Chapter 5

Peristaltic transport of a fourth grade fluid in an inclined asymmetric channel under the effect of a magnetic field

5.1 Introduction

Peristaltic motion appears in a wide variety of physiological and engineering applications such as urine transport in the ureter, motion of spermatozoa in the cervical canal, the movement of chyme in the gastrointestinal tract, swallowing of food through oesophagus, the vasomotion of small blood vessels, in roller and finger pumps and many others. After the seminal work of Latham [48], several researchers have analyzed the phenomenon of peristaltic transport under various assumptions.

In the past five decades fluid dynamics of magnetohydrodynamic (MHD) fluid has been the object of scientific and engineering research. Also, it is known that most of the physiological fluids are non-Newtonian fluids. Theoretical studies of non-Newtonian fluids have been conducted by various workers in this field (Fecteau and Fecteau, [20]; Fecteau and Fecteau, [21]; Chen et al., [8]). Specifically, the non-Newtonian fluids in the presence of a magnetic field are very useful in magnetotheraphy. The controlled application of low intensity and frequency pulsing magnetic fields are modify the cell and tissue behavior. Moreover, the non-invasive radiological test that uses a magnetic field (not radiation) to evaluate organs in abdomen prior to surgery in the small intestine (but not always). Hence, magnetically susceptible of chyme can be satisfied from the heat generated by magnetic field or the ions contained in the chime. The peristaltic flows of magnetohydrodynamic (MHD)
have been studied by (Stud et al., [84]; Srivastava and Agrawal, [80]; Agrawal and Anwaruddin, [3]; Mekheimer, [56]; El Shahed and Hourn, [11]; Siddiqui et al., [75]; and Hayat et al., [30]). Peristaltic transport of a third order fluid in a planar channel has investigated by Siddiqui et al. [73] and the corresponding axisymmetric tube results were studied by Hayat et al. [26]. Hayat and Ali [32] have analyzed the peristaltic motion of a third grade fluid in a tube with influence of magnetic field has analyzed by. Hayat et al. [33] have investigated the peristaltic flow of a MHD third order fluid under the effect of a magnetic field in a planar channel. Non-linear peristaltic flow of a fourth grade fluid in a planar channel under the effect of magnetic field has discussed by Hayat et al. [34]. Subba Reddy et al. [86] have investigated the peristaltic flow of a fourth grade fluid in an inclined channel under the effect of a magnetic field.

Eytan and Elad [16] have presented a mathematical model of wall-induced peristaltic fluid flow in two-dimensional channel with wave trains having a phase difference moving independently on the upper and lower walls to simulate intra-uterine fluid motion in a sagittal cross-section of the uterus. They have obtained a time dependent flow solution in a fixed frame by using lubrication approach. Recently, Haroun [23] studied the effect of wall compliance on peristaltic transport of a Newtonian fluid in an asymmetric channel. Srinivas and Pushparaj [77] have investigated the peristaltic pumping of MHD gravity flow of a viscous incompressible fluid in a two-dimensional asymmetric inclined channel. Ali and Hayat [5] have analyzed peristaltic transport of a micropolar fluid in an asymmetric channel. Haroun [24] analyzed peristaltic flow of a fourth grade fluid in an inclined asymmetric channel.
In view of these, we studied the effect of magnetic field on peristaltic flow of a fourth grade fluid in an inclined asymmetric channel under the assumption of long wavelength. Series solutions of axial velocity and pressure gradient are given by using regular perturbation technique when Deborah number is small. Numerical computations have been performed for the pressure gradient and pressure. The effects of various emerging parameters on the pressure gradient and pressure rise are studied in detail through graphs.

5.2 Mathematical formulation

We consider the peristaltic transport of an incompressible fourth grade fluid in a two-dimensional asymmetric channel of width \( a_1 + a_2 \). The fluid is conducting while the channel walls are non-conducting. The channel walls are inclined at an angle \( \alpha \) to the horizontal. The flow is induced by sinusoidal wave trains propagating with constant speed \( c \) along the channel walls. Fig. 5.1 shows the physical model of the problem.

The geometry of the wall surfaces is defined

\[
Y = H_1(X,t) = a_1 + b_1 \cos \left( \frac{2\pi}{\lambda} (X - ct) \right) \quad \text{(Upper wall)} \tag{5.2.1}
\]

\[
Y = H_2(X,t) = -a_2 - b_2 \cos \left( \frac{2\pi}{\lambda} (X - ct) + \theta \right) \quad \text{(Lower wall)} \tag{5.2.2}
\]

where \( b_1, b_2 \) are the amplitudes of the upper and lower waves, \( \lambda \) is the wavelength, \( \theta \) is the phase difference which varies in the range \( 0 \leq \theta \leq \pi \) and \( t \) is the time. Further, \( a_1, a_2, b_1, b_2 \) and \( \theta \) satisfies the condition \( b_1^2 + b_2^2 + 2b_1b_2\cos \theta \leq (a_1 + a_2)^2 \) so that walls will not intersect with each other.

