Chapter - 2: Pre - Processing

Objectives

• Understanding the noises involved in CT Liver images

• Mathematical expression for power spectral density of CT Liver images.

• Understanding the conventional methods for reducing the noise.
2.1. Overview

In this chapter the basics of the image processing techniques are discussed to explore the areas of study to be conducted thoroughly. This chapter covers fundamentals of image processing and various segmentation techniques and modalities are discussed in detail. This chapter includes lot of literature background related with the current research. Pre-processing techniques and different image processing methods are discussed in detail. In section – 2.2 the methods of image processing and fundamental steps of digital images processing were discussed. In section – 2.3 the image segmentation methods and the theory behind it were explained with appropriate examples. In section – 2.4 Image modalities are discussed along with various types of image scanning methods. All these methods and their implications, limitations were explored in with mathematical analysis. In section – 2.5 various types of noises in CT liver images were discussed to understand the role of noise played in digital image processing while considering the image for segmentation process. Finally in section – 2.6 various image de-noising methods were discussed in different domains. The influence of spatial and transform domain on the filtering process are discussed clearly.

2.2. Image processing

Signal processing methods includes image processing as a part of testing the images in detailed way to identify various research aspects in the modern technological world. In most of the image processing methods the input will be image from a picture or a video [12]. The role of image processing is vital for segmenting the images into small pieces and helps the analyst to understand the problem area in the human liver parts [22]. General consideration of this technique will be with a two dimensional image and various
signal processing methods will be applied for detailed study. This method will provide
the analyst to study the picture at pixel level to identify the scope of their interest areas as
shown in figure – 2.1.a. and the steps involved in the whole image processing are given
by figure – 2.1.b.

![Image Processing Application at Pixel Level](image)

**Figure – 2.1:** (a) Image processing application at pixel level

The figure – 2.1.b explains various steps involved in image processing include three
major steps. They are pre-processing, segmentation and post-processing. In the **pre-
processing** one has to consider another four steps to complete this processing step. One
has to find the region of interest and need to extract the same from the digital image. In
the second step a polar transformation has to be developed and third step deals with
filtering of noise. In the final step one has to consider the image quality towards next
process of segmentation by contrasting or enhancing the image quality. The
**segmentation process** can be done in three ways. They are edge detection and
thresholding, region growing [21] and dynamic cost functions [68]. This process will
improve the quality of desired image for the analysis of problem areas in hospitals or
other applications.
In the **post processing** method the substitution of knowledge base outline is submitted and later dropout interpolation is conducted. Spline Fitting and Visualization are the two final steps involved in this method to complete the image processing methods.

![Image Processing Steps Diagram]

**Figure – 2.1**: (b) Image processing Steps [68]

Most of the images are classified into various sub-images referred as ROI (region of interest). ROI will be a focused area of interest that undergoes with wide range of research aspects [43] [87] [89]. Only selected regions in this process will be studied at
pixel level and these parts will be having improvised colour combinations and rest of the image part will be mostly with a blur. However, image processing is also possible in optical analog systems also apart from only digital image processing [12]. The applications of image processing are used in wide range of applications includes entertainment for improving picture quality with different light intensities, medical researchers, military and civil applications. In all these areas the images will be captured at different scenarios and are processed with different software tools for understanding the level of risk/ properties of various physical materials in different conditions. The study using image processing at pixel level will help the health industry to utilize it in various medical applications with extensive scope at interested areas.

In another major discussion according to the authors in [23] stressed the point of using digitized images for the applications of segmentation using image processing. For digitizing the images they are initially converted into analog signals that will undergo for a scanning process for a display. Digitized form of images contain array of finite length with binary words and these given images will be sampled to a discrete grid [88] [90]. Generally the computers are used to view this digitized images after converting the same into an analog signal. Apart from that digitization also involves the sampling and quantization process for different sample values of images. Processing of the image is performed after converting it into bit information after which the processing techniques will be applied for enhancing the image or to reconstruction or compressing.
2.2.1. Digital image processing

Digital image processing (DIP) is considered to be a subset of electronic domain where the images are converted as an array of tiny integers (pixels) that represent physical quantities like scene radiance. It works as an enhancement for human observations towards performing various analyses for wide ranges of advantages in terms of cost, speed and flexibility. Indirectly it becomes a mean for reducing the cost by improving the performance by using a personal computer [12]. Generally the image in any form can be considered as a complex form of wide range of physical processes. These physical processes include intensity, distribution of radiation, and interaction of radiation with matter and geometry of projection of transferred radiation in various dimensions [70].

