Chapter 3

Recoil ranges proof of fusion in completeness

In order to disentangle the complete and in-complete fusion events in heavy ion interactions, the forward recoil ranges of residues have been measured in the interaction of $^{12}$C$^{+}$159Tb at three distinct above barrier energies $\approx 74, 80 \& 87$ MeV $^{*1,4}$. The measurement and analysis of forward recoil ranges is one of the most direct and irrefutable methods to disentangle these events. In the present measurements the recoil-catcher technique followed by off-line $\gamma$-ray spectroscopy has been employed. The complete and in-complete fusion events have been tagged by full and partial linear momentum transfer components, respectively. An attempt has been made to separate out the relative contributions of complete and in-complete fusion events. The in-complete fusion events may be understood on the basis of breakup fusion model, where, such events may be attributed to the fusion of $^{8}$Be and/or $^{4}$He from $^{12}$C projectile to the target nucleus. Analysis of the data indicates that the in-complete fusion has significant contribution at the studied energies and its contribution has been found to increase with the beam energy. An attempt has also been made to explain the observations of in-complete fusion reactions in the light of SUMRULE model, based on sharp-cut off model. This model has been found to underpredict the in-complete fusion cross-sections. The diffuseness in $\ell$-distribution has been suggested to explain the underlying reaction mechanism.

$^{1}$Abhishek Yadav et al., FIG*12 Conference Proceedings, (2012).
The measurement and analysis of excitation functions (EFs) reveal that there is a significant enhancement, in the production cross-section of \( \alpha \)-emitting channels, over the PACE4 predictions. The code PACE4 do not include the ICF into account in the calculations. As such, the enhancement has been attributed due to the contribution from ICF reactions. In order to understand the ICF reaction dynamics, it has been assumed that the residues produced in HI-induced reactions may originate via two different processes viz; (a) from the compound nucleus formed via full linear momentum transfer events, and (b) from the in-completely fused composite (IFC) nucleus formed due to partial momentum transfer from projectile to the target nucleus. For incident energies near and a little above the \( V_{\text{fus}} \), the complete momentum transfer events are dominant. However, at relatively higher projectile energies, both the complete and partial momentum transfer events contribute to the production cross-section of the \( \alpha \)-emitting channels. Therefore, it is desirable to separate out the relative contributions of CF and ICF processes.

In the Chapter-2 of this thesis, an attempt has been made to extract the relative contributions of CF and ICF processes from the analysis of experimentally measured EFs, in light of the predictions based on the statistical model calculations. It has been observed that the influence of in-complete fusion increases with the projectile energy. It may be pointed out that the procedure adopted to deduce the ICF-contribution in Chapter-2 is based on the statistical model predictions. From the physics point of view, it is desirable to have a model independent approach. Hence, in the present chapter, an attempt has been made to develop a model independent approach to separate out the CF and ICF contributions. In this approach, the degree of linear momentum transfer (\( \rho_{\text{LMT}} \)) from the projectile to the target nucleus has been deduced by measuring the forward recoil ranges (FRRs) of the residues produced in \(^{12}\text{C} + ^{159}\text{Tb} \) interactions. The forward recoil ranges of CF and ICF reaction products depend on the recoil velocity of composite system associated with the degree of linear momentum transfer (\( \rho_{\text{LMT}} \)) from the projectile to the target nucleus. In the case of CF reactions, the projectile completely fuses with the target nucleus transferring the full linear momentum. On the other hand, during the ICF reactions a part of the
projectile fuses with the target nucleus and thus partial linear momentum is transferred. The degree of linear momentum transfer may be given as:

\[ \rho_{LMT} = \frac{P_{frac}}{P_{proj}} \]  

(3.1)

where, \( P_{frac} \) is the linear momentum of the fused fraction of the projectile and \( P_{proj} \) is the entire linear momentum of the projectile. As already mentioned, \( \rho_{LMT} \) is proportional to the fused mass of the projectile, i.e., maximum LMT may give maximum recoil velocity to the reaction products. This is a promising way to disentangle the full and partial momentum transfer events corresponding to the CF and ICF processes, respectively. Since, in the CF process, the maximum \( \rho_{LMT} \) is transferred from the projectile to the target nucleus, therefore, for a given entrance channel the CN has predetermined mass, energy and momenta. While, in the case of ICF, a partial \( \rho_{LMT} \) results due to the formation of an incompletely fused composite system in the excited state, which may not have the unique values of the mass, energy and momenta. This may be because of the fluctuations in the fused mass from the projectile to the target nucleus and also due to various possible interaction trajectories. The velocity distribution of a given type of reaction products is expected to be symmetric about \( v_0 \), having a width which depends upon the evaporation process and, in particular, on the particles evaporated from CN. The mean velocity \( v_0 \), may be given as:

\[ v_0 = v_{CN} = \frac{\sqrt{2} M_P \cdot E}{M_{P+T}} \]  

(3.2)

where, \( M_P \) is the projectile mass, \( M_{P+T} \) is the total mass of the composite system (projectile+target), and \( E \) is the projectile energy in the laboratory frame.

Thus, the experimentally measured forward recoil ranges of final reaction products, in a stopping medium, may give information about the \( \rho_{LMT} \) involved. The measurement of forward recoil range distributions (FRRDs) can be used as one of the most direct methods to distinguish the different
ICF components, where the same kind of residues may be formed by more than one fusion channel. Though, the differences in the velocity/ranges of CF and ICF reaction products are not so significant, but by using very thin catcher foils (≈ µg/cm²), it is possible to separate the CF and ICF residues. Further, as already mentioned, for the nuclei which are formed via partial fusion of projectile, the recoil velocity will be less than that for complete momentum transfer events. Therefore, the reaction products populated via ICF will show relatively smaller range in the stopping medium as compared to CF reaction products. For a different $\rho_{LMT}$, the residues may have different recoil ranges in the stopping medium. The FRRD measurements may thus be used to separate out the relative contributions of various partial fusion components to the formation of particular reaction residue. In the present work, the recoil-catcher technique followed by off-line gamma-spectroscopy has been used. The details of these measurements are given in the following sections/subsections.

