Chapter 1

Introduction

1.1 Objective and Scope of the Thesis and Literature review

The hydrodynamic stability of shear flows forms an important part owing to its relevance to both atmosphere and oceanographic environments. The stability of shear flows is of enormous importance in dealing with laboratory and energy systems. The stability of some of the important shear flows like Poiseuille flow, Couette flow and boundary layer flow are investigated in greater detail. When a fluid is driven away from thermal and mechanical equilibrium, it will undergo a sequence of instabilities each of which leads, to a change in spatial or temporal structure. No single theory has emerged so far, to describe the complete phenomenon and remains mathematically challenging. Transition from laminar to turbulent flow begins with the instability of flow.

To understand the shear flow instability the theory of hydrodynamic stability is an important topic in fluid mechanics that became of great interest in the early part of the nineteenth century. The primary goal of hydrodynamic stability is to understand whether a laminar flow, when perturbed, will transition to new state. The flow is generally defined as stable if the flow returns to its original laminar state and unstable if the disturbance grows and causes the laminar flow to change into a different state. Stability theory deals with the mathematical analysis of the evolution
of disturbances superposed to a laminar base flow. In many cases one assumes the disturbances to be small so that further simplifications can be justified. In particular, a linear equation governing the evolution of disturbances is desirable. As the disturbance velocities grow above a few percent of the base flow, nonlinear effects become important and linear equations no longer accurately predict the disturbance evolution. Although the linear equations have a limited region of validity they are important in detecting physical growth mechanisms and identifying dominant disturbance types. The study of hydrodynamic stability has many practical applications in engineering such as calculations of aerodynamic drag, or nuclear power-plant heat exchangers. Within the hydrodynamic stability there are many different sub topics related to the different forms of hydrodynamic stability, such as stability of flows caused by convection, or stability of parallel shear flows. Excellent theoretical and experimental studies have been reported on the stability of shear flows in Newtonian fluids during the past decades, and a growing volume of work devoted to this area is well documented by Lin (1945), Tollmien (1947), Meksyn (1948), Lin (1955), Chandrasekhar (1961), and Orzag(1971) and Drazin and Reid (1981), Takashima (1996), Makinde (2009), Hill & Straughan (2010), Shankar et al. (2014).

The stability of shear flows in the presence of applied magnetic field is important in geophysics and astrophysics. The subject dealing with the motion of electrically conducting fluids in the presence of magnetic field is termed as magnetohydrodynamics (MHD) or hydromagnetics. Examples of such fluids include plasmas, liquid metals and salt water or electrolytes. The set of equations which describe MHD are a combination of Navier-Stokes equations of fluid dynamics and Maxwell’s equations of electromagnetism. The field of MHD was initiated by Hannes Alfvén (1942) for which he received the Nobel Prize in Physics in 1970. In MHD the body force acting on the fluid is the Lorentz force that arises when electric current flows at an angle to the direction of an impressed magnetic field. The interactions between conducting shear flows and transverse magnetic fields have many practical applications such as designing magnetohydrodynamic (MHD)flow meters , MHD
power generators, MHD pumps and nuclear fusion devices etc (Kukutami 1964). There are also many theoretical, astrophysical and geophysical problems such as accretion disks around black holes or neutron stars and how the earth’s magnetic field is influenced by the presence of hydrodynamic shear motions in the liquid core. The superimposition of transverse magnetic field on plane Poiseuille flow has two physical effects on the fluid. Firstly a weak electromagnetic damping force known as the Lorentz force is produced and secondly it causes the basic velocity profile to become flat in the core or centre of the flow due to the electromagnetic breaking effect. The breaking is caused by the interaction between the induced current and the applied magnetic field this in turn causes an electric current to flow down the boundary layers. This causes the boundary layer to become compressed against the boundary and has the aforementioned effect on the velocity profile. They were theoretically predicted and experimentally investigated in the seminal work of Hartmann and Lazarus (1937).

Shear flows encompass a wide variety of configurations commonly encountered in the engineering and natural sciences: wakes behind bluff bodies, jets, boundary layers near solid surfaces, etc. Such flows typically exhibit strong shear in the cross-stream direction and are spatially developing along the stream from a well defined spatial origin, say the trailing edge of a splitter plate, an obstacle, a nozzle exit plane, the leading edge of a plate, etc. The shear flows are known to be prone to instabilities: temporally or spatially growing wave are generated which travel along the stream and lead to the formation of unsteady vertical structures. It is important to predict whether a given shear flow displays one or the other type of dynamics solely from the properties of the instability waves it can support. Such an issue is not only of fundamental interest in order to understand the various processes leading from laminar flows to turbulence, it is also of crucial importance if one is to devise efficient control strategies well suited to the dynamical nature of the flow under consideration.

By definition, shear occurs whenever two adjacent fluid particles move in parallel directions, but at different velocities. The shear is defined as the amplitude of the local
velocity gradient perpendicular to the motion. Shear flows are ubiquitous in nature and can occur on any scale. Flows pumped through pipes by some pressure gradient along the pipe (called Poiseuille flows) are present everywhere in natural or engineered systems: blood flow through the body, from small capillaries to arteries, fluid flow through underground river systems, magma flows and pyroclastic flows through volcano chimneys, water flowing through a hose, a kitchen faucet, oil in a car engine, in a pipeline, etc.. These are often subject to strong shear if the wall boundaries are no-slip (so fluid is moving in the center of the pipe, but not on the sides. Shear flows can also be driven by differential pressure gradients (or any other forces) in open systems, and are found in the ocean, in the atmospheric wind patterns, in the surface and subsurface flows of the Sun, giant planets, other stars, in the orbital motion of gas in accretion disks, etc..

