Chapter -5

The Stability of the modified plane Poiseuille flow through horizontal porous layer in the presence of a longitudinal Magnetic field - A numerical study.

5.1 Introduction

The instability of a hydrodynamic shear flow in a horizontal channel has been studied extensively by many authors and copious literature available on this topic has been documented in the book by Drazin and Reid (2004). The study of Poiseuille flow in a horizontal channel in the presence of magnetic field has received considerable attention owing to its importance in the astrophysical and geophysical contexts (see for example, Hains 1965, Hunt 1966). The effect of magnetic field on the stability of laminar flows of an electrically conducting fluid has been found theoretically in a number of cases: it is known to be generally of a stabilizing nature, and this has been confirmed qualitatively by experiments also. Using this assumption, Stuart (1954) examined the stability of plane Poiseuille flow with a parallel magnetic field. Hains (1965) investigated the similar kind of problem to study the influence of a coplanar magnetic field on the stability of a conducting fluid flowing between parallel planes. Four sets of stability diagrams were presented so that each stability curve will represent the effect of a given applied magnetic field, as only one of the four quantities in the Reynolds number is changed. The flow is always stable for initial disturbances of the field produced by passage of a pulsating current through walls of finite conductivity. Potter and Kutchey (1973) dealt with the problem discussed
by Lock (1955) and found that the fluid flow becomes more stable as magnetic Prandtl number increases while Takashima (1996) reexamined this problem under the appropriate boundary conditions on the magnetic field perturbations and to check the validity of Lock’s simplification. Besides, Makinde and Mhone (2007) studied the temporal stability of magnetohydrodynamic Jeffery-Hamel flows at very small magnetic Reynolds number. Proskurin and Sagalakov (2008) studied the stability of Poiseuille flow in the presence of a longitudinal magnetic field and investigated about the dependence of the critical Reynolds number on the electrical conductivity. They have found at large Reynolds numbers, a new branch of instability and a sudden change in the critical Reynolds number.

In recent years, much work has been dedicated to the area of hydrodynamic stability with porous media because of its relevance to a variety of situations occurring in engineering and nature. In petroleum industries, porous medium is used for oil recovery, filtration and cleaning of oil spills. In nuclear industries, porous medium is used for effective insulation and for emergency cooling of nuclear reactors. Study of flow through a porous medium is also of immense use in geothermal studies and in biomedical engineering problems to understand the transport processes in lungs, kidneys, cartilages in synovial joints and so on. Makinde (2009) investigated the temporal development of small disturbances in a pressure driven fluid flow through a channel filled with a saturated porous medium. The critical stability parameters were presented for a wide range of porous medium shape factor parameter. Recently, Shankar et al. (2014a) investigated the problem of stability of fluid flow in a Brinkman porous medium numerically using Chebyshev collocation method and they presented the stability characteristics of the system in detail. Straughan and Harfash (2013) studied a model for Poiseuille flow instability in a porous medium of Brinkman type. They have analyzed the effect of slip boundary conditions on the onset of instability.

Nonetheless, many problems of practical importance involve electrically conducting fluids. In particular, an electrically conducting fluid saturating a porous medium in the presence of a magnetic field is of general interest from the viewpoint of many applications. For example, the study of the interaction of the geomagnetic field with the electrically conducting field of the Earth’s crust, which behaves as a porous medium, is of great interest to geophysicists. In addition, in metallurgical applications involving continuous casting, the solidification of strands
can be improved by electromagnetic stirring in order to obtain better final mechanical properties (cf. Ni et al. 1993). Porous materials used in many technological applications such as heat exchangers, chemical reactors and fluid filters possess high permeability and porosity. For example, for permeabilities of compressed foams as high as $8 \times 10^{-6} \text{ m}^2$ and for a 1 mm thick foam layer, the equivalent Darcy number is equal to 8. In such situations, the Darcy model does not give satisfactory results and there is a need to consider non-Darcian effects to study the problem. Then the consideration of inertial effects is inevitable and they exhibit profound influence on the stability of convective flows. In addition, for a high porosity porous medium, Givler and Altobelli (1994) determined experimentally that $5.1 \leq \mu_e / \mu \leq 10.9$, where $\mu_e$ is the effective viscosity or the Brinkman viscosity and $\mu$ is the fluid viscosity. Therefore, it is instructive to consider the ratio of these two viscosities is different from unity.