In laboratory frame, the equations governing two-dimensional motion of an incompressible MHD fourth grade fluid are

\[
\rho \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \right) U = -\frac{\partial p}{\partial x} + \frac{\partial S_{XX}}{\partial x} + \frac{\partial S_{XY}}{\partial y} - \sigma B_0^2 U + \rho g \sin \alpha \tag{5.2.3}
\]
\[
\rho \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \right) V = - \frac{\partial p}{\partial y} + \frac{\partial s_{xy}}{\partial x} + \frac{\partial s_{yy}}{\partial y} - \rho g \cos \alpha
\]  

(5.2.4)

and the equation of continuity is

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(5.2.5)

In which \( \rho \) is the density, \( U \) and \( V \) are the velocity components in X and Y directions, \( P \) is the pressure, \( \sigma \) is the electrical conductivity of the fluid, \( B_0 \) is the constant magnetic field, \( g \) is the acceleration due to gravity and \( S_{xx} \), \( S_{xy} \), \( S_{yy} \) are the components of extra stress tensor.

**Fig. 5.1.** The Physical model
The constitutive equations for an incompressible fourth grade fluid are

\[ T = -p I + S \]  \hspace{1cm} (5.2.6)

\[
S = \mu A_1 + \alpha_1 A_2 + \alpha_2 A_2^2 + \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (tr A_1^2) A_1 + \gamma_1 A_4
\]
\[
+ \gamma_2 (A_1 A_3 + A_3 A_1) + \gamma_3 A_2^2 + \gamma_4 (A_1^2 A_2 + A_2^2 A_1) + \gamma_5 (tr A_2) A_2 + 
\]
\[
+ \gamma_6 (tr A_2) A_2^2 + [\gamma_7 tr A_3 + \gamma_8 tr (A_2 A_1)] A_1 
\]  \hspace{1cm} (5.2.7)

Where \( \mu \) is constant viscosity and \( \alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7, \gamma_8 \) being material constants and \( A_n \) representing the Rivlin-Ericksen tensors defined by

\[
A_n = \frac{d A_{n-1}}{dt} + A_{n-1} (\text{grad} \ V) + (\text{grad} \ V)^T A_{n-1}, n > 1 \]  \hspace{1cm} (5.2.8)

\[
A_1 = (\text{grad} \ V) + (\text{grad} \ V)^T \]  \hspace{1cm} (5.2.9)

\[
x = X - ct, y = Y, u(x,y) = U - c, v(x,y) = V, p(x,y) = P(X,Y,t) \]  \hspace{1cm} (5.2.10)

Introducing the non-dimensional parameters and variables

\[
\bar{x} = \frac{x}{\lambda}, \bar{y} = \frac{y}{\alpha_1}, \bar{u} = \frac{u}{c}, \bar{v} = \frac{v}{c}, \bar{S} = \frac{a_1}{\mu} S, \bar{p} = \frac{a_1^3}{\mu c}, \bar{\ell} = \frac{ct}{\lambda}, h_1 = \frac{h_1}{a_1}, h_2 = \frac{h_2}{a_1} \]  \hspace{1cm} (5.2.11)

Using Eqs. (5.2.10), we obtain Eqs. (5.2.3)-(5.2.5) as (after dropping bars)

\[
\rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) u = - \frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \sigma B_{0}^2 (u + c) + \rho g \sin \alpha \]  \hspace{1cm} (5.2.12)

\[
\rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) v = - \frac{\partial p}{\partial y} + \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y} - \rho g \cos \alpha \]  \hspace{1cm} (5.2.13)

and the equation of continuity is

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  \hspace{1cm} (5.2.14)

where \((u, v)\) are velocity components in the wave frame.

Using Eqs. (5.2.9), we obtain Eqs. (5.2.12)-(5.2.14) as

\[
Re \delta \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) u = - \frac{\partial p}{\partial x} + \delta \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - M^2 (u + 1) + \frac{Re}{Fr} \sin \alpha \]  \hspace{1cm} (5.2.15)