Most shear flows are spatially developing, i.e. their velocity profile evolves as the flow proceeds downstream. Typically, the Reynolds number increases, the laminar shear flow undergoes a linear instability, followed by an often complicated, and not completely understood, route to turbulence. Notable exceptions of this behavior pattern are the flow through an infinite straight channel and that through an infinite straight pipe. The former is linearly stable up to a critical Reynolds number of 5772 while the latter is linearly stable at any Reynolds number $Re$. Both these flows usually undergo a transition to turbulence at Reynolds number of 1500 or 2000. Parallel shear flows with smooth velocity profiles are more difficult to analyze and usually require numerical techniques. A useful result, due to Rayleigh (see Drazin & Reid (1981)), is that an inviscid flow must contain an inflexion point in order to be unstable. A viscous flow, however, does not require an inflexion point in order to be unstable.

Fluids surrounding us are of two types. The first category of fluids like air, water is known as Newtonian fluids whose viscosity is independent of shear rate. However there exists another class of fluids known as Non Newtonian fluids whose viscosity depends on shear rate and shear history. Fluids like blood, syrup, paint or molasses are known as shear thinning liquids whose viscosity decreases with increasing shear whereas in cases of shear thickening fluids like suspensions of corn starch or quick sand viscosity
increases with shear. Newtonian fluids, such as air and water, transition to turbulence under the influence of inertia. For low Reynolds number (Re), the behavior of Newtonian fluids is dominated by viscous dissipation. As Re increases, the influence of inertia becomes more important and, at large Re, flows of these fluids become turbulent.

A magnetic field in the direction of the temperature and pressure gradient will hinder the transverse motion essential to convection, and make the convective "cells" narrower and less efficient, reducing the rate of energy transfer. In the Sun, this is seen by the relative darkness of sunspots, where a vertical magnetic field of thousands of gauss reduces the efficiency of convection of heat from below, so the surface cools below the general level of the photosphere. It was noted above that the only conducting fluids available for laboratory experiments are mercury and liquid sodium, both inconvenient for different reasons. It is very difficult to reach large magnetic Reynolds numbers in laboratory experiments, and so to verify important theoretical results with any accuracy. There is another MHD laboratory available, however, and that is the Sun. The surface of the sun is hot, relatively dense plasma where the magnetic Reynolds number is very large, so the magnetic field is well and truly frozen into this fluid. However, we cannot change the experimental parameters, and we do not know what is going on below the level that we can see, so it is a less than perfect laboratory. Nevertheless, there are many interesting and varied phenomena that show the influence of MHD very well.

Understanding of MHD finds applications in the fields of high-temperature energy conversion systems, new kinds of liquid metal batteries, the production of solar-grade silicon, CO₂ free production of hydrogen, liquid metal targets in modern neutron or particle sources and transmutation systems, casting and solidification of steel and light metals, welding and soldering processes, to basic laboratory experiments with relevance to liquid metal cooled systems, materials processing as well as to geo and astrophysics. The knowledge and experiences arising from the fundamental research activities provide a solid basis for the development of

Natural convection flows are inherent to a variety of problems, from the analysis of atmospheric flows to safety critical issues in fire control and nuclear power plants. In contrast, a free convection flow field is a self-sustained flow driven by the presence of a temperature difference, which is opposed to a forced convection where external means are used to cause the flow. Natural convection fluid motion is due solely to buoyancy force caused by the density differences as a result of the temperature difference. This force is a strong function of the temperature difference between the solid and the fluid. As such the buoyancy force will induce a flow current due to the gravitational field and the variation in the density field. Convection above a hot surface occurs because hot air expands, becomes less dense, and rises. In general, natural convection heat transfer is usually smaller compared to a forced convection heat transfer. Natural convection can be divided in two main branches namely external and internal natural convection. External natural convection may occur along different geometries such as free convection along vertical walls, inclined walls, and horizontal walls. In addition, natural convection may take place around horizontal and vertical cylinders as well as around spheres. Natural convection may take place around other immersed bodies such as cubes and spheroids.

The fluid motion generated by buoyancy due to density variations, resulting from a temperature difference, is referred to as natural convection. The stability of flow driven by the combined shear and buoyancy forces is one of the classical problems with significance for fundamental fluid mechanics as well as for geophysical and engineering applications. Among engineering applications there are materials processing, crystal growth, cooling systems for nuclear reactors, solar energy collectors, manufacturing and welding. The stability of a natural convection is
relevant to many industrial and manufacturing processes such as reactor core, extrusion, drawing etc. many geophysical phenomenon’s like motion in between earths tectonic plates are maintained by buoyancy forces, strongly modified by the coexisting shear. The recognition of high free convection heat transfer rates in atomic reactors, electrical transformers and other engineering applications prompted many to understand and study the natural convection. A growing volume of work on the stability of natural convection between two vertical parallel plates is well documented by Roberts (1967), Vest and Arpaci (1969), Potter and Kudchey (1973), Takashima (1996), Gelfgat et al (2001), Kaddeche et al (2003), Shankar et al (2014).

