Chapter 1

Introduction
1.1 An invitation to the problem

In financial management, risk management is an important concept. Financial Risk Management can be defined as an act of creating economic value of the firm by effectively managing the exposure to the risk, with judicious use of several financial instruments and sophisticated techniques. The different type of exposure to risk, a firm is subjected to mainly involve credit risk and market risk. Financial risk management can be qualitative and quantitative in approach.

In order to determine the optimal asset allocation strategies mathematical models are vastly used by the corporate investors and portfolio managers respectively. The key organizations in finance are households, business firms, financial intermediaries and capital markets. The tradition in neoclassical economics is to consider the existence of households, their tastes and their endowments as exogenous to the theory. But other economic organizations are regarded as primary because of the functions they serve and are therefore endogenous to the theory. Giving concentration more on household, there are two players – consumer and investor. The consumer chooses how much of her income and wealth to allocate to current consumption and thereby, how to save for
future consumption. As investor, the household solves the problem to determine the fractional allocation of her savings among the available investment opportunities.

Risk management plays a critical role in determining the financial dynamics of the corporate investors. However, risk management holds equal significance when individual investors and their investment objectives are considered. Therefore for this purpose asset management is important for the individual investors. If a person owns more than one asset for investment, she has an investment portfolio. A portfolio consists of more than one asset. The main aim of the portfolio owner is to enhance the value of portfolio by selecting investments that yield good returns.

1.2 Portfolio Management

Portfolio Management refers to the science of analyzing the strengths, weaknesses, opportunities and threats for performing wide range of activities related to the one’s portfolio for maximizing the return at a given risk. It helps in making selection of Debt Vs Equity, Growth Vs Safety, and various other tradeoffs. Portfolio management involves the task of taking decisions about investment policy and mix, matching
investments to the objectives of the investors, allocating assets for individuals and institutions and balancing risk against performance. Portfolio management has a great importance in theory of finance. Managing a portfolio involves inherent risks. Portfolio management is goal driven and target oriented. Constructing a portfolio involves making wide range of decisions regarding buying or selling of stocks, bonds, or other financial instruments. In portfolio management both time and magnitude are very important.

Portfolio optimization plays a critical role in determining portfolio strategies for investors. In portfolio optimization investors want to either maximize portfolio returns or minimize portfolio risks. Since return is compensated based on risk, investors have to balance the risk-return tradeoff for their investments and it depends upon investors risk-return preferences. So, a single optimized portfolio is not in a position to satisfy all investors.

The traditional mean variance optimization approach fails to meet the demand of investors who have multiple investment objectives. In order to achieve the multiple objective of the investor and to satisfy the aim of obtaining the optimal portfolio it is necessary to understand the proportion of investment of
different assets in the portfolio. That means fixation of asset weights correctly of a given portfolio is of prime importance from investors' point of view. The above discussed premises points at the ultimate necessity of fixation of assets' weight as the decision variable in the portfolio optimization problems. The use of portfolio weights for model generation may be especially important in a setting where the expected return and the variance are assumed to be correctly predicted and are expected to show same behavior in future. The use of portfolio weights to measure the performance of trading strategies was pioneered by Cornell (1979). Cornell's measure was modified by Copeland and Mayers (1982) to analyze Value Line rankings. The concept was applied to weight-based measure of mutual fund performance (Grinblatt and Titman, 1993).

The weight-based performance measures of portfolio are rather simple. For example, an investor if knows that the returns of assets are likely to be higher or lower than expected by the market, then, other things remaining same, the investor can earn profit by changing his portfolio weights toward those assets whose returns are likely to be higher than expected and away from those assets whose returns are likely to be lower. In other words the covariance between the change in a portfolio's
weights and subsequent abnormal asset returns may be used to measure performance of portfolio.

