STUDIES ON
RELIABILITY OPTIMIZATION PROBLEMS
BY GENETIC ALGORITHM

SYNOPSIS

Thesis submitted to
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1. Introduction

The subject “Reliability Optimization” appeared in the literature in due late 1940s and was first applied to communication and transportation systems. Most of the earlier works were confined to an analysis of certain performance aspects of an operating system. One of the goals of the reliability engineer is to find the best way to increase the system reliability. The reliability of a system can be defined as the probability that the system will be operating successfully at least up to a specified point of time (i.e., mission time) under stated conditions. As systems are becoming more complex, the consequences of their unreliable behavior have become severe in terms of cost, effort and so on. The interests in accessing the system reliability and the need to improve the reliability of products and system have become more and more important.

The primary objective of reliability optimization is to find the best way to increase the system reliability. This can be done by different ways. Some of these are as follows:

(i) Increasing the reliability of each component in the system.

(ii) Using redundancy for the less reliable components.

(iii) Using standby redundancy which is switched to active components when failure occurs.

(iv) Using repair maintenance where failed components are replaced.

(v) Using preventive maintenance such that components are replaced by new ones whenever they fail or at some fixed interval, whichever is earlier.

(vi) Using better arrangement for exchangeable components.
To improve the system reliability, implementation of the above steps will normally result in the consumption of resources. Hence, a balance between the system reliability of a system and resource consumption is an important task.

When, redundancy is used to improve the system reliability, the corresponding problem is known as redundancy allocation problem. The objective of this problem is to find the number of redundant components that maximizes the system reliability under several resource constraints. This problem is one of the most popular ones in reliability optimization since 1950s because of its potentiality for broad applications. When it is difficult to improve the reliability of unreliable components, system reliability can easily be enhanced by adding redundancies on those components. However, for design engineers improving of component reliability have been generally preferred over by adding redundancy, because, in many cases, redundancy is difficult to add to real systems due to technical limitations and relatively large quantities of resources, such as weight, volume and cost that are required.

Network reliability design problems have attracted many researchers, such as network designers, network analysts, and network administrators, in order to share expansive hardware and software resources and provide the access of main systems from different locations. These problems have many applications in the areas of telecommunications and computer networking and related domains in the electrical, gas sewer networks. During the designing of network systems, one of the important steps is to find the best layout of components to optimize some performance criteria, such as cost, transmissions delay or reliability. The corresponding optimal design problem can be formulated as a combinatorial problem.
However, recently developed advanced technologies such as semiconductor, integrated circuits and nano technology, however, have revived the importance of the redundancy strategy. The current downscaling trend in the semiconductor manufacturing has caused many inevitable defects and subsequent faults in integrated circuits. It is widely accepted that there are certain limitations on enhancing reliability or yield in semiconductor manufacturing by developing relevant physical technologies. Hence, various fault-tolerant and self-repairable techniques have been studied. These approaches are mainly based on adding redundancies on components and controlling the usage of redundancies. In fact, most memory integrated circuits and VLSI, which includes internal memory blocks, currently use a hierarchical redundancy scheme to increase the yield and reliability of the chip.

To efficiently constitute the fault-tolerant systems with redundancy, the number of redundancies should be optimized. However, for improving the system reliability the addition of redundant components to the system is a formidable task due to several resource constraints arising out of the size, cost and quantities of resources coupled with technical constraints. Thus, the redundancy allocation problem is a practical problem of determining the appropriate number of redundant components that maximize the system reliability under different resource constraints. Equivalently, the problem is a non-linear constrained optimization problem. To solve this type of problem, several researchers have proposed different approaches. In their works, the reliabilities of the system components are assumed to be known at a fixed positive level, which lies between zero and one. However in real-life situations, the reliabilities of these individual components may fluctuate due to different reasons. It is not always possible for a technology to produce different components with exactly identical reliabilities. Moreover the human factor, improper storage facilities and other
environmental factors may affect the reliabilities of the individual components. Hence, it is sensible to treat the component reliabilities as positive imprecise numbers between zero and one instead of fixed real numbers. To define the problem associated with such imprecise numbers, generally different approaches like stochastic, fuzzy and fuzzy-stochastic approaches are used. In stochastic approach, the parameters are assumed to be random variables with known probability distribution whereas in fuzzy approach, the parameters, constraints and goals are considered as fuzzy sets with known membership functions. On the other hand, in fuzzy-stochastic approach, some parameters are viewed as fuzzy sets and others as random variables. However, to select the appropriate membership function for fuzzy approach, probability distribution for stochastic approach and both for fuzzy-stochastic approach is a very complicated task for a decision-maker and it arises a controversy situation as to solve a decision-making problem, other decisions are to be taken intermediately. Therefore, to overcome the difficulties arisen in the selection of those, the imprecise numbers may be represented by interval numbers. As a result, the objective function of reliability optimization problem will be interval valued, which is to be optimized.