Although the problem of hydrodynamic stability has been extensively investigated for Newtonian fluids, relatively little attention has been devoted to this problem with non-Newtonian fluids and couple stress fluids. With growing importance of non-Newtonian fluids with suspended particles in modern technology and industries, the investigations of such fluids are desirable. The study of such fluids has applications in a number of processes that occur in industry, such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, exotic lubrication, nuclear slurries and colloidal and suspension solutions. Work on stability of natural convection in a vertical fluid layer by Gozum & Arpaci (1974), Takashima (1993), Shankar et al (2014). Jain and Stokes (1974), studied the effect of couple stresses in fluids on the hydrodynamic study of plane Poiseuille flow, while effect of couple stresses on thermal convective instability is analyzed by many researchers (see Malashetty et al 2006, Gaikawad et al 2007, Sunil et al 2011 and Rudraiah et al 2011).

A great deal of attention is also envisaged on the study of stability of non-Newtonian viscoelastic and couple stress fluids. The objective of this thesis is to investigate the stability of shear flow of non-Newtonian couple stress fluid. The couple stress fluid theory presents models for fluids whose microstructure is of mechanical significance. The effect of very small microstructure in a fluid can be felt if the characteristic geometric dimension of the problem considered is of the same order of
magnitude as the size of the microstructure. The main feature of couple stresses is to introduce a size dependent effect. Classical continuum mechanics neglects the size effect of material particles within the continua. This is consistent with ignoring the rotational interaction among particles, which results in symmetry of the force-stress tensor. However, in some important cases such as fluid flow with suspended particles, this cannot be true and a size dependent couple-stress theory is needed. The spin field due to micro rotation of freely suspended particles set up an antisymmetric stress, known as couple-stress, leading thus to couple-stress fluid. These fluids are capable of describing various types of lubricants, blood, suspension fluids etc. The study of couple-stress fluids has applications in a number of processes that occur in various industries such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, and colloidal solutions etc.

The most non-Newtonian fluids of practical interest are highly viscous and therefore, are often processed in the laminar flow regime. The fluids containing a microstructure such as those containing additives suspensions, granular matter or long-chained polymers will explain the peculiar behaviors of non-Newtonian fluids; the micro-continuum theory proposed by Stokes (1966) is the simplest generalization of the classical theory of fluids which allows for polar effects such as the presence of and anti-symmetric stress tensor, couple stresses and body couples. Moreover, the couple stress fluid model is one of the numerous models that were proposed to describe response characteristics of non-Newtonian fluids. The constitutive equations in these fluid models can be very complex and involve a number of parameters, also the resulting flow equations lead to boundary value problems in which the order of the equations is higher than the Navier – Stokes equations.

The dynamics of flows through porous media has been a topic of considerable interest for the last one and half centuries, since Darcy (1856) postulated his famous law describing the motion of a viscous fluid through a porous medium. These flows are of prime importance in various branches of Science and Technology. The porous
flows occur in absorption and filtration process in chemical engineering. The theory of flow through porous media is applied in petroleum engineering in the extraction of oil from the petroleum reservoirs. This subject has wide spread applications to specific problems used in civil engineering and agricultural engineering and in many industries. The porous flows have applications in geothermal engineering and in bio fluid mechanics also. It is applicable in the field of energy extraction from geothermal region and in the heat removal from nuclear fluid debris. In biomechanics the study of flows past a permeable bed is useful in explaining blood circulation in lungs. The space occupied by the blood in the lung is idealized into a two dimensional channel and the interstitial tissue into a porous medium. The epithelial tissues between the air and vascular space are less permeable and hence be treated as impermeable membranes.

Fundamental studies associated with natural convection in porous media have increased substantially over the past decades because of the importance of porous media in diverse technological and industrial applications such as migration of pollutants, storage of nuclear waste, oil recovery enhancement, thermal insulation, electronics cooling and packed bed chemical reactors. The coupling of free flow with porous media flow appears in environmental applications, for instance when the transport of a pollutant from a river into the ground water is modeled, as well as in technical applications to filters and catalyzers. The copious literature available on the problem of thermal convection in porous media has been well documented by Nield (1984), Vafai and Tien (1989), Kaviany (1991), Vafai (2000, 2005) , Ingham and Pop (2005) and Nield and Bejan (2013). The hydrodynamic stability of shear flows through a porous media has been analyzed by Nield (2003), Awartani and Hammad (2005), Lauga & Cossu (2005) Makinde & Mhone (2007), Makinde (2009), Hill and Straughan (2010), Rudraiah et al (2011) and Shankar et al. (2014, 2015).

The objective of this thesis is to investigate the stability of the plane Poiseuille flow by applying linear stability theory. In particular the study is made to investigate the effect of an externally applied transverse magnetic field on the stability of plane Poiseuille flow with and without porous media in Newtonian and non-Newtonian
fluids. To do this the Navier-Stokes equations are used as they describe the motion of fluids. These equations are coupled with Maxwell’s equations to study the magnetohydrodynamic side of the problem. In the case of natural convection flow, we use energy equation coupled with Navier-Stokes equations. The reason that Navier-Stokes equations are so useful is because their solutions are velocity fields which describe the velocity at any given point in space and time. To model the fluid motion more accurately appropriate boundary conditions must be applied along with certain assumptions such as the conservation of mass and fluid is a continuum, incompressible and is Newtonian/non-Newtonian.