Most of the portfolio measurement techniques are return-based, and involve regressing the return of a portfolio on some benchmark return. The measure of performance is the intercept in the regression. The minimal information requirement of the return methodologies is cited as one of the strength. An investor needs only returns on the managed portfolio and the benchmark in portfolio optimization problem. The overemphasis on the risk and return dimensions of the portfolio optimization often overlooks the necessity of the potential important information regarding the composition of the portfolio. Previously, portfolio weights were used with unconditional moments to measure portfolio performance. However, the inclusion of return-based measures into a conditional framework changed the results (Ferson and Schadt, 1996) and hence made it interesting to consider formulation and modeling in weight-based measures of performance.

The manner in which the choice between the different courses of action or inaction is made plays a critical role in deciding the effectiveness of a portfolio manager. The easiest way to make the decision is to evaluate from the array of
predicted risk and return of the securities in the portfolio and select the best combination. But the question which remains to be addressed is the search and applicability of a common model that can be used to capture the risk and returns, the different prediction mechanisms, the role played by different information dynamics which are supposed to influence the assessment of the risks and returns, and so many prior expectations that may be brought to influence the proportion of security in the design of the optimum portfolio.

Since the middle of twentieth century, financial economist or practitioner and statisticians had been measuring the performance of a managed portfolio from various angles. Even after years of research, several issues remain unsolved. The classical mean variance model is aimed at satisfying the optimizing needs of the risk averse investors. However, in real world investors exhibit a multitude of risk profiles which explicitly points to the fact and necessity of a model which can take into account the dynamics of the different categories of investors on the parameter of risk taking aptitude and attitude. Thus, it becomes evident that the investors are influenced by their nature or value system. According to Heller (1971) there are six
values of importance. Out of which, propensity to take risk is most important value system (Jauch and Glueck, 1988).

Nature of the investors may not be same and their investment needs depend on their nature. Some investors are risk takers, some are risk aversive, some other investors invest their wealth after a proper planning, who are known as risk planners and the rests are random selectors who randomly select the assets without giving any importance on expected return and risk. Thus, if an investor can correctly predict the proportion of amount to be invested in each assets of the portfolio then right decision can be taken to earn maximum utility.

An important unsolved issue is how to handle the dynamic behavior of a managed portfolio. Not only because of the existence of time-varying required returns in a portfolio but also due to investor’s strategy or other influencing factors, management of portfolios become difficult. Thus creation of optimal portfolio strategy for investors of various risk profiles becomes a lurking question in the financial risk management scenario. An attempt has been made in this work to formulize and propose a multi-objective approach to portfolio optimization problems. In this work portfolios risk and return are optimized
and various portfolio optimization models are integrated. Detailed analysis based on heuristic weight generation and subsequent optimization and application of the model are provided and compared to portfolio generated through the mean-variance approach and Sharpe’s approach.

The basics and ideas of investment portfolio management are used in various dimensions as discussed:

Application Portfolio Management: it involves management of complete group or subset of computer software applications in a portfolio. As the applications of software include maintenance cost and development cost, these can be considered as investment. The decisions regarding purchasing new software or modifying existing software are important parts in application portfolio management.

Product Portfolio Management: it means grouping of major products developed or sold by businesses into (Logical) portfolio. These products are arranged according to major line-of-business or business segment. In this case investment decision involves development of new product or modifying the existing product or discontinuation any other product.
Project Portfolio management or initiative portfolio management: it includes a specified beginning and end; precise and limited collection of desired results or work products and management team for taking the initiative and utilizing the resources.

1.3 Concept of Risk and Return

1.3.1 Concept of Risk

The earliest definition of risk was given by Knight (1921). According to him, risk is measurable uncertainty. Many authors have given various definitions of risk. Risk is a concept that denotes a potential negative impact to an asset. However, there was an attempt made by Holton (2004) to summarize the most relevant definition of risk to give a more general concept of risk. According to him, risk is the exposure to a proposition of which one is uncertain. A finance relevant definition given by Jorion (2000) defines risk as the volatility of the expected results on the value of assets and liabilities of interest. Portfolio management relevant definition given by Cool (1999) defines risk as the absolute value of probable loss.