3.1 Target preparation and irradiations

The $^{12}$C+$^{159}$Tb system has been chosen for the measurement of forward recoil range distributions. This system has been considered because of the fact that EFs for several reaction channels were already measured where a significant contribution of ICF has been observed. In the FRRD measurements, the thickness of target is decided in such a way that the recoiling nuclei are not stopped in the target thickness itself. As such, thin targets prepared using vacuum evaporation method with Al-backing are generally used for such measurements. However, in the present measurements, the self-supporting samples of $^{159}$Tb (Abundance = 99.99%) of thickness ≈ 190µg/cm² have been prepared by rolling method. In order to trap the recoiling residues, thin Al-catchers prepared by ultra-high vacuum evaporation technique have been used. The thicknesses of samples have been determined by the $\alpha$-transmission method. The thickness of Al-catchers used were ≈ 15-50µg/cm². The samples were pasted on rectangular Al-holders of size ≈2.5 × 2.1 cm², having concentric holes of 1.0 cm diameter. The
3.1 Target preparation and irradiations

Irradiations have been performed using $^{12}$C$^{6+}$-beam in the General Purpose Scattering Chamber (GPSC) having an invacuum transfer facility (ITF) [66]. Stack of the thin Al-catcher foils (sufficient to stop the compound nucleus (CN) formed via full linear momentum transfer) was placed just after the target, so that the heavy recoiling residues populated via CF and/or ICF could be trapped at their respective ranges in Al-catcher foil thicknesses. A typical target-catcher foils set-up used for the measurement of forward recoil ranges is shown in Fig.3.1.

![Figure 3.1: A typical stack arrangement to trap the recoiling residues for measuring the forward recoil range distributions.](image)

The effective projectile energy on the target has been estimated by calculating energy loss at the middle of the $^{159}$Tb target. The target-catcher foil assembly, made up of a thin target followed by a stack of thin Al-catcher foil to trap the recoiling reaction products, was irradiated at a given beam energy for sufficiently long time to achieve the desired statistics. The irradiations have been carried out for $\approx 12$ hrs, with beam currents $\approx 30$ nA. Such irradiations have been carried out at three beam energies i.e., at
\(\approx 74, 80 \& 87 \text{ MeV}\). The delay time between the stop of irradiation and the beginning of counting was minimized using invacuum transfer facility. The beam flux has been calculated using the total charge collected in the Faraday cup placed behind the target-catcher foil arrangement.

### 3.2 Post irradiation analysis

After the irradiation the stack was taken out from the scattering chamber using the ITF. The target-catcher foil assembly was disassembled to record the activities collected in each of the Al-catcher foils. The ERs populated via CF and/or ICF processes are expected to be trapped at different catcher foil thicknesses, depending on the ER’s recoil velocity and/or on the degree of linear momentum transfer associated with the mode of reaction channel. The activities produced in each catcher foil of the stack were counted separately using a pre-calibrated, high resolution HPGe-spectrometer of 100 c.c. active volume coupled to a CAMAC based CANDLE software [68]. The HPGe-spectrometer was pre-calibrated both for energy and efficiency. The resolution of \(\gamma\)-ray spectrometer was \(\approx 2 \text{ keV}\), for 1.33 MeV \(\gamma\)-ray of \(^{60}\text{Co}\) source. The characteristic \(\gamma\)-radiations of the residues have been used for their identification. The induced activities have been used to determine the production probability of various residues. The \(\gamma\)-ray spectra of each foil has been recorded at increasing times and the decay curve analysis has been done to measure the half-lives of the residues, used for their identification. A typical \(\gamma\)-ray spectrum of induced activity in Al-catcher foil after the interaction of \(^{12}\text{C} + ^{159}\text{Tb}\) at \(\approx 87 \text{ MeV}\) is shown in Fig.3.2. The various identified peaks which have been used for recoil range analysis are assigned to the different reaction products on the basis of their characteristic \(\gamma\)-ray energies and measured half-lives. Nuclear data, like half-lives, gamma-ray energies, etc., of identified residues have been taken from the Table of Isotopes[70, 71, 72]. The measured intensities of the characteristic \(\gamma\)-radiations were used to determine the production yield of various residues.
3.2 Post irradiation analysis

In order to obtain the normalized yield of the residues as a function of cumulative thickness of the Al-catcher foils, the cross-section of the reaction products in each catcher foil was divided by respective catcher foil thickness. The resulting normalized yield of reaction products have been plotted against the cumulative catcher foil thickness to obtain the differential recoil range distributions. In the present work the recoil range distributions of the residues, $^{168}$Lu (3$n$), $^{167}$Lu (4$n$), $^{165}$Lu (6$n$), $^{167}$Yb ($p$3$n$), $^{165}$Tm ($\alpha$2$n$), $^{163}$Tm ($\alpha$4$n$), $^{161}$Ho (2$\alpha$2$n$), $^{160}$Ho$^g$ (2$\alpha$3$n$) and $^{160}$Ho$^m$ (2$\alpha$3$n$), at three

Figure 3.2: Typical $\gamma$-ray spectrum of induced activity in an Al-catcher after the interactions of $^{12}$C+$^{159}$Tb system at $\approx$ 87MeV energy.
different beam energies $\approx 74$, 80 & 87 MeV have been measured and are plotted in Figs.3.3-3.11, respectively. In the recoil range measurements, the cross-section for the production of a given type of residues as a function of the range in any medium are affected by relative errors which depend essentially on the counting statistics and the uncertainty in the catcher thicknesses and in presently studied cases, are less than or at most are around 15%. The size of the circles in these figures include the uncertainty in the yield values.