In the literature, considerable work has been done to find the stability of parallel flows in ordinary fluids. But much attention has not been given to stability of Newtonian and non-Newtonian fluid flows with induced magnetic field through a vertical and horizontal porous layer. The scope of the thesis lies in determining the stability of shear flows in Newtonian and non-Newtonian electrically conducting fluids. In particular, the following problems have been investigated in this thesis:


3. The Stability of the modified plane Poiseuille flow through horizontal Porous layer in the presence of a longitudinal Magnetic field - A numerical study.


Chapter-1

The pressure or buoyancy driven flow instability either in a fluid layer or in a fluid saturated porous layer in the presence of vertical/horizontal uniform magnetic field has been a subject of extensive theoretical and practical importance in the recent past because of its comprehensive importance in various naturally occurring phenomena as well as in many science and engineering applications. In what follows, we briefly survey the literature pertaining to the studies undertaken in this thesis.

Hydrodynamic Stability of Shear Flows

The field of hydrodynamic stability has a long history, going back to Reynolds and Lord Rayleigh in the late 19th century. The pioneers in the theoretical study of hydrodynamic stability includes some of the most illustrious names in Physics such as Helmholtz (1868), Kelvin (1871), Rayleigh (1880), and Reynolds (1883). It is the first three that are generally credited with the development of classical linearly primary stability theory for their work on the inertial instability of incompressible fluids of constant density. Although it was Reynolds (1883) experimental work that really stimulated the systematic study of viscous shear flows.

Rayleigh (1880) did his most famous work on thermal instabilities, in which he modeled Benard’s experiments by determining the equations of motion and boundary conditions and deriving the linear equations for normal modes. From this he discovered the dimensionless number the bears his name, Rayleigh number which is associated with buoyancy driven flows and is the ratio of buoyancy forces and it is the product of the Grashof and Prandtl number. Below a critical Rayleigh number the primary form of heat transfer is conduction and above it is convection. Although his greatest insight was to come in his famous theorem on the role of inflexion points in inviscid flows, known as the Rayleigh criteria.

In Reynolds classic 1883 paper he writes of his experiments where he used three tubes of diameter 1, ½ and ¼ inches with all three being 4 feet 6 inches long submerged in a large glass tank full of water. Water was then drawn through the tubes
which were fitted with trumpet mouthpieces, so the water might enter without disturbance and arrangements made so a streak of dye can enter the tubes with the clear water. He observed at low velocities the streaks would remain in a straight line throughout the tube. If the water in the tank was disturbed before entering the tubes the streaks would move about in the tubes but with no apparent sinuosity that is a regular meandering pattern. As the intake velocity was incrementally increased he observed at some point the band of color would strongly diffuse throughout the water with the distinctive turbulence pattern of curls and eddies, he also found as the intake velocity was increased the point in the tube where the turbulence begins became closer to the trumpet. To quantify these results he formulated the dimensionless parameter known today as the Reynolds number which is the ratio of inertial forces to viscous forces. Reynolds observed there was no critical value of $Re$ below which the flow was stable and above the flow was unstable but rather the critical velocity was sensitive to the disturbance in the water before entering the tube. It was this that led Reynolds to realize that it was a stability problem. The method of normal modes for studying the oscillations and instability of dynamic systems of particles and rigid bodies was already highly developed in Reynolds time. The equations were linearized by neglecting the products of perturbations and resolved into their independent components. It was then Stokes, Kelvin and Rayleigh that developed the method of normal modes for inviscid flows in fluid dynamics by essentially using partial rather than ordinary differential equations. It is fair to say that inviscid theory is now reasonable complete, both physically and mathematically.

Due to the difficulties of integrating the equations of motion for a viscous fluid the viscosity term was neglected until 1905 (Prandtl). In Prandtl 1905 paper he considered the flow of a fluid with low viscosity with the clear recognition that the main effect was the shear forces in the boundary layer. The greatest recognized advance of the time in the context of the linear stability of parallel shear flows was made by Orr (1907) and Sommerfeld (1908) who independently solved the linearized form of the Navier-Stokes equation for the perturbation velocity field. They both
considered small travelling wave type disturbance of the steady parallel flow and derive the so called Orr-Sommerfeld equation. Early attempts to solve the Orr-Sommerfeld equation proved for more complicated than had been anticipated due to the partial differential equations and the underdeveloped methods of asymptotic analysis. One of the first to try and solved the Orr-Sommerfeld equation was Taylor for Couette flow, he found it to be stable for all Reynolds numbers in apparent disagreement with experiments although with modern apparatus this is known to be correct for infinitesimal disturbances. Next he turned his attention to parallel shear flows and found that viscosity plays a dual role. Firstly it has the expected stabilizing effect whereby it dissipates energy. He adumbrated that viscosity also has a more complicated effect of diffusing momentum, which in shear flows as a destabilizing effect.

The asymptotic solution of Heisenberg (1924) to the Orr-Sommerfeld equation showed that inviscidly stable flows can be unstable at large but finite Reynolds numbers. Tollmien (1929, 1935) and Schlichting (1933) developed Heisenberg’s theory and estimated, for the blasius boundary layer, a value for the critical Reynolds number and obtained the normal variation of the wave disturbances. These infinitesimal wave disturbances are called Tollmien-Schlichting waves.

