing models/theories fairly explain ICF data obtained at energies \( \approx 7-10 \text{ MeV/nucleon} \) up to some extend, but do not provide satisfactory reproduction of ICF data at lower incident energies. Due to the unavailability of reliable theoretical model to predict low energy ICF, the experimental study of ICF is still an active area of investigations.

The most debated and outstanding issues related to low energy ICF have been, (i) the localization of the \( \ell \) window, (ii) the usefulness of ICF to populate high-spin states in final reaction products, and (iii) the effect of entrance channel parameters on the onset and strength of ICF. In recent years, high quality data on excitation functions (EFs) [18,19,21,23], spin distributions (SDs) [17], and linear momentum distributions [20,22,24] of individual reaction products have been obtained at the Inter-University Accelerator Center (IUAC), New Delhi in a variety of experiments. These studies conclusively demonstrate the low energy ICF but limited only for a few projectile-target combinations. The EFs of individual reaction channels populated in \( ^{12}\text{C}+^{159}\text{Tb} \) system at energies \( \approx 4-7 \text{ MeV/nucleon} \) are presented in this work. Owing to the \( \alpha \)-like structure of the projectile, the ICF reaction mechanism is expected to influence the decay channels involving \( \alpha \) particles. The ICF strength function has been deduced from the analysis of experimental EFs in the framework of statistical model code PAC4E based on equilibrated compound nucleus decay. In order to generate some systematics, the values of percentage ICF fraction obtained for nearby systems, i.e., \( ^{12}\text{C}+^{103}\text{Rh} \) [37], \( ^{12}\text{C}+^{115}\text{In} \) [38], \( ^{12}\text{C}+^{128}\text{Te} \) [39], \( ^{12}\text{C}+^{160}\text{Tb} \) (this work), \( ^{12}\text{C}+^{165}\text{Ho} \) [40], and \( ^{12}\text{C}+^{169}\text{Tm} \) [41,42], have been compared. This paper is organized as follows. The experimental details and data reduction procedure are discussed in Sec. II. The results obtained in the present work are discussed in connection with the existing data in Sec. III. The outcome of the present work is summarized in the last section of this paper.

II. EXPERIMENTAL DETAILS AND DATA REDUCTION PROCEDURE

The experiment has been performed at the Inter University Accelerator Center (IUAC), New Delhi, India, using off-line \( \gamma \)-ray spectroscopy. Natural \( ^{159}\text{Tb} \) targets \( (t_{\text{in}} \approx 1.2-2.5 \text{ mg/cm}^2) \) and Al-catcher foils \( (t_{\text{in}} \approx 1.5-2.5 \text{ mg/cm}^2) \) were prepared by rolling technique. An Al-catcher foil of sufficient thickness has been placed behind the target foil so that the recoiling products during the irradiations may be trapped in the catcher foil thickness. To cover a wide energy range in an irradiation, an energy degradation technique has been used. In this experiment, five stacks (each made by three target-catcher foil assemblies) were prepared. The irradiations have been carried out in the general purpose scattering chamber (GPSC) at energies \( \approx 59, \ 70, \ 73, \ 85, \ \text{and} \ 88 \text{ MeV} \). An in-vacuum transfer facility has been used to minimize the lapse time between the stop of the irradiation and beginning of the counting of the activity induced in a target-catcher assembly. The incident beam energy on each target foil in a stack has been estimated using the code SRIM [43]. For an example, at the highest incident energy \( (i.e., \approx 88 \text{ MeV}) \), the uncertainty in the energy is estimated to be \( \approx \pm 0.69 \text{ MeV} \), and, at the lowest incident energy \( (i.e., \approx 54.83) \) is estimated to be \( \approx \pm 0.52 \text{ MeV} \). Considering the half-lives of interest, the irradiations have been carried out for \( \approx 8-10 \text{ h} \) duration for each stack. A Faraday cup has been installed behind the target-catcher foil assembly to measure the beam current. The beam current has been maintained \( \approx 25-30 \text{ nA} \) during all the irradiations.

The radio activity produced in the target-catcher foil assemblies have been followed by a precalibrated high resolution HPGe detector coupled to a CAMAC based data acquisition system CANDLE [44]. The HPGe detector used in this experiment has been calibrated using standard \( \gamma \) sources, e.g., \( ^{60}\text{Co}, ^{133}\text{Ba}, \) and \( ^{152}\text{Eu} \). The efficiency of the detector has been determined using same sources at various source (target-catcher foil assembly)—detector separations to wash out the solid angle effect. The energy resolution of the detector has been estimated \( \approx 2.5 \text{ keV for } 1408 \text{ keV } \gamma \) line of \( ^{152}\text{Eu} \) source. A 50 Hz pulser was used to determine the dead time of the detector. The source (target-catcher foil assembly)—detector separation has been adjusted to keep the dead-time below 10% during the counting. Reaction residues have been identified by their characteristic \( \gamma \) lines, and confirmed by the decay-curve analysis. As a representative case, a typical \( \gamma \)-ray spectrum obtained at incident energy \( \approx 87.31 \pm 0.69 \text{ MeV} \) is shown in Fig. 1, and some of the \( \gamma \) peaks corresponding to different CF and/or ICF residues are labeled. The reaction residues expected to be populated via CF and/or ICF in \( ^{12}\text{C}+^{159}\text{Tb} \) system are given in Table I with their spectroscopic properties taken from Refs. [45,46]. The production cross section \( (\sigma) \) have been determined using the standard formulation as given in Ref. [21]. Experimentally measured production cross sections \( (\sigma) \) (mb) of individual reaction residues are given in Tables II and III. It may be pointed out that the errors in the measured production cross sections may arise due to (i) the nonuniformity of target foils, (ii) fluctuations in the beam current during the irradiations, (iii) the uncertainty in geometry dependent efficiency of HPGe detector, and (iv) due to the dead time of the spectrometer. Detailed discussion on the error analysis is given elsewhere [21]. In the present work, the overall error including statistical errors is estimated to be \( \leq 15\% \), excluding the uncertainty in branching ratio, decay constant, etc., which have been taken from the Table of Radioactive Isotopes [45].

III. OBTAINED RESULTS, ANALYSIS AND THEIR INTERPRETATION

The EFs of \( ^{168}\text{Lu}^\gamma \) (3n), \( ^{167}\text{Lu}(4n) \), \( ^{165}\text{Lu}(6n) \), \( ^{167}\text{Yb}(3n) \), \( ^{165}\text{Tm}(2\alpha 2n) \), \( ^{163}\text{Tm}(4n) \), \( ^{165}\text{Ho}(2\alpha 2n) \), and \( ^{160}\text{Ho}(2\alpha 3n) \) radionuclides expected to be populated via CF and/or ICF of \( ^{12}\text{C} \) and \( ^{159}\text{Tb} \) have been measured at energies \( \approx 4-7 \text{ MeV/nucleon} \). In order to understand up to what extent
the decay of these radionuclides can be justified by equilibrated CN decay, experimentally measured EFs are analyzed within the framework of the statistical model code PACE4 [48]. The code PACE4 is based on the Hauser-Feshbach theory of CN decay, and uses statistical approach of CN de-excitation by Monte Carlo procedure. In this code, the angular momentum projections are calculated at each stage of de-excitation, which enables the determination of angular distribution of the emitted particles and angular momentum conservation is explicitly taken into account. The BASS models were used to calculate CF cross sections [47]. The default optical model parameters for neutrons, protons, and $\alpha$ particles are used [48]. The $\gamma$-ray strength functions for $E1$, $E2$, and $M1$ transition were taken from tables of Endt [49]. In this code, the level density parameter ($\alpha = A/K$ MeV$^{-1}$, where $A$ is the mass number of the nucleus and $K$ is a free parameter) is one of the important parameters. The value of free parameter $K$ can be varied to reproduce experimental EFs with in the physically justified limits. It may be pointed out that any enhancement in the EFs predicted by PACE4 may be attributed to some physical effect which is not included in this code.

A. $x$ and $pxn$ channels

Figure 2(a) shows the experimentally measured EFs of $^{168}$Lu$^{α+}$($t_{1/2} = 6.7$ min, $5.5$ min), $^{167}$Lu($t_{1/2} = 51.5$ min), $^{165}$Lu($t_{1/2} = 10.74$ min), and $^{167}$Yb($t_{1/2} = 17.5$ min) evaporation residues expected to be populated via $3n$, $4n$, $6n$, and $p3n$ emission from the excited $^{171}$Lu$^*$ nucleus formed in CN reactions. Self-explanatory notations are used to explain the decay channels in this figure. The solid lines through the data points are drawn to guide the eyes. During the decay-curve analysis, the evaporation residue $^{167}$Yb($p3n$) is found to be strongly fed from its higher charge isobar (precursor hereafter) $^{167}$Lu($4n$) through $\beta^+$ emission. The half-life of of precursor (i.e., $^{167}$Lu $\rightarrow t_{1/2}^{\text{pre}} = 51.5$ min) is larger than the half-life of the daughter nuclei (i.e., $^{167}$Yb $\rightarrow t_{1/2}^{\text{d}} = 17.5$ min). In this case, the independent production cross section ($\sigma_{\text{ind}}$) of $^{167}$Yb has been deduced using the following successive radioactive decay formulation [50]:

$$N_d(t) = C_{\text{iso}}e^{-\lambda_d t} + \frac{(P_{\text{pre}}\lambda_{\text{pre}})}{(-\lambda_d - \lambda_{\text{pre}})}N_{\text{pre}}(t)e^{-\lambda_{\text{pre}} t},$$

(1)

where $N_d(t)$ and $N_{\text{pre}}(t)$ are the number of daughter and precursor nuclei produced at time $t$. $C_{\text{iso}}$ is the cumulative (precursor + daughter) number of nuclei produced at the end of the irradiation, $\sigma_{\text{pre}}$ and $\sigma_d$ are the production cross sections of precursor and daughter nuclei; and $\lambda_{\text{pre}}$ and $\lambda_d$ are the decay constants of precursor and daughter nuclei, respectively. The value of $N_{\text{pre}}(t)$ has been deduced from the experimentally measured decay curve of $^{167}$Lu. The value of $N_d(t)$ ($^{167}$Yb) has been obtained by solving Eq. (1), which has been translated to its production cross section ($\sigma_{\text{ind}}$), and is plotted in Fig. 2(a) as independent production of $^{167}$Yb($3n$).

To reproduce the experimental EFs of $xn/pxn$ channels using statistical model code PACE4, and to identify right level density parameter for the analysis of $\alpha$-emitting channels, different values of the level density parameter have been tested by varying the free parameter $K$ (i.e., $K = 8-12$). Values of the level density parameter $K$ significantly higher than $K = 8$ are expected to not be appropriate for the excitation energies involved in the studied reactions. Nevertheless, we have considered them in the calculations to better enlighten the dependence of the calculated excitation functions on this

\[\text{TABLE I. List of identified reaction residues (channels) with their spectroscopic properties.}\]

<table>
<thead>
<tr>
<th>Residue</th>
<th>$T_{1/2}$</th>
<th>$J^*$</th>
<th>$E_x$(keV)</th>
<th>$I^r$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{168}$Lu$(3n)$</td>
<td>5.5 min</td>
<td>$3^+$</td>
<td>198.86</td>
<td>180.0$^a$</td>
</tr>
<tr>
<td>$^{168}$Lu$(α3n)$</td>
<td>6.7 min</td>
<td>$6^-$</td>
<td>198.86</td>
<td>70.0$^a$</td>
</tr>
<tr>
<td>$^{167}$Lu$(4n)$</td>
<td>51.5 min</td>
<td>$7/2^+$</td>
<td>213.21</td>
<td>3.5</td>
</tr>
<tr>
<td>$^{165}$Lu$(6n)$</td>
<td>10.74 min</td>
<td>$1/2^+$</td>
<td>120.58</td>
<td>25</td>
</tr>
<tr>
<td>$^{167}$Yb$(p3n)$</td>
<td>17.5 min</td>
<td>$5/2^-$</td>
<td>176.2</td>
<td>20.4</td>
</tr>
<tr>
<td>$^{165}$Tm$(α2n)$</td>
<td>30.06 h</td>
<td>$1/2^+$</td>
<td>242.85</td>
<td>35</td>
</tr>
<tr>
<td>$^{163}$Tm$(α4n)$</td>
<td>1.81 h</td>
<td>$1/2^+$</td>
<td>346.75</td>
<td>3.9</td>
</tr>
<tr>
<td>$^{161}$Ho$(2α2n)$</td>
<td>2.48 h</td>
<td>$7/2^-$</td>
<td>103.03</td>
<td>3.6</td>
</tr>
<tr>
<td>$^{160}$Ho$(2α3n)$</td>
<td>25.6 min</td>
<td>$5^+$</td>
<td>645.25</td>
<td>16.20</td>
</tr>
<tr>
<td>$^{160}$Ho$(α3n)$</td>
<td>5.02 h</td>
<td>$2^-$</td>
<td>645.25</td>
<td>16.20</td>
</tr>
<tr>
<td>$^{167}$Yb$(α3n)$</td>
<td>6.7 min</td>
<td>$5/2^-$</td>
<td>728.18</td>
<td>30.8</td>
</tr>
<tr>
<td>$^{167}$Yb$(p3n)$</td>
<td>17.5 min</td>
<td>$5/2^-$</td>
<td>879.39</td>
<td>20.2</td>
</tr>
</tbody>
</table>

$^a$These intensities are relative.
paramet...r (i.e., $^{165}\text{Tm} \rightarrow t_{1/2}^{d} = 30.06\ h$). As demonstrated by Cavinato et al. [50], the independent production cross section ($\sigma_{\text{ind}}$) of daughter nuclei may be defined in terms of cumulative ($\sigma_{\text{cum}}$) and precursor ($\sigma_{\text{pre}}$) cross sections as follows:

$$\sigma_{\text{ind}} = \sigma_{\text{cum}} - F_{\text{pre}}\sigma_{\text{pre}}.$$

Here $F_{\text{pre}}$ is the precursor coefficient which depends on the branching ratio of precursor decay ($P_{\text{pre}}$) to the final nucleus as

$$F_{\text{pre}} = \frac{t_{1/2}^{d}}{P_{\text{pre}}t_{1/2}^{d} + t_{1/2}^{f}}.$$

Here $t_{1/2}^{d}$ and $t_{1/2}^{f}$ are the half-lives of the precursor and final nuclei, respectively. The values of half-lives and branching ratio of precursor decay ($P_{\text{pre}}$) have been taken from Refs. [45,46]. After the inclusion of these observables, the

### Table II. Experimentally measured production cross sections $\sigma$ (mb) of $^{168}\text{Lu}^{4+}$, $^{167}\text{Lu}$, $^{167}\text{Yb}$, $^{165}\text{Tm}$, and $^{163}\text{Tm}$ residues.