### 3.3 Most probable recoil ranges ($R_{P}^{exp}$)

The CF and ICF processes lead to the characteristic velocity distribution of the reaction residues. As such, the distribution of measured yields of the residues as a function of velocity and/or the ranges in a stopping medium may give an insight into the reaction mechanism involved. Though, the differences in the velocity/ranges of CF and ICF reaction products are not so significant, but by using very thin Al-catcher foils ($\approx \mu g/cm^2$), it is possible to disentangle the contribution due to the CF and ICF processes. A precise measurement of the LMT component corresponding to the different reaction processes is very difficult, due to the recoil velocity distribution of evaporation residues and also because of straggling effects. Further, it may be pointed out that, the particle(s) emission from the forward moving recoiling residues may change the energy/momentum of the final residues, depending on the direction of emission. This is reflected in the width (FWHM) of the experimentally measured recoil range distributions. The width may also arise because of the contributions from straggling effects. As such, in order to get a reliable value for the degree of linear momentum transfer from the experimental data, a careful deconvolution of the measured FRRD's is required. The relative contributions of linear momentum transfer in the production of particular reaction products via CF and/or ICF processes may be computed by fitting the experimentally measured FRRD data with Gaussian peaks using the ORIGIN software [69].
Figure 3.3: Experimentally measured FRRDs for $^{168}$Lu-residues populated via $3n$-channel at $\approx 74$ & $80$ MeV beam energies.
The yield curves of evaporation residues obtained from FRRDs are assumed to be Gaussian in nature and may be given as:

\[ Y = Y_0 + \frac{A}{\omega_A^2 \sqrt{2\pi}} e^{- \frac{(R - R_P)^2}{2\omega_A^2}} \]

(3.3)

where, \( R_P \) is the most probable mean range, \( \omega_A \) is the width parameter (FWHM) of the distribution, and \( A \) is the area under the RRD peak. Further, the normalized yield \( Y \) may be estimated by the chi square fit (\( \chi^2 \)) of the experimentally determined range distributions and may be represented as follows,

\[ \chi^2 = \frac{1}{(m - p - 1)} \left\{ Y(A) - Y_0(A) \right\}^2 \]

(3.4)

The value of the chi-square (\( \chi^2 \)) was minimized in this analysis using a non-linear least-square fit routine, keeping the width parameter (\( \omega_A \)) and most probable mean range (\( R_P \)) in the FRRDs as free parameters. As indicated in the Figs.3.7-3.11, the residues involving \( \alpha \)-particles in the exit channel, show more than one LMT components. In such cases, the experimentally measured normalized yields have been fitted by assuming multi-peaks in the similar way as mentioned above. The contribution of different fusion components (\(^{12}\text{C} \) and \(^{8}\text{Be} \)-fusion) have been obtained by dividing the area under the peak of the corresponding fusion component by the total area associated with the distribution. The percentage contributions, coming from different CF and/or ICF components, deduced in such a way for the residues populated in \(^{12}\text{C} + ^{159}\text{Tb} \) system are also indicated in Figs.3.7 to Fig.3.11, along with their corresponding channels.
3.3 Most probable recoil ranges ($R_{P}^{\text{expt}}$)

Figure 3.4: Experimentally measured FRRDs for $^{167}$Lu-residues populated via 4$n$-channel at $\approx$ 74, 80 & 87 MeV beam energies.
Figure 3.5: Experimentally measured FRRDs for $^{165}\text{Lu}$-residues populated via $6n$-channel at $\approx 74, 80 \& 87$ MeV beam energies.
3.3 Most probable recoil ranges ($R_{P}^{\text{expt}}$)

Figure 3.6: Experimentally measured FRRDs for $^{167}$Yb-residues populated via $p3n$-channel at $\approx 74, 80 \& 87$ MeV beam energies.
Figure 3.7: Experimentally measured FRRDs for $^{165}\text{Tm}$-residues populated via $\alpha 2n$-channel at $\approx 74, 80$ & $87$ MeV beam energies.
3.3 Most probable recoil ranges \( (R^\text{expt}_P) \)

Figure 3.8: Experimentally measured FRRDs for \(^{163}\text{Tm}\)-residues populated via \( \alpha 4n \)-channel at \( \approx 74, 80 \) & \( 87 \) MeV beam energies.
Figure 3.9: Experimentally measured FRRDs for $^{161}$Ho-residues populated via $2\alpha 2n$-channel at $\approx 74, 80$ & $87$ MeV beam energies.
Figure 3.10: Experimentally measured FRRDs for $^{160}$Ho$^\alpha$-residues populated via $2\alpha 3n$-channel at $\approx$ 74, 80 & 87 MeV beam energies.
Figure 3.11: Experimentally measured FRRDs for $^{160}\text{Ho}^m$-residues populated via $2\alpha3n$-channel at \(\approx 74, 80 \& 87\) MeV beam energies.
3.4 Interpretation of experimental results

The measured FRRDs presented in Figs. 3.3-3.11 clearly indicate the different linear momentum transfer components, depending on the fused mass of the projectile with the target nucleus. In case of the $xn \ (x=3,4,6)$ & $p3n$-channels (Figs. 3.3-3.6), the measured RRDs show only a single peak, at all the three bombarding energies, indicating only one linear momentum transfer component, a characteristic of the CF process, involved in the production of $^{168}$Lu, $^{167}$Lu, $^{165}$Lu and $^{167}$Yb residues, respectively. A close observation of the range distribution of these FRRDs reveals that FRRD peak shifts towards relatively higher cumulative catcher thickness with increase in the beam energy, as expected. Further, as already mentioned, the neutron emission from the forward recoiling residues may change the energy/momentum of the final residue, depending on the direction of emission. This is reflected in the width (FWHM) of the experimentally measured recoil range distributions. The width may also arise because of the contributions from straggling. The identified reaction products and their experimentally measured most probable ranges $R_{p}^{expt}$, for all the CF residues along with the theoretically estimated (using the code SRIM) mean ranges $R_{p}^{theo}$, are given in Table 3.1. The most probable recoil ranges ($R_{p}^{theo}$) have been calculated, assuming that in the case of CF, the incoming ion completely fuses with the target nucleus and transfers its total linear momentum to the fused system, which recoils for the conservation of linear momentum. However, in case of ICF reactions, the linear momentum is transferred in the ratio of the mass transferred from the projectile to the target nucleus. Thus, the theoretical range calculations are, based on the recoil energy transferred to the composite system, done by SRIM code [67] based on the range energy formalism.