Squire (1933) showed that the first wave to become unstable as the Reynolds number was increased was always two-dimensional. Consequently the two-dimensional perturbations had been extensively studied. This stability problem of ordinary hydrodynamics has received considerable attention over several decades, and, in particular, the work of Heisenberg (1924) and Tollmien (1929, 1935) must be mentioned.

Later, Tollmien (1947), Lin (1945), Meksyn (1946, 1948) and Holstein (1950) have examined the equations of hydrodynamical stability in considerable mathematical detail. The particular problem of the stability of parabolic flow between
parallel planes has been solved by Heisenberg, Lin and Meksyn using asymptotic series methods.

The differences between the ideas and results of these workers and those of Pekeris (1948) have been resolved analytically by Tatsumi (1952) in favour of the work of Heisenberg, Lin and Meksyn. Thomas (1952) has confirmed Lin's numerical results with a solution on a high-speed computing machine. The above work is concerned only with infinitesimal disturbances, but by taking into account certain non-linear terms, Meksyn and Stuart have shown that the critical Reynolds number decreases as the amplitude of the disturbance increases.

Thomas (1953) has used numerical methods to obtain the eigenvalues with high accuracy for a number of values of the disturbance wave-number and flow Reynolds number. All of these solutions, however, are for the least stable eigenmode only, and most of them are restricted to values of the wave-number and Reynolds number which are close to the stability boundary.

To our knowledge, only Southwell & Chitty (1930), Grohne (1954) and Gallagher & Mercer (1964) have reported calculations of any of the higher eigenvalues for any plane parallel flow. Grohne has also calculated the first four eigenvalues for plane Poiseuille flow. The particular problem of the stability of parabolic flow between parallel planes has been solved by Heisenberg, Lin and Meksyn using asymptotic series methods.

Orszag (1971) reconsider the problem of stability of plane Poiseuille flow, using expansions in Chebyshev polynomials to approximate the solutions of the Orr-Sommerfeld equation. He obtained the results that are considerably more accurate than those obtained previously and concluded that the critical Reynolds number is 5772.22 and critical wave number is 1.02056 for instability of plane Poiseuille flow.
The experiments of Nishioka, Iida and Ichikawa (1975) were the first to experimentally verify the linear stability theory, in fact they were able to maintain Laminar flow up to $Re = 8000$ for plane Poiseuille flow, far higher than linear theory predicts. The reason that laminar flow was possible at such Reynolds numbers was attributed to the fact that the growth rate of unstable special disturbances is small. Consequently their channel was too short for the disturbances to grow sufficiently to cause transition. They experimentally determined the amplification rate and plotted it against the angular frequency and concluded $Re_c \approx 6000$, in agreement with the linear theory. The width to depth ratio or aspect ratio they used was 27.4 and the background turbulence was kept down to 0.05% They found that the linear transition is preceded intermittent irregular velocity fluctuations similar to the turbulent spots or bursts observed in Blasius flow transition. As a prelude to each burst there was always a sinusoidal velocity fluctuation, with frequencies just outside the upper branch of the linear neutral curve. Before the remarkable experiments of Nishioka et al the experimental attempts to verify linear theory for plane Poiseuille flow had all failed due to technical difficulties of minimizing the background turbulence.

Herbert (1977) computed the threshold amplitudes above which two-dimensional disturbances grow for a various sub critical Reynolds numbers ($Re < 5772$). His results agree qualitatively with the experimental data of Nishioka et al (1975) at low disturbance frequencies. At high frequencies, the computation showed that the threshold increases monotonically with frequency. On the other hand, the experiment showed that, at high frequencies, the threshold reached a local maximum before dropping rapidly on further increase of the frequency. This may have been due to three-dimensional effects present in the experiment. Later in 1980 Kozlov and Ramazonov were able to obtain similar results to those of Nishioka et al even though with a longer channel.
Plane MHD flow

In recent years, the subject of magneto-hydrodynamics has received increasing attention. The unified velocity and magnetic field equations for an incompressible, viscous and electrically conducting fluid in motion in the presence of a magnetic field have been derived by Batchelor (1950), and he and Chandrasekhar (1951) have examined the subject of magneto-turbulence. Problems of magneto-hydrodynamic stability, that is, problems concerning the growth or decay of small disturbances in a fluid-flow-magnetic-field system, are of obvious importance, since comparison is readily made with the vast amount of information on the stability of non-conducting fluids.

The effect of a magnetic field on thermal instability has been attacked independently by Thompson (1951) and Chandrasekhar (1952). In addition, the latter (Chandrasekhar 1953) has extended Taylor's work on the stability of viscous flow between rotating cylinders to the case when there is a magnetic field along the axis of the cylinders. The above-mentioned work on magneto-hydrodynamic stability shows that the magnetic field has a stabilizing influence. This is a point which was made by Bullard (1949) and by Batchelor (1950). It is proposed in this paper to examine generally the stability of parallel flows and particularly the stability of parabolic flow between parallel planes.