From the economic point of view, risk is any event or action that may adversely affect an organization’s ability to
achieve its objectives and execute its strategies. In finance, financial risk is essentially any risk associated with any form of financing. The existence of risk means that the investor can only associate a single number or payoff with investment in any asset with certain probability associated with it. A common property of investment opportunities is that their actual returns might differ from what has been expected; or in short: they are risky. When the actual return is lower than the expected outcome, it is known as downside risk whereas when the deviation from the actual return is more than the expected outcome is known as upside risk.

Risk reflects not only the dangers associated with an investment, but also the chances. Therefore, a risky situation is one in which surprises and unexpected developments might occur. Volatility of return is this type of risk measure which is one of the foundations of portfolio theory. Risk can be measured in different ways; viz. semi-variance which measures only the negative deviations from the expected value. More recently, Value at Risk (VaR) has been used by the authors. It means the maximum loss within a certain period of time with a given probability. A third type of risk refers to a situation of danger or peril. In finance, this concept is applied as an aspect of risk.
measure as applied in circumstances where catastrophes ought to be prevented; investors ought to be saved from hazardous situations. However, in finance there are two types of risk—systematic risk or un-diversifiable risk and unsystematic risk or diversifiable risk. Systematic risk is the market risk or the risk that cannot be diversified away. Systematic risk affects the whole economy of a country. A perfectly diversifiable portfolio also carries some systematic risk which we cannot avoid. Unsystematic risk or diversifiable risk is the risk which is associated with individual assets and it differs from asset to asset. It is industry or company specific risk. Unsystematic risk can be diversified away by including a number of assets in the portfolio. However, unsystematic risks do not present enough information about the overall risk of the entire portfolio. The impact of risk has two components: uncertainty and exposure. The chance of facing risk is uncertainty and exposure is the amount of the possible loss if the risk has been faced.

1.3.2 Concept of Return

Return in its optimistic outlook can be defined as any potential gain which is supposed to be over and above than the amount invested in any asset. In other words, return is the ratio of money gained or lost (whether realized or unrealized) on an
investment relative to the amount of money invested. A general investor always tries to earn some return from any investment. The return can be calculated over a single period, or expressed as an average over multiple periods of time. Return can be calculated in various ways viz. arithmetic return, geometric return, logarithmic return. Return may be annual return and annualized return. An annual return is a single-period return, while an annualized return is a multi-period or arithmetic average return.

Personalized investment returns are of recent origin and are in much demand amongst the investor's community. The demand for personalized investment returns holds the argument that the fund returns may not be the actual account returns which are based upon the actual investment account transaction history. This occurs because investments may have been made on various dates and additional purchases and withdrawals may have incurred on various dates and the related amount is varying and thus is unique to the particular account. The fund returns may be more or less than the account return. More and more fund and brokerage firms have begun providing personalized account returns on investor's account statements in response to this need.
1.4 Existing Approaches

1.4.1 Mathematical Programming Approach

The selection of a best element (with regard to some criteria) from some set of available alternatives in parlance of mathematics, computer science, or management science is known as mathematical optimization (alternatively, optimization or mathematical programming). In the simplest case, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function. The generalization of optimization theory and techniques to other formulations comprises a large area of applied mathematics. More generally, optimization includes finding "best available" values of some objective function given a defined domain, including a variety of different types of objective functions and different types of domains (Wikipedia, 2012).

The following are applications of mathematical optimization and include the following:

- **Convex programming** studies the case when the objective function is convex (minimization)
or concave (maximization) and the constraint set is convex. This can be viewed as a particular case of nonlinear programming or as generalization of linear or convex quadratic programming (Wikipedia, 2012).

**Linear programming (LP),** a type of convex programming, studies the case in which the objective function $f$ is linear and the set of constraints is specified using only linear equalities and inequalities. Such a set is called a polyhedron or a polytope if it is bounded (Wikipedia, 2012).