On the basis of the above description, it is clear that the population of reaction products $^{168}$Lu produced via $3n$ channel is associated with the entire LMT from projectile to the target nucleus, and may be represented as:

\[ ^{12}\text{C} + ^{159}\text{Tb} \Rightarrow ^{171}\text{Lu}^* \Rightarrow ^{168}\text{Lu} + 3n \]
Chapter 3. Recoil ranges

In the similar way, the FRRDs for the residues $^{167}$Lu (4$n$), $^{165}$Lu (6$n$) and $^{167}$Yb (p3$n$) are found to have a single peak associated with complete linear momentum transfer from projectile to the composite nucleus, indicating the production of these residues via the CF process only.

Table 3.1: Experimentally measured most probable ranges $R_{p(\text{exp})}$ deduced from RRD curves, and theoretically calculated forward mean ranges $R_{p(\text{the})}$ in Al in units of $\mu$g/cm$^2$ for CF and ICF components using the range energy relation, along with the reaction products produced in the interaction of $^{12}$C with $^{159}$Tb at $\approx 74$ MeV.

<table>
<thead>
<tr>
<th>Residues</th>
<th>$R_{p(\text{exp})}^{\text{CF}}$</th>
<th>$R_{p(\text{the})}^{\text{CF}}$</th>
<th>$R_{p(\text{exp})}^{\text{ICF-8Be}}$</th>
<th>$R_{p(\text{the})}^{\text{ICF-8Be}}$</th>
<th>$R_{p(\text{exp})}^{\text{ICF-4He}}$</th>
<th>$R_{p(\text{the})}^{\text{ICF-4He}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{168}$Lu</td>
<td>315±43</td>
<td>321</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{167}$Lu</td>
<td>312±48</td>
<td>321</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{165}$Lu</td>
<td>314±52</td>
<td>321</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{167}$Yb</td>
<td>330±28</td>
<td>321</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{165}$Tm</td>
<td>340±32</td>
<td>321</td>
<td>163±23</td>
<td>150</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{163}$Tm</td>
<td>333±61</td>
<td>321</td>
<td>158±19</td>
<td>150</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{161}$Ho</td>
<td>334±53</td>
<td>321</td>
<td>150±21</td>
<td>150</td>
<td>22±8</td>
<td>21</td>
</tr>
<tr>
<td>$^{160}$Ho$^g$</td>
<td>-</td>
<td>321</td>
<td>174±28</td>
<td>150</td>
<td>23±9</td>
<td>21</td>
</tr>
<tr>
<td>$^{160}$Ho$^m$</td>
<td>-</td>
<td>321</td>
<td>178±26</td>
<td>150</td>
<td>23±11</td>
<td>21</td>
</tr>
</tbody>
</table>

Further, in case of $\alpha$-emitting channels, the residues $^{165}$Tm, $^{163}$Tm, $^{161}$Ho, $^{160}$Ho and $^{160}$Ho are expected to be populated, respectively via $\alpha 2n$, $\alpha 4n$, $2\alpha 2n$ and $2\alpha 3n$ channels. The observed FRRDs were resolved into two Gaussian peaks, for $\alpha xn$-channels, using the ORIGIN software [69]. The FRRDs for the residues, $^{165}$Tm ($\alpha 2n$) and $^{163}$Tm ($\alpha 4n$) have been plotted at three different energies in Figs.3.7-3.8. As can be seen from these figures, the FRRDs may be fitted with two Gaussian peaks, one at higher cumulative depth, indicating the complete momentum transfer events, however, another peak at lower cumulative depth corresponds to the fusion of $^8$Be (if $^{12}$C is assumed to break-up into $^8$Be + $\alpha$ and $^8$Be fuses) with $^{159}$Tb target nucleus. The FRRDs of $\alpha xn$-channels clearly indicate the presence of
more than one linear momentum transfer components. It is observed that the complete as well as the incomplete momentum transfer peaks in the range spectra are centered at the expected position shown by arrow. It may be observed from Figs.3.7-3.8, that the mean range, $R_p^{\text{exp}}$ shifts towards slightly higher cumulative catcher thickness as the beam energy increases, as expected. It may be inferred that the residues $^{165}\text{Tm}$ populated through $\alpha2n$ channel may be populated via two ways:

(i) Fusion of $^{12}\text{C}$

$$^{12}\text{C} + ^{159}\text{Tb} \Rightarrow ^{171}\text{Lu}^* \Rightarrow ^{165}\text{Tm} + \alpha2n \text{ and/or } 2p4n$$

or:

(ii) Fusion of $^8\text{Be}$ ($\alpha$ as spectator)

$$^{12}\text{C}(^8\text{Be} + \alpha) \Rightarrow ^8\text{Be} + ^{159}\text{Tb} \Rightarrow ^{167}\text{Tm}^* + \alpha \text{ (as spectator)}$$