\[
Re \delta^3 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) v = - \frac{\partial p}{\partial y} + \delta^2 \frac{\partial S_{xy}}{\partial x} + \delta \frac{\partial S_{yy}}{\partial y} - \frac{Re}{Fr} \delta \cos \alpha \]  \hspace{1cm} (5.2.16)
and the equation of continuity is
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\] (5.2.17)
\[
S = A_1 + \lambda_1 A_2 + \lambda_2 A_1^2 + \xi_1 A_3 + \xi_2 (A_1 A_2 + A_2 A_1) + \xi_3 (trA_1^2) A_1 + \eta_1 A_4
\]
\[
+ \eta_2 (A_1 A_3 + A_3 A_1) + \eta_3 A_2^2 + \eta_4 (A_1^2 A_2 + A_2 A_1^2) + \eta_5 (trA_2) A_2 +
\]
\[
+ \eta_6 (trA_2) A_1^2 + [\eta_7 trA_3 + \eta_8 tr(A_2 A_1)] A_1
\] (5.2.18)
\[
A_n = \left( u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) A_{n-1} + A_{n-1} (\text{grad } V) + (\text{grad } V)^T A_{n-1}
\]
Here the wave number \( \delta \), Reynolds number \( Re \), Hartman number \( M \), Froude number \( Fr \), the material coefficients \( \lambda_1, \lambda_1, \xi_1, \xi_2, \xi_3, \eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6, \eta_7, \eta_8 \) are
\[
\delta = \frac{a}{\lambda}, \quad Re = \frac{\rho a_1 c}{\mu}, \quad M^2 = \frac{\sigma a_1^2}{\mu}, \quad Fr = \frac{c^2}{\mu a_1},
\]
\[
\lambda_1 = \frac{a_1 c}{\mu a_1}, \quad \lambda_2 = \frac{a_2 c}{\mu a_1}, \quad \xi_1 = \frac{a_1 c^2}{\mu a_1^2}, \quad \xi_2 = \frac{a_2 c^2}{\mu a_1^2},
\]
\[
\xi_3 = \frac{a_3 c^2}{\mu a_1^2}, \quad \eta_1 = \frac{a_1 c^3}{\mu a_1^3}, \quad \eta_2 = \frac{a_2 c^3}{\mu a_1^3}, \quad \eta_3 = \frac{a_3 c^3}{\mu a_1^3}, \quad \eta_4 = \frac{a_4 c^3}{\mu a_1^3}, \quad \eta_5 = \frac{a_5 c^3}{\mu a_1^3}, \quad \eta_6 = \frac{a_6 c^3}{\mu a_1^3}, \quad \eta_7 = \frac{a_7 c^3}{\mu a_1^3}, \quad \eta_8 = \frac{a_8 c^3}{\mu a_1^3}, \quad (5.2.19)
\]
The corresponding dimensionless boundary conditions in wave frame of reference are given by
\[
u = -1 \quad \text{at} \quad y = h_1 = 1 + \phi_1 \cos 2\pi x \\
u = -1 \quad \text{at} \quad y = h_2 = -d - \phi_2 \cos (2\pi x + \theta)
\] (5.2.20) (5.2.21)
Here \( \phi_1 = \frac{b_1}{a_1}, \phi_2 = \frac{b_2}{a_1}, d = \frac{a_2}{a_1} \).

Adopting long wave length procedure, the Eqs. (5.2.15) & (5.2.16), become
\[
0 = -\frac{\partial p}{\partial x} + \frac{\partial s_{xy}}{\partial y} - M^2 (u + 1) + \frac{Re}{Fr} \sin \alpha
\] (5.2.22)
\[
0 = -\frac{\partial p}{\partial y}
\] (5.2.23)
where \( S_{xy} = \frac{\partial u}{\partial y} + 2 \Gamma \left( \frac{\partial u}{\partial y} \right)^2 \), in which the Deborah number \( \Gamma = \xi_2 + \xi_3 \).

Eqs. (5.2.22) \& (5.2.23) can be rewritten as

\[ \frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left[ \frac{\partial u}{\partial y} + 2 \Gamma \left( \frac{\partial u}{\partial y} \right)^2 \right] - M^2 (u + 1) + \frac{Re}{Fr} \sin \alpha \]  (5.2.24)

\[ \frac{\partial p}{\partial y} = 0 \]  (5.2.25)

The volume flow rate in wave frame of reference is given by

\[ q = \int_{h_2}^{h_1} u \, dy \]  (5.2.26)

The instantaneous flux \( Q(X,t) \) in the laboratory frame is

\[ Q(X,t) = \int_{h_2}^{h_1} (u + 1) \, dy = \int_{h_2}^{h_1} u \, dy + \int_{h_2}^{h_1} dy = q + h_1 - h_2 \]  (5.2.27)

The average flux over one period \( (T = \lambda/c) \) of the peristaltic wave is

\[ \bar{Q} = \frac{1}{T} \int_0^T Q \, dt = \frac{1}{T} \int_0^T (q + h_1 - h_2) \, dt = q + 1 + d \]  (5.2.28)