2.2.2. Fundamental steps of image processing

There are many steps in image processing and they include few more sub steps. They are image acquisition, image enhancement, image restoration, colour image processing, wavelets and multi-resolution processing, compression, morphological processing, segmentation, representation and description, object recognition and knowledge base. All these steps involved in DIP are shown the figure – 2.2. A brief explanation of each component is provided here under and the detailed study is covered in different parts of this research thesis according to the demand of research flow.
Image acquisition is the first step to be followed in DIP and it can be as simple as to get an image from already existing digital form. This stage generally involves pre-processing like scaling. It also defines the problem domain and will be inputting the images from different resources. Image enhancement is considered to be the simplest areas of image processing. It is used to bring a detailed study of hidden elements by highlighting certain features on image to be investigated. The parameters generally tested using this enhancement is to study the brightness and contrast levels [25]. In this method even small image is enhanced with resolutions and body weights. Image restoration on the other hand will improve the appearance of selected picture. However, it will not be similar to enhancement but purely based on mathematical or probabilistic models of image degradation. Colour image processing got an importance due to the increasing demand for digital images over internet.
It is generally included in digital domain including colour modelling and processing [12]. Wavelets are considered to be a foundation to present an image in different degrees of resolution. The subjected image will be divided into subdivisions for data compression and pyramidal representation. Compression technique will be dealing with various techniques to reduce the storage needed for saving an image for transmitting it within the bandwidth available especially when, data is to transferred by compressed data. Morphological processing considers extracting image components applied for representing and describing the shape. Segmentation makes the partitions of images into ingredient parts and objects. However, this part is the most difficult task of digital image processing. But after a rigid process it brings the successful solutions for image problem by identifying the individuality. Recognition assigns the labels to each object based on the descriptions. Finally, the knowledge base contains the detailed regions of an image with the information of interested area [24] [69].

DIP uses computer algorithms for performing image processing on digitized images. Being the sub category of Digital Signal Processing (DSP), DIP is having wide ranges of advantages over analogue image processing method. One can implement wide range of algorithm for the input data using Digital image processing for avoiding problems that are tend to increase due to noise and signal distortions during the processing periods [12]. Similarly they are convertible into a multi-dimensional system as the images are defined in two dimensions. DIP also uses complex algorithm that offers sophisticated performance in a simple way, which are not possible by using any kind of analog systems. Generally DIP is considered to be a technology that used for classification, extraction, pattern recognition, for projection and for multi – scale signal analysis.
There are few more techniques that are widely used in DIP are pixilation, linear filtering, hidden Markov models [14], anisotropic diffusion, self-organizing maps, wavelets and neural networks.

2.3. Image segmentation

As discussed in above sections image segmentation splits the digital images into multiple regions with respect to different interest areas. Based on the requirements the image is used to be divided and corresponding technique is applied to study. Though there are many advantages using image segmentation it is also noticed that many disadvantages are also seen in this method for the following reasons.

![Figure 2.3](image.png)

**Figure – 2.3:** Original Image to be segmented

In the first case, the splitting of images are in ambiguous nature and they demands an algorithms with generalized or with specific parameters. In the second case, many parameters like noise, in homogeneity, acquisition artefacts are poor in contrast.
These parameters will create a problem in terms of accountability for segmentation algorithm due to lack of high level interactivity with the user. However, in most of the computer applications edge detection[3] and image segmentation needs to take a critical step before a high level task is to be performed [17]. Image segmentation can also be based on the image colour and texture. However, progress has been considered in both segmentation methods separately [16].

Figure – 2.4: Final Segmented Image

Segmented images used by medical industry for studying the anatomical structures, diagnosis and assist surgical planning. Segmentation of medical images used to be a tedious process before the existence of computational algorithms. And segmentation was done by hands of clinical experts. However this process given positive results with accurate outcomes though it was a slow process. The segmentation process conducted by experts used to form gold standards with which segmentation algorithms will validates.
There are many techniques that exist with own strengths and limitations as particular solutions were not available for image segmentation problems.

*The common techniques in image segmentation are:*

1. Thresholding based image segmentation,
2. Region based segmentation,
3. Edge based segmentation; and
4. Deformable active contour models (Snake and Geodesic Active contour models).

In the next chapter a detailed study will be covered about image segmentation. A brief introduction is given in this place to draw the attention on the necessities of image segmentation at this juncture.