The steady two-dimensional motion of a conducting fluid between parallel planes under a transverse magnetic field was the first problem in magnetohydrodynamics to be solved, by Hartmann and Lazarus (25). Stuart (102) then investigated the effect of a co-planer magnetic field and found that a parallel magnetic field led to a steady rise in $Re_c$ as the magnetic strength was increased. Lock (51) then investigated the effect of a perpendicular magnetic field and found it had a far stronger stabilizing effect because the principal effect of the magnetic field is to modify the velocity distribution where as in the parallel field case it only adds a Lorentz force term to the Orr-Sommerfeld equation. In all this early work as in this study the magnetic Prandtl number $Pr_m << 1$, thereby eliminating all the magnetic

- 16 -
fluctuation terms from the Navier-Stokes equations which also removes the magnetic boundary conditions, considerably simplifying the calculations. The Alfvén number \( Al \) must also be large enough for the Hartmann number \( M \) to retain its finite value when it is defined as \( M = Prm^{1/2} Re Al \) (see Roberts, 1967). This approximation has been made by several authors for various stability problems such as the aforementioned Stuart, Lock, Takashima (1996), as well as Kakutani (1964), Nagata (1998) to mention but a few. The magnetic Prandtl number is a dimensionless parameter that approximates the ratio of momentum diffusivity and magnetic diffusivity. Although this simplification had been called into question by Lock himself and later Takashima (1996) for large \( M \) values due to the high \( Re \) involved making the \( Re_m \) not so small compared with unity.

Potter and Kudchey (1973) repeated Locks work but without the simplifications and found that the fluid flow became more stable as \( Pr_m \) is increased although there do appear to be mistakes in their working. They also used the incorrect boundary conditions for the magnetic field perturbations. Takashima (1996) eliminated the mistakes and found that the transverse magnetic field had both stabilizing and destabilizing effect on the fluid flow when \( Pr_m \) is sufficiently small and that at a fixed value of \( M \) the flow becomes more unstable as \( Pr_m \) is increased.


The problem of buoyancy driven flow in a two dimensional shallow cavity with a transverse magnetic field has been proposed by Garandet et al (1992). It was
demonstrated that, in the high Hartmann number limit, the velocity gradient in the core was constant outside the two Hartmann layers at the vicinity of the walls normal to the magnetic field. The re-circulating part of the flow was studied by means of a series expansion. The instability in either a couple stress fluid layer or couple stress fluid saturated porous layer heated from below has been investigated including the external constraints such as magnetic field and/or rotation.

In 1994 Takashima analyzed the problem of stability of natural convection in a vertical layer of electrically conducting fluid which is confined between two parallel plates maintained at constant and different temperatures and is permeated by a transverse magnetic field. He found that the magnetic field has a stabilizing effect on the convection flow against both stationary and travelling wave disturbances. A recent numerical study (Gelfgat & Bar-Yoseph 2001) is also devoted effect of a magnetic field on the onset of oscillatory instability, but concerns the convective flows in a two-dimensional rectangular cavity.

**Natural convection**

Natural convection is the study of heat transport processes by fluid motion carrying energy with it as a result of the temperature difference between the fluid and the solid. Convection heat transfer consists of two main mechanisms due to both diffusion (random molecular motion) and bulk motion of the fluid. Convection heat transfer can be classified according to the nature for the flow as forced convection and free convection. Forced convection investigates the heat transfer between a moving fluid and a solid surface. The fluid flow is caused by an external means such as a fan, a pump, or atmospheric winds. Thus the flow has a nonzero streaming motion in the far field away from the solid surface. There are various types of forced convection such as duct flows and bodies immersed in a uniform stream.

One of the earliest studies of laminar-turbulent natural convection, by Saunders (1936), inferred the presence of turbulence in a flow adjacent to a heated
vertical surface from measured heat transfer characteristics. Interferograms of this same flow configuration, photographed by Eckert and Soehngen (1951), suggested for first time that the advent of turbulence in such flow was, in all likelihood, the amplification of initially small disturbances. The disturbances were seen to amplify in two-dimensional form, initially as a sinusoidal disturbance and later as a more complicated wave.

Natural convection in a vertical slot of finite height was first investigated analytically by Batchelor (1954). It was concluded that at low Rayleigh numbers heat is transferred across the slot primarily by conduction. At higher Rayleigh numbers, however, the existence of a new regime consisting of a thin boundary layer around an isothermal core was suggested, with convection the predominant mode of heat transfer. Hereafter these will be referred to as the conduction regime and boundary layer regime, respectively. Interferometric temperature measurements, performed with air by by Eckert and Carlson (1961) confirmed the existence of two such flow regimes, however in the boundary –layer regime a vertical temperature gradient was observed in the core. The same behavior was found in high Prandtlumber fluids by Elder (1965), who measured the velocity field as well as the temperature field.

The stability of natural convection in the conduction regime was first considered by Gershuni (1953), who obtained highly approximate curves of neutral stability for the case of stationary disturbances. Later this work has been extended by Birickh (1967) and by Rudakov (1967). The most complete work up to date is that Rudakov, who obtained results for Prandtl numbers up to ten. For this range of Prandtl number he found the instability to set in as stationary convection at a Grashof number of 7700 with a variation of at most 5 percent as a function of Prandtl number. Ostrach and Maslen (1961) discussed the stability of same problem with respect to travelling waves of the Tollmien-Schlichting type but did not present a complete solution. However, the instability of this flow with respect to travelling waves of very long wavelength was shown by Yuan (1966).
Convection stability in a vertical slot with differentially heated sidewalls has been studied by several authors. The results showed that the stability limit is a function of the Grashof and Prandtl numbers. For a small Prandtl number fluid ($Pr < 12.7$), the parallel flow undergoes a transition to a stationary multicell flow pattern when the Grashof number exceeds a critical value. This transition has been observed experimentally by Vest and Arpac (1969). The critical Grashof number is weakly dependent on the Prandtl number, having the approximate value $Gr = 7700\pm5\%$. For a high Prandtl number fluid ($Pr > 12.7$), the unstable parallel flow becomes a pair of oscillatory travelling waves moving in opposite directions, and the critical Grashof number decreases as the Prandtl number increases. Several researchers concluded that the instability of the basic flow for a small Prandtl number fluid is induced by the shear mode, while for a high Prandtl number fluid, instability is caused by the buoyant mode.