These types of optimization problems with interval objective can be solved by a well known powerful computerized heuristic search and optimization method, viz. genetic algorithm (GA), which is based on the mechanics of natural selection (depending on the evolution principle “Survival of the fittest”) and natural genetics. It is executed iteratively on the set of real/binary coded solutions called population. In each iteration (which is called generation), three basic genetic operations, viz. selection/reproduction, crossover and mutation are performed.
2. Historical Review of Reliability Optimization Problems

Now-a-days, our society is mostly dependent on modern technological systems and there is no doubt that these technological systems have improved the productivity, health and affluence of our society. However, this increasing dependence on modern technological systems requires dealing with the complicated operations and sophisticated management. For each of the complex/complicated systems, the system reliability plays an important role. The reliability of any system is very important to manufacturers, designers and also to the users. During the design phase of a product, reliability engineers/designers are called upon to measure the reliability of that product. They desire the larger reliability of their products which raise the production cost of the items. In such a case, there arises a question as to how to meet the goal for the system reliability. As a result, the increase in the production cost has negative effects on the user's budget. Therefore, the design reliability optimization problem is phrased as reliability improvement at a minimum cost. In this connection, a widely known method for improving the system reliability of a system is to introduce several redundant components. For better designing a system using components with known cost, reliability, weight and other attributes, the corresponding problem can be formulated as a combinatorial optimization problem, where either system reliability is maximized or system cost is minimized. Therefore both the formulations generally involve constraints on allowable weight, cost and/or minimum targeted system reliability level. The corresponding problem is known as the reliability redundancy allocation problem. The primary objective of the reliability redundancy allocation problem is to select the best combination of components and levels of redundancy either to maximize the system reliability and/or to minimize the system cost subject to several constraints.
To solve these problems, several researchers have developed different optimization methods which include exact methods, approximate methods, heuristics, meta-heuristics, hybrid heuristics and multi-objective optimization techniques etc. Dynamic programming, branch and bound, cutting plane technique, implicit enumeration search technique are exact methods which provide exact solution to reliability optimization problems. The variational method, least square formulation and geometric programming and Lagrange multiplier give an approximate solution. A detailed review of the different optimization approaches to determine the optimal solutions is presented in Tillman, Hwang and Kuo (1977a, 1980), Sakawa (1978b, 1981b), Kuo, Prasad, Tillman and Hwang (2001) and Kuo and Wan (2007a, 2007b). On the other hand, heuristic, meta-heuristic and hybrid heuristic have been used to solve complicated reliability optimization problems. They can provide optimal or near optimal solution in reasonable computational time. Genetic algorithm, simulated annealing, tabu search, ant colony optimization and particle swarm optimization are some of the approaches in those categories. For detailed discussion, one may refer to the works of Kuo, Hwang and Tillman (1978) Coit and Smith (1996), Hansen and Lih (1996), Ravi, Murty and Reddy (1997), Zhao and Song (2003), Liang and Smith (2004), Coelho (2009a) and others.

Bellman (1957) and Bellman and Dreyfus (1958, 1962) used dynamic programming to maximize the reliability of a system with single cost constraint. In their works, the problem was to identify the optimal levels of redundancy for only one component in each subsystem.

In the year 1968, Fyffe, Hines and Lee (1968) considered a system having 14 subsystems with both cost and weight constraints and solved the corresponding reliability optimization problem by dynamic programming approach. In their work, for
each subsystem there are three or four different choices of components each with
different reliability, weight and cost. They used Lagrange's multiplier technique to
accommodate the multiple constraints.

Nakagawa and Miyazaki (1981) used a surrogate constraints approach, by
showing the inefficiency of the use of a Lagrange multiplier with dynamic programming.
Their algorithm was tested for 33 different Fyffe’s problems of which feasible solutions
were obtained only for 30 problems.

Redundancy allocation problem can be solved by another important approach
i.e., integer programming approach. Ghare and Taylor (1969) first used the branch and
bound method to maximize the system reliability under given non-linear but separable
constraints. Bulfin and Liu (1985) formulated the problem as a knapsack problem using
surrogate constraints (approximated by Lagrangian multipliers found by subgradient
optimization (Fisher (1981)) and used integer programming to solve it. Nakagawa and
Miyazaki (1981) investigated the Fyffe problem and formulated 33 variances of the
problem as integer programming problem. Bulfin and Liu (1985) solved the same
problem.