The hydrodynamic stability of flow of an incompressible fluid through a plane-parallel channel or circular duct filled with a saturated sparsely packed porous medium has been discussed on the basis of an analogy with a magnetohydrodynamic problem by Nield (2003). Hill and Straughan (2010) clearly mention that stability of flow in a porous channel cannot be discussed on the basis of an analogy with a magneto-hydrodynamic problem. This is because, in the resulting stability equation the terms containing porous parameter and Hartmann number are not same unlike in thermal convection problems. Recently, Harfash (2015) studied convective movement of a reacting solute in a viscous incompressible occupying a plane layer in a saturated porous medium and subjected to a vertical magnetic field.

Motivated by scarcity of literature on the stability of MHD flows in porous media, the temporal development of small disturbances in a porous channel saturated with an electrically conducting fluid in the presence of a uniform longitudinal magnetic field is investigated. This chapter is structured as follows. In Section 5.2, the problem is formulated and the solution of the steady basic flow is obtained in section 5.3. The eigenvalue problem for temporal development of small disturbances is derived in section 5.4. The Galerkin technique is employed to solve the resulting eigenvalue problem is presented in section 5.5 and the results are discussed quantitatively in section 5.6.
5.2 Mathematical Formulation

We consider a horizontal sparsely packed porous medium saturated by an incompressible electrically conducting fluid in the presence of a uniform longitudinal magnetic field \( B_0 \) between two rigid parallel plates at \( z = h \) and \( z = -h \) (see Fig. 5.1). We take the origin midway between the plates and use rectangular coordinates \( x \) and \( z \), with the \( x \)-axis in the direction of the flow and the \( z \)-axis perpendicular to the plates. The fluid layer is permeated by a uniform externally applied magnetic field \( \vec{B}_e = (B_e, 0, 0) \) is along the channel axis. Figure 5.1 show the geometric arrangement of the problem described above.

The governing equations are:

Flow continuity:

\[
\nabla \cdot \vec{q} = 0. \tag{5.2.1}
\]

Momentum equation:

\[
\frac{1}{\varepsilon} \frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon^2} (\vec{q} \cdot \nabla) \vec{q} = -\frac{1}{\rho} \nabla P + \frac{\mu_e}{4\pi\rho} (\vec{B} \cdot \nabla) \vec{B} + \frac{\mu}{k\rho} \nabla^2 \vec{q} - \frac{\mu_f}{k\rho} \vec{q}. \tag{5.2.2}
\]

Magnetic continuity:

\[
\nabla \cdot \vec{B} = 0. \tag{5.2.3}
\]

Magnetic field equation:

\[
\frac{\partial \vec{B}}{\partial t} + \frac{1}{\varepsilon} (\vec{q} \cdot \nabla) \vec{B} = -\frac{1}{\varepsilon} (\vec{B} \cdot \nabla) \vec{q} + \lambda \nabla^2 \vec{B}. \tag{5.2.4}
\]

where \( \vec{q} = (u, v, w) \) denotes the velocity vector, \( \vec{B} = (B_x, B_y, B_z) \) denote the magnetic field vector, \( \rho \) the fluid density, \( P = p + \mu B^2 / 8\pi \) the total pressure, \( \lambda = 1/4\pi \mu \sigma \) the magnetic viscosity, \( \mu \) the magnetic permeability, \( \mu_f \) the fluid viscosity, \( \mu_e \) the effective viscosity, \( k \) the permeability, \( \varepsilon \) the porosity of the porous medium, \( \sigma \) the electrical conductivity and \( t \) the time. Let us render the above equations dimensionless using the quantities

\[
\vec{q}^* = \frac{\vec{q}}{u_0}, \quad \nabla^* = h \nabla, \quad t^* = \frac{t}{\varepsilon u_0}, \quad P^* = \frac{P}{\rho u_0^2}, \quad \vec{B}^* = \frac{\vec{B}}{B_0}, \tag{7.2.5}
\]