<table>
<thead>
<tr>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>$^{168}\text{Lu}^{4+}$ (CF)</th>
<th>$^{167}\text{Lu}$ (CF)</th>
<th>$^{167}\text{Yb}$ (CF)</th>
<th>$^{165}\text{Tm}$ (CF+ICF)</th>
<th>$^{163}\text{Tm}$ (CF+ICF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>54.83 + 0.52</td>
<td>184.5 ± 29.68</td>
<td>12.12 ± 2</td>
<td>–</td>
<td>0.2 ± 0.03</td>
<td>–</td>
</tr>
<tr>
<td>58.51 + 0.50</td>
<td>177.5 ± 24.2</td>
<td>155.23 ± 23.25</td>
<td>1 ± 0.13</td>
<td>13.66 ± 1.9</td>
<td>–</td>
</tr>
<tr>
<td>61.37 + 0.63</td>
<td>118 ± 15.27</td>
<td>340.15 ± 51</td>
<td>–</td>
<td>10 ± 1.9</td>
<td>16.65 ± 2.5</td>
</tr>
<tr>
<td>62.63 + 0.80</td>
<td>94.5 ± 12.85</td>
<td>545.2 ± 79</td>
<td>–</td>
<td>15 ± 2.19</td>
<td>19.61 ± 2.65</td>
</tr>
<tr>
<td>65.49 + 0.69</td>
<td>47.5 ± 6.95</td>
<td>635.2 ± 93.45</td>
<td>–</td>
<td>19.5 ± 2.45</td>
<td>17.45 ± 2.13</td>
</tr>
<tr>
<td>67.24 + 1.17</td>
<td>45 ± 6.25</td>
<td>649.3 ± 85</td>
<td>–</td>
<td>49 ± 6.5</td>
<td>12.52 ± 1.56</td>
</tr>
<tr>
<td>69.15 + 0.85</td>
<td>14 ± 3.01</td>
<td>699.5 ± 102.9</td>
<td>–</td>
<td>55.34 ± 7.9</td>
<td>10.16 ± 1.3</td>
</tr>
<tr>
<td>72.24 + 0.76</td>
<td>7.5 ± 1.26</td>
<td>499.6 ± 70.68</td>
<td>0.03 ± 0.005</td>
<td>45.8 ± 6.62</td>
<td>8.76 ± 1.5</td>
</tr>
<tr>
<td>74.97 + 0.57</td>
<td>2.5 ± 0.29</td>
<td>298.4 ± 42.1</td>
<td>2 ± 0.3</td>
<td>19.2 ± 2.59</td>
<td>8.21 ± 0.96</td>
</tr>
<tr>
<td>77.77 + 0.62</td>
<td>1 ± 0.15</td>
<td>149.6 ± 19.5</td>
<td>36 ± 4.5</td>
<td>29 ± 4.2</td>
<td>8.98 ± 1.12</td>
</tr>
<tr>
<td>79.62 + 0.61</td>
<td>0.5 ± 0.08</td>
<td>150 ± 19.8</td>
<td>58 ± 8.5</td>
<td>11 ± 1.5</td>
<td>9.98 ± 1.38</td>
</tr>
<tr>
<td>82.47 + 0.82</td>
<td>0.15 ± 0.03</td>
<td>89.9 ± 12.65</td>
<td>301 ± 46</td>
<td>8 ± 1.02</td>
<td>12.04 ± 1.56</td>
</tr>
<tr>
<td>84.44 + 0.56</td>
<td>0.1 ± 0.02</td>
<td>35.3 ± 4.5</td>
<td>408 ± 58.9</td>
<td>4.5 ± 0.5</td>
<td>17.79 ± 2.67</td>
</tr>
<tr>
<td>87.31 + 0.69</td>
<td>0.01 ± 0.0015</td>
<td>23.2 ± 3.9</td>
<td>745 ± 109.5</td>
<td>4 ± 0.45</td>
<td>18.9 ± 2.34</td>
</tr>
</tbody>
</table>

### Table III. Experimentally measured production cross sections $\sigma$ (mb) of $^{160}\text{Ho}$ and $^{160}\text{Ho}^{4+}$ residues along with the $\Sigma\sigma_{\text{CF}}$, $\Sigma\sigma_{\text{ICF}}$, $\sigma_{\text{TF}}$ and $F_{\text{ICF}}$ (%).

<table>
<thead>
<tr>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>$^{160}\text{Ho}$ (CF+ICF)</th>
<th>$^{160}\text{Ho}^{4+}$ (CF+ICF)</th>
<th>$^{160}\text{Ho}^{4+}$ (CF+ICF)</th>
<th>$\Sigma\sigma_{\text{CF}}$</th>
<th>$\Sigma\sigma_{\text{ICF}}$</th>
<th>$\sigma_{\text{TF}}$</th>
<th>$F_{\text{ICF}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>54.83 + 0.52</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>173</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>58.51 + 0.50</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>385</td>
<td>6.02</td>
<td>394.06</td>
<td>1.54</td>
</tr>
<tr>
<td>61.37 + 0.63</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>529</td>
<td>12.40</td>
<td>539.96</td>
<td>2.29</td>
</tr>
<tr>
<td>62.63 + 0.80</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>588</td>
<td>16.78</td>
<td>602.97</td>
<td>2.78</td>
</tr>
<tr>
<td>65.49 + 0.69</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>710</td>
<td>30.79</td>
<td>731.08</td>
<td>4.16</td>
</tr>
<tr>
<td>67.24 + 1.17</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>779</td>
<td>42.91</td>
<td>808.68</td>
<td>5.22</td>
</tr>
<tr>
<td>69.15 + 0.85</td>
<td>1.37 ± 0.18</td>
<td>–</td>
<td>–</td>
<td>849</td>
<td>59.12</td>
<td>913.39</td>
<td>6.51</td>
</tr>
<tr>
<td>72.24 + 0.76</td>
<td>4.49 ± 0.56</td>
<td>–</td>
<td>–</td>
<td>953</td>
<td>92.02</td>
<td>1066.37</td>
<td>8.81</td>
</tr>
<tr>
<td>74.97 + 0.57</td>
<td>6.25 ± 1.10</td>
<td>2.37 ± 0.29</td>
<td>1.41 ± 0.21</td>
<td>1040</td>
<td>122.29</td>
<td>1170.42</td>
<td>10.52</td>
</tr>
<tr>
<td>77.77 + 0.62</td>
<td>7.50 ± 0.95</td>
<td>5.42 ± 0.75</td>
<td>1.55 ± 0.23</td>
<td>1110</td>
<td>162.06</td>
<td>1304.84</td>
<td>12.42</td>
</tr>
<tr>
<td>79.62 + 0.61</td>
<td>6.34 ± 0.89</td>
<td>5.71 ± 0.84</td>
<td>1.83 ± 0.25</td>
<td>1160</td>
<td>173.64</td>
<td>1336.42</td>
<td>12.99</td>
</tr>
<tr>
<td>82.47 + 0.82</td>
<td>5.31 ± 0.75</td>
<td>6.45 ± 0.91</td>
<td>2.24 ± 0.36</td>
<td>1230</td>
<td>185.65</td>
<td>1419.18</td>
<td>13.08</td>
</tr>
<tr>
<td>84.44 + 0.56</td>
<td>4.50 ± 0.65</td>
<td>7.89 ± 1.25</td>
<td>2.56 ± 0.34</td>
<td>1270</td>
<td>199.83</td>
<td>1453.13</td>
<td>13.75</td>
</tr>
<tr>
<td>87.31 + 0.69</td>
<td>2.10 ± 0.32</td>
<td>6.09 ± 0.86</td>
<td>2.00 ± 0.31</td>
<td>1320</td>
<td>207.8</td>
<td>1501.73</td>
<td>13.84</td>
</tr>
</tbody>
</table>
independent production cross section ($\sigma_{\text{ind}}$) can be written as

$$\sigma_{\text{ind}} = \sigma_{\text{cum}} - P_{\text{pre}} \frac{t_{1/2}}{t_{1/2}^p} \sigma_{\text{pre}}. \quad (4)$$

As has already been mentioned, the production of $^{165}\text{Tm}(\alpha2n)$ is substantially fed (with a branching ratio $P_{\text{pre}} = 1$) from its precursor $^{165}\text{Yb}$. The value of $F_{\text{pre}}$ is found to be $1.0055 \pm 0.001$ for the given combination of precursor and final nuclei, providing

$$\sigma_{\text{ind}}(^{165}\text{Tm}) = \sigma_{\text{cum}} - 1.0055. \sigma_{\text{pre}}. \quad (5)$$

The value of $\sigma_{\text{ind}}$ of $^{165}\text{Tm}(\alpha2n)$ as given above is plotted in Fig. 3(a).

It may be pointed out that the residues populated via $\alpha$-emitting channels may arise from both CF and/or ICF processes. In the case of CF, the incident projectile ($^{12}\text{C}$) entirely fuses with target nucleus ($^{159}\text{Tb}$) to form a fully
equilibrated CN, which may eventually decay via an $\alpha xn$ channel. However, in the case of ICF, only a part of incident projectile (i.e., $^{12}\text{C} \rightarrow ^8\text{Be} + \alpha$) fuses with the target nucleus to form an incompletely fused composite system, and the remnant $\alpha$ or $^8\text{Be}$ goes on moving in the forward cone as a spectator.

The fraction of ICF in $\alpha$-emitting channels can be accounted by analyzing EFs of evaporation residues in the framework of statistical model code PACE4. As mentioned in the previous section, the code PACE4 does not take ICF into account, therefore, any enhancement in the experimental EFs over the PACE4 predictions may be attributed to contribution coming from ICF. The experimental EFs of individual $\alpha$-emitting channels (expected to be populated via both CF and/or ICF processes) are compared with PACE4 predictions in Fig. 3(a)–3(d). The PACE4 calculations has been performed using same set of input parameters which has been used to reproduce $xn/pxn$ channels. Solid black curves are the best fit to the PACE4 predictions for level density parameter $a = A/K \text{MeV}^{-1}$. As can be seen in Fig. 3(a)–3(d), in general, PACE4 underpredicts the experimental EFs of these residues. The experimentally observed higher production cross section over PACE4 predictions may be attributed to contribution coming from ICF processes, its contribution being distributed over the full range of energy of the EFs, with a different behavior depending on the residue. Particularly interesting is the trend of the EF measured for $^{165}\text{Tm}$, which appears to reflect the interplay between CF and ICF processes through three different decay channels:

(i) CF-1: the CF of $^{12}\text{C}$ with $^{159}\text{Tb}$ leads to an excited nucleus $^{171}\text{Lu}^*$ which may decay via two protons and four neutrons ($2p4n$ channel) as

$$^{12}\text{C} + ^{159}\text{Tb} \rightarrow ^{171}\text{Lu}^* \rightarrow ^{165}\text{Tm} + 2p4n,$$

$Q$ value $= -53.47 \text{ MeV},$

$E_{\text{thr}} = 57.50 \text{ MeV}.$

(ii) CF-2: the excited $^{171}\text{Lu}^*$ nucleus formed in a CF reaction may decay through an $\alpha$ cluster and two neutrons ($\alpha2n$ channel) as

$$^{12}\text{C} + ^{159}\text{Tb} \rightarrow ^{171}\text{Lu}^* \rightarrow ^{165}\text{Tm} + \alpha2n,$$

$Q$ value $= -25.17 \text{ MeV},$

$E_{\text{thr}} = 27.07 \text{ MeV}.$

(iii) ICF: only a part of projectile $^{12}\text{C}$ (i.e., $^8\text{Be}$) fuses with $^{159}\text{Tb}$ to form an incompletely fused composite system ($^{165}\text{Tm}^*$) while an $\alpha$ cluster flows in the forward direction as a spectator. The excited $^{167}\text{Tm}^*$ may then decay via two neutrons ($2n$) as

$$^{12}(^8\text{Be} + \alpha) \rightarrow ^8\text{Be} + ^{159}\text{Tb} \rightarrow ^{165}\text{Tm}^*$$

$$\rightarrow ^{165}\text{Tm} + \alpha + 2n,$$

($\alpha$ particle’ as a spectator),

$Q$ value $= -17.80 \text{ MeV},$

$E_{\text{thr}} = 18.70 \text{ MeV}.$
FIG. 3. (Color online) Experimentally measured EFs of evaporation residues $^{165}\text{Tm}(\alpha 2n)$, $^{163}\text{Tm}(\alpha 4n)$, $^{161}\text{Ho}(2\alpha 2n)$, and $^{160}\text{Ho}\pm(2\alpha 3n)$ are compared with the PACE4 predictions. Solid black curves represent PACE4 predictions performed for $a = A/8$ MeV$^{-1}$. In (a), red dash dotted and blue dotted lines through the data points are drawn to explain the trend of excitation function. See text for explanation.

In this figure, the contributions coming from the type CF1 and CF2 (i.e., the contributions of $2p4n$ and/or $\alpha 2n$) are peaking at $\approx 63$ MeV. However, for the energies above $\approx 70$ MeV, PACE4 predicts very low cross-section as compared to the experimental data points. As such, it can be inferred that the ICF significantly contributes to the production of $^{165}\text{Tm}$ residue through $\alpha$(as a spectator) + 2n channel.

Further, for better visualization of ICF fraction in $\alpha$-emitting channels, the sum of all identified $\alpha$-emitting channels ($\Sigma\sigma_{\alpha x n+2\alpha x n}$) is compared with that estimated by statistical model code PACE4 ($\Sigma\sigma_{\text{PACE4}}$) in Fig. 4. As shown in this figure, the experimentally measured EFs of $\alpha x n/2\alpha x n$ channels are significantly higher than PACE4 predictions for the same value of level density parameter (i.e., $a = A/8$ MeV$^{-1}$), which has been used to reproduce CF residues in the present work. Since, the statistical model code PACE4 does not take ICF into account, therefore, the observed enhancement in the experimentally measured EFs over the theoretically predicted ones, points toward the contribution of ICF in the production of these residues.

In order to deduce ICF contribution in $\alpha x n/2\alpha x n$ channels, the same data reduction procedure has been used as given in Refs. [15,19,21]. The contribution of ICF in the production of $^{165}\text{Tm}$, $^{163}\text{Tm}$, $^{161}\text{Ho}$, and $^{160}\text{Ho}\pm$ residues has been accounted as $\Sigma\sigma_{\text{ICF}} = \Sigma\sigma_{\text{exp}} - \Sigma\sigma_{\text{PACE4}}$. In recent reports [20,22,24], the fraction of ICF deduced using above data reduction procedure has been found to be in good agreement with that estimated from the analysis of forward ranges and angular distributions of heavy recoils. In order to see how does ICF contributes to the total reaction cross section ($\sigma_{\text{TF}} = \Sigma\sigma_{\text{CF}} + \Sigma\sigma_{\text{ICF}}$), systematically deduced ICF cross section ($\Sigma\sigma_{\text{ICF}}$) is plotted with the sum of all CF channels ($\Sigma\sigma_{\text{CF}}$) and $\sigma_{\text{TF}}$ as a function of incident projectile energy in Fig. 5. For better visualization of increasing ICF contribution with energy, the value of $\Sigma\sigma_{\text{ICF}}$ is plotted in the inset. As shown in this figure, the increasing separation between $\Sigma\sigma_{\text{CF}}$ and $\sigma_{\text{TF}}$ with incident projectile energy indicates energy dependence of ICF.

FIG. 4. (Color online) Experimentally measured and theoretically predicted EFs of all $\alpha$-emitting channels are compared. Physically justified level density parameter $a = A/8$ MeV$^{-1}$ is used in PACE4 calculations. The value of $(\Sigma\sigma_{\alpha x n+2\alpha x n})$ is significantly higher than that predicted by PACE4, which may be attributed to the contribution of ICF. Lines through the data points are drawn to guide the eyes.

FIG. 5. (Color online) The total fusion cross section ($\sigma_{\text{TF}}$), the sum of all CF ($\Sigma\sigma_{\text{CF}}$), and ICF ($\Sigma\sigma_{\text{ICF}}$) channels are plotted as a function of incident projectile energy. Lines through the data points are drawn to guide the eyes.
Fig. 6. (Color online) The reproduction of Coulomb barrier ($V_b$) of $^{12}$C+$^{159}$Tb system from the analysis of the experimentally measured CF cross sections. The dashed line through the data points is achieved by best fitting procedure of data.