Misra and Sharma (1991) presented a fast algorithm to solve integer
programming problems like those of Ghare and Taylor (1969). The problem was
formulated as a multi-objective decision-making problem with distinct goals for
reliability, cost and weight and also solved by integer programming by Gen, Ida,

To solve this type of problem several other methodologies have been proposed
by researchers, like, Kuo, Lin, Xu and Zhang (1987), Hikita, Nakagawa and Narihisa
Among these methodologies, applications of GA in reliability optimization problems have been received warm reception among the researchers. In this connection one may refer to the work of Painton and Campbell (1994, 1995). They used GA in solving an optimization model that identifies the types of component improvements and the level of effort spent on those improvements to maximize one or more performance measures (e.g., system reliability or availability) subject to the constraints (e.g., cost) in the presence of uncertainty. In the year 1994, a redundancy allocation problem with several failure modes was solved by Ida, Gen and Yokota (1994) with the help of GA. Coit and Smith (1996) have solved a redundancy optimization problem by applying GA to a series-parallel system with mix of components in which each subsystem is a \(k\)-out-of-\(n\): \(G\) system. In the year 1998, Coit and Smith (1998) used GA-based approach to solve the redundancy allocation problem for series-parallel system, where the objective is to maximize a lower percentile of the system time to failure distribution. Yun and Kim (2004) and Yun, Song and Kim (2007) solved multi-level redundancy allocation in series-parallel system using genetic algorithm.

From the earlier-mentioned discussion, it may be observed that all the problems solved by several researchers are of single objective. However, in most of the real-world design or decision-making problems involving reliability optimization, there occurs the simultaneous optimization of more than one objective function. When designing a reliable system, as formulated by multi-objective optimization problem, it is always desirable to simultaneously optimize several objectives such as system reliability, system cost, volume and weight. For this reason multi-objective optimization problem attracts a lot of attention from the researchers. The objective of this problem is to

3. Objectives and Motivation of the Thesis

In reliability engineering, the reliability optimization is an important problem. As mentioned earlier this problem came into the existence in the late 1940s and was first applied to communication and transport system. After that a lot of works has been done
by several researchers incorporating different factors. To solve those problems, a number of methods/techniques has been proposed. In most of these works, the reliability of a component was considered as precise value i.e., fixed lying between zero and one. However, due to some factors mentioned in Section 1, it may not be fixed though it may vary between zero and one. So, to represent the same, some of the researchers have used either stochastic or fuzzy or fuzzy-stochastic approaches. On the other hand, it may be represented by an interval which is significant. To the best of our knowledge, very few works have been done considering interval valued reliabilities. Even today, there is a lot of scope to work in this area considering interval valued reliabilities of components. The detailed scheme of works along with the works presented in this thesis and also the further scope of research has been shown in Figure 1.

It may be noted from Figure 1 that the objectives of the thesis is

(i) to formulate the different types of redundancy allocation problems involving reliability maximization and cost minimization considering component reliabilities as interval valued numbers.

(ii) to formulate chance constraints reliability stochastic optimization problem, network reliability design problem and multi-objective reliability optimization with fixed and interval valued values of reliability of components.

(iii) to solve the problems mentioned in (i) and (ii) by real coded genetic algorithm, interval mathematics and order relations proposed in the thesis.
**Figure 1:** Organization of research work
4. Organization of the Thesis

In this thesis, some reliability optimization problems have been formulated and solved in interval environment with the help of interval mathematics, our proposed interval order relations and real coded genetic algorithm. The entire thesis has been divided into nine chapters as follows:

**Chapter 1**  
Introduction

**Chapter 2**  
Solution Methodologies

**Chapter 3**  
Reliability Redundancy Allocation Problems in Interval Environment

**Chapter 4**  
Reliability Optimization under High and Low-level Redundancies for Imprecise Parametric Values

**Chapter 5**  
Reliability Optimization under Weibull Distribution with Interval Valued Parameters

**Chapter 6**  
Stochastic Optimization of System Reliability for Series System with Interval Component Reliabilities

**Chapter 7**  
Reliability Optimization with Interval Parametric Values in the Stochastic Domain

**Chapter 8**  
Multi-objective Reliability Optimization in Interval Environment

**Chapter 9**  
General Conclusion and Scope of Future Research

**Chapter 2** deals with an overview of existing finite interval mathematics, interval order relations and real coded genetic algorithm. In this chapter, we have also proposed new definition of interval power of an interval and new order relations of intervals irrespective of decision-makers’ value system.