5.3 Solution

Eq. (5.2.24) is non-linear differential equation, so that it is not possible to obtain a closed form solution, so we seek a perturbation solution. We expand the flow quantities in a power series of the small parameter Deborah number \( \Gamma \) as follows:

\[ u = u_0 + \Gamma u_1 + O(\Gamma^2) \, , \]  (5.3.1)

\[ p = p_0 + \Gamma p_1 + O(\Gamma^2) \, , \]  (5.3.2)

\[ q = q_0 + \Gamma q_1 + O(\Gamma^2) \, , \]  (5.3.3)

Substituting these equations into the Eqs. (5.2.24)-(5.2.25), (5.2.20) \& (5.2.21) we obtain

5.3.1 System of order zero

\[ \frac{\partial^2 u_0}{\partial y^2} - M^2 (u_0 + 1) = \frac{dp_0}{dx} + \frac{Re}{Fr} \sin \alpha \, , \]  (5.3.4)
\[ \frac{dp_0}{dy} = 0 , \quad (5.3.5) \]

Together with the boundary conditions
\[ u_0 = -1 \text{ at } y = h_1 \quad (5.3.6) \]
\[ u_0 = -1 \text{ at } y = h_2 \quad (5.3.7) \]

### 5.3.2 System of order one

\[ \frac{\partial^2 u_1}{\partial y^2} - M^2 u_1 = \frac{dp_1}{dx} - 2 \frac{\partial}{\partial y} \left( \frac{\partial u_0}{\partial y} \right)^3 , \quad (5.3.8) \]
\[ \frac{dp_1}{dy} = 0 , \quad (5.3.9) \]

Together with the boundary conditions
\[ u_1 = 0 \text{ at } y = h_1 \quad (5.3.10) \]
\[ u_1 = 0 \text{ at } y = h_2 \quad (5.3.11) \]

### 5.3.3 Zeroth-order solution

Solving Eq. (5.3.4) using boundary conditions (5.3.6) & (5.3.7), we get
\[ u_0 = \frac{1}{M^2} \left( \frac{dp_0}{dx} - \frac{Re}{Fr} \sin \alpha \right) [A_1 \cosh My + A_2 \sinh My - 1] - 1 . \quad (5.3.12) \]

where \( A_1 = \frac{\sinh M h_2 - \sinh M h_1}{\sinh M (h_2 - h_1)} \) and \( A_2 = \frac{\cosh M h_1 - \cosh M h_2}{\sinh M (h_2 - h_1)} \).

and the volume flow rate \( q_0 \) is given by
\[ q_0 = \int_{h_2}^{h_1} u_0 dy \]
\[ = \frac{1}{M^2} \left( \frac{dp_0}{dx} - \frac{Re}{Fr} \sin \alpha \right) \left[ \frac{2-2 \cosh M (h_1 - h_2) - M (h_1 - h_2) \sinh M (h_2 - h_1)}{\sinh [M (h_2 - h_1)]} \right] - (h_1 - h_2) \quad (5.3.13) \]

From Eq. (5.3.13)
\[ \frac{dp_0}{dx} = \frac{[q_0 + (h_1 - h_2)] M^2 \sinh [M (h_2 - h_1)]}{2 - 2 \cosh M (h_1 - h_2) - M (h_1 - h_2) \sinh M (h_2 - h_1)} + \frac{Re}{Fr} \sin \alpha \quad (5.3.14) \]
5.3.4 First order solution

Substituting Eq. (5.3.12) into Eq. (5.3.8) and Solving Eq. (5.3.8) using boundary conditions (5.3.10) & (5.3.11), we obtain

\[ u_1 = \]

\[ \frac{1}{M^2} \left( \frac{dp_1}{dx} \right) [A_1 \cosh M y + A_2 \sinh M y - 1] + \frac{1}{4M^4} \left( \frac{dp_0}{dx} - \frac{Re}{Fr} \sin \alpha \right)^3 [A_9 \cosh M y + A_{10} \sinh M y - 3A_3 \cosh 3M y - 3A_4 \sinh 3M y - 4A_5 M y \sinh M y - 4A_6 M y \cosh M y] \]

\[ (5.3.15) \]

where \( A_3 = \frac{1}{4} [A_1^3 + 3A_1 A_2^2] \), \( A_4 = \frac{1}{4} [A_2^3 + 3A_2^2 A_1] \), \( A_5 = \frac{3}{4} [A_1 A_2^2 - A_1^3] \), \( A_6 = \frac{3}{4} [A_2^3 - A_1^2 A_2] \), \( A_7 = 3A_3 \cosh 3Mh_1 + 3A_4 \sinh 3Mh_1 + 4A_5 Mh_1 \sinh Mh_1 + 4A_6 Mh_1 \cosh Mh_1 \), \( A_8 = 3A_3 Mh_2 + 3A_4 \sinh 3Mh_2 + 4A_5 Mh_2 \sinh Mh_2 + 4A_6 Mh_2 \cosh Mh_2 \), \( A_9 = \frac{A_7 \sinh Mh_2 - A_8 \sinh Mh_1}{\sinh [M(h_2-h_1)]} \), \( A_{10} = \frac{A_9 \cosh Mh_1 - A_7 \cosh Mh_2}{\sinh [M(h_2-h_1)]} \),