**Second order cone programming (SOCP)** is a convex program, and includes certain types of quadratic programs (Wikipedia, 2012).

**Semidefinite programming (SDP)** is a subfield of convex optimization where the underlying variables are semi definite matrices. It is generalization of linear and convex quadratic programming (Wikipedia, 2012).

**Conic programming** is a general form of convex programming. LP, SOCP and SDP can all be viewed as conic programs with the appropriate type of cone (Wikipedia, 2012).
Geometric programming is a technique whereby objective and inequality constraints expressed as polynomials and equality constraints as monomials can be transformed into a convex program (Wikipedia, 2012).

Integer programming studies linear programs in which some or all variables are constrained to take on integer values. This is not convex, and in general much more difficult than regular linear programming (Wikipedia, 2012).

Quadratic programming allows the objective function to have quadratic terms, while the feasible set must be specified with linear equalities and inequalities. For specific forms of the quadratic term, this is a type of convex programming (Wikipedia, 2012).

Fractional programming studies optimization of ratios of two nonlinear functions. The special class of concave fractional programs can be transformed to a convex optimization problem (Wikipedia, 2012).

Nonlinear programming studies the general case in which the objective function or the constraints or both contain nonlinear parts. This may or may not be a convex program. In
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general, whether the program is convex affects the difficulty of solving it (Wikipedia, 2012).

**Stochastic programming** studies the case in which some of the constraints or parameters depend on random variables (Wikipedia, 2012).

**Robust programming** is, like stochastic programming, an attempt to capture uncertainty in the data underlying the optimization problem. This is not done through the use of random variables, but instead, the problem is solved taking into account inaccuracies in the input data (Wikipedia, 2012).

**Combinatorial optimization** is concerned with problems where the set of feasible solutions is discrete or can be reduced to a discrete one (Wikipedia, 2012).

**Infinite-dimensional optimization** studies the case when the set of feasible solutions is a subset of an infinite-dimensional space, such as a space of functions (Wikipedia, 2012).

**Heuristics and metaheuristics** make few or no assumptions about the problem being optimized. Usually, heuristics do not guarantee that any optimal solution need be
found. On the other hand, heuristics are used to find approximate solutions for many complicated optimization problems (Wikipedia, 2012).

**Constraint satisfaction** studies the case in which the objective function $f$ is constant (this is used in artificial intelligence, particularly in automated reasoning) (Wikipedia, 2012).

**Dynamic programming** studies the case in which the optimization strategy is based on splitting the problem into smaller sub-problems. The equation that describes the relationship between these sub-problems is called the Bellman equation (Wikipedia, 2012).

**Mathematical programming** with equilibrium constraints is where the constraints include variational inequalities or complementarities (Wikipedia, 2012).

A subset of mathematical programming approach, the linear programming is a class of optimization problems. The linear programming problems have one objective function and the set of constraints with linear equalities and inequalities. Because of the effectiveness and robustness of linear program
solving algorithms this techniques are useful for portfolio rebalancing problems (Wikipedia, 2012).

Chekhlov et al (2004) stated that portfolio allocation problems can efficiently be handled with linear programming based algorithms. The techniques are attractive to the investors as it demonstrates the problems with thousands of instruments and scenarios. The portfolio mean-variance optimization techniques are a class of quadratic programming problems. The quadratic programming Optimization models can lead to non-convex multi extrema problems.

**Modern Portfolio theory: Markowitz, Sharpe, Tobin and more**

The modern portfolio theory introduced by Harry Markowitz (1952) was based on linear programming problem. In the model he defined the linear programming as either by maximizing the return subject to certain amount of risk or minimizing the risk subject to certain amount of return. He proposed that investors should focus on selecting portfolios based on their joint risk-reward feature. The expected return of any portfolio can be measured by using the historical returns of each asset on the portfolio. Various statistical measures such as
average (return), standard deviation and linear correlation are used to measure the volatility of the portfolio. Markowitz (1959) used volatility and expected return as proxies for risk and reward. Markowitz defined an optimal way of selecting a portfolio by balancing the risk and reward features of the portfolio. According to him a rational investor should select a portfolio that lies on the efficient frontier. Modern Portfolio Theory as propounded by Harry Markowitz has introduced the concept of efficient frontier. "Efficient Frontier" can be defined as the combination of assets, i.e. a portfolio, if it has the best possible expected level of return for its level of risk (usually proxied by the standard deviation of the portfolio's return).