**2.4. Image Modalities**

In the modern era the imaging technology is playing a key role to diagnosis patient condition with ease. These methods are allowing the medical practitioners to peck into the body without using olden invasive methods. This is not only helping the patient to be more comfortable and safe but it gives wide range of opportunity for the doctors for testing all possible solutions. Imaging was evolved from the time when the x-ray image of a hand [26]. Using this method motion of an organ can be visualized like heart beating and etc. There is a lot of demand for image modalities since last 100 years from the time of discovery of X-rays [26]. The X-ray is one of the medical imaging modalities method used by most of the Radiologist's along with Ultrasonography, Computerised Tomography (CT).
Magnetic Resonance Imaging (MRI), Single Photon Emission Computed Tomography (SPECT) and Position Emission tomography (PET) [4]. Imaging will address two issues and they are structure and function. By using image modalities one can view structures inside the body of a patient and in other cases one can also observe the chemical process in Biochemistry.

2.4.1. Ultrasonography

There are different methods for computing and diagnosing the symptoms of various diseases and Ultrasonography is used widely in medical industry at an extensive way [49]. This method is considered to be an easy assessable and cheap method. Its internal structure makes this method more dynamic to apply at selected location using Radio Frequency Ablation (RFA). This method is having the ability of monitoring thermal ablation is having some limitations as it provides quality information about the treatment. However this method does not provide reliable information of temperature of a patient [27]. Usage of this method initially assesses the features of Hemangioma or Cysts. The usage of Ultrasonography found to be excellent modality in terms of initial detection of lesions and to identify the solid cystic lesion [48]. However, this often comes out to be not specific in detecting the lesions and additional hepatic imaging was needed in most of the cases. CT is found to be more reliable and one can reproduce this method easily to evaluate the focal hepatic lesions. In the latest medical treatment the multiphase CT scans are used as an updated version of equipments which are much capable to show vascularity of hepatic tumours. These latest machinery are capable of showing nodules with clear view. At the same time this will also help to see the vicinity of lesion that
represents changes in inflammatory. Figure - 2.5 (a) shows the sample image of Ultrasonography and the figure - 2.5 (b) shows the Liver CT demonstrates faint segmented egg shell.

**Figure - 2.5 (a):** Image from Ultrasonography

**Figure - 2.5 (b):** The Liver CT demonstrates faint segmented egg shell [49]
2.4.2. Computerized Tomography

Computerized Tomography (CT) is a method used to examine the body organs using scanning methods by X rays and construct series of scans across the cross sections in different angles using a computer. This was considered to be a revolution of Radiology in Computerized Tomography [50]. This method depends on the method of building the two dimensional projection from a three dimensional object. It is called as computer assisted tomography (CAT) or in simple words it is Computerized Tomography (CT). Generally in this method the scan will be a natural progression from x - ray. However, this method based much on mathematics with n – dimensional space to (n – 1) dimensional space. The scan taken in this method will be having an x – ray picture taken from various angles and many such things will be combined to construct a three dimensional structure of a body [26] [41] [50]. In this method a patient is kept stable and a rotating tube and detector will move across the patient and a beam of fan with x – rays will determine pieces of patient’s body. Figure – 2.6 shows a slice representing the horizontal line and the data connected from different fixed positions of sources and detectors can be seen. In this figure – 2.6 (a) shows the radiography with horizontal line marking, (b) shows the Sinogram of projected data, (c) shows the convolved projection data and (d) shows the reconstructed data from projected data. All the detectors have individual readings and are represented by individual sonograms. Generally the intensities will be proportional to the integral lines of x-ray attenuation co-efficients between the respective sources and detector positions [50]. Two dimensional images with X – ray attenuation coefficients are considers the integral lines and image reconstruction happens. However, the CT images the attenuation co-efficients will change according to
different tissues and these will change even with respect to a healthy tissue to weaker tissue. The human body scanning by a CT scan will produce slices of different cross sectional areas without any surgical intervention.

**Figure 2.6:** (a) shows the Radiography with horizontal line marking, (b) Sinogram of projected data, (c) convolved projection data and (d) reconstructed data from projected data
Figure – 2.7: Image from Computerised Tomography with excellent diagnosis [26]

Due to reconstruction of image is done using computer it will be in a digital form and will be analysed quantitatively [34]. In the mean time this method is very much easier to separate the tissues from bones when compared to simple x – ray scans as the data collected in this method is more compared with other methods.

Reconstruction algorithms in this method have advantage of subtle differences among tissue absorption coefficients and it allows demarcation of various structures of a body. Generally CT Images considers being the superior when compared with single x – ray scans. However, it suffers for having similar to that of soft tissue x – ray absorption. MRI scan is used for more detailed imaging over CT method [26].