Gill and Kirkham (1970) analyzed the limiting case of infinite $Pr$ and also found travelling waves to be the cause of instability, irrespective of the level of stratification. Also, the numerical solutions of the steady state Boussinesq equations for $Pr = 1000$ have confirmed the existence of a steady, multicellular, secondary flow at values of $Gr$ much less than the critical value for travelling wave instability. In contrast, the computational and experimental work of Hart (1971) indicates that travelling-wave instability occurs in water ($Pr = 6.5$) if the vertical temperature gradient is sufficiently large. Birikh et al. (1969) and Gotoh & Mizushima (1973) found that the critical Grashof number for stationary instability increases with increasing vertical stratification, but their calculations were done for $Pr$ not greater than 7.5 and only for low to moderate levels of stratifications.

Bergholz (1978) studied the instability of steady natural convection of a steady stratified fluid between vertical surfaces maintained at different temperatures. The energetics of the critical disturbance modes also are investigated. The numerical results shows that, if the value of the Prandtl number is in the low to moderate range,
there is a transition from stationary to travelling-wave instability if the stratification exceeds a certain magnitude. However, if the Prandtl number is large, the transition, with increasing stratification, is from travelling-wave to stationary instability. The theoretical predictions are in excellent agreement with the experimental observations of Elder and of Vest & Arpaci for stationary instability, and in fair to good agreement with the experimental results of Hart, for travelling-wave instability.

Direct simulations of the multicellular flow between two vertical parallel plates, after the onset of instability have been carried out, with the aid of a computer, by Lee & Korpela (1983) and Lauriat & Desrayaud (1985), among others. In their researches the aspect ratio of the cavity was taken to be finite, and their goal was to elucidate the influence of the aspect ratio on the structure of the flow and the resulting heat transfer. Except for the study by Nagata & Busse (1983), the only study that take advantage of the property that in the vertical direction the multicellular flow in a very tall cavity is specially periodic, is by Gershuni & Zhukhovitskii (1976). Later, Arnon Chait & Korpela analyze the multicellular flow between two vertical parallel plates using a time-splitting pseudo spectral method. The steady flow of air, and the time periodic flow of oil are investigated and the dynamics of the process was discussed. The spectra of kinetic energy and thermal variance for air found to be smooth and viscously dominated. Similar spectra of oil are bumpier, and the dynamics of the time dependent flow are determined to be confined to the lower end of the spectral alone.

Takashima and Hamabata (1984) found that the transition from stationary to travelling-wave mode occurs at a certain value of $Pr$ between 12.4 and 12.5 for the stability of natural convection in a vertical slot. This transition value of $Pr$ has also been supported by Chen and Pearlstein (1989). Later, Fizumura (1990) showed that the value of $Pr$ is given by 12.45425644. The characteristics and the stability of two-dimensional buoyant-thermo capillary-driven flows in finite shallow cavities have been investigated for low-Prandtl-number fluids by Ben Hadid & Roux (1992). This
study has shown the existence of complex flow structures in the form of multicellular steady states and the transition to oscillatory convection has been observed.

Nonlinear stability analysis was performed by Fizumura and Mizushima (1991) for vertical layer and by Fuzimura and Kelly (1993) for an inclined layer. Takashima (1994) studied the effect of transverse magnetic field on the stability of natural convection of an electrically conducting fluid is confined between two parallel vertical plates. He concluded that the magnetic field stabilizes the convective flow due to the value of Prandtl number $Pr$ at the point of transition from stationary to travelling wave mode was found to decrease with increasing Hartmann number $M$. Hence the magnetic field effects are dominant if the fluid is highly electrically conducting.

Recently Shankar et al. (2014) studied the stability of natural convection in a vertical couple stress fluid layer and showed that the couple stress parameter has a destabilizing effect on the convective flow against stationary mode, while it exhibits a duel behavior if the instability is via travelling -wave mode. Shankar et al (2015) have studied the effect of horizontal alternating current field on the stability of natural convection in a dielectric fluid saturated vertical porous layer.

**Flow through Porous medium**

Natural convection in porous media has been widely studied and well documented in the literature both experimentally and numerically. Major developments have been made in modeling natural convection gas transport in porous media including several important physical aspects. Some of the studies have used what is now commonly known as Brinkman-Forchheimer-extended Darcy or the generalized model. Significant advances have been made in developing the momentum equation that governs the fluid flow in porous media starting from Darcy’s law to the generalized model. Darcy’s law revealed proportionality between the velocity and the applied pressure difference for low speed flow in an unbounded
porous medium. As such Darcy’s law does not account for inertial effects or no-slip condition at the wall. To account for the solid boundary, Brinkman’s equation which also known as Brinkman’s extension of Darcy’s law was developed. Brinkman’s equation incorporated two viscous terms. The first is the typical Darcy term and the second is similar to the Laplacian term. Darcy’s law is linear in the Darcy velocity, which holds for a sufficiently small velocity. At higher velocities, inertial effects become appreciable causing an increase in the form drag. Forchheimer equation was developed as an extension to the Darcy’s law to account for a quadratic drag. A generalized model for the fluid flow through a porous media was developed during the past couple of decades which accounts for the inertial and boundary effects, and the quadratic drag. These effects are incorporated by using this general flow model known as the Brinkman-Forchheimer-extended Darcy model.