$$\Rightarrow ^{167}\text{Tm}^* \Rightarrow ^{165}\text{Tm} + 2n$$

In the same way the production of $^{163}\text{Tm}$ residues may be attributed due to (i) fusion of $^{12}\text{C}$ with $^{159}\text{Tb}$ forming $^{171}\text{Lu}$ in the excited state and then emitting an $\alpha$ and 4 neutrons and (ii) fusion of $^8\text{Be}$ with $^{159}\text{Tb}$ forming $^{167}\text{Tm}$ in excited state which then emit $\alpha$ and 4 neutrons. Further, in case of $2\alpha2n$-channels, the measured FRRDs have been found to be resolved into three Gaussian peaks. The measured FRRDs for $2\alpha2n$-channel have been plotted in Figs.3.9, at three beam energies. In this figure the observation of three peaks may be understood assuming the break-up of $^{12}\text{C}$ into possible $\alpha$-clusters. The peaks at $\approx 334\pm53$, $378\pm47$ and $396\pm67$ $\mu$g/cm$^2$ depths for three beam energies respectively are attributed to the complete momentum transfer i.e., fusion of $^{12}\text{C}$ with the target nucleus. However, the peaks at $\approx 150\pm21$, $193\pm37$ and $207\pm29$ $\mu$g/cm$^2$ for three beam energies belongs to the partial linear momentum transfer ($\frac{2}{3}\rho^{CF}_{\text{LMT}}$) events i.e., the fusion of $^8\text{Be}$. Another peak at the lower cumulative depths corresponds to the fusion of $\alpha$-particle with the target nucleus; involving $\frac{1}{3}\rho^{CF}_{\text{LMT}}$. As such, it may be inferred that the residues $^{161}\text{Ho}$ produced through $2\alpha2n$-channel have the contribution from both the processes, viz., CF as well as ICF, which may
be represented as;

(i) Fusion of $^{12}\text{C}$

\[ ^{12}\text{C} + ^{159}\text{Tb} \rightarrow ^{171}\text{Lu}^* \rightarrow ^{161}\text{Ho} + 2\alpha 2n \text{ and/or } 4p6n \]

(ii) Fusion of $^8\text{Be}$ (\(\alpha\) as spectator)

\[ ^{12}\text{C}(^8\text{Be} + \alpha) \rightarrow ^8\text{Be} + ^{159}\text{Tb} \rightarrow ^{167}\text{Tm}^* + \alpha \text{ (as spectator)} \]
\[ \rightarrow ^{167}\text{Tm}^* \rightarrow ^{161}\text{Ho} + \alpha 2n \]

(iii) Fusion of \(\alpha\) ($^8\text{Be}$ as spectator)

\[ ^{12}\text{C}(^8\text{Be} + ^4\text{He}) \rightarrow ^4\text{He} + ^{159}\text{Tb} \rightarrow ^{163}\text{Ho}^* + ^8\text{Be} \text{ (as spectator)} \]
\[ \rightarrow ^{163}\text{Ho}^* \rightarrow ^{161}\text{Ho} + 2n \]

The above description is based on break-up fusion model, where it is assumed that the incident \(^{12}\text{C}\) ion breaks up into fragments (e.g., \(^8\text{Be} + \alpha\) or \(\alpha + ^8\text{Be}\)) as it enters in the nuclear field of the target nucleus. The fragments so produced are assumed to move nearly with the same velocity as that of incident ion. One of the fragments (\(^8\text{Be}\) or \(\alpha\)) fuses with the target nucleus forming an incompletely fused composite system, which recoils in the forward direction to conserve the input linear momentum. Similar kind of range distributions have been observed in the production of \(^{160}\text{Ho}\) residues populated via 2\(\alpha3n\)-channel.

An attempt has also been made to check the consistency in the FWHM of the observed recoil range distributions. The normalized FWHM values have been deduced using the relation FWHM/\(R_{\text{exp}}\) for the observed distributions and are tabulated in Table 3.2. The normalized FWHM values have been found to be consistent for the CF and ICF residues individually. As can be seen from this table that for \(\alpha\)-emitting channels, the average peak resolution for CF is \(\approx 0.28\), while for ICF-\(\alpha\) and ICF-2\(\alpha\) the average peak resolution increases to \(\approx 0.69\) and 1.78, respectively, as expected.

In order to compare the range integrated yields of CF and ICF reactions, the statistical model calculations have been done using the code PACE4 [62]. This code is based on the statistical approach of CN de-excitation by Monte
Table 3.2: Comparison of normalized FWHM of the range distributions

<table>
<thead>
<tr>
<th>Residues</th>
<th>$^{174}$CF</th>
<th>$^{80}$ICF</th>
<th>$^{87}$ICF</th>
<th>$^{174}$ICF-α</th>
<th>$^{80}$ICF-α</th>
<th>$^{87}$ICF-α</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{168}$Lu (3$n$)</td>
<td>0.62</td>
<td>0.77</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{167}$Lu (4$n$)</td>
<td>0.56</td>
<td>0.70</td>
<td>0.65</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{165}$Lu (6$n$)</td>
<td>0.67</td>
<td>0.65</td>
<td>0.67</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{177}$Yb (p3$n$)</td>
<td>0.58</td>
<td>0.63</td>
<td>0.71</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{165}$Tm (α2$n$)</td>
<td>0.31</td>
<td>0.25</td>
<td>0.22</td>
<td>0.83</td>
<td>0.72</td>
<td>0.81</td>
</tr>
<tr>
<td>$^{163}$Tm (α4$n$)</td>
<td>0.20</td>
<td>0.30</td>
<td>0.34</td>
<td>0.87</td>
<td>0.80</td>
<td>0.83</td>
</tr>
<tr>
<td>$^{161}$Ho (2α2$n$)</td>
<td>0.32</td>
<td>0.20</td>
<td>0.34</td>
<td>0.69</td>
<td>0.75</td>
<td>0.63</td>
</tr>
<tr>
<td>$^{160}$Ho$^9$ (2α3$n$)</td>
<td>-</td>
<td>0.22</td>
<td>0.26</td>
<td>0.74</td>
<td>0.79</td>
<td>0.73</td>
</tr>
<tr>
<td>$^{160}$Ho$^m$ (2α3$n$)</td>
<td>-</td>
<td>0.24</td>
<td>0.22</td>
<td>0.69</td>
<td>0.77</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Carlo procedure. In the code PACE4, angular momentum projections are calculated at each step of de-excitation. The angular momentum conservation is explicitly taken into account. The CF cross-sections are calculated using BASS formula [74]. The details of the PACE4 code are given in the section 2.4, of Chapter-2. It has been observed that the contribution of CF satisfactorily matches with that predicted by PACE4 code. However, the contribution of ICF reactions (given in Table-3.3) could not be reproduced by calculations using the same set of parameters, since, PACE4 code does not take ICF into account.