2.4.3. Magnetic Resonance Imaging

MRI provides detailed anatomical data of various structures of imaging modalities. It is the only method that does not use high energy radiation. Nuclear resonance principle is
used in this MRI scan method and it is purely based on the resonance properties of protons. However, when considered with chemical research another atom will be applied for the same purpose [26]. Generally used atoms are carbon, nitrogen and phosphorous. But one cannot use all the atoms as they are all not eligible for imaging. The atoms with non-zero nuclear spin and atoms with odd number of protons and odd number of neutrons are eligible.

Figure – 2.8: Image from Computerised Tomography [26]

Generally the MRI scanners use good magnets for polarizing and exciting the hydrogen nuclei from the water molecule and then into human tissue. This produces a detectable signal that is encoded spatially and the output results will be an image of human body. Generally MRI uses 3 electro-magnetic fields [32] [75] [77] as follows:

The first one will be static field in which a strong magnetic field will polarize the hydrogen nuclei. The second one is called gradient field with weak time varying field used for spatial encoding and finally the third one uses a weak RF Frequency field for
manipulating the hydrogen nuclei for producing signals that can be measured after collecting from RF antenna.

![Figure 2.9: Segmentation of MRI brain image by replacing 1 with δ function [62]](image)

In a MRI scan the objects placed in magnetic field gradient will generate the resonance signals of magnetic field from the nuclei and all the unpaired electrons will depend on the applied magnetic field [50] [76] [78]. In a uniform linear field gradient the plot between signals and frequencies will provide a one-dimensional projection that is perpendicular to gradient axis out of entire signal intensity. The projection areas are produced by varying the field gradient and based on these variations only the two or three dimensional
pictures are reconstructed. In the technical aspects of MRI Scan, it will be generally based on the Fourier Transform of the image to be scanned.

Figure – 2.10: The Slice through a Spine shows after a MRI Scanner shows the bulging disc pressing the spine nerve that requires surgical intervention [50]

Most of the MRI scans will have a strong dependency on the Fourier Transforms while scanning the images and then into pixels. In the process of retrieving the images and analysis of images the inverse Fourier transformation is essentially needed to generate an output of the image. However, both the above methods (CT and MRI) are harmful due to
ionizing the radiation while collecting the data for image scans [29]. To avoid such problems due to radiation the ultrasounds are used in other scanning methods[10] [42].

2.4.4.Single Photon Emission Computed Tomography

It is called as SPECT in simple words and largely used technique for diagnosing. By using this method the images of human body can be created by distribution of radio pharmaceutical tracers [51]. Advantage of this method is the ability of transmitting the gamma rays over $2\pi$ crystal geometry very efficiently. Another major advantage of this method is to have a stationary design which is friendlier for the patient. This stationary design also improves the system stability and eliminated the rotation ring artefacts that come from crystal non-uniformity [52]. Generally the SPECT uses thin 10 mm crystal to restrict the light that spreads from high resolution images and triad of parallel holed collimators for sampling Gamma Ray projections.

The latest collimators are incorporated with continuously varying converging focuses for providing high sensitivity and resolutions by increasing gamma ray projections. Due to these varying converging the sampling of gamma rays are considered to be more efficient. It uses the idea of traces for image chemical process. Generally the tracer will be an analogue biologic active compound where the atoms will be replaced by other radio-active atom. After introducing such tracer into body specific uptake will be traced by means of labelled atom. Generally SPECT will be using TC-99m (Technetium) for bones, liver and brain, Iodine (I-123) for brain and Gallium (Ga-67) for tumours. These radioisotopes are gamma emitters and most of the times a patient gets a dose of up to 25mCi (milliCurie) from the activity.
Figure 2.11: SPECT Annular Camera of Brain of three segmented parallel hole rotating collimator system [52]

Figure 2.12: Image from Single Photon Emission Computed Tomography (SPECT) [26]
2.4.5. Position Emission Tomography

This method of scanning of an image test will help the functioning of tissues and organs in a human body. It uses very less radioactive material for performing this activity. As in case of SPECT this method also uses the idea of trace for image chemical process. The type of radioactive material to be used will depend on the organ or tissue that is under study by the PET scan. The usage of PET according to Vansteenkiste in [53] is based on the ability to visualize differences between glucose metabolisms of different tissues. In a human body the neoplastic cells will be having higher glycolysis rate when compared to non-neoplastic cells.