Theoretical consideration of fluid flow in porous media has received great attention in recent decades. Most of the earlier studied (Straus & Schubert 1977, Storesletten & Pop 1996,) were based on Darcy’s, which states that the volume-averaged velocity is proportional to the pressure gradient. The Darcy model is shown to be valid under the conditions of low velocities and small porosity (Kassoy & Zebib, 1975). However, in many practical situations the porous medium is bounded by an impermeable wall, has higher flow rates, and reveals non-uniform porosity distribution in the near wall region, making Darcy’s law inapplicable.

To model a real physical situation better the Brinkman flow model is employed, since it can predict hydraulics through such hyper porous media as noted by Nield and Bejan (2006). The Brinkman model also takes into account the presence of a solid boundary through the addition of a viscous term in Darcy’s law and, furthermore, it is generally applicable for porous media with both low and high permeabilities (Brinkmann, 1984). The effects of variable viscosity in the instability of flow and temperature fields in a water saturated porous medium are discussed by Kassoy and Zebib (1975), Straus and Schubert (1977) and Grey et al. (1982). In all
these studies, theoretical investigation on the temporal stability of fluid flow in a saturated porous medium with respect to the Brinkmann model has not been discussed.

The hydrodynamic stability of flow of an incompressible fluid through a plane parallel channel or circular duct filled with a saturated sparsely packed porous medium modeled by Brinkmann has been discussed on the basis of analogy with a magnetohydrodynamic problem by Nield (2003). Awartani and Hamdan (2005) considered the stability of plane, parallel fully developed flow through porous channels and studied the effects of porous matrix and the microscopic inertia.

The influence of slip boundary conditions on the modal and nonmodal stability of pressure driven channel flows was studied by Lauga and Cossu (2005). By employing the Brinkmann model with fluid viscosity same as effective viscosity, Makinde (2009) investigated the temporal development of small disturbances in a pressure driven fluid flow through a channel filled with a porous media. The Brinkmann flow model is employed in order to obtain the basic flow velocity distribution. In some limiting cases he solved the eigenvalue problem by a spectral collocation technique with expansions in Chebyshev polynomials and found that a decrease in porous medium permeability have the effect of damping the disturbances and therefore eliminating the growth of any small disturbances in the flow field.

Recently Adesanya (2014) have investigated the problem of hydrodynamic stability analysis for variable viscous fluid flow through a porous medium. By assuming a periodic solution of the Squire form, a linearized fourth-order eigenvalue problem is obtained and solved using Adomian decomposition method (ADM). The results of the computation showed that increase in viscosity variation parameter stabilizes the flow while increase in porous permeability parameter has a destabilizing effect on the flow.
Couple stress fluid flow

Several models have been proposed to describe the physical behavior and properties of non-Newtonian fluids. These models exhibit a non linear relationship between the stress and rate of strain. Among these, couple stress fluids introduced by Stokes (1961) have distinct features, such as the presence of couple stresses, body couples and non-symmetric stress tensor. Stokes (1966) generalized the classical model to include the effect of the presence of the couple stresses and this couple stress fluid model has been widely used because of its relative mathematical simplicity compared with other models developed for the couple stress fluid. Stokes (1968) discussed the hydromagnetic steady flow of a fluid with couple stress effects.

Sharma and Thakur (2000) investigated thermal stability of an electrically conducting couple stress fluid saturated porous layer in the presence of magnetic field. They reported that the couple stress postpone the onset of stationary convection. Sunil et al (2002) have studied the stability of superposed couple stress fluids in a porous medium with magnetic effect. They derived a sufficient condition for the nonexistence of over stability.

In another work Sunil et al (2004) investigated the effect of suspended particles on double diffusive convection in a couple stress fluid saturated porous medium. They reported that for stationary convection, the stable solute gradient and couple stress have stabilizing effects. Sharma and Sharma (2004) studied the onset of convection in a couple stress fluid saturated porous layer in the presence of rotation and magnetic field. Shivakumara (2009) has studied onset of convection in a couple stress fluid saturated porous medium with nonuniform temperature gradients.

Work on stability of natural convection in a vertical fluid layer subsequently extended to non-Newtonian fluids is concerned only with viscoelastic fluids (see Gozum & Arpaci 1974, Takashima 1993). Jain and Stokes (1974) studied the effect of couple stresses in fluids on the hydrodynamic study of plane Poiseuille flow, while
effect of couple stresses on thermal convective instability is analyzed by many researchers (see Malashetty et al 2006, Gaikawad et al 2007, Sunil et al 2011 and Rudraiah et al 2011). Goel et al. (1999) have studied the hydro magnetic stability of an unbounded couple-stress binary fluid mixture under rotation with vertical temperature and solute concentration gradients. Recently Shankar et al (2014) studied the stability of natural convection in a vertical couple stress fluid layer and concluded that the couple stress parameter shows destabilizing effect on the convective flow against stationary mode, while it exhibit a dual behavior if the instability is via travelling-wave mode.