Markowitz work was expanded by James Tobin (1958) by adding a risk-free asset to the analysis. He showed that by using leverage or deleverage on the portfolios on the efficient frontier it was possible to outperform them in terms of their risk and reward relation. To prove it he introduced the concepts of “Capital Market Line” and “super-efficient portfolio”.

In the Capital Asset Pricing Model (CAPM) Sharpe (1964) pointed out that the market portfolio lies on the efficient frontier and is also actually Tobin's super-efficient portfolio. CAPM first introduced the concept of “beta” and relates an asset's expected
return to its beta. He showed that according to the risk appetite, all investors should hold the market portfolio where it is leveraged or de-leveraged with positions in the risk-free asset.

1.4.2 Model Approach

The description of a system using mathematical concepts and language is known as a mathematical model. Mathematical modeling is the process of developing a mathematical model. In order to explain a system and to study the effects of different components, and also to make predictions about behavior mathematical models comes handy. Mathematical models can take many forms, including but not limited to dynamical systems, statistical models, differential equations, or game theoretic models. A mathematical model usually describes a system by a set of variables and a set of equations that establish relationships among the variables. There are six basic groups of variables namely: decision variables, input variables, state variables, exogenous variables, random variables, and output variables. Since there can be many variables of each type, the variables are generally represented by vectors. Decision variables are sometimes known as independent variables. Exogenous variables are sometimes known as parameters or constants. The variables are not independent of
each other as the state variables are dependent on the decision, input, random, and exogenous variables. Furthermore, the output variables are dependent on the state of the system (represented by the state variables).

Objectives and constraints of the system and its users can be represented as functions of the output variables or state variables. The objective functions will depend on the perspective of the model's user. Depending on the context, an objective function is also known as an index of performance, as it is some measure of interest to the user. Although there is no limit to the number of objective functions and constraints a model can have, using or optimizing the model becomes more involved (computationally) as the number increases (Wikipedia, 2012).

**Classifying mathematical models**

Many mathematical models can be classified in some of the following ways:

**Linear vs. nonlinear:** Mathematical models are usually composed by variables, which are abstractions of quantities of interest in the described systems, and operators that act on these variables, which can be algebraic operators, functions, differential operators, etc. If all the operators in a mathematical...
model exhibit linearity, the resulting mathematical model is defined as linear. A model is considered to be nonlinear otherwise (Wikipedia, 2012).

**Deterministic vs. probabilistic (stochastic):**

A deterministic model is one in which every set of variable states is uniquely determined by parameters in the model and by sets of previous states of these variables. Therefore, deterministic models perform the same way for a given set of initial conditions. Conversely, in a stochastic model, randomness is present, and variable states are not described by unique values, but rather by probability distributions (Wikipedia, 2012).

**Static vs. dynamic:** A static model does not account for the element of time, while a dynamic model does. Dynamic models typically are represented with difference equations or differential equations (Wikipedia, 2012).

**Discrete vs. Continuous:** A discrete model does not take into account the function of time and usually uses time-advance methods, while a Continuous model does. Continuous models typically are represented with $f(t)$ and the changes are reflected over continuous time intervals (Wikipedia, 2012).
Deductive, inductive, or floating: A deductive model is a logical structure based on a theory. An inductive model arises from empirical findings and generalization from them. The floating model rests on neither theory nor observation, but is merely the invocation of expected structure. Application of mathematics in social sciences outside of economics has been criticized for unfounded models. Application of catastrophe theory in science has been characterized as a floating model (Wikipedia, 2012).