and the volume flow rate \( q_1 \) is given by

\[ q_1 = \int_{h_2}^{h_1} u_1 \, dy \]

\[ = \frac{1}{M^3} \frac{dp_1}{dx} \left[ \frac{2-2 \cosh M(h_1-h_2)-M(h_1-h_2) \sinh M(h_2-h_1)}{\sinh [M(h_2-h_1)]} \right] + \frac{1}{4M^4} \left( \frac{dp_0}{dx} - \frac{Re}{Fr} \sin \alpha \right)^3 A_{11} \]

\[ (5.3.16) \]

From Eq. (5.3.16), we have

\[ \frac{dp_1}{dx} = \left[ q_1 - \frac{1}{4M^4} \left( \frac{dp_0}{dx} - \frac{Re}{Fr} \sin \alpha \right)^3 A_{11} \right] \frac{M^3 \sinh [M(h_2-h_1)]}{2-2 \cosh M(h_1-h_2)-M(h_1-h_2) \sinh M(h_2-h_1)} \]

\[ (5.3.17) \]
where \( A_{11} = \frac{(A_0 + 4A_2)}{M} [\sinh(Mh_1) - \sinh(Mh_2)] + \frac{(A_0 + 4A_6)}{M} [\cosh(Mh_1) - \cosh(Mh_2)] - \frac{A_2}{M} [\sinh(3Mh_1) - \sinh(3Mh_2)] - \frac{A_4}{M} [\cosh(3Mh_1) - \cosh(3Mh_2)] - 4A_5 [h_1 \cosh(Mh_1) - h_2 \cosh(Mh_2)] - 4A_6 [h_1 \sinh(Mh_1) - h_2 \sinh(Mh_2)] \) .

(5.3.18)

Substituting from equations (5.3.14) and (5.3.17) into the Eq. (5.3.2) and using the relation

\[
\frac{dp}{dx} = \frac{dp_0}{dx} + \Gamma \frac{dp_1}{dx} \quad \text{and neglecting terms greater than } O(\Gamma^2) \text{, we get}
\]

\[
\frac{dp}{dx} = \left( \frac{[q+(h_1-h_2)]M^3 \sinh(M[h_2-h_1])}{2-2 \cosh(M(h_1-h_2)-M(h_1-h_2) \sinh(M(h_2-h_1))} \right) + \frac{\Gamma}{4} \left( \frac{[q+(h_1-h_2)]^3 M^9 A_{11} \sinh(M[h_2-h_1])}{[2-2 \cosh(M(h_1-h_2)-M(h_1-h_2) \sinh(M(h_2-h_1))^4]} \right) + Re \frac{Fr}{Fr} \sin \alpha
\]

(5.3.19)

The pressure rise per wave length \( \Delta p \) and is defined as

\[
\Delta p = \int_0^1 \frac{dp}{dx} \, dx,
\]

(5.3.20)

The above integral numerically evaluated using the MATLAB software.

5.4 Discussion of the results

In order to see the effects of \( Wi, M, \theta, \phi_1, \phi_2, d, \alpha, Fr \) and \( Re \) on the axial pressure gradient and the pumping characteristics, we have plotted Figs.5.2-5.19.

Fig. 5.2 depicts the variation of axial pressure gradient \( dp/dx \) with Debaroh number \( \Gamma \) for \( \phi_1 = 0.5, \phi_2 = 0.7, d = 1.2, \bar{Q} = -1, M = 1, \theta = \frac{\pi}{4}, \alpha = \frac{\pi}{4}, Fr = 2 \) and \( Re = 10 \). It is found that the axial pressure gradient \( dp/dx \) increases with increasing Deborah number \( \Gamma \). Further, it is observed that, the axial pressure gradient is more for fourth grade fluid \((0 < \Gamma < 1)\) than that of Newtonian fluid \((\Gamma = 0)\).
The variation of axial pressure gradient $dp/dx$ with Hartmann number $M$

\[
\phi_1 = 0.5, \quad \phi_2 = 0.7, \bar{Q} = -1, d = 1.2, \Gamma = 0.01, \theta = \frac{\pi}{4}, \alpha = \frac{\pi}{4}, Fr = 2 \quad \text{and} \quad Re = 10
\]

is shown in Fig. 5.3. It is observed that the axial pressure gradient $dp/dx$ increases with increasing Hartmann number $M$.