To support our measurement and the adopted data reduction procedure, an attempt has been made to deduce the value of fusion barrier ($V_b$) from the analysis of experimentally measured CF excitation functions. According to Gutbrod et al. [51], the CF probability may be given as

$$\Sigma \sigma_{\text{CF}} = \pi R^2 (1 - V_b/E_{\text{lab}}).$$  \hspace{1cm} (6)

If the normalized value of $\Sigma \sigma_{\text{CF}}$ is plotted as a function of $1/E_{\text{lab}}$, it should show a linear decrease. The normalized value of $\Sigma \sigma_{\text{CF}}$ has been plotted as a function of $1/E_{\text{lab}}$ in Fig. 6. As shown in this figure, the data points can be fitted by a linear equation which intersects the x axis at $E_{\text{lab}}$ corresponding to $\approx$52 MeV. This confirms the value of fusion barrier ($V_b \approx$52 MeV) of $^{12}$C+$^{159}$Tb system, and strengthen the present measurements and the data reduction procedure. Further, the percentage fraction of ICF ($F_{\text{ICF}}$) has been deduced as a function of various entrance channel parameters, and is discussed in the following sections.

C. ICF strength function

For better insights into the onset and influence of ICF, the percentage fraction of incomplete fusion ($F_{\text{ICF}}$) has been deduced from the analysis of data as demonstrated in Ref. [19]. The $F_{\text{ICF}}$ is a measure of relative strength of ICF to the total fusion, and defined as $F_{\text{ICF}}(\%) = (\Sigma \sigma_{\text{ICF}}/\Sigma \sigma_{\text{TF}}) \times 100$. The value of $F_{\text{ICF}}$ is plotted as a function of reduced incident projectile energy ($E_{\text{lab}}/V_b$) in Fig. 7, i.e., termed as ICF strength function. The ICF strength function defines empirical probability of ICF at different incident projectile energies. As shown in this figure, the value of $F_{\text{ICF}}$ is found to be $\approx$1.5% at $1.12V_b$ (i.e., $\approx$12% above the barrier), and increases smoothly up to $\approx$14% at the highest measured energy (i.e., $1.64V_b$) for $^{12}$C+$^{159}$Tb system. The observed increasing trend of $F_{\text{ICF}}$ with energy indicates that the breakup probability of incident projectile increases under the influence of increasing input angular momenta. The incident projectile ($^{12}$C) may break up into several combinations of $\alpha$ clusters to yield ICF. Some of the breakup combinations observed in previous studies are (i) $^{12}$C may break up into $^{8}$Be and $^{4}$He($\alpha$) clusters, and/or (ii) three $\alpha$ fragments [38–40]. Depending on the favorable input angular momentum conditions, one or a group of fragments (in successive mode) may fuse with the target nucleus to form an incompletely fused composite system [17].

Further, it may not be out of place to mention that some of the reaction channels could not be measured due to the short-half-lives of these missing channels in the present work, which is the limitations of employed technique. In order to incorporate the missing CF channels, the value of $\Sigma \sigma_{\text{ICF}}$ has been corrected using PACE4 predictions. However, no correction could made to incorporate missing ICF channels. Therefore, the quoted value of $\Sigma \sigma_{\text{ICF}}$ in Fig. 5 should be treated as the lower limit of ICF for this system. The inclusion of missing ICF channels may modify slightly the picture presented in Fig. 7.

D. ICF dependence on entrance channel mass asymmetry

As suggested by Morgenstern et al. [36], the ICF contributes significantly above $v_{\text{rel}} \approx 0.06$ (6% of c) for more mass-asymmetric systems. In order to test Morgenstern’s mass-asymmetry systematics, and to understand how the ICF...
systems is expected to go up as discussed in the text. However, the values of Morgenstern’s mass-asymmetry systematics. In general, the $^{12}$C, $^{16}$O systems are slightly away from the increasing trend drawn to guide the eyes. As shown in this figure, the data plotted for six target nuclei for only $^{12}$C beam agree reasonably well with Morgenstern’s mass-asymmetry systematics [36]. On the basis of results and analysis presented, it may be concluded that apart from CF, the ICF is also a process of greater importance at incident projectile energies $\approx 4–7$ MeV/nucleon. The ICF fraction strongly depends on the entrance channel properties, such as mass asymmetry (or the Coulomb factor $Z_P Z_T$), and incident projectile energies. It may, however, be pointed out that the system studied in the present work is rather light, which may not cater to the requirement of synthesizing super heavy elements. However, a rich data set from medium to heavy targets may help to develop some systematics to understand the probability of involved reaction processes at these energies, which may be useful in the super heavy element research. For further refinement of the outcome of this work, the measurement of forward recoil ranges and spin distributions from near barrier energies to well above it are in order.

ACKNOWLEDGMENTS

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IV. SUMMARY AND CONCLUSIONS

In this work, the EFs of several radionuclides populated via CF and/or ICF in $^{12}$C+$^{159}$Tb system have been measured for the energy range $\approx 4–7$ MeV/nucleon, and analyzed in the framework of equilibrated CN decay using statistical model code PACE4. During the decay curve analysis for the identification of different reaction products, it has been found that some of the $\alpha xn$ and $\alpha xn$ channels have contribution from precursor decay of higher charge isobar. An attempt has been made to deduce the independent production cross section from cumulative and precursor decay contribution. The experimentally measured EFs of $\alpha xn/\alpha xn$ channels have been found to agree reasonably well with the predictions of statistical model code PACE4, indicating their production via CF only. However, in the case of all $\alpha$-emitting channels, a significant enhancement in the production cross sections has been observed as compared to the PACE4 predictions. This enhancement has been attributed to the ICF of $^{12}$C with $^{159}$Tb. It has been observed that the probability of ICF increases with incident projectile energy, and mass asymmetry and/or target mass. The results presented in this work are found to follow Morgenstern’s mass-asymmetry systematics [36]. On the basis of results and analysis presented, it may be concluded that RT, the ICF is also a process of greater importance at incident projectile energies $\approx 4–7$ MeV/nucleon. The ICF fraction strongly depends on the entrance channel properties, such as mass asymmetry (or the Coulomb factor $Z_P Z_T$), and incident projectile energies. It may, however, be pointed out that the system studied in the present work is rather light, which may not cater to the requirement of synthesizing super heavy elements. However, a rich data set from medium to heavy targets may help to develop some systematics to understand the probability of involved reaction processes at these energies, which may be useful in the super heavy element research. For further refinement of the outcome of this work, the measurement of forward recoil ranges and spin distributions from near barrier energies to well above it are in order.
LARGE INFLUENCE OF INCOMPLETE FUSION IN...

Effect of alpha-Q-value on reaction dynamics at $\approx 4\text{-}7$ AMeV

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Effect of alpha-Q-value on reaction dynamics at ≈ 4-7 AMeV

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Abstract. This paper deals with the dependence of incomplete fusion reaction dynamics on various entrance channel parameters, specially, the alpha-Q-value effect. The excitation functions for several radio-nuclides formed in ¹⁵C+⁰²Trb interactions at ≈ 4-7 AMeV have been measured using activation technique followed by γ-spectroscopy. The experimentally measured excitation functions have been analyzed in the framework of statistical model code PACE4. A sizeable contribution of incomplete fusion has been delineated in the production of α-emitting channels in reference to the Monte Carlo simulation based statistical model code PACE4. For better insights into the onset and strength of incomplete fusion, the incomplete fusion strength function has been deduced as a function of various entrance channel parameters. A significant amount of incomplete fusion contribution has been observed at slightly above barrier energies, and found to increase smoothly with incident projectile energy. Present results have also been compared with the results obtained in the interactions of ¹⁰⁶O and ¹⁵C with same target ¹⁰⁰Trb, to probe the dependence of incomplete fusion on projectile, specially in the binding energy & alpha-Q-value. The present work in light of previous data hints that instead of binding energy, the alpha-Q-value of the projectile is a parameter which influences the in-complete fusion reaction dynamics.

1 Introduction

Incomplete fusion (ICF) reactions, as recognized since mid-sixties by the very first experimental observation[1] of energetic α-particles emitted very copiously in heavy-ion (HI)-induced reactions at energies > 10.6 AMeV, are the collisions where, besides the formation of a fusion-like nucleus and its decay, forward peaked “fast α-particles” have been observed. The ICF is still an active area of investigations due to complex nature of in-complete mass transfer and its ambiguous dependence on various entrance channel parameters viz., projectile type/energy, imparted input angular momentum (ℓ) to the system, α-breakup energy (Qα), mass-asymmetry of the interaction partners (μ,=A₁/A₂+), deformations of interaction partners etc. An outstanding theoretical challenge, emerged after the observation of “fast α-particles”, is to model the ICF processes. Several approaches have been adopted to understand the ICF-reaction processes [2-16]. In the most commonly used model, proposed by Siwek-Wilczynski et al.[21], the concept of generalized critical angular momentum (ℓ) has been used. According to this model, if the entrance channel angular momentum exceeds the critical limit ℓcrit for complete fusion, the fusion can not occur unless a part of the projectile (P²) is emitted which carries away the excess of angular momentum. The remainder of the projectile (P¹) has now a resulting angular momentum lower than its own critical limit for fusion with the target (ℓ₁ ≥ ℓcrit). This model has been completed by Wilczynski et al.[25], by the addition of a “sum-rule” formalism applied to complete and incomplete fusion reactions. Quantum mechanical models such as the continuum discretized coupled channels model (CDCC) and the time-dependent wave packet method have also been proposed, which are unable to separate out the incomplete and complete fusion contributions. However, A. Diaz-Torres et al. in his recent paper[3] has presented a classical dynamical model that treats breakup stochastically for low energy reactions of weakly bound nuclei. It may, further, be pointed out that the aforementioned models/theories, generally, have been used to fit the experimental data obtained at energies ≥ 10.5 AMeV or so. However, in general, none of the proposed models is able to fit the experimental data obtained at relatively low bombarding energies i.e., ≈ 4-7 AMeV. As such, due to the unavailability of any reliable theoretical model to ex-

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plain the emission of fast PLFs associated with ICF at energies ≈ 4-7 AMeV, the study of ICF is still an active area of investigations.

The $^{12}$C, $^{16}$O and $^{20}$Ne, which are considered to have $\alpha$-cluster structure, beams have been used in most of the previous studies. In fact the cluster structure has been suggested as one of the factors leading to forward peaked alpha particles in ICF reactions. Some amount of experimental data is now available but no systematic studies have been carried out to ascertain this aspect and more results are necessary. As such, a program has been undertaken to carry out some conclusive experiments using $^{12}$C, $^{14}$N and $^{16}$O beams on different targets, which may provide us a rich data set to understand the underlying dynamics.

The present work is the first step in this direction, where, the excitations functions of $^{12}$C+$^{159}$Tb [14] and $^{13}$C+$^{159}$Tb (present work) systems at energies ≈ 4-7 AMeV have been measured and analyzed in the frame-work of equilibrated compound nucleus decay. The ICF cross sections for other systems have been compared to study convincingly the effect of different entrance-channel parameters on the said reaction dynamics. The structure of the paper is as follows; the section-2 deals with the experimental methodology, while the analysis of experimental excitation functions in terms of CN-model is given in section-3. While, the summary of the present work is given in last section of the paper.

2 Experimental details and methodology

The experiments have been performed using the $^{13}$C beam delivered from the 15UD-Pelletron at the Inter University Accelerator Center (IUAC), New Delhi, India using the off-line $\gamma$-ray spectroscopy [11,12]. The isotopically pure, self-supporting $^{159}$Tb targets ($t_m$ ≈ 1.2-2.8 mg/cm²) and the Al-foils ($t_m$ ≈ 1.5-2.5 mg/cm²), served as energy degrader as well as catcher foils of sufficient thickness to stop the recoiling residues produced during the interaction, were prepared using the rolling technique. In order to achieve wide energy range in a single irradiation, energy degration technique has been used. In the present experiment five stacks, each having three target-catcher foil assemblies were irradiated at energies ≈ 58, 70, 73, 85 and 88 MeV. Keeping the half-lives of interest in mind, irradiations were carried out for ≈ 9-11 h duration for each stack. The Pelletron crew provided a nearly constant beam current ≈ 30 nA throughout the irradiations. A Faraday cup, which was placed behind the target-catcher foil assembly, was used to measure the integrated beam current, for every 120 s, so as to correct for the variation in the beam intensity during the irradiation time, which are particularly significant for short-lived radio-nuclides. The activities produced in each target-catcher foil assembly have been recorded using a high resolution HPGe detector coupled to a PC through CAMAC based data acquisition software CANDLE (Collection & Analysis of Nuclear Data using Linux nEtwork)[28]. A typical $\gamma$-ray spectrum obtained at ≈ 87.62 ± 0.40 MeV beam energy is shown in Fig.1. The detector used in these experiments was pre-calibrated both for energy as well as efficiency using various standard $\gamma$-sources such as $^{60}$Co, $^{153}$Ba and $^{152}$Eu. It may, however, be pointed out that most of the $\gamma$-rays of interest for the expected residues lie in this energy range. The energy resolution of the detector was ≈ 2.5 keV at the 1408 keV $\gamma$-line of $^{152}$Eu. A 50Hz pulser was used to determine the dead time. The detector-sample separation was adjusted such as to keep the dead-time below 10% during the counting so as to minimize the pile up effects. The efficiency calibration of the detector in the specified geometry was carried out using a standard $^{152}$Eu source of known strength at various source-detector separations in order to wash out the solid angle effect and to increase the accuracy in the measurement of cross-section.

2.1 Identification of reaction products

In order to determine the production cross-section of these residues, first of all it is desirable to identify the various reaction products via their characteristic $\gamma$-lines. However, the confirmation of the evaporation residues were made by measuring their decay half-lives. The reaction residues identified using the assignment of characteristic $\gamma$-radiations and decay curve analysis are given in Tables-I, along with their spectroscopic properties which has been taken from the Table of Isotopes[29].
3 Obtained results & their interpretation

The EFs of $^{167}$Lu(3n), $^{168}$Lu(4n), $^{166}$Lu(6n), $^{167}$Yb(p4n), $^{165}$Tm($\alpha$2n), $^{163}$Tm($\alpha$5n), $^{162}$Ho(2n2n), $^{162}$Ho(2n2a), $^{161}$Ho(2n3n), $^{160}$Ho(2n4n), and $^{160}$Ho(2n3a) radio-nuclides expected to be populated via CF and/or ICF of $^{13}$C with $^{159}$Tb have been measured in the energy range $\approx 4$–7 MeV. Information regarding the reaction mechanism in the $^{13}$C+$^{159}$Tb interactions may be obtained by comparing the experimentally measured EFs to the theoretical ones. In the present work, the theoretical calculations have been performed using code PACE4[31] to check whether the reaction products are populated only via CF process. The code PACE4, which is a revised version of PACE2, is based on Hauser-Feshbach theory of CN-decay, uses statistical approach of CN de-excitation by Monte Carlo procedure. The angular momentum projections are calculated at each stage of de-excitation, which enables the determination of angular distribution of the emitted particles. The angular momentum conservation is explicitly taken into account. The code uses the BASS formula for CF cross sections calculation[32]. The default optical model parameters for neutron, proton and $\alpha$-emission were used in the code. The $\gamma$-ray strength functions for E1, E2 and M1 transitions were taken from tables of Endt [33]. This code has been modified to take into account the excitation energy dependence of level density parameter using the prescription of Kataria et al. [34]. In this code, the level density parameter $\alpha = A/K$, is one of the important parameters, where, $A$ is the mass number of the nucleus and $K$ is a free parameter. The value of $K$ may be varied to match the experimental data. In the present work, a value of level density parameter ($\alpha$) was taken as $A/8$ MeV$^{-1}$ i.e. $K=8$ is used for the calculations as it gives best fit to the experimental data of complete fusion channels. It may, however, be pointed out that the ICF and PE-emission are not taken into consideration in this code.