In portfolio management a lot of the research has been done in modeling the uncertainty of the value of assets on a portfolio and the relations between them, often heavily relying in probability theory and statistics. These models are very often used to simulate possible future scenarios through extensive computer programs. These models may play a vital role in the decision making process of the investors and they represent solely another tool of analysis. Although the use of a model constitutes another risk by itself, it may enable portfolio and risk managers to explicitly take into consideration some of the uncertainty they face and to quantify and estimate as accurately as possible the risks they take (Wikipedia, 2012).
1.5 Research gap and Research Problem

Markowitz (1952) is credited to proclaim the Numero Uno position in successfully quantifying the two basic conflicting objectives of investing in a portfolio viz. maximizing expected return and minimizing risk. Since the formulation of the modern portfolio theory, his work has attracted the attention of the academic world and has been instrumental in providing the fundamental direction to address the issue of portfolio optimization. Unfortunately, in the real world of investment management, the Markowitz framework has had surprisingly little impact. The reasons are, first, investors tend to focus on small segments of their potential investment universe. They select the undervalued assets and finds assets with positive momentum, or identifying relative value trades. Unfortunately, the Markowitz model needs expected returns to be specified for every component of the relevant universe which is unrealistic. But in practice this is typically defined by a broad benchmark. Secondly, the investors put emphasis on the weights in a portfolio. They are not much involved in balancing the expected returns against the contribution to portfolio risk which is the relevant margin in the Markowitz framework. The application of Markowitz model to ascertain the weights of the assets to construct the portfolio often results in extreme values which seems to be computationally justified but lacks logical and
intuitive appeal to the investor community. Thus, in practice the situation demands that substantial amount of energy needs to be invested in quest for reasonable numbers to make logical appeal and intuitive acceptance by manipulating the original model. The basic motivation for framing the research is driven by the above discussed premise.

Choosing a single asset for investment is not a difficult task for an investor. However, the investor's decision becomes difficult when faced with virtually innumerable investable choices and more importantly an infinite number of combinations of assets. The investor chooses those portfolios which lie on the efficient frontier. The choice may be affected by investor's beliefs, objectives, preferences, expectations, risk aversion, time and budget constrains, estimations among other. In addition, external factors will also affect the choice. Due to all the external and internal factors, the investor faces a dynamic decision problem in selection of optimal portfolio. Also, it is quite evident that the interactions of risk and reward are stated in portfolio theory in a very broad framework and has deeply influenced the way institutional portfolios are managed, and is also successful in motivating the “passive management” investment techniques. The mathematics of portfolio theory is widely used in risk
management and is basic for more recent risk measures. But when the investors use the models to solve real world problem, every assumption of the model becomes its limitation and often become obvious and thus is expected to have deep implications on the actual risk and reward that the portfolio’s holders will bear. Most recent works have shown that practically one can not select an optimal portfolio by considering the mean-variance portfolio theory. And above it, the gains from portfolio optimization are seen to have been nullified by the error explicit on the most common model's parameters estimators (Uppal, DeMiguel, Garlappi and Nogales, 2007). Uppal, DeMiguel, Garlappi and Nogales (2007) have put forward their argument by demonstrating how a naively diversified portfolio with equal weights in every asset, can out-perform out-of-sample on a risk adjusted basis (Sharpe-ratio in this case) an “optimally” diversified portfolio. However, some parameters of mean-variance model have been improved in several other studies. Sharpe’s (1963) study is one of the examples where he observed the market portfolio in order to improve the estimations of the expected return and covariance matrix.

Thus, though Markowitz model is the pioneer work in portfolio management, it suffers from some other serious
limitations. To overcome the computational complexity, it has to rely on a number of strict technical assumptions which are more or less away from reality viz. markets are assumed to be perfect that means there are neither taxes nor transactions costs and assets are infinitely divisible; investors make their decisions at exactly one point in time for a single-period horizon; and the means, standard deviations and correlation coefficients are sufficient to describe the assets’ returns. This theory needs a large number of data. W. Sharpe introduced Single Index Model to reduce huge data need. This model can be used with relatively few data as compared to Markowitz’s mean – variance – efficient portfolio theory. Single Index Model assumes that the only reason of security movement is a common co-movement with the index and the index is unrelated to a security’s unique return. The model does not consider other factors which affect the security return such as company performance, economy of the concerned industry, economic condition of the company etc.