The percentage ICF contributions of different fusion components have been obtained by dividing the area under the ICF peak of the corresponding fusion component by the total area associated with the experimental data. The values of $F_{ICF}$ deduced from FRRDs data are compared with the $F_{ICF}$ deduced from the excitation function measurements $^8$, as a function of beam energy ($E_{lab}$). This comparison is shown in Fig.3.12. As can be seen from this figure that the ICF fraction increases rapidly with energy at lower energies, however, at relatively higher energies the $F_{ICF}$ increases with slow rate.

Table 3.3: Experimentally measured range integrated cross-section $\sigma_{RRD}^{exp}$ deduced from RRD curves, and theoretically calculated cross-section $\sigma_{PACE}^{theo}$

<table>
<thead>
<tr>
<th>Residues</th>
<th>$\sigma_{RRD}^{exp}$</th>
<th>$\sigma_{PACE}^{theo}$</th>
<th>$\sigma_{RRD}^{exp}$</th>
<th>$\sigma_{RRD}^{exp}$</th>
<th>$\sigma_{RRD}^{exp}$</th>
<th>$\sigma_{RRD}^{exp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{168}$Lu($3n$)</td>
<td>3.20</td>
<td>3.18</td>
<td>1.10</td>
<td>0.97</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{167}$Lu($4n$)</td>
<td>297</td>
<td>314</td>
<td>96</td>
<td>90.2</td>
<td>12.0</td>
<td>15.9</td>
</tr>
<tr>
<td>$^{165}$Lu($6n$)</td>
<td>1.4</td>
<td>0.61</td>
<td>120</td>
<td>109</td>
<td>510</td>
<td>298</td>
</tr>
<tr>
<td>$^{167}$Yb($p3n$)</td>
<td>33</td>
<td>29</td>
<td>13.3</td>
<td>12.05</td>
<td>2.82</td>
<td>3.07</td>
</tr>
<tr>
<td>$^{165}$Tm($\alpha2n$)</td>
<td>6.79</td>
<td>0.83</td>
<td>11.15</td>
<td>0.28</td>
<td>17.32</td>
<td>0.24</td>
</tr>
<tr>
<td>$^{163}$Tm($\alpha4n$)</td>
<td>165.32</td>
<td>10.98</td>
<td>260.9</td>
<td>38.05</td>
<td>280.44</td>
<td>53.09</td>
</tr>
<tr>
<td>$^{161}$Ho($2\alpha2n$)</td>
<td>7.27</td>
<td>0.20</td>
<td>5.94</td>
<td>0.23</td>
<td>3.34</td>
<td>0.14</td>
</tr>
<tr>
<td>$^{160}$Hg$^g$(2$\alpha3n$)</td>
<td>2.04</td>
<td>0.06</td>
<td>6.26</td>
<td>0.33</td>
<td>7.0</td>
<td>0.9</td>
</tr>
<tr>
<td>$^{160}$Hg$^m$(2$\alpha3n$)</td>
<td>1.64</td>
<td>0.06</td>
<td>5.03</td>
<td>0.33</td>
<td>5.69</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Nevertheless, it may also be observed from Fig.3.12, that the measurements of FRRDs and the EFs both give nearly same $F_{ICF}$ values, which strengthen the present deduction procedure and indicates the self-consistency of the data. In Table 3.4, comparison of range integrated cross-sections with that obtained from EF measurements for $xn$ and $pxn$-channels has also been done, and found to agree reasonably well.

3.4.1 Comments on mass-asymmetry & projectile structure effects

In order to have a better understanding about the dependence of underlying reaction dynamics on the mass-asymmetry and on the projectile structure, the presently deduced $F_{ICF}$ values have been compared with the $F_{ICF}$ obtained in the case of $^{16}$O induced reactions on the same target $^{159}$Tb [23]. The Fig.3.13, shows the comparison of $F_{ICF}$ values for both $^{12}$C+$^{159}$Tb and $^{16}$O+$^{159}$Tb systems at different reduced projectile energies.
Table 3.4: Comparison of range integrated cross-sections with EF.

<table>
<thead>
<tr>
<th>Residues</th>
<th>≈ 74 MeV RRD (EFs)</th>
<th>≈ 80 MeV RRD (EFs)</th>
<th>≈ 87 MeV RRD (EFs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{168}$Lu(3n)</td>
<td>3.2 (3.4)</td>
<td>1.1 (0.76)</td>
<td>-</td>
</tr>
<tr>
<td>$^{167}$Lu (4n)</td>
<td>297 (320)</td>
<td>96 (90)</td>
<td>12 (10)</td>
</tr>
<tr>
<td>$^{165}$Lu (6n)</td>
<td>1.4 (1.05)</td>
<td>120 (124)</td>
<td>510 (550)</td>
</tr>
<tr>
<td>$^{167}$Yb (p3n)</td>
<td>33 (32.2)</td>
<td>13.3 (11.6)</td>
<td>2.8 (3.6)</td>
</tr>
</tbody>
</table>

Figure 3.12: The values of $F_{ICF}$ deduced from RRD measurements for the system $^{12}$C+$^{159}$Tb, as a function of the reduced projectile energy. The $F_{ICF}$ values deduced from EF measurements are compared.
It may be observed from this figure that the $^{16}\text{O}$ as projectile has relatively higher ICF contribution than for the $^{12}\text{C}$, at the same normalized projectile energies. It may, however, be pointed out that according to Morgenstern’s mass-asymmetry systematics [39, 40] a more asymmetric system would have more ICF probability. The Fig.3.13, shows the comparison of $F_{ICF}$ values for both $^{12}\text{C}^{+159}\text{Tb}$ and $^{16}\text{O}^{+159}\text{Tb}$ systems at different reduced projectile energies.