Fig. 5.4 illustrates the variation of the axial pressure gradient $dp/dx$ with phase shift $\theta$ for $\phi_1 = 0.5$, $\phi_2 = 0.7$, $\bar{Q} = -1$, $d = 1.2$, $M = 1$, $\Gamma = 0.01$, $\alpha = \frac{\pi}{4}$, $Fr = 2$ and $Re = 10$. It is observed that the axial pressure gradient $dp/dx$ decreases with increasing phase shift $\theta$.

The variation of the axial pressure gradient $dp/dx$ with upper wave amplitude $\phi_1$ for $\Gamma = 0.01$, $\phi_2 = 0.7$, $\bar{Q} = -1$, $d = 1.2$, $M = 1$, $\theta = \frac{\pi}{4}$, $\alpha = \frac{\pi}{4}$, $Fr = 2$ and $Re = 10$ is depicted in Fig. 5.5. It is found that the axial pressure gradient $dp/dx$ increases with an increase in amplitude of the upper wave $\phi_1$. The same trend is observed for $\phi_2$ as shown in Fig. 5.6.

Fig. 5.7 shows the variation of the axial pressure gradient $dp/dx$ with width of the channel $d$ for $\Gamma = 0.01$, $\phi_1 = 0.5$, $\phi_2 = 0.7$, $\bar{Q} = -1$, $M = 1$, $\theta = \frac{\pi}{4}$, $\alpha = \frac{\pi}{4}$, $Fr = 2$ and $Re = 10$. It is noted that the axial pressure gradient $dp/dx$ decreases on increasing $d$.

The variation of the axial pressure gradient $dp/dx$ with an inclination angle $\alpha$ for $\Gamma = 0.01$, $\phi_1 = 0.5$, $\phi_2 = 0.7$, $\bar{Q} = -1$, $M = 1$, $\theta = \frac{\pi}{4}$, $d = 1.2$, $Fr = 2$ and $Re = 10$ is presented in Fig. 5.8. It is found that the axial pressure gradient $dp/dx$ increases with an increase in $\alpha$.

Fig. 5.9 depicts the variation of the axial pressure gradient $dp/dx$ with Froude number $Fr$ for $\Gamma = 0.01$, $\phi_1 = 0.5$, $\phi_2 = 0.7$, $\bar{Q} = -1$, $M = 1$, $\theta = \frac{\pi}{4}$, $\alpha =$
\[ \frac{\pi}{4}, \ d = 1.2 \text{ and } Re = 10. \] It is observed that the axial pressure gradient \( dp/dx \) decreases on increasing Froude number \( Fr \).

The variation of the axial pressure gradient \( dp/dx \) with Reynolds number \( Re \) for \( \Gamma = 0.01, \ \phi_1 = 0.5, \ \phi_2 = 0.7, \ \tilde{Q} = -1, \ M = 1, \ \theta = \frac{\pi}{4}, \ \alpha = \frac{\pi}{4}, \ d = 1.2 \) and \( Fr = 2 \) is shown in Fig. 5.10. It is found that the axial pressure gradient \( dp/dx \) increases with increasing Reynolds number \( Re \).

Fig. 5.11 represents the variation of pressure rise \( \Delta p \) with time averaged flux \( \tilde{Q} \) for different values of Deborah number \( \Gamma \) with \( \phi_1 = 0.5, \ \phi_2 = 0.7, \ d = 1.2, \ M = 1, \ \theta = \frac{\pi}{4}, \ \alpha = \frac{\pi}{4}, \ Fr = 2 \) and \( Re = 10 \). It is observed that, in the pumping region \( (\Delta p > 0) \) the time-averaged flux \( \tilde{Q} \) increases with increasing Deborah number \( \Gamma \), while it decreases with increasing Deborah number \( \Gamma \) in both the free-pumping \( (\Delta p = 0) \) and co-pumping \( (\Delta p > 0) \) regions.

The variation of \( \Delta p \) with time-averaged flux \( \tilde{Q} \) for different values of Harmann number \( M \) with \( \phi_1 = 0.5, \ \phi_2 = 0.7, \ d = 1.2, \ \Gamma = 0.01, \ \theta = \frac{\pi}{4}, \ \alpha = \frac{\pi}{4}, \ Fr = 2 \) and \( Re = 10 \) is depicted in 5.12. It is found that any two pumping curves intersecting in the first quadrant, to the left of this point of intersection, the time-averaged flux \( \tilde{Q} \) increases with increasing Hartmann number \( M \) and to the right of this point of intersection, the \( \tilde{Q} \) decreases with increasing \( M \).