![Fig. 2. (a) Experimental EFs of xn ($x=3, 4, 5$) and p4n-channels populated in the $^{13}$C+$^{159}$Tb system. The solid lines through the data points are drawn to guide the eyes. (b) Sum of experimentally measured EFs of all xn/p4n-channels ($\Sigma\sigma_{CF}$) are compared with that predicted by PACE4 using physically reasonable parameters for mass region $A=150$. Experimental data is in good agreement with the PACE4 calculations for $a = A/8$ MeV$^{-1}$.

Table 1. List of identified reaction residues (channels) with their spectroscopic properties

<table>
<thead>
<tr>
<th>Residue</th>
<th>T$_{1/2}$</th>
<th>J</th>
<th>E$_{x}$ (keV)</th>
<th>I (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{167}$Lu(3n)</td>
<td>3.06 hrs</td>
<td>7/2$^+$</td>
<td>191.21</td>
<td>20.6</td>
</tr>
<tr>
<td>$^{168}$Lu(4n)</td>
<td>5.5 min</td>
<td>3$^+$</td>
<td>198.86</td>
<td>18.0$^*$</td>
</tr>
<tr>
<td>$^{168}$Lu(4n)</td>
<td>6.7 min</td>
<td>6$^-$</td>
<td>198.86</td>
<td>18.0$^*$</td>
</tr>
<tr>
<td>$^{167}$Lu(5a)</td>
<td>51.5 min</td>
<td>7/2$^+$</td>
<td>218.21</td>
<td>3.5</td>
</tr>
<tr>
<td>$^{167}$Yb(p4n)</td>
<td>17.5 min</td>
<td>5/2$^-$</td>
<td>176.2</td>
<td>20.4</td>
</tr>
<tr>
<td>$^{165}$Tm($\alpha$2n)</td>
<td>7.70 hrs</td>
<td>2$^+$</td>
<td>184.41</td>
<td>16.1</td>
</tr>
<tr>
<td>$^{165}$Tm($\alpha$3n)</td>
<td>30.06 hrs</td>
<td>1/2$^+$</td>
<td>242.85</td>
<td>35</td>
</tr>
<tr>
<td>$^{163}$Tm($\alpha$5n)</td>
<td>1.81 hrs</td>
<td>1/2$^+$</td>
<td>190.07</td>
<td>1.28</td>
</tr>
<tr>
<td>$^{162}$Ho(2n2n)</td>
<td>15.0 hrs</td>
<td>1$^+$</td>
<td>185.20</td>
<td>28.6</td>
</tr>
<tr>
<td>$^{162}$Ho(2n2a)</td>
<td>67.0 hrs</td>
<td>6$^-$</td>
<td>185.20</td>
<td>0.42</td>
</tr>
<tr>
<td>$^{161}$Ho(2n3n)</td>
<td>2.48 hrs</td>
<td>7/2$^-$</td>
<td>103.03</td>
<td>3.6</td>
</tr>
<tr>
<td>$^{160}$Ho(2n4n)</td>
<td>25.6 min</td>
<td>5$^+$</td>
<td>645.25</td>
<td>16.20</td>
</tr>
<tr>
<td>$^{160}$Ho(2n3a)</td>
<td>5.02 hrs</td>
<td>2$^-$</td>
<td>645.25</td>
<td>16.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* these intensities are relative.
3.1 xn and pxn-channels

Fig.2(a) shows the experimentally measured EFs of $^{169}$Lu($t_{1/2}=34.06$ h), $^{168}$Lu($t_{1/2}=5.5$ min, $6.7$ min), $^{167}$Lu($t_{1/2}=51.5$ min), and $^{167}$Yb($t_{1/2}=17.5$ min) evaporation residues expected to be populated via 3n, 4n, 5n, and p4n emission from the excited $^{172}$Lu* nucleus formed in CF reactions. Self explanatory notations are used to explain the decay channels in this figure. The solid lines through the data points are drawn to guide the eyes. During the decay-curve analysis, the evaporation residue $^{167}$Yb(p3n) is found to be strongly fed from its higher charge isobar (pre-cursor here after) $^{167}$Lu(5n) through $\beta^+$ emission. The half-life of of pre-cursor (i.e., $^{167}$Lu $\rightarrow t_{1/2}^{pre} = 51.5$ min) is larger than the half-life of the daughter nuclei (i.e., $^{167}$Yb $\rightarrow t_{1/2}^d = 75.5$ min). In this case, the independent production cross-section ($\sigma_{\text{ind}}$) of $^{167}$Yb has been deduced using the following successive radio-active decay formulation [27]:

$$N_d(t) = C_{t=0} \cdot e^{-\lambda t} \cdot \left( \frac{P_{\text{pre}} \cdot \lambda_{\text{pre}}} {\lambda_d - \lambda_{\text{pre}}} \right) \cdot \left( N_{\text{pre}}(t) \cdot e^{-\lambda_{\text{pre}} \cdot t} \right)$$

Where: $N_d(t)$ and $N_{\text{pre}}(t)$ are the number of daughter and pre-cursor nuclei produced at time ‘t’. $C_{t=0}$ is the cumulative (pre-cursor + daughter) number of nuclei produced at the end of the irradiation, $\sigma_{\text{pre}}$ and $\sigma_d$ are the production cross-sections of pre-cursor and daughter nuclei; and $\lambda_{\text{pre}}$ and $\lambda_d$ are the decay constants of pre-cursor and daughter nuclei, respectively. The value of $N_{\text{pre}}(t)$ has been deduced from the experimentally measured decay-curve of $^{167}$Lu.

The value of $N_d(t)$ ($^{167}$Yb) has been obtained by solving equation (1), which has been translated to its production cross-section ($\sigma_{\text{ind}}$), and is plotted in Fig.2(a) as independent production of $^{167}$Lu.

Fig.2(b) shows the experimental EFs of all xn/pxn channels (i.e., $\Sigma \sigma_{\text{EXP}}$) compared with PACE4 predictions. As shown in this figure, the value of $\Sigma \sigma_{\text{EXP}}$ is very well reproduced by PACE4 for the level density parameter ‘$\nu$’ = A/8 MeV$^{-1}$, which is suitable for A=150 mass-region[11–14]. This indicates the production of these residues through the de-excitation of fully equilibrated compound nucleus formed in a CF reaction.

3.2 $\alpha$-emitting channels

The experimental EFs of $^{166}$Tm($\alpha$2n), $^{166}$Tm($\alpha$3n), $^{166}$Ho($\alpha$5n), $^{164}$Ho($\alpha$2n3n) and $^{164}$Ho($\alpha$4n+2n) residues are shown in Fig.3(a). Due to the involvement of $\alpha$-emission in the exit channel, these residues are expected to be populated via both CF and/or ICF processes. It has been noticed that the evaporation residue $^{165}$Tm is strongly fed from its pre-cursor $^{165}$Yb. In this case, the half-life of pre-cursor (i.e., $^{165}$Yb $\rightarrow t_{1/2}^{pre} = 9.9$ min) is smaller than the daughter nuclei (i.e., $^{165}$Tm $\rightarrow t_{1/2}^d = 30.06$ hrs). Hence, the pre-cursor contribution has been subtracted using the prescription given by Cavinato et al. [27], and the deduced value of $\sigma_{\text{ind}}$ of $^{165}$Tm($\alpha$3n) is plotted in Fig.3(a).

Further, the sum of all identified $\alpha$-emitting channels ($\Sigma \sigma_{\text{EXP}}$) is compared with that estimated by statistical model code PACE4 ($\Sigma \sigma_{\text{PACE4}}$) in Fig.3(b). As can be seen from this figure, the experimentally measured EFs of o2nx/2oxn - channels are significantly higher than PACE4 predictions for the
same set of parameter, which has been used to reproduce CF-residues in the present work. Since the statistical model code PACE4 does not take ICF into account, therefore, the observed enhancement in the experimentally measured EFs over the theoretically predicted ones, points towards the contribution of ICF in the production of these residues. For example, the production of $^{165}\text{Tm}(a2n)$ via both CF and/or ICF can be justified as:

(i) CF: entire projectile ($^{13}$C) fuses with $^{152}$Tb to form an excited CN ($^{172}$Lu$^+$), which may decay through a3n or 2p5n channel leaving behind $^{165}\text{Tm}$ residual nucleus.

$^{13}$C+$^{150}$Tb$\rightarrow^{172}$Lu$^+\rightarrow^{165}$Tm+a3n or 2p5n

(ii) ICF: the projectile ($^{13}$C) breaks up into its constituent α-clusters (i.e., $^2\text{Be}+\alpha$). In this case, only a part of projectile ($^2\text{Be}$) fuses with $^{159}$Tb to form an IFC system ($^{168}\text{Tm}^*$), while the remnant behaves like a spectator. The excited IFC system $^{168}\text{Tm}^*$ further decay through three neutrons (3n) to reach final reaction products $^{165}\text{Tm}$.

$^{13}$C($^2\text{Be}+\alpha$)$\rightarrow^{159}$Be+$^{159}$Tb$\rightarrow^{168}$Tm$^*$+$^{165}$Tm+$\alpha$ (spectator)+3n

In order to deduce ICF contribution in αxn/2αxn - channels, the same data reduction procedure has been used as given in refs.[11–13]. In recent reports [11–13], the fraction of ICF deduced using above data reduction procedure has been found to be in good agreement with that estimated from the analysis of forward ranges and angular distributions of heavy recoils. In order to see how does ICF contributes to the total fusion cross-section ($\sigma_F = \Sigma\sigma_{CF} + \Sigma\sigma_{ICF}$), systematically deduced ICF cross-section ($\Sigma\sigma_{ICF}$) is plotted with the sum of all CF-channels ($\Sigma\sigma_{CF}$) and $\sigma_F$ as a function of projectile energy in Fig.4. As shown in this figure, the increasing separation between $\Sigma\sigma_{CF}$ and $\sigma_F$ with incident projectile energy indicates energy dependence of ICF.

4 Incomplete fusion strength function

As is evident from Figs.3 (b) & 4 that ICF-processes contribute significantly to the evaporation residue cross-sections. It can be noticed from Fig.4 that the CF component has significant contribution at $\approx 52$ MeV projectile energy, while ICF contribution seems to start from $\approx 63$ MeV. Further, it may also be noted from Fig.4, that the separation between the plots for $\sigma_F$ and $\Sigma\sigma_{CF}$ increases with the projectile energy, which indicates larger contribution from ICF at relatively higher projectile energies. From Fig.4, it is clear that the ICF starts competing with CF from $\approx 26\%$ above the Coulomb barrier ($V_b \approx 52$ MeV) and increases towards higher energies.

Fig. 4. The total fusion cross-section ($\sigma_F$), the sum of all CF ($\Sigma\sigma_{CF}$), and ICF ($\Sigma\sigma_{ICF}$) channels are plotted as a function of incident projectile energy. Solid lines through the data points are drawn to guide the eyes. The value of $\Sigma\sigma_{ICF}$ significantly contributed to the value of $\sigma_F$.

4.1 Dependence of $F_{ICF}$ on projectile energy and α-Q-value

For better insights into the onset and influence of ICF, the percentage fraction of incomplete fusion ($F_{ICF}$) has been deduced from the analysis of data as demonstrated in ref.[11,12]. The $F_{ICF}$ is a measure of relative strength of ICF to the total fusion, and defined
as: $F_{ICF}(\%) = (\Sigma_{ICF}/\Sigma_{CF}) \times 100$. The value of $F_{ICF}$ is plotted as a function of reduced incident projectile energy ($E_{ab}/N_b$) in Fig. 5, i.e., termed as ICF strength function.

The value of $\% F_{ICF}$ for two systems: (i) $^{13}\text{C}^{\text{150}}\text{Tb}$ (present work) and (ii) $^{12}\text{C}^{\text{150}}\text{Tb}$ [14] has been deduced using above formulation and is plotted as a function of reduced incident projectile energy in Fig. 5. As can be seen from this figure, the value of $F_{ICF}$, for $^{13}\text{C}$ as projectile, is found to be $\approx 1.0$ % at $1.20N_b$ (20 % above the barrier), and increases smoothly up to $\approx 7$ % at highest measured energy (i.e., 1.68$N_b$). However, for $^{12}\text{C}^{\text{150}}\text{Tb}$ system the $F_{ICF}$ is found to be $\approx 1.5$ % at $1.12N_b$ (12 % above the barrier), and increases smoothly up to $\approx 14$ % at highest measured energy (i.e., 1.66$N_b$). From this figure it is evident that the ICF contribution for both the systems increases with the projectile energy, as expected.

It is not out of place to mention that the binding energy of $^{12}\text{C}$ is higher than the $^{13}\text{C}$; however, it is surprising that though $^{12}\text{C}$ gives large ICF fraction. Hence, in order to ascertain this surprising picture, the $F_{ICF}$ have been plotted against the alpha-Q-value of the projectile for the three different target-projectile combinations: $^{16}\text{O}^{150}\text{Tb}$, $^{12,13}\text{C}^{150}\text{Tb}$ in Fig. 6. From this figure it is evident that as the alpha-Q-value of the projectile increases, the $F_{ICF}$ decreases. As such, the alpha-Q-value is a more reasonable parameter to understand ICF-reaction dynamics as compared to binding energy of the projectile.

### Summary

In the present work, the EFs for several radio-nuclides produced via CF and/or ICF in $^{13}\text{C}^{\text{150}}\text{Tb}$ interactions at energies $\approx 4.7$ MeV/nucleon have been measured and analyzed in light of statistical model predictions. Some pnn and $\alpha$nn-channels are found to be populated directly and/or via pre-cursor decay of higher charge isobars. An attempt has been made to deduce the independent production cross-section from cumulative and pre-cursor decay contribution of different radio-nuclides. The experimentally measured EFs have been found to agree reasonably well for $\alpha$n/pnn-channels indicating their production via CF only, as expected. However, in case of all the $\alpha$-emitting channels, significant enhancement in the production cross-sections has been observed as compared to theoretical model predictions and is attributed due to the ICF of $^{13}\text{C}$ i.e. the break-up of projectile into clusters ($^8\text{Be} + ^4\text{He}$ and/or $^4\text{He} + ^6\text{Be}$) leading to the various ICF processes. From the analysis it has, also, been observed that the projectile break-up probability increases with projectile energy, which reveals the dependence of ICF processes sensitively on projectile energy. The present results in light of the previously studied systems has concluded that the alpha-Q-value is an important parameter. As the alpha-Q-value increases the incomplete fusion contribution to total fusion decreases, which is justified. Further, it may also be concluded that apart from CF, the ICF is also found to contribute significantly to the total reaction cross-section even at projectile energies as low as $\approx 4.7$ MeV/nucleon. Therefore, while predicting the total reaction cross-section for a projectile-target combination, the contribution from ICF should also be taken into consideration. Further, the better insight of underlying processes can be obtained by comparing a rich set of experimental data for various projectile-target combinations. However, the measurement of recoil range distribution and spin-distribution of residues populated by CF as well as ICF using particle-$\gamma$ coincidence technique both at relatively low and higher bombarding energies may also provide a more clear and conclusive picture of the incomplete fusion processes.