Multi Index Model was introduced to capture some of the non-market influences that cause securities to move together. Non-market influences means a set of economic factors or structural groups that account for common movement in security prices beyond that accounted for by the market index itself. It
uses extra indices in the hope of capturing additional information which are not present in Sharpe’s Single Index Model. The problem of introducing additional indices is that they may pick up random noise rather than real influences. To eliminate the problem of picking up random noise averaging techniques were introduced. But the disadvantage of averaging technique is that real information may be lost in the averaging process.

In spite of the classical mean – variance- optimum portfolio theories, some other approaches are also used frequently by the authors such as stochastic dominance, geometric mean return and analysis in terms of characteristics of the return distribution etc. Later many works have been reported in the literature on portfolio management. Many authors have suggested selection of portfolio in many ways. These have established a close relationship with statistic of modeling. Various stochastic formulations (see Marton, 1980; Sahalia & Brandt, 2001; Detemple, Garcia and Rindisscher, 2003; Beliakov and Bagirov, 2006; Okhrin and Schmid, 2008 etc.) have been used to discuss the problem of selection of portfolio and various complicated statistical tools have been used to discuss the problem of selection of weight of the portfolio (the proportion of wealth invested in each individual asset). However,
the adoption of more sophisticated risk measures like value at risk and constraints including restrictions on the maximum number of different assets in a portfolio and minimum holding size, have made it all but impossible to optimize portfolios with classical techniques.

Earlier attempt by Elton and Gruber (1973) and recent attempt by Ledoit and Wolf (2004) has come up with the finding that the output of optimally portfolios can be improved by imposing a structure to the covariance matrix as opposed to its sample estimator. Although this recent research gives some insight for the mean-variance approach, it fails to give a well-known robust estimate for the expected return of most assets and also fails to achieve the benefits promised by portfolio optimization in its conception. Hence, there is a need to delve deep into the nuances of mean-variance approach to design an integrated approach to address the issue of portfolio optimization in more simplistic terms and churn out results from the proposed model which will be more attractive intuitively. To address the issue, the need to frame integrated robust model motivates the present research to integrate different mathematical and statistical models along with heuristics which allow us to estimate and emulate the risk features of a given
portfolio and to use simulation techniques to generate scenarios and weights which enable us to perform portfolio optimization taking into account explicitly the role of value system in the decision making framework of the investor.

1.6 Preview of the work

Selection of the optimum portfolio is a complex task for the general investors as choice of optimum weight is very difficult to make. There may be basically two ways of arriving at an optimum portfolio – one by minimizing the risk and the other by maximizing the return. This doctoral work proposes to strike a balance between these two. In chapter 3 optimum portfolios have been constructed subject to minimum return constraints. This minimum return constraint starts from the minimum return of the security in the portfolio and increases step by step to maximum return of the security in the portfolio. Then a heuristic procedure for arriving at security weights has been introduced based on the investors’ propensity to take risk. For this purpose, two extreme situations have been chosen – risk taker and risk aversive investor. After constructing heuristic portfolio the extent of closeness between the ideal portfolio constructed on the basis of optimization method and portfolio constructed on the basis of heuristic methods has been examined. For this purpose
Euclidian distance is considered. After detailed analysis, a point of change have been identified beyond which the optimum portfolio is closer to optimistic portfolio than to pessimistic portfolio before which the optimum portfolio is closer to pessimistic portfolio than optimistic portfolio.