Fig. 5.13 shows the variation of \( \Delta p \) with time-averaged flux \( \tilde{Q} \) for different values of phase shift \( \theta \) with \( \phi_1 = 0.5, \ \phi_2 = 0.7, \ d = 1.2, \ \Gamma = 0.01, \ M = 1, \ \alpha = \frac{\pi}{4}, \ Fr = 2 \) and \( Re = 10 \). It is observed that the time-averaged flux \( \tilde{Q} \) decreases with increasing \( \theta \) in the pumping region, while it increases with increasing \( \theta \) in both the free pumping and co-pumping regions.
The variation of $\Delta p$ with time-averaged flux $\bar{Q}$ for different values of amplitude of the upper wave $\phi_1$ with $M = 1, \phi_2 = 0.7, d = 1.2, \Gamma = 0.01, \theta = \frac{\pi}{4}, \alpha = \frac{\pi}{4}, Fr = 2$ and $Re = 10$ is presented in Fig. 5.14. It is found that the time-averaged flux $\bar{Q}$ increases with increasing $\phi_1$ in the pumping region, while it decreases with increasing $\phi_1$ in the free-pumping and co-pumping regions. The same phenomenon is observed for the amplitude of the lower wave $\phi_2$ as presented in Fig. 5.15.

Fig. 5.16 shows the variation of $\Delta p$ with time-averaged flux $\bar{Q}$ for different values of width of the channel $d$ with $M = 1, \phi_1 = 0.5, \phi_2 = 0.7, \Gamma = 0.01, \theta = \frac{\pi}{4}, \alpha = \frac{\pi}{4}, Fr = 2$ and $Re = 10$. It is observed that the time-averaged flux $\bar{Q}$ decreases with increasing $d$ in the pumping region, while it increases with increasing $d$ in the free-pumping and co-pumping regions.

The variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of inclination angle $\alpha$ with $M = 1, \phi_1 = 0.5, \phi_2 = 0.7, \Gamma = 0.01, \theta = \frac{\pi}{4}, d = 1.2, Fr = 2$ and $Re = 10$ is shown in Fig. 5.17. It is noted that, the time-averaged flux $\bar{Q}$ increases with increasing inclination angle $\alpha$ in all the three regions.

Fig. 5.18 depicts the variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of Froude number $Fr$ with $M = 1, \phi_1 = 0.5, \phi_2 = 0.7, \Gamma = 0.01, \theta = \frac{\pi}{4}, \alpha = \frac{\pi}{4}, d = 1.2$ and $Re = 10$. It is noted that the time-averaged flux $\bar{Q}$ decreases with increasing Froude number $Fr$ in all the three regions.

The variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of Reynolds number $Re$ with $M = 1, \phi_1 = 0.5, \phi_2 = 0.7, \Gamma = 0.01, \theta = \frac{\pi}{4}, \alpha = \frac{\pi}{4}, d = 1.2$ and $Fr = 2$. It is noted that the time-averaged flux $\bar{Q}$ decreases with increasing Reynolds number $Re$ in all the three regions.
\( \alpha = \frac{\pi}{4}, d = 1.2 \) and \( Fr = 2 \) is shown in Fig. 5.19. It is found that, the time-averaged flux \( \bar{Q} \) increases with an increase in Reynolds number \( Re \) in all the three regions.