Figure – 2.13: Reduction of radiation treatment volume using PET is compared with a CT Image (a) shows the CT Image (b) shows the PET corrected Image.
The radioactive material will be injected into vein inhaled or swallowed. However, only difference is about combining the principles of CT and SPECT. Lat of radioactive material will be accumulated at the area with high level of chemical activity. This method will generally show the disease areas as brighter parts after the scanning using PET [52].

This method is helpful to evaluate various conditions which include problems related with neurology, heart and cancer [52]. The major advantage of this method is to have more sensitivity that is obtained from naturally collimating photons through some physical process, without using absorbing collimators. The method shown in figure 2.13 reduces the treatment by a volume of 37% by identifying the tissues easily from a human body [53]. Other major difference from SPECT is that traces used in this method are labelled with positron emitting isotopes like fluorine – 18 and carbon – 11. These are having short life that reduces overall time of exposure for the patient. Fluorine – 18 has half – life time, which is just under 2 hours and carbon – 11 in having only life time of 20 minutes [26].

![Image from Photon Emission Tomography](image)

*Figure – 2.14: Image from Photon Emission Tomography [26]*
The usage of PET in oncology will be mainly based on the resolution levels of the camera used for scanning the images of a body. Higher the resolution of camera gives higher performance of the pictures quality and sensitivity. Due to high cost of these PET cameras most of the hospitals prefer to use Gamma Camera Co-incidence Images (GCI) as an alternative. The other pitfall of PET is in terms of sensitivity due to the requirement of critical mass of metabolic malignant during the diagnosis. Hence, probability of finding false – negative can occur in lesions that are less than 1 cm with less metabolic activity.

2.5. Types of Noises in CT LIVER Image

The following sections will discuss types of noise disturbing the quality of a digital image and various sources for disturbance are discussed with the help of mathematical models. In real time applications the noise signals are contaminated during the time of transmitting the signal, during acquisition, while storing and retrieving process. The mostly known noise in digital signals are acquisition noise and is also called as additive white Gaussian Noise (AWGN) with low level variance value [64]. Most of the real time applications face a negligible acquisition noise, which is mostly due to the usage of high quality sensors for image capturing. Examples for such applications are remote sensing, instruments with biomedical applications and etc are having quite a large number of acquisition noise within their process. A transmission channel will be comprised within such type of equipments for image acquisition. In such scenarios the acquisition noise can be ignored or considered as negligible as the noise is considered to be due to transmission noise. However there is another strong reason for considering this acquisition noise negligible as the human visual system (HVS) in real life cannot recognize large and
dynamic range of images. Due to this reason the quantization of an image is done at 256 levels and every pixel is represented by 8 bits (known as a byte). The sensors available in the modern day usage are having high quality of sensing sensors which don’t have the noise levels greater than half of resolution of an ADC (Analog to Digital Converter). Hence the magnitude of noise in time domain can be expressed as $n(t) < \frac{1}{2} \frac{V}{2^{8}}$. $n(t)$ is the amplitude of noise at any time $t$, and $V$ is considered to be the out of sensor and in general it will be maximum for voltage of ADC input level. For an example, if $V = 3.3$ volts, amplitude of noise must not be less than $6.5\text{mV}$ [64]. Most of the real time applications will be having low level of acquisition noise compared to the actual margin and it comes under the consideration. The mostly assumed noise in real time by most of the people is the noise generated due to transmission system and these channels will be linear and are dispersive due to the limits imposed on bandwidth. The transmission systems will transmit the images/ image signals either in digital or analogue form. The image edges tend to get blur when an analogue signal is transmitted by using a linear dispersive channel. In such scenarios the images will get contaminated as none of real time AWGNs are not noise free. The poorer the channel higher the noise variance for making signal excursion to a high negative or positive level compared to thresholding value at the end of receiver will involve the saturated maximum and minimum values. In these conditions the noise pixels are seen like spots with black or white colors. The name given to this kind of noise is salt pepper noise (SPN). There will be a mixed effect of both AWGN and SPN during the transmission of analogue image.
Inter symbol interference (ISI) takes place when the transmission of digital image is considered using a linear dispersive channel. Addition to that AWGN cannot be ignored in real time environment. Due to these two (ISI and AWGN) the situations will be worst to get an error in recognizing the level 0 as 1 or vice-versa [64]. Under these situations the values of pixels are changed to random values with varying random positions within the frame of the image. This type of noise in real time known as random – valued impulse noise (RVIN).