The objective of **Chapter 3** is to develop and solve the reliability redundancy allocation problems of series-parallel, parallel-series and complex/complicated systems.
considering the reliability of each component as interval valued number. For optimization of system reliability and system cost separately under resource constraints, the corresponding problems have been formulated as constrained integer/mixed-integer programming problems with interval objectives with the help of interval arithmetic and interval order relations. Then the problems have been converted into unconstrained optimization problems by two different penalty function techniques. To solve these problems, two different real coded genetic algorithms (GAs) for interval valued fitness function with tournament selection, whole arithmetical crossover and boundary mutation for floating point variables, intermediate crossover and uniform mutation for integer variables and elitism with size one have been developed. To illustrate the models, some numerical examples have been solved and the results have been compared. As a special case, taking lower and upper bounds of the interval valued reliabilities of component as same, the corresponding problems have been solved and the results have been compared with the results available in the existing literature. Finally, to study the stability of the proposed GAs with respect to the different GA parameters (like, population size, crossover and mutation rates), sensitivity analyses have been shown graphically.

Chapter 4 deals with redundancy allocation problem in interval environment that maximizes the overall system reliability subject to the given resource constraints and also minimizes the overall system cost subject to the given resources including an additional constraint on system reliability where reliability of each component is interval valued and the cost coefficients as well as the amount of resources are imprecise and interval valued. These types of problems have been formulated as an interval valued non-linear integer programming problem (IVNLIP). In this work, we have formulated two types of redundancy, viz. component level redundancy known as
low-level redundancy and the system level redundancy known as high-level redundancy. These problems have been transformed as an unconstrained problem using penalty function technique and solved using genetic algorithm. Finally, two numerical examples (one for low-level redundancy and another for high-level redundancy) have been presented and solved and the computational results have been compared.

Chapter 5 presents the reliability optimization problem of a complex /complicated system where time-to-failure of each component follows the Weibull distribution with imprecise parameters. In the earlier work, either both the scale and shape parameters of Weibull distribution or the scale parameter as a random variable with known distribution are considered as fixed. However, in reality, both the parameters may vary due to some factors and it is sensible to treat them as imprecise numbers. Here, this imprecise number is represented by an interval number. In this chapter, we have formulated the reliability optimization problem with Weibull distributed time-to-failure for each component. The corresponding problem has been formulated as an unconstrained mixed-integer programming problem with interval coefficients using penalty function technique and solved by genetic algorithm. Finally, a numerical example has been solved for different types of scale and shape parameters of Weibull distribution.

Chapter 6 deals with chance constraints based reliability stochastic optimization problem in the series system. This problem can be formulated as a non-linear integer programming problem of maximizing the overall system reliability under chance constraints due to resources. In this chapter, we have formulated the reliability optimization problem as a chance constraints based reliability stochastic optimization problem with interval valued reliabilities of components. Then, the chance constraints of the problem are converted to the equivalent deterministic form. The transformed
problem has been formulated as an unconstrained integer programming problem with
interval coefficients by Big-M penalty technique. Then to solve this problem, we have
developed a real coded genetic algorithm (GA) for integer variables with tournament
selection, intermediate crossover and one neighborhood mutation. To illustrate the
model, two numerical examples have been considered and solved by our developed GA.
Finally to study the stability of our developed GA with respect to the different GA
parameters, sensitivity analyses have been carried out and presented graphically.

In Chapter 7, the problem of reliability optimization has been examined in the
stochastic domain with respect of resource constraints and the concept of interval
valued parameters has been integrated with the stochastic setup so as to increase the
applicability of the resultant solutions. In particular, the five-link bridge network system
has been studied under a normal setup with Genetic Algorithm as the optimization tool.
Deterministic solution and non-interval valued parametric solutions follow from the
general optimization results.

In Chapter 8, we have solved the constrained multi-objective reliability
optimization problem of a system with interval valued reliability of each component by
maximizing the system reliability and minimizing the system cost under several
constraints. For this purpose, five different multi-objective optimization problems have
been formulated in interval environment with the help of interval mathematics and our
newly proposed order relations of interval valued numbers. Then these optimization
problems have been solved by advanced genetic algorithm and the concept of Pareto
optimality. Finally, for the purpose of illustration and comparison, a numerical example
has been solved.

In Chapter 9, general concluding remarks drawn from our studies and further
scope of research have been presented.
References


Publications

The thesis includes the following published works and a few works communicated for publication:

**Published/Accepted papers**


**Communicated papers**