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Chapter 7. Reprints of the Published Papers


Entrance channel effect in the incomplete fusion reactions

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Entrance channel effect in the incomplete fusion reactions

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Abstract: In the present work the effect of various entrance channel parameters on incomplete fusion strength and the reaction dynamics in \textsuperscript{12}C+\textsuperscript{159}Tb system at energies \(\approx 4\text{-}7\text{MeV}/\text{A}\) have been investigated by measuring the excitation functions of individual reaction channels. Experimental excitation functions have been analyzed in the framework of compound nucleus decay using statistical model code PACE4. Analysis of data suggests the production of \(\alpha\text{-}p\text{xn}\)-channels via complete fusion of \textsuperscript{12}C with \textsuperscript{159}Tb, as these are found to be well reproduced by PACE4 predictions, while, a significant enhancement in the excitation functions of \(\alpha\)-emitting channels has been observed over the theoretical ones. This enhancement has been attributed due to incomplete fusion. For better insight into the underlying dynamics, fraction of incomplete fusion to the total fusion has been deduced and compared with \textsuperscript{16}O+\textsuperscript{159}Tb and other nearby systems as a function of various entrance channel parameters. The fraction of incomplete fusion has been found to be sensitive to the projectile type, energy and entrance-channel mass-asymmetry.

1 Introduction

Recently, the unexpected presence of incomplete fusion (ICF) at low projectile energies (i.e. \(\approx 4\text{-}7\text{MeV}/\text{A}\)), where only complete fusion (CF) is expected to be the sole contributor to the total fusion cross-section has attracted both experimentalist as well as theorists [1-9]. In recent studies [5-10] a substantial fraction of ICF has been observed at slightly above barrier energies. The concept of incomplete mass transfer in HI-reactions has originated after the pioneering experimental observation of “fast-\(\alpha\)-particles” by Britt and Quinton [11] at relatively higher energies \(\approx 10.5\text{MeV}/\text{A}\). Since then, several theoretical models have been proposed to understand the ICF-reaction dynamics [12-16], however, these models explain the experimental data up to some extent only, that too at energies \(\geq 10\text{MeV}/\text{A}\) or so. Further, the study of ICF reactions got resurgent interest after the observation of large ICF contribution at energies \(\approx 4\text{-}7\text{MeV}/\text{A}\) [5-10]. In one of our recent letters [7], it has been observed that the ICF component at fixed incident energy is associated to increased mean input angular momenta (\(\ell\)-values) for successively opened \(\alpha\)-emitting channels. As per the generalized concept of input angular momenta, all \(\ell\)-values from \(\ell = 0\) to \(\ell _{\text{crit}}\) are associated with central and/or near central interactions, where projectile-target nuclei amalgamate and thus, form a completely fused composite system, which may eventually decay via usual modes [15]. However, for the higher \(\ell\)-values ICF sets in, where, the sum of repulsive Coulomb and centrifugal potential overcomes the attractive nuclear potential. As a result, pocket in the effective potential disappears and fusion of interacting nuclei is hindered due to the higher input angular momenta (\(\ell\)-values) than the fusion limit (\(\ell _{\text{crit}}\)). To release excess input angular momenta imparted into the system the projectile breaks up into its constituents. It may be pointed out that the projectile break up modifies the picture of fusion of two strongly bound nuclei as; (i) complete fusion (CF), where projectile and/or its all fragments may fuse with the target nucleus after projectile breakup and (ii) incomplete fusion (ICF), defined as the fusion of only one of the fragments with target nucleus and remnant moving in forward cone with projectile velocity and (iii) no capture breakup (NCBU), where no breakup fragment is captured by the target nucleus. As such, the total fusion cross-section (TF here after) may be defined as the sum of CF and ICF cross-sections. Earlier experiments [5-8] have conclusively demonstrated the presence of substantial ICF contribution even at low energies but limited only to a few projectile-target combinations. Hence, in order to investigate the role of various entrance channel parameters, e.g., (i) projectile type/energy, and (ii) mass-asymmetry (\(\mu _{A_{\text{p}}A_{\text{T}}}\))

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residues produced during the interaction of projectile-target combination \(^{12}\text{C}+^{159}\text{Tb}\) have been measured at \(\approx 87.31\) MeV, indicating different CF and/or ICF residue \(\gamma\)-lines. Further, confirmation of the identified reaction products has been made by the decay curve analysis. Nuclear data on radio-nuclides, such as the corresponding \(\gamma\)-ray abundances and half lives were taken from ref [18]. The production cross sections of the reaction products have been determined using the standard formulation (for details see ref. 7-8). In the present work the EFs for \(^{168,168m}\text{Lu}\) (3n), \(^{166}\text{Lu}\) (4n), \(^{166}\text{Lu}\) (6n), \(^{159}\text{Yb}\) (p3n), \(^{165}\text{Tm}\) (n2n), \(^{165}\text{Tm}\) (n4n), \(^{161}\text{Ho}\) (2n2n), \(^{160}\text{Ho}\) (2n3n) and \(^{159}\text{Ho}\) (2o4n) residues have been measured. The errors in the measured production cross-sections of different radio-nuclides may arise due to various factors and the overall errors including statistical errors are estimated to be \(\leq 15\%\), excluding the uncertainty in branching ratio, decay constant, etc.

### 2 Experiment and Data Reduction

The experiments have been performed at the Inter University Accelerator Center (IUAC), New Delhi, India, using activation technique followed by off-line \(\gamma\)-ray spectroscopy [5-8]. The irradiations of the samples have been carried out in the General Purpose Scattering Chamber having an in-vacuum transfer facility, which has been used to minimize the lapse time between the stop of the irradiation and beginning of the counting of the activity induced in the target-catcher assembly. The isotopically pure, self-supporting \(^{159}\text{Tb}\) targets of thickness \(\approx 1.2-2.5\) mg/cm\(^2\) were prepared by rolling technique. The Al-foils were also prepared by the rolling technique, which served as energy degrader as well as catcher foils \((\approx 1.5-2.5\) mg/cm\(^2\)) to trap the recoiling residues produced during the interaction of \(^{12}\text{C}+^{159}\text{Tb}\). In order to cover a wide energy region in a single irradiation, energy degradation technique has been used. In the present experiment five stacks, each having three target-catcher foil assemblies were irradiated at energies \(\approx 59, 70, 73, 85\) and \(88\) MeV. Keeping the half-lives of interest in mind, irradiations were carried out for \(\approx 8-10\) h duration for each stack. The Pelletron crew provided a nearly constant beam current \(\approx 30\) nA throughout the irradiations. A Faraday cup, which was placed behind the target-catcher foil assembly, was used to measure the integrated beam current, for every \(120\) s, so as to correct for the variation in the beam intensity during the irradiation, which is particularly significant for short-lived radio-nuclides. The activities produced in each target-catcher foil assembly have been recorded using a high resolution pre-calibrated HPGe detector coupled to a PC through CAMAC based data acquisition software CANDLE [17]. A 50Hz pulser was used to determine the dead time. The detector-sample separation was adjusted to keep the dead-time below 10\% during the counting so as to minimize the pile up effects. The efficiency calibration of the detector in the specified geometry was carried out using a standard \(^{152}\text{Eu}\) source of known strength at various source-detector separations. A typical \(\gamma\)-ray spectrum for the \(^{12}\text{C}+^{159}\text{Tb}\) interaction at \(\approx 87.31\) \(\pm 0.69\) MeV is shown in the Fig.1, where some of the prominent \(\gamma\)-lines have assigned to different reaction products, populated via CF and/or ICF channels.

#### 3 Interpretation of excitation functions

Information regarding the reaction mechanism in the \(^{12}\text{C}+^{159}\text{Tb}\) interactions may be obtained by comparing the experimentally measured EFs with the theoretical ones calculated using code PACE4 [19]. The code PACE4 is based on Hauser-Feshbach theory for CN-decay and uses statistical approach of CN de-excitation by Monte Carlo procedure. The code uses the BASS model [20] for CF cross section calculation. The default optical model parameters for neutrons, protons and \(\alpha\)-particles are used. This code has been modified to take into account the excitation energy dependence of the level density parameter using the prescription of Kataria et al. [21]. It should be pointed out that the ICF and PE-emission are not taken into consideration in this code.\\

#### 3.1 The \(xn\) \& \(pxn\)-channels: mainly populated via CF

From the analysis of experimental data, activities corresponding to nuclide \(^{168,168m}\text{Lu}\) (\(T_{1/2}=6.7\) min \& 5.5 min), \(^{167}\text{Lu}\) (\(T_{1/2}=51.5\) min), \(^{166}\text{Lu}\) (\(T_{1/2}=10.74\) min) and \(^{167}\text{Yb}\) (\(T_{1/2}=17.5\) min), which are populated via 3n, 4n,
6n and p3n-channels, respectively, have been determined. It may be pointed out that the residue corresponding to 5n-channel could not be measured due to its short half-life. However, p2n-channel produces stable residue. The experimentally measured and theoretically calculated EFs for residues populated via x n & pxn-channels are shown in Fig.2. In the present work the theoretical calculations have been done with code PACE4. The measured cross-sections have been done with code PACE4. The level density parameter ‘a’ (=A/K), is one of the important parameters, where, A is the mass number of the nucleus and K is a free parameter. The value of K may be varied to match the experimental data. In the present work as well as in our previous studies, the measured EFs have been reproduced satisfactorily using ‘a’ = A/8 MeV⁻¹ for this mass-region. Since, the theoretical calculation of PACE4 for the residues populated via x n-channels (x = 3, 4, 6) and the production cross-section for 166Yb (p3n) residue deduced after the subtraction of precursor contribution [22] satisfactorily reproduce the experimentally measured excitation functions, it may be inferred that these channels are populated via CF only (see Fig.2). In this figure the ))) has been corrected for the contribution of 5n & p2n-channels obtained from PACE4 predictions.

3.2 α-emitting channels: enhancement over PACE4 predictions

As the EFs for x n & pxn-channels are satisfactorily reproduced via PACE4 calculations and hence, it is reasonable to assume that the parameters used in these calculations are suitable for the present system. Therefore, the same set of input parameters has also been used to fit the EFs of α-emitting channels. Note that in the case of α-emitting-channels, the residue may be formed via two ways: (i) by CF of 12C followed by the formation of an excited CN from which evaporation of neutrons and α-particle may take place, or (ii) the partial fusion of 12C ion when it breaks into 8Be and α, where 8Be fuses with the target nucleus leaving the α-particle as spectator and vice-versa. The measured cross-sections corresponding to αxn (167Tm (x=2, 4)) and α2xn-channels (165Ho (x=2, 3, 4)) are shown in Fig.3 (a-b). In this Fig.3 (a-b) the black line are the theoretical calculations done with the parameters as used to reproduce the xn/pxn-channels. Again, the residue 164Tm (α3n) could not be measured experimentally due to its short half-life. It is very much evident from the Fig.3 (a-b) that there is a significant enhancement in experimental cross-section ))(expt) over theoretical ]]((PACE4) for both αxn and 2αxn-channels. Hence, it may be concluded that all the α-emitting channels (αxn & 2αxn) have significant contributions from ICF reaction processes.

3.3 Incomplete fusion fraction

Since, it is evident from analysis of EFs that ICF-reactions contribute significantly to the evaporation residue cross-sections. As such, an attempt has been made to deduce the ICF contribution from experimentally measured EFs, using the prescription of Gomes et al., [3]. The ICF-contribution for individual channels has been deduced by subtracting CF cross-sections σxn (predicted by code) from the experimentally measured cross sections σexpt at each studied energy. The ICF contributions so deduced for all measured ICF-channels (ΣΣICF) and the sum of cross-sections for all CF-channels (ΣΣCF) obtained from theoretical model predictions are plotted along with the total fusion cross-section (σF = ΣΣCF+ΣΣICF) in Fig. 4(a). The projectile energy normalized with Coulomb barrier (Eproj/Vb) in Fig.4 (a), has been chosen to compare different projectile-target combinations in a plot, as
increases with mass-asymmetry, individually for each
Moreover, it may be observed from this figure that $F_{ICF}$
the $F_{ICF}$ relative velocity ($\alpha$)
for different projectile-target combinations at a constant
function of mass-asymmetry of the interacting partners
that for the same target
16
of projectile, in Fig.4 (b) the $F_{ICF}$
ICF
fusion, may be defined as, $F_{ICF}$
measure of relative strength of ICF contribution to total
In the present work, the percentage $F_{ICF}$
normalized projectile energy (upper panel); however, the lower
Fig.4. The TF with CF & ICF cross-sections as a function of
normalized projectile energy (upper panel); however, the lower
panel contains the comparison of near-by system at same
relative velocity (details see in text).

4 Remarks on the effect of projectile type

In the present work, the percentage $F_{ICF}$, which is the
measure of relative strength of ICF contribution to total
fusion, may be defined as, $F_{ICF}$ (%) = ($\Sigma \sigma_{ICF}/\sigma_{TF}$) ×100
have been deduced. In order to see the effect of the type
of projectile, in Fig.4 (b) the $F_{ICF}$ have been plotted as a
function of mass-asymmetry of the interacting partners
for different projectile-target combinations at a constant
relative velocity ($v_{rel} = 0.053$). It is clear from this figure,
that for the same target $^{169}$Tm the $F_{ICF}$ is $\approx 18\%$
larger for $^{16}$O than for $^{12}$C as projectile. Similarly, for $^{159}$Tb target
the $F_{ICF}$ is $\approx 6\%$ more for $^{16}$O beam as compared to $^{12}$C.
Moreover, it may be observed from this figure that $F_{ICF}$
increases with mass-asymmetry, individually for each $^{16}$O
and $^{12}$C projectiles. It may also be observed that for both
$^{169}$Tm and $^{159}$Tb target nuclides, the ICF-contribution for
$^{16}$O projectile is larger than the ICF-contribution for $^{12}$C
projectile. Besides the excess of alpha cluster in $^{16}$O, one of
the possible reasons for higher $F_{ICF}$ in case of $^{16}$O may
be the fact that it has relatively less $Q_{ICF}$ (-7.16 MeV) as
compared to $^{12}$C, $Q_{ICF}$ (-7.37 MeV) value. From Fig.4 (b) it
may be concluded that $F_{ICF}$ depends not only on the mass-
asymmetry of the interacting partners but also on the type
of projectile, which plays an important role in the
underlying reaction dynamics.

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Role of high $\ell$-values in the onset of incomplete fusion
I. INTRODUCTION

The dynamics of incomplete fusion were intensively investigated in recent years using light heavy ion (\( A \leq 16 \)) beams at energies \( \approx 4–7 \) MeV/nucleon [1–10]. At these energies, the interaction trajectories (\( \ell \) bins) lying within the target dimensions prominently lead to the reaction modes, namely (i) complete fusion (CF), which is defined as the capture of an entire projectile by the target nucleus, and (ii) incomplete fusion (ICF), where only a part of the projectile fuses with the target nucleus [11–19]. In a qualitative way, both the reaction modes can be disentangled on the basis of driving angular momenta at different interaction trajectories (\( \ell \) bins) [20]. Central and/or near-central trajectories (\( 0 \leq \ell \leq \ell_{\text{crit}} \)) lead to CF, where an excited composite system forms after intimate contact and transient amalgamation of the projectile and the target nucleus. In this case, the attractive nuclear potential influences the sum of repulsive Coulomb and centrifugal potentials. Eventually, the projectile’s kinetic energy and driving angular momenta are equally distributed among all the accessible internal degrees of freedom of a composite to form an equilibrated compound nucleus (CN) [21–25]. However, at the relatively higher \( \ell \) values (above \( \ell_{\text{crit}} \)) associated with noncentral interactions, the centrifugal potential overwhelms the attractive nuclear potential, so the pocket in the entrance channel potential disappears [26–29]. As a consequence, fusion of the entire projectile (the case of CF) with the target nucleus hinders and gives way to ICF. In the case of ICF, a part of the projectile emits as a spectator (\( P^\ast \)) to release excess driving angular momenta. After such an emission, the remnant (participant \( P^P \)) is supposed to have an effective driving angular momenta less than or equal to its own critical limit (\( \ell_{\text{eff}} \leq \ell_{\text{crit}}^{P^P+T} \)) for fusion to occur, which leads to an incompletely fused composite (IFC) and direct projectile-like fragments (PLF’s) centered in the forward cone [30,31]. It may further be pointed out that the CN formed in the case of CF is expected to have a predetermined mass/charge, excitation energy, and angular momenta. However, the IFC system forms with relatively less mass/charge and excitation energy (due to partial fusion of the projectile), but at high angular-momenta (imparted into the system due to noncentral interactions) as compared to the CF population(s).

