In recent past, variational inequality theory has appeared as an elegant and fascinating branch of applicable mathematics. Variational inequalities arise in various models for a large number of mathematical, physical, regional and other problems. Till now a considerable interest has been shown in developing various extensions and generalizations of variational inequalities related to multi-valued operations, nonconvex optimization, nonmonotone operators and structural analysis. There are three different aspects to study variational inequalities (i) Mathematical Modelling: To convert the problems of real life or the problems from science, engineering and social science into a variational inequality problem. (ii) Existence Theory: To study the existence of solutions of variational inequalities. (iii) Numerical Methods: To find the algorithms for computing the approximate solutions of variational inequalities, which converge to the exact solutions.

Chapter 1 deals with the brief introduction of variational inequalities, variational-like inequalities, quasi-variational inequalities, variational inclusions and system of variational inequalities (inclusions) besides some definitions and results from functional analysis which will be used in the subsequent chapters.

In Chapter 2, we study a generalized quasi-variational-like inclusion problem with \((A, \eta)-\text{accretive and relaxed cocoercive mappings in } q\)-uniformly smooth Banach spaces. We also introduce and study a variational-like inclusion problem and its resolvent equation involving infinite family of set-valued mappings in real Banach spaces. In the last section, we consider a mixed variational inclusion problem involving infinite family of fuzzy mappings in uniformly smooth Banach spaces.

In Chapter 3, we define \(H(\cdot, \cdot)-\phi-\eta\)-accretive operator and discuss some of its properties. By applying the concept of \(H(\cdot, \cdot)-\phi-\eta\)-accretive operator and its properties, we solve a generalized variational-like inclusion problem in \(q\)-uniformly smooth Banach spaces. Further, we also study the fuzzy version of
generalized variational-like inclusion problem and a fuzzy resolvent equation problem.

In Chapter 4, we introduce and study a new $H(\cdot, \cdot)$-cocoercive operator in Hilbert spaces with some of its properties. We apply this new concept to solve a generalized variational inclusion problem, a system of variational inclusions and a resolvent equation problem.

In Chapter 5, we generalize the concept of $H(\cdot, \cdot)$-cocoercive operator discussed in Chapter 4 to $H(\cdot, \cdot)$-$\eta$-cocoercive operator in Banach spaces. The existence results for a variational-like inclusion problem, a generalized variational-like inclusion problem and a system of generalized variational-like inclusion problem are proved.