In chapter 4 the optimum portfolio is obtained through a mathematical programming framework so as to minimize the portfolio risk subject to return constraint expressed in terms of coefficient of optimism ($\alpha$), where $\alpha$ varies from 0 to 1. Simultaneously, four heuristic portfolios have been developed for optimistic and pessimistic investors, risk planners and random selectors. Given the optimum portfolio and a heuristic portfolio City Block Distance has been calculated to measure the departure of the heuristic solution from the optimum solution.

In chapter 5 coefficient of optimism has been introduced in the weight of risk planner to observe the change of the behavior of the heuristic portfolio. The City Block Distance is used to calculate the distance between the optimum portfolio and the heuristic portfolios. For moderate values of the coefficient of optimism a heuristic investor’s decision nearly coincides with the corresponding optimum portfolio. However,
for extreme situations i.e. optimistic and pessimistic situations heuristic portfolio differs from optimum portfolio.

Chapter 6 states the comparison between Sharpe’s cut off principle portfolio and proposed near optimum portfolio with that of optimum portfolio under Sharpe’s Single Index Model. Here also the coefficient of optimism in the decision making process has been considered to compare Sharpe’s approach under optimality principle and cut off principle and the proposed near optimum portfolio, based on Single Index Model and to examine the suitability of near optimum portfolio over Sharpe’s cut off principle portfolio. These optimum portfolios have been obtained through a mathematical programming framework so as to minimize the portfolio risk subject to return constraint expressed in terms of coefficient of optimism. To know the similarity between the cut off principle portfolio and the near optimum portfolio with that of optimum portfolio under Sharpe’s Single Index model, City Block Distance has been considered. Up to moderate value and very high value of coefficient of optimism, near optimum portfolio shows better result. However, for moderate to high value of coefficient of optimism, the cut off principle portfolio shows closer result. This put forward the admissibility of the near optimum portfolio.
Chapter 7 concludes the work and presents some limitation of the present work. It also provides a direction for future research which can be carried out to make the proposed approach more robust and practically applicable.

1.7 Scope of the work

The portfolio optimization problem is mainly concerned with selecting the optimal investment strategy of an investor. In other words, the investor looks for an optimal decision on how many shares of which security should be purchased to maximize the expected utility. If the investor knows the securities that may give maximum expected return or minimum expected risk, it is easy to take optimal decision. But in real world it is difficult to find out those securities due to presence of efficient market. The statistical models used in behavioural finance are not very easily understandable to the general investor. When a general investor wishes to invest money in any portfolio of securities they are more concerned about the expected return and risk of the portfolio not about the various statistical models. The present study mainly focuses on the weight of the securities in the portfolio and proposes a simple heuristic tool to help those investors so that they can get a near optimum portfolio for
investment. Heuristic method is not universally accepted but is having intuitive appeal.

The present study aims to identify the objectives, background, methodology, and proposes a model which proposes to simplify the portfolio optimization problem. The academic endeavor provides a rigorous treatment to the weight as a decision variable in the optimization framework. The decision variable obtained heuristically is also factored into the optimization framework so as to provide an all inclusive dimension to the simplified approach. The work also provides a framework and analysis of the allocation decisions of the linear programming model and non-linear programming model. The interactions of the different value system with different decision-making systems of the investors have been well captured in the heuristic model generation process and its applicability has been ensured by comparing with the performance of the optimal solution generated by classical models.

One can get number of values for weight, risk and return in the optimization framework depending on the computational techniques used to compute weight, risk and return. The resultant weight values churned out form optimized solutions
provides a complex set which thereby makes it practically impossible to determine the global optima. Hence, in order to simplify the process, weights have been considered as the only decision variable in the optimization framework considering the assumption that for different set of investors the risk and return are held constant but the proportion of total investment in different securities in the portfolio can be manipulated and hence controlled to reach at the optimum solution. Therefore, according to the class of investors and their corresponding risk appetite heuristic weights can be generated. Heuristic portfolios are designed to compute the heuristic risk and return to compare with the optimum portfolios’ risk and return to ascertain their closeness and make informed decisions.