After the first experimental observation of direct PLF’s (associated with ICF) [32,33], several dynamical models were proposed to understand the underlying dynamics. Udagawa and Tamura defined ICF in their Break-up fusion (BUF) model [34] based on the distorted-wave Born approximation (DWBA). In this model, the projectile is assumed to break up into constituent \( \alpha \) clusters within the nuclear field of the target nucleus. One of the fragments fuses with the target nucleus (depending on the available \( \ell \) window) and the remnant assumed to be ejected in the forward cone with an almost projectile velocity as spectator. Wu and Lee [35,36] described the production of direct PLF’s by employing a simple plane-wave-projectile-breakup model, where the projectile breakup is described as a fast process governed by the distribution of nucleon momenta in the projectile before the interaction and the fast PLF’s are assumed to be produced from the coupling of Fermi-momentum and the center-of-mass momentum. A similar description for the production of direct PLF’s was
presented in Refs. [37,38], Wilczynski et al. [26], in their sum-rule model, inferred that ICF is confined to the ℓ space above the ℓ_{crit} for CF and mainly originated from noncentral interactions. This model was later extended by Brûnceal et al. [39]. In the promptly emitted particles (PEP’s) model, Bondorf et al. [40], elaborated that the nucleons transferred from the projectile to the target nucleus may get accelerated in the nuclear field of the target nucleus, and consequently, acquire extra velocity to escape before equilibration. In addition, Fermi-jet [41,42], moving-source [43], exciton [44,45], and overlap [46,47] models were proposed. Morison et al. [48] and Mohring et al. [49] also proposed dynamical models for projectile breakup and ICF. Despite the aforementioned studies, ICF dynamics are not fairly understood at energies where both CF and ICF were observed below and above the ℓ_{crit} [57,58]. Almost similar conclusions were drawn by Tserruya et al. [59] and Oeschler et al. [29], where both CF and ICF were observed below and above the value of ℓ_{crit}. Apart from the above studies, the most debated and still outstanding issues about the ICF dynamics at energies ≈4–7 MeV/nucleon. Further, Morgenstern et al. [50] correlated the probability of ICF with mass asymmetry of interacting partners. Recently, we presented a supplement on Morgenstern’s systematics in Refs. [7,8]. The advances in the understanding of ICF dynamics took place after the particle-γ-coincidence measurements by Inamura et al. [51] and Zolnowski et al. [52]. Apart from that, the correlation of energies and angles of charged-particle(s) along with the γ multiplicity were measured by Geoffroy et al. [53], where the origin of PLF’s was investigated from undamped noncentral interactions (high ℓ values). The noncentral nature of ICF was also emphasized by Trautmann et al. [54] and Inamura et al. [51,55]. In a review on ICF, Gerschel [56] inferred that the localization of the ℓ window also depends on the target deformation. For rare-earth targets, the emission of PLF’s was found to originate from high ℓ values [23,26,51,53], but the results obtained by Tricoire et al. [42] with semimagic targets suggested that the origin of direct PLF’s from the ℓ values is even smaller than 0.5 ℓ_{crit} [57,58]. Almost similar conclusions were drawn by Tserruya et al. [59] and Oeschler et al. [29], where both CF and ICF were observed below and above the value of ℓ_{crit}. Apart from the above studies, the most debated and still outstanding issues about the ICF dynamics at energies ≈4–7 MeV/nucleon are (i) the localization of the ℓ window, and (ii) to examine the possibility of populating high-spin states via ICF. However, ICF was used as a tool to produce high-spin states recently [60–63]. As such, to understand the above issues, a particle-γ-coincidence experiment is performed to draw some correlation between the driving angular momenta and successively opened ICF channels. This article is organized as follows. The experimental and data reduction procedures are given in Sec. II. Section III deals with the results and comparison with similar data from Refs. [8,10]. Summary and conclusions of the present work are given in the last section of this article.

II. EXPERIMENTAL AND DATA REDUCTION METHODOLOGY

Aiming to investigate the role of high ℓ values in successively opened ICF channels and to examine the possibility of populating high-spin states in final reaction products via ICF, a particle-γ-coincidence experiment was performed at the Inter-University Accelerator Center (IUAC), New Delhi, India. In the present work, spin distributions and feeding intensity profiles of CF and ICF products were measured at ≈5.6 A and 6.5 A MeV. An isotopically pure Tm (169Tm, abundance = 100%) target of thickness ≈1.83 mg/cm² was bombarded by 12C^{5+}(E_{lab} ≈ 5.6 A and 6.5A MeV, beam current ≈30–35 nA) beam delivered from the 15UD-Pelletron accelerator. Target thickness was measured by an α-transmission method. This technique is based on the measurement of the energy loss per unit path length by 5.487 MeV α particles obtained from a standard 244Am source, while passing through the target material. The experimental setup and technique are similar to the previous ones [10]. However, a short account of the experimental conditions and data reduction procedure are given in the following sections for ready reference.

A. Experimental setup

In the present work, particle (Z = 1, 2)-γ coincidences are recorded using the γ-detector array (GDA) along with an array of charged particle detectors (CPD’s) to identify the different reaction channels. The GDA consists of 12 Compton-suppressed high resolution high-purity germanium (HPGe) γ spectrometers arranged at three angles with respect to the beam axis, that is, 45°, 99°, and 153°, and four detectors were installed at each of these angles. While the array of CPD’s is an assembly of 14-phoswich detectors arranged in two truncated hexagonal pyramids, the bases of these pyramids lie in a horizontal plane with each having a trapezoidal shape. The top and bottom spaces are filled by two hexagonal detectors that, together with trapezoids, cover ≈90% of the total solid angle. To employ different gating conditions and to detect particles (Z = 1, 2) in coincidence with prompt γ rays at various angles, the array of CPD’s was divided into three angular zones (i) forward (F) 10°–60°, (ii) sideways (S) 60°–120°, and (iii) backward (B) 120°–170°. The coincidences were demanded between particles (Z = 1, 2) and prompt-γ rays by employing three gating conditions corresponding to the given angular zones for each value of Z. Depending on the fast and slow components of the CPD’s, particles (a sum of protons and α’s) and α’s in each angular zone were detected in coincidence with prompt-γ rays. As a matter of fact, the CPD’s at forward (F) angles (10°–60°) are expected to detect both slow and fast α components; that is, (i) slow α component: fusion-evaporation (CF) α particles, and (ii) fast α component: direct α particles associated with ICF. To record only the fast α component (associated with ICF) in the forward cone, it is essential to stop the slow α component in the forward (F) cone by putting an absorber onto the forward CPD’s. As such, the energy profile of slow α particles (emitted from fully equilibrated CN) was generated by the theoretical model code PACE4 [64] (see Fig. 1). This code is based on the statistical approach of CN de-excitation by Monte Carlo procedure and was extensively used for CN-related calculations in past years. Similar input parameters, as in Ref. [8], are used for this calculation. As can be seen in Fig. 1, the theoretically estimated most probable energy of slow α particles (E_{CF-s}) is found to be ≈18.5 A MeV at ≈5.6 A MeV. However, as per the definition
of ICF, the energy of the fast \( \alpha \) component \((E_{\text{ICF}-\alpha})\) can be calculated as projectile energy times ejectile-projectile mass ratio, and comes out to be \( \approx 22.5 \) MeV at 67.5 MeV for the \( ^{12}\text{C} \) beam. As such, to stop \( \approx 18.5 \) MeV slow \( \alpha \) particles, an Al absorber of appropriate thickness \((\text{i.e., } \approx 3 \text{ mg/cm}^2 \text{ estimated by the SRIM08 code [65] based on the range-energy formulation})\) was kept onto the forward (F) CPD’s so that only the fast \( \alpha \) component in the forward cone can be detected. Multiparameter, particle-\( \gamma \)-coincidence data are recorded in list mode, which includes different gating conditions such as particle(s)/\( \alpha \) detected in backward (B), forward (F), and sideways (S) angles. Singles data are also collected to identify \( xn \) channels predominantly populated via CF.

### B. Data reduction

Off-line data analysis was performed in steps using the nuclear physics data analysis software INGAsort [66]. In the first step, the energy calibration and gain matching of the HPGe detectors are carried out by counting standard radioactive \( \gamma \) sources \((^{155}\text{Eu and } ^{133}\text{Ba}) \) before and after the experiment precisely at the target position. In the second step, particle \((Z = 1, 2)\)-\( \gamma \)-coincidence spectra are generated to identify different reaction channels. Various gating conditions are projected onto the \( \gamma \) spectra (after proper gain-matching and energy calibration) to generate particle \((Z = 1, 2)\)-gated spectra for each angular zone. Assuming isotropic \( \gamma \) emission, all gated spectra for a particular gating condition are summed up to improve the event statistics. The different reaction products populated via CF and/or ICF are identified on the basis of their characteristic \( \gamma \) lines by looking into the particle-gated and/or singles spectra. The \( pxn \) channels \((^{181-185}\text{Re})\) are identified from singles spectra recorded by two coaxial detectors. In order to identify \( pxn \) channels populated via CF, backward (B)-\( \alpha \)-gated spectra are subtracted from backward (B) particles and \((Z = 1, 2)\)-gated spectra to generate pure backward (B)-proton-gated spectra. The CF \( pxn \) channels (consisting of slow \( \alpha \) component) are identified from the backward (B)-\( \alpha \)-gated spectra. Further, as expected in ICF, the direct \( \alpha \) particles (associated with ICF) are supposed to be concentrated only in the forward (F) cone. It may further be pointed out that the slow \( \alpha \) component (associated with CF) coming from the de-excitation of CN may also be emitted in the forward cone due to the isotropic nature of the particle emission in CN reactions. As was already mentioned in the previous section, the slow \( \alpha \) component is filtered out by putting an Al absorber on forward cone CPD’s. However, to remove any contamination from the slow \( \alpha \) component in the forward (F) cone, backward (B)-\( \alpha \)-gated spectra are subtracted from the forward (F)-\( \alpha \)-gated spectra. The ICF \( pxn /2pxn \) channels are identified from forward (F)-\( \alpha \)-gated spectra (slow-\( \alpha \)-component corrected). The relative production yield of the identified reaction products are deduced from the intensity and the area under the photopeak of characteristic prompt \( \gamma \) transitions assigned to a particular reaction product. Spectroscopic data, such as prompt \( \gamma \) energies and their intensities, are taken from the RADWARE level scheme directory [67].

Further, it may be pointed out that the relative number of statistical and “yrast”-like transitions depend on the entry state angular momenta and the available excitation energy \((E^\ast)\). The CF reaction products are formed at high \( E^\ast \) and low angular momenta leading to more statistical transitions, where “yrast” states are expected to be fed by statistical \( \gamma \) transitions. However, the ICF reaction products may achieve low \( E^\ast \) (due to the involvement of partial degrees of excitations) and high angular momenta (relatively higher values of impact parameters contribute to the high-spin states) at a given projectile energy. In such a case, the number of “yrast”-like transitions are expected to be much larger than that of the statistical ones, where less or no feeding is expected. Therefore, the spin distributions of CF and ICF products are expected to be entirely different in nature and can be used as a sensitive tool to probe reaction dynamics by looking into the entry state spin population [10]. As such, to have an insight into the decay patterns of CF and ICF reaction products, spin distributions of different reaction products are generated. Relative production yields are plotted as a function of experimentally observed spin \((J_{\text{exp}}^\ast)\), and normalized with their highest yield values \((Y_{\text{exp}}^\ast)\) at the lowest observed spin \((J_{\text{min}}^\ast)\) to compare different energy data in one panel. For the simplest analytical representation of data, experimentally measured spin distributions of ER’s are fitted by a function of the following type:

\[
Y = Y_c/[1 + \exp(J - J_c)/\Delta],
\]

where \( \Delta \) is related to the width of mean input angular momenta \((J_c)\) and \( Y_c \) is the normalization constant. Here \( J_c \) provides the qualitative information about the mean value of the driving angular momenta involved in the production of different reaction products [10].
FIG. 2. (Color online) Experimentally measured spin distributions for (a) CF-5n channel (identified from singles spectra) and (b) CF-p4n channel (identified from backward-proton-gated spectra) are plotted along with the spin distributions of the same channels from Ref. [10]. Reaction products are labeled by self-explanatory notations and emission cascades. The nomenclature shows that the exit channels are composed by the one given residual nucleus, neutron(s), and/or proton(s). Lines through the data points are the result of the best-fit procedure explained in the text.

III. RESULTS AND DISCUSSION

A. Spin distributions: Comparison with $^{16}$O + $^{169}$Tm

As mentioned in the previous section, the qualitative behavior of the population yield (intensity) of different reaction products (CF/ICF) with $J_{\text{obs}}$ are studied in the present work. Fig. 2 shows the spin distribution of 5n and p4n channels, that is, (a) $^{176}$Re(5n) identified from singles spectra, and (b) $^{176}$W(p4n) identified from backward (B)-proton-gated spectra. In Figs. 3 and 4, the spin distributions of $\alpha$-emitting-channels $^{173,175}$Ta(2$\alpha$2n) are identified from (a) backward (B)-$\alpha$-gated spectra (associated with CF), and (b) forward (F)-$\alpha$-gated spectra (associated with ICF) are given. While, in Fig. 5, the spin distribution of $^{171}$Lu(2$\alpha$2n) identified from forward (F)-$\alpha$-gated spectra is given. Further, to have a comparison with similar data, the spin distributions of $^{180}$Ir(5n), $^{180}$Os(p4n), $^{177,178}$Re(2$\alpha$4n), and $^{175}$Ta(2$\alpha$2n) populated via similar emission channels in the $^{16}$O + $^{169}$Tm system at $\approx$5.6 A MeV are plotted in Figs. 2–5. The lines and curves through the data points represent least-squares fits to a function as discussed in the earlier section. Different reaction products are labeled by self-explanatory notations and corresponding emission channels. At first sight, it can be noticed from Figs. 2–5 that the trend(s) of spin distribution for $\alpha$-emitting channels (ICF products), identified from forward (F)-$\alpha$-gated spectra, are found to be distinctly different as compared to CF products. This striking feature indicates the involvement of an entirely different de-excitation pattern in CF and ICF products. It may further be pointed out that the spin distributions of $^{176}$Re(5n), $^{176}$W(p4n)-B, and $^{177,178}$Re(2$\alpha$4n)-B identified to be populated via CF in $^{16}$O + $^{169}$Tm system. As shown in Figs. 2, 3(a), and 4(a),...
B. Feeding intensity profiles

It was already mentioned in the previous section that the CF products are found to be strongly fed over a broad spin range as compared to ICF products on the basis of spin-distribution trends. In addition to this, it is useful to have direct evidence of the feeding probability of the $\gamma$ population in CF and/or ICF channels. As such, the feeding intensity profiles (FIP’s) for different reaction products populated via CF and ICF are generated from the best fitting procedure of experimentally measured spin distributions. To generate FIP’s, the feeding probability of each observed $\gamma$ transition for different reaction products are plotted as a function of $J_{\text{obs}}$ in Figs. 6 and 7. As can be observed from Fig. 6(a), the feeding intensity for CF channels ($xn/pxn/\alpha xn$-B) is showing a sharp exponential rise toward low-spin states, which indicates a regular population with a strong feeding contribution for each $\gamma$ transition up to $J_{\text{obs}}^{\text{min}}$. Further, to have a better comparison of direct-$\alpha$-emitting channels (ICF) and fusion-evaporation $\alpha$-emitting channels (CF), the feeding intensity profiles for CF and ICF products are plotted in a single panel [see Figs. 6(b) and 7(a)]. As can be observed from Figs. 6(b) and 7(a), the feeding intensity of $\alpha xn$ channels ($^{173,174}$Ta-$\alpha 4n/\alpha 3n$-F) identified from backward (B-$\alpha$-gated spectra shows a similar trend as that which was observed for $xn/pxn$-B channels, where the band is fed over a broad spin range. Apart form that, as shown in Figs. 6(b) and 7(a) and 7(b), the feeding intensity for forward $\alpha$-emitting channels ($\alpha 4n/\alpha 3n$-F) falls off sharply for projectile energy $\approx 5.6$ A MeV ($^{12}$C + $^{169}$Tm system) as compared to 6.5 A MeV at higher-spin side (entry side), indicating the involvement of a rather small driving angular momenta at 5.6 A MeV, as expected.