There are many parameters that affect the quality of the CT image. Spatial& contrast resolutions, noise, linearity and artifacts are general basic characteristics that affect CT Images quality [28]. Generally enhancing and suppressing of any of the above mentioned characteristics will depend on the interests and region of body that is under scan. On the other hand the section thickness, algorithms and field of view are having profound effect on overall quality of CT image. Balancing all these characteristics is very important for producing better image for anatomical region scanning. According to Reddinger in [28] image noise and artifacts are biggest enemies for the quality of CT images. Generally these kinds of characteristics can be manipulated for decreasing the noise levels or by eliminating worst effects of the image characteristics.

Noise explained by Krishnan and Viswanathan in [54] as an undesired signal contaminating the image that results to have errors in images during the process of acquisition. In a noisy image the true pixel values will not reflect from the nature of original scene. In a broad classification of noise there are two types of noises: Substitutive Noise and Additive Noise [54] [55]. In some of the image processing
applications the vector processing operations use spectral and spatial information for removing the noise in localize array spots [57]. This method can be implemented either by using software or hardware tools.

Substitutive nose includes impulsive noises like salt noise, pepper noise, random value impulse noise and etc. Example for Additive noise is Additive white Gaussian noise (in simple it is also called as AWGN) also. However, Roger et al. classified noise on the shape of probability density function and histogram of an image under study [55].

2.5.1. Uniform Noise

It is one of the common kinds of noises found in digital images that change the actual values of pixels from the original values [56]. In this noise the gray level values of noise will be distributed evenly across a specific range of digital image for a defined specific range. Generally the quantization of this noise will be uniform all over the distribution.

The corresponding equation will give analytical expression for the uniform noise and the histogram of a uniform noise is represented by the following figure – 2.15.

- Uniform Noise: 
  \[ \text{Histogram}_{\text{Uniform}} = \begin{cases} \frac{1}{b-a} & \text{for } a \leq g \leq b \\ 0 & \text{elsewhere} \end{cases} \]  
  \[ 2.1 \]

- And the mean and variance value of this will be given as: 
  \[ \text{Mean} = \frac{a+b}{2} \]; and

- \[ \text{Variance} = \frac{(b-a)^2}{12} \]

Uniform noise in general used to produce other type of noise distributions to reduce the quality of image for evaluating the image restoration algorithm as it provides unbiased and neutral noise model at most of the scenarios [55]. Various filters are proposed to...
reduce this noise from the images. Median filter is generally used to address the uniform noise in real time applications.

\[ \frac{1}{b-a} \]

\[ a \quad b \]

**Figure - 2.15:** The histogram of a uniform noise

**Figure - 2.16:** (a) Original Image (b) uniform noise distributed image and (c) the Histogram
2.5.2. Gaussian Noise

Gaussian Noise in image processing can cause a serious damage to the quality of the image by highlighting the false information from a scanned image. This noise contains probability density function (PDF) of normal distribution which is also known as Gaussian distribution [55]. This is also considered to be the larger part of red noise from image sensor as shown in figure – 2.17. In simple at dark areas of an image the noise level will be constant. Most of the situations are accumulated with additive nature of noise in most of the digital images that contain uniform power thought the bandwidth with Gaussian probability distribution. Mathematically, the AWGN noise is represented by \( \eta[54] \).

\[
N_{AWGN}(t) = \eta_G(t). \sigma \quad ... \quad 2.2
\]

\[
f_{AWGN} = f(x, y) + \eta_G(x, y) \quad ... \quad 2.3
\]

The random variable with Gaussian probability is shown by \( \eta_G(t) \). This additive noise is characterised by its variance \( \sigma \). Generally the noise image contains the sum of original image (not corrupted) plus the Gaussian distributed random noise \( \eta_G \). The lower the value of \( \eta_G \) results to a lower value of \( \eta_G(x, y) \) and it can be also zero at certain low levels. In such situation the value of \( f_{AWGN} \) will be almost equal original image \( f(x, y) \) with no noise [54]. These kinds of Gaussian noises are generally observed to be generated while exposing the film or while developing the image in image processing. Treatment for the Gaussian noise is not properly given for the images corrupted by mixed Gaussian noise.

Gaussian noise:
\[
p(z) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(z-\mu)^2}{2\sigma^2}}
\]
This noise can affect the edges, and actual values of the tissues in a scanned picture to a reasonable level. There are many approaches to suppress these noises like fuzzy logic base approach and etc. The fuzzy based logic is capable of handling the images with vague and uncertain picture quality [54]. By using fuzzy image filters using the fuzzy rules Gaussian noise can be minimized by making use of membership functions. This method will help to improve the edges for sharpening them to an appropriate level for comparing the existing image by reducing the noise levels by visual inspection as well.
2.5.3. Salt and Pepper Noise

This kind of noise is generally caused due to contaminated impulse noise, basically due to malfunctioning of pixels in camera/ fault locations of memory/ transmission of images by noisy channels [59]. For an image corrupted by salt and pepper noise will be having only two noise pixel values $d_{\text{min}}$ and $d_{\text{max}}$. Generally the typical probability of each value is less than 0.2 and if the values are greater than 0.2 the noise identified will be out of the image range. According to Sonka et al. the pepper noise value is 0 and salt noise is 255 for a 8 - bit image [55] [59].