Figure 3.13: The percentage incomplete fusion fraction ($F_{ICF}$) deduced for $^{16}\text{O}^{+159}\text{Tb}$ [23] and $^{12}\text{C}^{+159}\text{Tb}$ (present work) systems as a function of reduced projectile energy.
3.4 Interpretation of experimental results

It may be observed from this figure that the $^{16}$O as projectile has relatively higher ICF contribution than for the $^{12}$C, at the same normalized projectile energies. It may, however, be pointed out that according to Morgenstern’s mass-asymmetry systematics [39, 40] a more asymmetric system would have more ICF probability. The mass-asymmetry of the interaction partners may be defined as; $\mu = A_T / A_{T+p}$. So, $^{12}$C+$^{159}$Tb ($\mu=0.9298$) should have more ICF contribution than $^{16}$O+$^{159}$Tb system ($\mu=0.9086$), which is contrary to the present observation. In addition to this, the binding energy aspect ($E_{^{16}$O binding} > E_{^{12}$C binding}$) is also unable to explain the present picture. One of the possible explanations may be the excess of alpha-clusters in $^{16}$O nucleus than in the $^{12}$C. In addition to this the alpha-Q-value for $^{16}$O is less than the $^{12}$C, i.e., $^{16}$O requires less energy to break-up into alpha-clusters than $^{12}$C and thus may give rise to large ICF contributions. As such, the alpha-Q-value of the projectile seems to be a reasonable parameter to explain the presently observed large $F_{ICF}$ values for $^{16}$O as projectile than for the $^{12}$C.

3.4.2 SUMRULE calculations: sharp cut-off in $\ell$-distribution

In the present work, an attempt has also been made to compare the results with calculations done with SUMRULE model [11, 12, 32], which is based on the partial statistical equilibrium and on the idea of generalized concept of critical angular momentum. According to this model, the transfer of mass may occur only if the angular momentum of relative motion of the captured fragment $P^p$ ($P^p$: participant, $P^s$: spectator) with respect to the target nucleus is smaller than the critical angular momentum for the in-completely fused system, i.e. $\ell_{eff} \leq \ell_{crit}^{P^p+T}$. The limiting angular momentum in the reference frame of the entrance channel $\ell_{limit}$, is related to the critical angular momentum $\ell_{crit}^{P^p+T}$ of fused part as [11, 12],

$$
\ell_{limit} = \frac{A_p \cdot A_T}{(A_{ps} \cdot A_p + A_{pp} \cdot A_T)} \cdot \ell_{crit}^{P^p+T}
$$

(3.5)
Chapter 3. Recoil ranges proof of fusion in–completeness

However, for a mass-asymmetric projectile-target combination, the limiting angular momentum may be re-written as:

\[ \ell_{\text{limit}} \sim \frac{A_p}{A_{pp}} \cdot \ell_{\text{crit}}^{pp+T} \] (3.6)

By assuming the smooth cut-off in the \( \ell \)-space the transmission coefficient for each individual reaction channel is given as:

\[ T_{\ell}(i) = \left[ 1 + \exp\left( \frac{\ell - \ell_{\text{limit}}(i)}{\Delta} \right) \right]^{-1} \] (3.7)

where, \( \Delta \) gives the diffuseness in the \( \ell \)-distribution. For smaller \( \ell \)-values the transmission coefficients \( T_{\ell} \), are almost unity for all the channels. The different reaction channels open up one after the other with increasing angular momentum and depending upon their corresponding limiting angular momenta \( \ell_{\text{limit}}(i) \). Hence, the reaction probabilities for a given partial wave \( \ell \);

\[ N_{\ell} \sum_i T_{\ell}(i) \times \exp\left( \frac{Q_{gg}(i) - Q_c(i)}{T} \right) = 1 \] (3.8)

where, \( N_{\ell} \) is the \( \ell \)-dependent normalization factor common for all reaction channels. Thus, absolute cross-section for the individual reaction channels is defined as;

\[ \sigma(i) = \pi \lambda^2 \sum_{\ell=0}^{\ell_{\text{max}}} (2\ell + 1) \times \frac{T_{\ell}(i) \cdot p(i)}{\sum_j T_{\ell}(j) \cdot p(j)} \] (3.9)

where \( \lambda = h/\sqrt{(2\mu E)} \) is the reduced wavelength and \( p(i) \) is the reaction probability for a given channel \( i \), which is proportional to \( \sim \exp[\{Q_{gg}(i) - Q_c(i)/T] \). Here, \( T \) is the effective temperature, and \( Q_c(i) \) is the change of the Coulomb interaction energy due to the transfer of charge. The \( \ell_{\text{max}} \) is
3.4 Interpretation of experimental results

defined as the largest $\ell$ for which the colliding system penetrates into the region where the total nucleus-nucleus potential is attractive and/or the distance of closest approach is smaller than the sum of the half-density radii. However, the critical angular momenta $\ell_{\text{crit}}$, which determine the magnitude of the "transmission coefficients" $T_\ell$, for individual reaction channels were calculated from a simplified formula as:

$$\ell_{\text{crit}}^2 = \frac{M_{\text{red}}(C_1 + C_2)^3}{h^2} \left[ 4\pi \gamma \frac{C_1 C_2}{C_1 + C_2} - \frac{Z_1 Z_2 e^2}{(C_1 + C_2)^2} \right]$$

(3.10)