On the basis of the aforementioned discussion on the trend of spin distributions, it can be concluded that the CF products are strongly fed over a broad spin range, however, ICF products are likely to be associated with a narrow-spin population (for high-spin side) and/or less feeding probability for low-spin states during the de-excitation.
FIG. 6. (Color online) Deduced feeding intensities of $\gamma$ cascades of different reaction products expected to be populated via (a) CF-$5n$, CF-$p4n$, and (b) CF-$\alpha3n$ (identified from backward-$\alpha$-gated spectra) and CF-$\alpha3n$ (identified from forward-$\alpha$-gated spectra) channels. Lines and curves are drawn just to guide the eyes.

FIG. 7. (Color online) Deduced feeding intensities of $\gamma$ cascades of different reaction products expected to be populated via (a) CF-$\alpha4n$ (identified from backward-$\alpha$-gated spectra), ICF-$\alpha4n$ (identified from backward-$\alpha$-gated spectra), and (b) ICF-$2\alpha2n$ (identified from forward-$\alpha$-gated spectra) channels. Lines and curves are drawn just to guide the eyes.

C. Remark on associated $\ell$ values

In the present work, the main stress is to figure out the $\ell$ values involved in the production of different CF and ICF channels and to examine the possibility of populating high-spin states via ICF. As such, the value of $J^\circ$ corresponding to the mean value of driving angular momenta ($\langle \ell \rangle$) is deduced from the best fitting procedure (as explained in the earlier section) of spin distributions for different reaction products populated via CF and ICF. The value of $\langle \ell \rangle$ involved in the production of different CF and ICF products is plotted as a function of various modes of a reaction in Fig. 8. To get some systematics for two almost similar systems, data from Ref. [10] are also plotted in this figure. As indicated in this figure, the value of $\langle \ell \rangle$ involved in the production of CF-$xn/pxn$-B and CF-$\alpha2xn-F$ channels is found to be approximately $7.5\hbar$, $10\hbar$, and $13.5\hbar$, respectively, at projectile energy $\approx5.6A$ MeV. However, at projectile energy $\approx6.5A$ MeV, the value of $\langle \ell \rangle$ for (F)-$\alpha xn/2\alpha xn$ channels is found to increase up to a certain value of $J_{obs}$ and then decreases gradually toward the band head. At energy $\approx5.6A$ MeV, the feeding intensity for the forward (F)-$\alpha xn$ channel increases up to $\approx9\hbar$ from the higher spin states (entry side). While at energy $\approx6.5A$ MeV it is increasing only up to $\approx14\hbar$. This trend indicates that the high-spin states are strongly fed even in the case of ICF channels. However, as the residual nucleus de-excites, the feeding intensity decreases gradually with available excitation energy and/or angular momenta, which indicates the absence of feeding to the lowest members of the “yrst” band, or the low-spin states are less populated in ICF-$\alpha xn/2\alpha xn$-F channels. Such a feeding intensity pattern is expected to arise from the narrow $\ell$ window, localized near and/or above the critical angular momentum for CF. Furthermore, it may be pointed out that the feeding intensity is found to be less at $\approx6.5A$ MeV as compared to $\approx5.6A$ MeV due to the high angular momenta imparted into the system.
between the value of channels (ICF products) indicates their origin from high $\ell$ values as compared to CF channels. A very useful correlation between the value of $\langle \ell \rangle$ and the successively opened ICF channels can be obtained from the data presented in Fig. 8. Following the approach presented in Ref. [10], the value of $\langle \ell \rangle$ associated with ICF in contrast with CF can be represented as

\begin{equation}
\langle \ell \rangle_{(\text{ICF-xn})} \approx 1.33 \langle \ell \rangle_{(\text{CF-xn})},
\end{equation}

\begin{equation}
\langle \ell \rangle_{(\text{ICF-2xn})} \approx 1.35 \langle \ell \rangle_{(\text{ICF-xn})} \approx 1.8 \langle \ell \rangle_{(\text{CF-xn})},
\end{equation}

and

\begin{equation}
\langle \ell \rangle_{(\text{ICF-2xn})} \approx 1.4 \langle \ell \rangle_{(\text{CF-xn})},
\end{equation}

\begin{equation}
\langle \ell \rangle_{(\text{ICF-2xn})} \approx 1.2 \langle \ell \rangle_{(\text{ICF-xn})} \approx 1.7 \langle \ell \rangle_{(\text{CF-xn})}.
\end{equation}

From the correlations presented above, it is interesting to note that the values of $\langle \ell \rangle$ involved in the production of various ICF-xn/2xn channels are found to be $\approx$30 to 70% higher as compared to the CF-xn/xn/2xn channels at both energies. This clearly indicates the involvement of high $\ell$ values in the production of ICF products at a constant projectile energy, essentially due to noncentral interactions, where a significant amount of orbital angular momentum between the projectile and target nucleus transformed into high-spin states of the final reaction products. Further, the correlation obtained for the $^{12C} + ^{169}\text{Tm}$ system at energies $\approx$5.6A and 6.5A MeV strongly supports the systematics presented in Ref. [10]. It can also be observed from Fig. 8 that the involved $\ell$ values in different reaction channels are found to increase linearly with the projectile energy and are almost the same (within $\approx$0.5$\hbar$) for each set of reaction channels at a given projectile energy. As indicated in Fig. 8, the value of $\langle \ell \rangle$ involved in the production of CF and ICF channels in the $^{12C} + ^{169}\text{Tm}$ system at $\approx$6.5A MeV is found to be almost similar to that observed in the case of the $^{16O} + ^{169}\text{Tm}$ system at $\approx$5.6A MeV. This may be due to the fact that the excitation energy of the composite system formed in the $^{12C} + ^{169}\text{Tm}$ system at $\approx$6.5A MeV ($E^* \approx 0.32$A MeV) comes out to be almost the same (slightly higher) as compared to the composite system formed in the $^{16O} + ^{169}\text{Tm}$ system at $\approx$5.6A MeV ($E^* \approx 0.305$A MeV). This seems to give the indication about the involvement of the same amount to driving angular momenta for the production of the same residues at the same excitation energy in the case of a nearby system. This trend may also be an indication that the heavy projectile(s) can pump more angular momenta into the system even at low projectile energy. However, to lead to a definite conclusion it is necessary to perform experiments with similar experimental conditions using a different projectile at a constant excitation energy.

Further, concerning the usefulness of ICF as a tool to populate high-spin states, it can be noted from Figs. 3 and 4 that the same residue can be populated at high-spin states via ICF as compared to CF. As a more precise example, as can be seen from Figs. 3 and 4, the value of $\langle \ell \rangle$ at $\approx$5.6A MeV is found to be $\approx$7.5$\hbar$ involved in the production of $^{173,174}\text{Ta}$ ($\alpha4n/\alpha3n$-B) identified from backward (B)-$\alpha$-gated spectra (associated with CF). However, if the same residue (at the same projectile energy) identified from forward (F)-$\alpha$-gated spectra that is supposed to be populated via ICF, the value of $\langle \ell \rangle$ is found to be $\approx$10.5$\hbar$ [refer to Fig. 2(b)]. Apart from that, the value of $\langle \ell \rangle$ associated with the production of $^{173,174}\text{Ta}$ via CF at $\approx$6.5A MeV (i.e., $\approx$9.5$\hbar$) is achieved via ICF even at lower projectile energy $\approx$5.6A MeV (i.e., $\approx$10.5$\hbar$). Similar characteristics are observed in cases of other reaction channels populated via both CF and ICF. As such, the approximate, but quite useful correlation emerged from these measurements about the possibilities to populate high-spin states can be represented as

\begin{equation}
\langle \ell \rangle_{(\text{CF-4n})} \approx 1.33 \langle \ell \rangle_{(\text{CF-4n})}, \quad \text{at } E_{\text{lab}} \approx 5.6A \text{ MeV},
\end{equation}

\begin{equation}
\langle \ell \rangle_{(\text{ICF-4n})} \approx 1.4 \langle \ell \rangle_{(\text{CF-4n})}, \quad \text{at } E_{\text{lab}} \approx 6.5A \text{ MeV};
\end{equation}

\begin{equation}
\langle \ell \rangle_{(\text{ICF-4n})} \approx 1.1 \langle \ell \rangle_{(\text{CF-4n})}(\approx 6.5A \text{ MeV}).
\end{equation}

As mentioned in the correlations, at $\approx$5.6A MeV, ICF can populate $^{173,174}\text{Ta}(\alpha4n)$ with $\approx$33% more angular momentum as compared to that populated via CF and $\approx$40% more at $\approx$6.5A MeV. Further, it is also clear from the second correlation that the CF is not able to populate the same amount of angular momenta even at $\approx$6.5A MeV, which is populated via ICF in $^{173,174}\text{Ta}(\alpha4n)$ at relatively low projectile energy $\approx$5.6A MeV. This striking feature strongly supports the possibility of populating high-spin states via ICF in final reaction products even at low projectile energy.
IV. SUMMARY AND CONCLUSIONS

In the present work, spin distributions and feeding intensity profiles of different CF and ICF products populated via \(xn/pxn/αxn/2αxn\) channels in the \(^{12}\text{C} + ^{169}\text{Tm}\) system are measured at energies \(\approx 5.6\text{A}\) and \(6.5\text{A}\text{MeV}\). The results of the present experiment are compared with similar data obtained in the \(^{16}\text{O} + ^{169}\text{Tm}\) system at \(\approx 5.6\text{A}\text{MeV}\) (from Ref. [10]). The spin distributions of ICF-\(αxn/2αxn\) channels are found to be distinctly different from those observed for CF-\(xn/pxn/αxn\) channels, which indicates entirely different de-excitation patterns in CF and ICF products toward the band head. The spin distribution(s) of CF products are found to reflect strong feeding through broad range spin population toward the band head. However, the spin distribution(s) associated with ICF are found to arise from the narrow spin population, localized near and/or above the critical angular momentum for CF, where a given PLF is emitted to release excess driving angular momenta. This indicates the competition from successively opened ICF channels for each value of \(\ell\) above \(\ell_{\text{crit}}\) for normal fusion (CF) at respective projectile energies. Moreover, the population of low-spin states are observed to be hindered and/or less fed in the case of ICF. This reveals the occurrence of ICF due to the influence of centrifugal potential in peripheral interactions, where driving angular momentum limits do not allow CF. It is also shown that the direct \(\alpha\) multiplicity increases in the forward cone with the value of \(\ell\) at a particular projectile energy. For example, at \(\approx 5.6\text{A}\text{MeV}\) the value of \(\langle \ell \rangle\) for CF-\(xn/pxn-\alpha xn-B/\alpha xn\) and ICF-\(2\alpha xn\)-F channels is found to be \(\approx 7.5\hbar\), \(\approx 10\hbar\), and \(\approx 13.5\hbar\), respectively. On the basis of these results, it can be concluded that the high \(\ell\) values associated with noncentral interactions essentially contribute to open up direct-\(\alpha\)-emitting channels. It may further be pointed out that the value of \(\langle \ell \rangle\) associated with a \(2\alpha\)-emitting channel is likely to originate from higher impact parameters than that associated with the production of a single direct-\(\alpha\)-emitting channel. Further, from the comparison of \(\ell\) values involved in the production of direct-\(\alpha\)-emitting (ICF products) and normal-\(\alpha\)-emitting channels (CF products), we present direct evidence that ICF can populate high-spin states in final reaction products, which is not possible to achieve via CF at a given projectile energy and/or even higher energy. As an extension of this work, the experiment for the same projectile-target combination was carried out at ten different energies to understand the obtained systematics in a better way, which will be presented in a forthcoming article.

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[67] RAdWARE, the level scheme directory on http://radware. phy.ornl.gov/agsdir1.html.
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Signature of pre-equilibrium-emission in forward-to-backward yield ratio measurement

SIGNATURE OF PRE-EQUILIBRIUM-EMISSION IN FORWARD-TO-BACKWARD YIELD RATIO MEASUREMENT

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In this paper, pre-equilibrium-emission has been investigated using particle-γ coincidence technique. Forward-to-backward yield ratios of different reaction products have been measured in $^{16}$O + $^{169}$Tm system at 5.6 MeV/nucleon. Coincidences between prompt-γ-rays and charged particles emitted in various angular zones have been recorded to identify different reaction products. Several gating conditions have been projected onto the energy spectra to achieve the information about mode of reaction. Yield profiles of fusion/fusion-like channels identified from forward/backward/sideways particle ($Z = 1, 2$)-gated-spectra have been measured. High yield in forward cone has been observed from the experimentally measured forward-to-backward yield ratios. The enhanced forward cone yield has been attributed to the pre-equilibrium-emission. The maximum observed spin ($J_{\text{max}}$) is found to be decreased with the number of proton emitted from the composite nucleus during the equilibrium and/or pre-equilibrium decay.

Keywords: Pre-equilibrium-emission; heavy-ion reactions; particle-gamma coincidence technique.

1. Introduction

Several experimental and theoretical studies indicate the existence of pre-equilibrium (PEQ)-emission at moderate excitation energies,$^{1-13}$ which is intermediate in character between direct (DIR) and equilibrium (EQ) reactions. In EQ-reactions, an excited compound nucleus (CN) is formed as a consequence of

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complete amalgamation of projectile and target nucleus. In this case, emission of light-nuclear-particles (LNPs) may occur from any stage of CN relaxation process. The CN de-excites either by emission of LNPs (if the excitation energy of CN is higher than the particle emission threshold) and/or by characteristics γ-radiations, giving birth to a residual nucleus. While, in case of DIR-reactions, LNPs are emitted because of the large momentum transfer from the very first projectile-target interactions. As a result, the ejectile carries high energy, leaving the residual at lower excitation. When the incoming energy of the projectile is shared by all the nucleons of the composite system, an equilibrium condition is reached. The LNPs are then emitted from the CN because of statistical fluctuations in energy. Between the two extremes, there is an intermediate stage of reaction that exhibits both DIR and EQ-like features, which is referred to as PEQ-emission. The emission of LNPs in PEQ-reaction take place after the first stage of DIR-reaction but from the partially equilibrated composite system (long before the establishment of statistical equilibrium). However, at later stage, a fully equilibrated compound nucleus (CN) supposed to form, which may undergo usual modes of de-excitation. Some of the important signatures of PEQ-emission are; (i) the presence of relatively larger number of energetic LNPs in the exit channel as compared to that emitted in EQ-reactions, (ii) forward peaked angular distribution of LNPs, (iii) slowly descending tails of the excitation functions (EFs).