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where $u_0$ is the average base velocity. Equation (5.2.5) is substituted in Eqs. (5.2.1) to (5.2.4) to obtain (after discarding the asterisks for simplicity)

$$\nabla \cdot \bar{q} = 0,$$

(5.2.6)

$$\frac{\partial \bar{q}}{\partial t} + (\bar{q} \cdot \nabla) \bar{q} = -\varepsilon^2 \nabla \rho - \frac{Al}{2} \nabla \bar{B}^2 + A(\bar{B} \cdot \nabla) \bar{B} + \frac{\Lambda}{Re} \nabla^2 \bar{q} - \frac{\sigma^2}{Re} \bar{q},$$

(5.2.7)

$$\nabla \cdot \bar{B} = 0,$$

(5.2.8)

$$\frac{\partial \bar{B}}{\partial t} + (\bar{q} \cdot \nabla) \bar{B} = (\bar{B} \cdot \nabla) \bar{q} + \frac{1}{Rm} \nabla^2 \bar{B},$$

(5.2.9)

Here, $Re = u_0 h / \nu$ is the Reynolds number, $\nu = \mu / \rho$ is the kinematic viscosity, $Rm = 4 \pi \mu h u_0 / \nu \varepsilon^2$ is the magnetic Reynolds number, $Al = B_0^2 \varepsilon^2 / 4 \pi \rho u_0^2$ is the Alfvén number, $\Lambda = \varepsilon^2 \mu_c / \mu_f$ the ratio of effective and fluid viscosity and $\sigma = \varepsilon h / \sqrt{k}$ is the porous parameter.

### 5.3 Base flow

The base flow is steady, laminar and fully developed, that is, it is a function of $z$ only. With these assumptions, Eq. (5.2.7) reduces to

$$Re \varepsilon^2 \frac{dp_b}{dx} = \frac{d^2 u_b}{dz^2} - \sigma^2 u_b.$$  

(5.3.1)

The associated boundary conditions are

$$u_b = 0 \text{ at } z = \pm 1.$$  

(5.3.2)

Solving Eq. (5.3.1) using the above boundary conditions, we get

$$u_b = \frac{\cosh(\sigma)}{\cosh(\sigma_\rho)} \frac{\cosh(\sigma \rho z)}{\cosh(\sigma_\rho)} - 1.$$  

(5.3.3)
5.4 Linear Stability Analysis

To study the linear stability of fluid flow, we superimpose an infinitesimal disturbance on the base flow in the form

\[ u = u_b(z) + \varepsilon \frac{\partial \hat{\phi}}{\partial z} (x,z,t), \quad w = -\varepsilon \frac{\partial \hat{\phi}}{\partial x} (x,z,t), \quad p = p_b(x) + \varepsilon \hat{p}(x,z,t), \]

\[ B_z = 1 + \varepsilon \frac{\partial \hat{\psi}}{\partial z} (x,z,t), \quad B_z = -\varepsilon \frac{\partial \hat{\psi}}{\partial x} (x,z,t), \]  \hspace{1cm} (5.4.1)

where \( \hat{\phi} \) the stream function, \( \hat{\psi} \) is the magnetic stream function, \( u_b(z) \) and \( p_b \) is the steady state solution of the system and \( \varepsilon \) is a small quantity. These relations automatically satisfy the continuity equations (5.2.6) and (5.2.8). Substituting equation (5.4.1) into equations (5.2.7) and (5.2.9) and restricting our attention to two-dimensional disturbances, equating the coefficients of leading order \( (\varepsilon) \), we obtain

\[ \hat{\phi}_z + u_b(z)\hat{\phi}_{xx} - \tilde{u}(z)\hat{\phi}_x = -\frac{\partial \hat{\psi}}{\partial z} + \frac{\Lambda}{Re} \left( \hat{\psi}_{x} + \hat{\phi}_{xx} \right) - \frac{\sigma_p^2}{Re} \hat{\phi}_x, \] \hspace{1cm} (5.4.2)