Where, $M_{\text{red}}$ is the reduced mass of the interacting partners, $\gamma$ is the surface tension coefficient, $Z_1$ & $Z_2$ and $C_1$ & $C_2$ are the atomic numbers and half-density radii of projectile and target nuclei, respectively. With this model one may calculate absolute cross-sections for CF, ICF channels, and other binary reactions which presumably proceed via the formation of a dinuclear system. The model contains three free parameters: the effective temperature $T$, the effective Coulomb interaction radius $R_c$, and the diffuseness in $\ell$ distribution, $\Delta$. Wilczynski et al. [11, 12], to fit the experimental data in the $^{14}\text{N}+^{159}\text{Tb}$ reaction at $E_{\text{lab}} = 140$ MeV, used $T = 3.5$ MeV, $R_c/(A_1^{1/3} + A_2^{1/3}) = 1.5$ and $\Delta = 1.7\hbar$. In order to obtain the magnitude of ICF reaction cross-sections in the present work, the same parameters as given above have been retained. Using these parameters, the SUMRULE model calculations are found to highly underestimate the measured cross-sections for the residues of interest, i.e., ICF channels. As a typical example the experimentally measured cross-sections for the $^{159}\text{Tb}(^{12}\text{C}, \alpha 2n)^{165}\text{Tm}$ and $^{159}\text{Tb}(^{12}\text{C}, 2\alpha 3n)^{160}\text{Ho}$ channels are $\approx 64.0 \pm 9.6$ mb and $5.0 \pm 0.7$ mb, however, the theoretically calculated SUMRULE values are found to be $1.32$ mb and $0.02$ mb at 86 MeV beam energy. These substantial discrepancies indicate the need for refinement in the assumptions of the SUMRULE model. Similar, deviations have also been found by Parker et al. [15] in their study on the $^{12}\text{C}+^{51}\text{V}$ system up to 100 MeV.
3.4.3 Diffuseness in $\ell$-distribution: Observation of ICF at $\ell < \ell_{\text{crit}}$

In order to have a better understanding about the diffuseness in $\ell$-distribution, the fusion $\ell$-distributions for the compound nucleus in $^{12}\text{C} + ^{159}\text{Tb}$ interactions at three studied energies, have been calculated using the code CCFULL [91], and are plotted in Fig.3.14.

Figure 3.14: Fusion spin-distributions calculated using the code CCFULL [91] for $^{12}\text{C} + ^{159}\text{Tb}$ system, where $\ell$ is in units of $\hbar$. 

\[
\ell_{\text{max}} = 36\hbar, \quad \ell_{\text{crit}} = 46\hbar, \quad \ell_{2\text{max}} = 40\hbar, \quad \ell_{3\text{max}} = 44\hbar
\]
The critical angular momentum $\ell_{\text{crit}}$, for the presently studied system $^{12}\text{C} + ^{159}\text{Tb}$, at which the pocket in the entrance channel potential vanishes has been calculated using the prescription given by Wilzyński et al., [11, 12]. The calculation gives $\ell_{\text{crit}} \approx 46 \hbar$. The values of $\ell_{\text{max}}$ at three respective energies in the present work are $\approx 36 \hbar, 40 \hbar, \text{and} 44 \hbar$, respectively, which are less than $\ell_{\text{crit}}$ for fusion for this system. The SUMRULE model assumes sharp cut-off $l$ values for the CF and ICF processes. The underestimation of the ICF cross-sections by the SUMRULE model may be due to the assumption that a major contribution to the ICF reactions comes from the collision trajectories with the angular momentum $l$ greater than the critical angular momentum for complete fusion ($\ell_{\text{crit}}$). It is evident from figure 3.14 that even at the highest energy studied in the present work, the population of $l$-values are less than $\ell_{\text{crit}}$ for fusion. As such, the ICF contributions are less probable at these energies as per the SUMRULE model assumptions. However, the present FRRD measurements clearly reveal the significant ICF contributions at these energies, and hence suggest that a significant number of $\ell$-waves below $\ell_{\text{crit}}$ may contribute to ICF. The present findings clearly suggest a broad diffused boundary that may penetrate close to the barrier.

### 3.5 Summary

The recoil range distributions of a large number of radio-nuclides viz; $^{168}\text{Lu}$ ($3n$), $^{167}\text{Lu}$ ($4n$), $^{165}\text{Lu}$ ($6n$), $^{167}\text{Yb}$ ($p3n$), $^{165}\text{Tm}$ ($\alpha 2n$), $^{163}\text{Tm}$ ($\alpha 4n$), $^{161}\text{Ho}$ ($2\alpha 2n$), $^{160}\text{Ho}^9$ ($2\alpha 3n$) and $^{160}\text{Ho}^m$ ($2\alpha 3n$) populated in $^{12}\text{C} + ^{159}\text{Tb}$ interactions at three above barrier energies have been measured. The analysis of the measured FRRDs of the reaction products presented, strongly reveal a significant contribution from the partial LMT of the projectile associated with ICF in several $\alpha$-emitting channels. Different partial LMT components are attributed to the fusion of $^8\text{Be}$ and $\alpha$-particle from the $^{12}\text{C}$ projectile to the target nucleus. The percentage ICF contributions for $^{12}\text{C} + ^{159}\text{Tb}$ system are found to have onset from $\approx 12\%$ above $V_b$. It has been found, in general, that the residues are populated not only via CF but ICF is also found to play an important role in the production of different reac-
tion products involving direct $\alpha$-cluster emission. The present results have also been compared with literature results, and reveal that instead of Morgenstern's mass-asymmetry systematics the projectile structure along with alpha-$Q$-value is an important parameter in the energy range of interest. On the other hand, the SUMRULE model calculations are found to highly underestimate the ICF cross-sections which may be due to the assumption that a substantial contribution to ICF comes from the collision trajectories with $\ell > \ell_{\text{crit}}$. In the energy range of the present study, $\ell_{\text{max}} < \ell_{\text{crit}}$, thus significant cross-sections for ICF at these beam energies indicate the contribution from collision trajectories with $\ell < \ell_{\text{crit}}$. The results obtained from the FRRDs give valuable information for establishing the CF and ICF yields at relatively low bombarding energies and also indicate that the $\ell$-values lower than $\ell_{\text{crit}}$ significantly contribute to the ICF reactions. More data on such reactions is required to explore the above aspects, so that the assumptions of the SUMRULE model for energies near the barrier, where $\ell < \ell_{\text{crit}}$, may be improved upon to explain the experimental data.