5.5 Conclusions

We studied the effect of magnetic field on the peristaltic flow of an incompressible fourth grade fluid in an inclined asymmetric channel under assumption of long wavelength. A perturbation technique is obtained for the case in which Deborah number \( \Gamma \) is small. It is found that the axial pressure gradient and the pumping increase with increasing \( \Gamma, M, \phi_1, \phi_2, \alpha \) and \( Re \), while they decrease with increasing \( \theta, d \) and \( Fr \).
Fig: 5.2 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of Deborah number $\Gamma$ for $\phi_1 = 0.5$, $\phi_2 = 0.7$, $d = 1.2$,
$\tilde{Q} = -1$, $M = 1$, $\theta = \frac{\pi}{4}$, $\alpha = \frac{\pi}{4}$, $Fr = 2$ and $Re = 10$. 
Fig: 5.3 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of Hartmann number $M$ for $\phi_1 = 0.5$, $\phi_2 = 0.7$, $d = 1.2$, $\bar{Q} = -1$, $\Gamma = 0.01$, $\theta = \frac{\pi}{4}$, $\alpha = \frac{\pi}{4}$, $Fr = 2$ and $Re = 10$. 
Fig: 5.4 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of phase shift $\theta$ for $\phi_1 = 0.5$, $\phi_2 = 0.7$, $d = 1.2$, $M = 1$, $\bar{Q} = -1$, $\Gamma = 0.01$, $\alpha = \frac{\pi}{4}$, $Fr = 2$ and $Re = 10$. 
Fig: 5.5 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of
$\phi_1$ for $\Gamma = 0.01$, $\phi_2 = 0.7$, $d = 1.2, M = 1, \theta = \frac{\pi}{4}$,
$\frac{\hat{Q}}{Q} = -1, \alpha = \frac{\pi}{4}$, $Fr = 2$ and $Re = 10$. 
Fig: 5.6 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of $\phi_2$ for $\Gamma = 0.01$, $\phi_1 = 0.5$, $d = 1.2$, $M = 1$, $\theta = \frac{\pi}{4}$, $Q = -1$, $\alpha = \frac{\pi}{4}$, Fr = 2 and Re = 10.
Fig: 5.7 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of $d$ for $\Gamma = 0.01$, $\phi_1 = 0.5$, $\phi_2 = 0.7$, $M = 1$, $\theta = \frac{\pi}{4}$, $\bar{Q} = -1$, $\alpha = \frac{\pi}{4}$, $Fr = 2$ and $Re = 10$. 
Fig: 5.8 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of $\alpha$ for $\Gamma = 0.01$, $\phi_1 = 0.5$, $\phi_2 = 0.7$, $M = 1$, $\theta = \frac{\pi}{4}$, $\tilde{Q} = -1$, $d = 1.2$, $Fr = 2$ and $Re = 10$. 
Fig 5.9 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of Froude number $Fr$ for $\Gamma = 0.01$, $\phi_1 = 0.5$, $\phi_2 = 0.7$, $\tilde{Q} = -1$, $M = 1$, $\theta = \frac{\pi}{4}$, $\alpha = \frac{\pi}{4}$, $d = 1.2$ and $Re = 10$. 
Fig: 5.10 Profiles of axial pressure gradient $\frac{dp}{dx}$ for different values of Reynolds number $Re$ for $\Gamma = 0.01$, $\phi_1 = 0.5$, $\phi_2 = 0.7$, $\vec{Q} = -1$, $M = 1$, $\theta = \frac{\pi}{4}$, $\alpha = \frac{\pi}{4}$, $d = 1.2$ and $Fr = 2$. 
Fig: 5.11 The variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of Deborah number $\Gamma$ with $\phi_1 = 0.5$, $\phi_2 = 0.7$, $d = 1.2$, $M = 1$, $\theta = \frac{\pi}{4}$, $\alpha = \frac{\pi}{4}$, $Fr = 2$ and $Re = 10$. 
Fig: 5.12 The variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of Hartmann number $M$ with $\phi_1 = 0.5$, $\phi_2 = 0.7$, $d = 1.2$, $\Gamma = 0.01$, $\theta = \frac{\pi}{4}$, $\alpha = \frac{\pi}{4}$, $Fr = 2$ and $Re = 10$. 
Fig: 5.13 The variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of phase shift $\theta$ with $\phi_1 = 0.5$, $\phi_2 = 0.7$, $d = 1.2$, $\Gamma = 0.01$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Fr = 2$ and $Re = 10$. 
Fig: 5.14 The variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of $\phi_1$ with $M = 1, \phi_2 = 0.7, d = 1.2, \Gamma = 0.01, \theta = \frac{\pi}{4}, \alpha = \frac{\pi}{4}, Fr = 2$ and $Re = 10$. 
Fig: 5.15 The variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of $\phi_2$ with $M = 1, \phi_1 = 0.5, d = 1.2, \Gamma = 0.01, \theta = \frac{\pi}{4}, \alpha = \frac{\pi}{4}, Fr = 2$ and $Re = 10$. 

$\phi_2 = 0.9, 0.7, 0.5, 0.3$
Fig: 5.16 The variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of $d$ with $M = 1, \phi_1 = 0.5, \phi_2 = 0.7, \Gamma = 0.01, \theta = \frac{\pi}{4}, \alpha = \frac{\pi}{4}, Fr = 2$ and $Re = 10$. 
Fig: 5.17 The variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of $\alpha$ with $M = 1, \phi_1 = 0.5, \phi_2 = 0.7, \Gamma = 0.01, \theta = \frac{\pi}{4}, d = 1.2, Fr = 2$ and $Re = 10$. 

$\alpha = \frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{6}, 0$
Fig: 5.18 The variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of Froude number $Fr$ with $M = 1$, $\phi_1 = 0.5$, $\phi_2 = 0.7$, $\Gamma = 0.01$, $\theta = \frac{\pi}{4}$, $\alpha = \frac{\pi}{4}$, $d = 1.2$ and $Re = 10$. 
Fig: 5.19 The variation of pressure rise $\Delta p$ with time averaged flux $\bar{Q}$ for different values of Reynolds number $Re$ with $M = 1$, $\phi_1 = 0.5$, $\phi_2 = 0.7$, $\Gamma = 0.01$, $\theta = \frac{\pi}{4}$, $\alpha = \frac{\pi}{4}$, $d = 12$ and $Fr = 2$. 