In order to have insight into PEQ-emission, a variety of dynamical models viz; Inter-Nuclear Cascade (INC) model, the quasi-free scattering model (QFS), the HYBRID model, the EXCITON model, etc., have been proposed to reproduce experimental data related to PEQ-emission. It may, however, be pointed out that from the aforementioned models, the EXCITON model has been considered to provide most suitable description of PEQ-emission. In this model the excitons i.e. the excited particles (p) and holes (h) are assumed to be produced through the interaction between projectile nucleons and target nucleus. These excitons interact with the nucleons in the target nucleus, eventually some of the energetic particles may escape out of the nucleus. Since, the average exciton energy of the composite system decreases due to the PEQ-emission of energetic LNPs, therefore, the equilibrated CN de-excites via slow LNPs and/or characteristic γ-radiations. Generally speaking, these models have been used to describe various experimental data obtained in light ion (LI)-induced reactions. The experimental data on PEQ-emission using heavy ion (HI)-beams is still limited for few projectile target combination and so not well understood at moderate excitation energies. This may be because of the fact that the distinction of PEQ and EQ-emission is difficult in case of HI-induced reactions than LI-induced reactions due to rather large momenta carried in by the HI-beams, and also due to the presence of incomplete fusion. In case of incomplete fusion, projectile breaks up into fragments. One of the fragments fuses with target nucleus, while rest part of the projectile behaves like a spectator. It may be pointed out that during the amalgamation of a part of projectile and target nucleus, PEQ-emission of LNPs may take place. As such, in the HI-induced reactions,
the LNPs are supposed to be originated from both complete and/or incomplete fusion. However, the PEQ-data using HI-beams is also important for better understanding of reaction dynamics. Apart from that, the data on PEQ-emission for various projectile-target combinations may provide a data base for energy systems, particularly for the recently proposed accelerator driven sub-critical-reactor system (ADSS)\textsuperscript{36} and/or also for the waste management.\textsuperscript{37} Nonetheless, a rich data set on different mode of reactions may be useful to determine optimum irradiation conditions to produce medically important radio-nuclides. This has lead to a renewed interest to the study of nuclear reactions. In order to study the origin of energetic LNPs in nuclear reactions, Zhu \textit{et al.},\textsuperscript{38} compared the population probability of LNPs emitted in forward cone in contrast with that emitted in backward cone. They observed higher emission probability of energetic LNPs in forward cone as that of backward cone, and referred to as the characteristic of PEQ-emission. Other investigations\textsuperscript{39–44} have also reported similar observations. As such, the process of PEQ-emission can be investigated by the angular co-relation of $\gamma$-rays in coincidence with LNPs emitted during the de-excitation of composite system. The fact that the probability of energetic LNPs emitted in forward cone supposed to be relatively large (due to their origin from partially equilibrated composite system) as compared to that emitted in backward cone (emitted dominantly via EQ-emission). Since, the PEQ-emission events are mostly concentrated in forward cone, therefore, the yield is expected to be relatively large as compared to that in the backward cone. As such, with a motivation to have insight into the interplay of EQ and PEQ-emission processes, an experiment has been performed at the inter-university accelerator center (IUAC), New Delhi, India. In the present work, forward-to-backward yield ratios $[R_{Y(F/B)}]$ of different reaction products have been measured in $^{16}$O + $^{169}$Tm system at $E/A \approx 5.6$ MeV. This paper is organized as follows; the experiment and data reduction procedure are given in Sec. 2, while Sec. 3 deals with the discussion on backward-to-forward yield ratios. The summary and conclusions of the present work are presented in Sec. 4 of this paper.

2. Experimental and Data Reduction Procedure

As mentioned above, forward-to-backward yield ratios $[R_{Y(F/B)}]$ of different reaction products have been measured in $^{16}$O + $^{169}$Tm system at $E/A \approx 5.6$ MeV to probe PEQ-emission. Isotropically pure (abundance = 100%), self-supporting $^{169}$Tm target of thickness 0.93 mg/cm$^2$ has been bombarded by $^{16}$O$^{7+}$ ($E/A \approx 5.6$ MeV, beam current $\approx 30$ nA) beam delivered from 15UD-Pelletron Accelerator. The thickness of target has been measured by $\alpha$-transmission method based on energy loss per unit path length by mono-energetic ($E_\alpha = 5.486$ MeV) $\alpha$-particles passing through the target material. The above projectile-target combination has been chosen because of the fact that the possible reaction products are well known rotational nuclei, and the prompt-$\gamma$-transitions are available in the literature. In the present work, gamma detector array (GDA) setup along with an array of charged
particle detectors (CPDs) has been used for the identification of prompt $\gamma$-rays in coincidence with charged particles. Schematic representation of experimental setup is given in Fig. 1. The GDA consists of 12 Compton suppressed, high resolution HPGe-detectors installed in three angular rings (i.e. 45°, 99°, 153°, four detectors in each ring) with respect to the beam axis. While, the array of CPDs is an assembly of 14 Phoswich detectors arranged in two truncated hexagonal pyramids. The bases of these pyramids lies in a horizontal plane with each other having trapezoidal shape. However, the top and bottom spaces are filled by two hexagonal detectors which together with trapezoids cover 90% of the total solid angle. The array of 14 CPD’s has been divided into three angular zones; i.e., (i) forward (F) 10°–60°, (ii) sideways (S) 60°–120° and (iii) backward (B) 120°–170° zones. Depending on the fast and slow components of the CPDs, particles (a sum of protons and $\alpha$’s) and $\alpha$’s in each angular zone can be identified. To remove scattered beam, CPDs have been covered by Al-absorbers of appropriate thickness. The coincidences were demanded between particles ($Z = 1, 2$) and prompt-$\gamma$-rays employing three gating conditions corresponding to given angular zones for each value of $Z$. Multi-parameter data in event-by-event (LIST) mode have been collected.

Data analysis has been performed using INGAsort. The efficiency determination and gain matching of HPGe detectors have been done by using standard radioactive ($^{152}$Eu, $^{133}$Ba) $\gamma$-sources of known strength to cover entire energy range of

![Schematic representation of the Charged Particle Detector Array (CPDA), and gamma detector array (GDA) set-up.](image-url)
interest. Gating conditions for particles (a sum of protons and α’s) and for α’s have been projected onto the prompt γ-ray spectra. In first step of analysis, the forward-α-gated spectra have been subtracted from the forward particle (Z = 1, 2)-gated spectra to generate forward-proton-gated spectra. Similarly, proton-gated-spectra for backward cone and for the sideways have been obtained. In order to improve the statistics, assuming the angular distribution of the observed γ-rays to be isotropic, all gated spectra for a particular angular zone or gating condition have been summed up. Reaction residues have been identified by their characteristic prompt-γ-lines by looking into the gated spectra. However, channel selection has been made on the basis of different gating conditions. The intensity and area under the photo-peak (efficiency corrected) of characteristic prompt-γ-transitions were used to determine the relative production yield for the observed spin (J_{obs}) of different reaction products. The γ-ray energies and their intensities used in the analysis have been taken from RADWARE and/or NNDC. In order to draw the yield profiles, yield of different reaction products identified from forward (F), backward (B) and sideways (S) proton-gated spectra have been plotted as a function of observed spin (J_{obs}).

3. Discussion on Forward-to-Backward Yield Ratios

In order to draw yield profiles, the yield of individual reaction products have been plotted as a function J_{obs}. For better comparison of yields identified from forward (F), backward (B) and sideways (S) angular zones, the yield profiles of Os (populated via p3n-channel) have been plotted in Fig. 2(a). Reaction channel has been labeled by self-explanatory notation of corresponding emission cascade. The overall errors from different factors have been estimated to be less than 10%. Lines through the data points are drawn to guide the eyes. As can be seen from this figure, the yield values for Os identified from backward (B)-proton-gated spectra is almost similar to that identified from sideways (S)-proton-gated spectra within the experimental uncertainties. However, a significant enhancement can be noticed in case of the residue identified from forward (F)-proton-gated spectra. This yield enhancement in the forward (F) cone may be attributed to the contribution coming from PEQ-emission process. This enhancement can be justified as the particles emitted in the PEQ-emission are supposed to be focused pre-dominantly in the forward cone, and are expected to show high yield as compared to that formed via EQ-reaction. Further, if the residue(s) is/are populated via de-excitation of fully equilibrated CN (formed in an EQ-reaction) then the forward-to-backward yield ratio \( R_{Y(F/B)} \) should be equal to one, and it should be more than one in case of PEQ-reaction for the whole range of observed transitions. As such, in order to have better insight into this characteristic of PEQ-reaction, forward (F)-to-backward (B) \( [R_{Y(F/B)}] \), forward (F)-to-sideways (S) \( [R_{Y(F/S)}] \), and backward (B)-to-sideways (S) \( [R_{Y(B/S)}] \) yield ratios have been plotted as a function of J_{obs} in Fig. 2(b) for Os isotope. As shown in this figure, the value of \( R_{Y(F/S)} \) is found to be almost one (within the experimental uncertainties) for the whole range of observed transitions. How-
Fig. 2. (a) Experimentally measured yield profiles for $^{181}$Os residue. Nomenclature used in this figure indicate the identification of this residue from different angular zones, e.g. forward (F), sideways (S) and backward (B), (b) yield ratios ($R_Y$) of $^{181}$Os residue for different angular zones such as $R_Y(F/B)$, $R_Y(F/S)$ and $R_Y(S/B)$ are plotted as a function of $J_{obs}$ (refer to text for details). Enhancement in the yield ratio over horizontal dotted line indicates contribution from PEQ-reaction. Lines through the data points are drawn to guide the eyes.

ever, a significant enhancement in case of $R_Y(F/B)$, and $R_Y(F/S)$ can be noticed for almost all transitions. This striking difference in the yield ratios seems to be the indication of PEQ-emission. On the basis of this observation, it can be inferred that the production probability of $^{181}$Os (p3n) have contribution from both EQ- and PEQ-emission processes. The fluctuations in the yield value with $J_{obs}$ indicate the variation of PEQ-emission contribution for different transitions. These fluctuations may also be due to the contribution from both direct as well as unknown feeding to the yrast line transitions.
Fig. 3. Experimentally measured forward to backward yield ratios for (a) Re-isotopes (populated via 2pxn-channels), (b) W-isotopes (populated via 3pxn-channels), and (c) Ta-isotopes (populated via 4pxn-channels) are plotted as a function of $J_{\text{obs}}$. Explanation of the figure is same as Fig. 2.

It may, however, be pointed out that in the laboratory frame even an equilibrated source will exhibit forward (F)-to-backward (B) asymmetry in the LNPs emission due to the transfer of linear momentum in the course of nuclear interactions. This is an important aspect to consider to characterize a PEQ-emission event. This effect may be incorporated by comparing experimental results with the predictions of theoretical model codes. To compare the experimental data with the theoretical predictions, it is required to theoretically estimate the yield of individual reaction channels at different values of $J_{\text{obs}}$ which is not straightforward. Therefore, an alternative attempt has been made to probe the forward (F)-to-backward (B) asymmetry in the LNPs emission. The energy spectra of protons emitted in different angular zones (forward-F, backward-B and sideways-S) from the fully equilibrated compound nucleus have been generated from the predictions of statistical model code PACE4. Physically justified input parameters have been used in these calcula-
The distribution of protons has been found to be Gaussian in nature and peaking $\approx 10–12$ MeV. From the analysis of theoretically generated proton spectra, overall $\approx 20\%$ enhancement has been observed in proton emission from the equilibrated compound nucleus in the forward-F cone as compared to that observed in backward-B cone. The relative contribution of forward (F)-to-backward (B) particle emission asymmetry is incorporated in the identified reaction products. Figure 2(a) shows the corrected yield profile(s) of $^{181}$Os (p3n), where the contribution from forward (F)-to-backward (B) particle emission asymmetry is marked by a dotted horizontal line. As shown in this figure, the inclusion of the contribution from the forward (F)-to-backward (B) particle emission asymmetry do not reproduce the enhancement in the values of $R_{Y(F/B)}$ and $R_{Y(F/S)}$. These values are found to be significantly higher for the entire range of $J_{\text{obs}}$. As such, it can be inferred that the enhancement above the dotted line is attributed to the PEQ-emission of protons in the forward cone.

Similarly, the forward (F)-to-backward (B) yield ratios [$R_{Y(F/B)}$] of different reaction products (populated via pxn, 2pxn, 3pxn and 4pxn-channels) have been plotted as a function of $J_{\text{obs}}$ in Figs. 3(a) and (b), and Fig. 4. As can be seen from these figures, the $R_{Y(F/B)}$ is found to be higher than one and showing a significant enhancement above horizontal dotted lines, which indicates the contribution from PEQ-emission in the production of these reaction products. It can also be noticed that the enhancement in the yield ratios is large for higher values of $J_{\text{obs}}$, in general. This seems to indicate the origin of different reaction products from both central or non-central (peripheral) interactions. In non-central interactions, the centrifugal field may increase the driving input angular momenta ($\ell$-values) imparted into the system. As such, the composite(s) system formed at large $\ell$-values may undergo...
the emission of PEQ-nucleons leaving behind the composite nucleus in high spin states. It may, however, be pointed out that the maximum observed spin \( J_{\text{max}}^{\text{obs}} \) is found to be decreasing as the proton multiplicity increases, in general. As can be seen from Figs. 3(a) and (b) and Fig. 4, the value \( J_{\text{max}}^{\text{obs}} \) is found to be \( \approx 25\hbar \) for 2pxn-channels. However, in case of 3pxn [Fig. 3(b)] and 4pxn-channels (Fig. 4), the values of \( J_{\text{max}}^{\text{obs}} \) are found to be \( \approx 20\hbar \) and \( \approx 13\hbar \), respectively. This feature of yield ratios indicates the removal of angular momenta from the composite nucleus is proportional to the number of protons emitted during the EQ and/or PEQ-decay, as expected.

4. Summary and Conclusion

In summary, it can be inferred that the forward (F)-to-backward (B) yield ratios are showing significant enhancement over the unity line, which signify finite contribution from PEQ-emission in \(^{16}\text{O} + ^{169}\text{Tm}\) system at energy as low as 5.6 MeV/nucleon. The observed fluctuations in the yield ratios may be because of feeding from unknown transitions, which could not be corrected in this measurement. The maximum observed spin \( J_{\text{max}}^{\text{obs}} \) is found to be decreased with the number of proton emitted from the composite nucleus during EQ and/or PEQ-decay. As such on the basis of results presented in this paper, it may be concluded that PEQ-emission in HI-induced reactions is a process of importance at energy as low as 5.6 MeV/nucleon, which may be investigated by yield ratio measurement. However, the result presented in this paper can be supplemented by more conclusive measurements. Additional information about the PEQ-emission can be obtained by measuring the multiplicity, the angular distribution, and energy spectra of emitted light nuclear particle(s), which can serve as extra degrees of freedom to the understanding of underlying process.

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References

Do not believe in a thing because you have read about it in a book...... Do not believe in a thing because another man has said it was true...... Do not believe in words because they are hallowed by tradition...... Find out the truth for yourself...... Reason it out...... that is realization......

- Swami Vivekananda