Analytically the Salt and Pepper is written as [55]:

$$\text{Histogram}_{\text{Salt & Pepper}} = \begin{cases} 
A & \text{for } g = a(\text{ pepper }) \\
B & \text{for } g = b(\text{salt}) \ldots \ldots .2.4
\end{cases}$$

In general adaptive median filter (AMF) is used to detect these impulse noise levels. These filters will detect most of the pixels that are damaged by salt and pepper impulse noise and the same is tougher to identify corrupted images by random valued impulse noise [58].
From the figure – 2.19 it can be seen that this noise seems to be an impulse kind of noise and due to the same reason it is called as impulse noise or shot noise or spike noise. Generally it is the result of malfunctioning of pixel elements of camera sensors, wrong memory locations or due to errors in timings while processing the digitization [55]. The noise density will be very high if the images are highly corrupted by these impulse noises and resultant outputs for different percentages of corrupted values (50% and 80 %) are shown in figure – 2.20.
Figure – 2.20: Image restoration for (a) 50% noise corrupted image (c) 80% noise corrupted image and (b), (d) outputs of the applied salt and pepper noise methods [59]
2.5.4. Rayleigh Noise

The probability density function of Rayleigh noise is given by the following equation:

\[
\text{RayleighNoise}(z) = \begin{cases} 
\frac{z}{b} (z - a) e^{-\frac{(z-a)^2}{b}} & z \geq a \\
0 & \text{for } Z < a 
\end{cases} \quad 2.5
\]

Where \( \mu = a + \sqrt{\frac{\pi b}{4}} \sigma^2 = \frac{b(\mu - \mu)}{4} \) \quad 2.6

For \( \sigma > 0 \), and \( x \geq 0 \) and \( \sigma \) controls the amount of spread [60].

**Figure - 2.21:** Probability Density Function of Rayleigh Noise [60]
2.5.5. Erlang/ Gamma Noise

- \( P(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az}, & z \geq 0 \\ 0 & z < 0 \end{cases} \) .......................... 2.7

Where \( \mu = \frac{b}{a} \sigma^2 = \frac{b}{a^2} \)

![Probability Density Function of Gamma Noise](image)

**Figure – 2.22:** Probability Density Function of Gamma Noise

2.5.6. Their mathematical expressions for power spectral density

The main occurrence of the noise components in the spatial domain is due to the inefficiency of the acquisition system and during transmission.
Here different types of noise and their distributions are summarized as follows.

- **Uniform Noise**: 
  \[ \text{Histogram}_{\text{Uniform}} = \begin{cases} \frac{1}{b-a} & \text{for } a \leq z \leq b \\ 0 & \text{elsewhere} \end{cases} \]  
  \[2.8\]

- **Gaussian Noise**: 
  \[ p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \]  
  \[2.9\]

- **Rayleigh Noise**: 
  \[ p(z) = \begin{cases} \left(\frac{z}{b}\right) e^{-\frac{(z-a)^2}{b}}, & z \geq a \\ 0, & \text{otherwise} \end{cases} \]  
  \[2.10\]

- **Erlang/Gamma Noise**: 
  \[ p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az}, & z \geq 0 \\ 0, & z < 0 \end{cases} \]  
  \[2.12\]

  Where \( \mu = a + \frac{\pi b}{4} \sigma^2 = \frac{b(4-\mu)}{4} \)  
  \[2.11\]

- **Impulse or salt & Pepper noise**: Usually this value can be large with respect to image and can be positive or negative
  \[ p(z) = \begin{cases} P_a, & z = a \\ P_b, & z = b \\ 0, & \text{otherwise} \end{cases} \]  
  \[2.13\]

The above equations show the probability distribution function (PDF) for different types of noises.
Figure – 2.23: Summary of different types of noise and their distributions
2.6. Curvelet based image denoising

Image processing is facing a serious problem due to the fundamental problem of image de-noising. Due to the properties like sparsity and multi-resolution structures of wavelet they provide a superior performance in image de-noising. Due to the popularity of using wavelet transformations denoising has been introduced in the domain of wavelets. Recently due to above reasons the focus of researchers shifted to wavelet domain from spatial and Fourier domains. This method was introduced by Donoho’s in 1995 and his methods don’t demand any kind of tracking nor correlation of wavelet maxima and minima [65]. Simple approaches were explained by Donoho in wavelet based denoising technique to solve most of the image processing problems that deal with real time applications. To get the optimum value of threshold the author introduced data adaptive thresholds which are considered to be giving more substantial improvements in terms of quality. To reduce the artifacts most of these techniques are applied to a non-orthogonal wavelet coefficient. Classification of image de-noising can be seen in figure– 2.24 and basically they are classified into two methods: spatial filtering method and transform domain filtering method [65].

2.6.1. Spatial Filtering method

This is considered to be a traditional method or eliminating the noise from an image by using spatial filters. These filters are classified into two types: Non–linear and Linear Filters. In the first case the noise will be removed without giving an attempt for identifying it explicitly. Low pass filters (LPF) will be used for filtering on a group of pixels with higher region of frequency spectrum.
Figure – 2.24: Image Denoising Methods Classification [65]
These spatial filters will remove noise at larger extent but it leaves the image with blur and the edges of image will be most of the times invisible. For avoiding such problem in recent times weighted median, rank conditioned rank selection and relaxation median methods are developed according to Motwani et al. [65]. In case of linear filters, they also tend to output the images with sharp edges with blur, destroy lines and few more image details. However, they perform poorly during the present of noise which is signal-dependent. A mean filter is used (also considered to be an optimal linear filter) to avoid Gaussian noise. The information of spectrum of noise and original signal is needed by Wiener Filter method when the signal is smooth. This filter performs spatial smoothing and the complexity of this model controls to select the window size. Wavelet based denoising was introduced to over the Wiener filtering weaknesses.

2.6.2. Transform domain Filtering Method

This method is subdivided into many ways based on the functions that are applied at different levels. Basically it is classified into two types: adaptive and non-adaptive (very popular in real time). Non-adaptive methods include spatial frequency filtering comprises of LPFs using Fast Fourier Transforms (FFT). The noise removal is achieved by designing an appropriate frequency domain filter for suitable cut-off frequency when noise components are de-correlated from input signals in frequency domain [65] [72]. There are several digital approximations irrespective of having curvelet transform as original transformation. Two transformations were introduced recently and the first one is based on unequal spaced fast Fourier transforms (USFFT) and second is based on wrapping of specially selected Fourier samples [31] [71]. The second one is exhibits with
faster runtime especially for those with inverse transform. Generally the curvelet transforms are decompose images at various frequency bands. Isotropic wavelets will be applied as basic function at different frequency bands. This approach is widely used at various fields due to its ability for obtaining high quality of output images. But some of the applications are prevented in such fields as in case of medical images [61].

However, these methods consume a lot of time and are heavily dependent on cutoff frequency. The performance also depends on the behavior of filter functions. The wavelet domain is again subdivided into two methods: linear and non-linear. Wiener filter in wavelet domain gives optimal results if the signal corruption is modeled as Gaussian process and accuracy is the result of mean square error (MSE). The filter designing will be based on many assumptions and hence the resultant image will be more displeasing when compared to original noisy image irrespective of reducing the MSE successfully. At the higher frequencies of finest scales one will be opt to use between wavelet and curvelet decompositions. Using the curvelet most of the scales are bound to transform and it provide approximate rotation invariance that is sharp directional selectivity that is benefitted for most of the de-noising applications [30]. The applications of non-linear thresholding in wavelet domain are used extensively and this method exploits sparsity property mapping white noise in signal domain. If signal energy is focused with few coefficients in transfer domain noise energy does not. This method allows the signal to be separated from the noise.
Figure 2.25: De-noised image with various Denoise methods of Curvelet Transforms

(a) Noisy Image (b) CTD (c) HWTD and (d) Pixel fusion methods[61]
2.7. Summary

The usage of image processing in most of the sensitive areas like army, hospitals and etc made this area of research more interesting and essential to analyse the real time problems carefully. The images captured will be tested very closely at pixel levels and various noise parameters are summarized that are influencing the digital images. These techniques are extensively used to enhance the quality of picture or to analyse the picture with quality information to avoid the risks for patients in hospitals. These methods are also alarming the doctors and patients by detecting the tissue particles in advance from the inner part of a human body. Segmentation methods are introduced to improve the quality of testing the properties of images by using various methods like thresholding, gradient vector method and etc.