\[ \hat{\psi}_z + u_b(z)\hat{\psi}_{xx} = -\frac{\partial \hat{\phi}}{\partial z} + AL(\hat{\psi}_{zz} + \hat{\psi}_{xx}) + \frac{\Lambda}{Re} \left( \hat{\phi}_{xx} + \hat{\phi}_{zz} \right) - \frac{\sigma_p^2}{Re} \hat{\psi}_x, \] \hspace{1cm} (5.4.3)

\[ \hat{\psi}_z + u_b(z)\hat{\psi}_{xx} = \hat{\phi}_x + \frac{1}{Rm} \left( \hat{\psi}_{xx} + \hat{\psi}_{zz} \right), \] \hspace{1cm} (5.4.4)

\[ \hat{\psi}_z + u_b(z)\hat{\psi}_{xx} = \hat{\phi}_x - u_b(z)\hat{\psi}_x + \frac{1}{Rm} \left( \hat{\psi}_{xx} + \hat{\psi}_{zz} \right). \] \hspace{1cm} (5.4.5)

We seek solution of the linearized system of equations (5.4.2) to (5.4.5) in the form

\[ \hat{\phi}(x,z,t) = \phi(x) e^{i(\alpha x - \omega t)}, \]

\[ \hat{\psi}(x,z,t) = \psi(z) e^{i\alpha x - \omega t}, \] \hspace{1cm} (5.4.6)

where \( \omega = \omega_c + i\omega_i \) is the wave speed, \( \omega_c \) is the phase velocity and \( \omega_i \) is the growth rate and \( \alpha \) is the horizontal wave number which is real and positive. The sign of \( \omega_i \) is indicative of growth or decay of the infinitesimal disturbances. If \( \omega_i > 0 \), then the system is linearly unstable and if
$c_i < 0$, then the system is linearly stable. Substituting equation (5.4.6) into equations (5.4.2) - (5.4.5), and eliminating the pressure term, we get the following stability equations.

\[
(u_b - c)(D^2 - \alpha^2)\phi - (D^2u_b)\phi - Al(D^2 - \alpha^2)\psi = \frac{\Lambda}{\iota \alpha \text{Re}} (D^2 - \alpha^2)^2 \phi - \frac{\sigma_p^2}{\iota \alpha \text{Re}} (D^2 - \alpha^2)\phi , \tag{5.4.7}
\]

\[
\frac{i}{\alpha \text{Re} \text{Pm}} (D^2 - \alpha^2)\psi = \phi - (u_b - c)\psi . \tag{5.4.8}
\]

where $\text{Pm} = \text{Rm} / \text{Re}$ is the magnetic Prandtl number.

The boundaries are rigid and the appropriate boundary conditions are:

\[
\phi = D\phi = \psi = 0 \quad \text{at} \quad z = \pm 1 . \tag{5.4.9}
\]

### 5.5 Numerical solution

Equations (5.4.7) and (5.4.8) together with the boundary conditions (5.4.9) constitute an eigenvalue problem which has to be solved numerically. The resulting eigenvalue problem is solved using Galerkin method. Accordingly, $\phi(z)$ and $\psi(z)$ are expanded in terms of Legendre polynomials in the form

\[
\phi(z) = \sum_{n=0}^{N} a_n \xi_n(z), \quad \psi(z) = \sum_{n=0}^{N} b_n \zeta_n(z), \tag{5.5.1}
\]

with the corresponding base functions

\[
\xi_n(z) = (1-z^2)^2 P_n(z), \quad \zeta_n(z) = (1-z^2)P_n(z),
\]

where, $P_n(z)$ is the Legendre polynomial of degree $n$ and $a_n$ and $b_n$ are constants. It may be noted that $\phi(z)$ and $\psi(z)$ satisfies the boundary conditions. Eq. (5.5.1) is substituted into Eqs. (5.4.7) and (5.4.8) and the resulting error is required to be orthogonal to $\xi_m(z)$ and $\zeta_m(z)$ for $m = 0,1,2,\ldots,N$. This gives

\[
\frac{\Lambda}{\text{Re}} \sum_{n=0}^{N} a_n \int_{-1}^{1} \left(\xi_n\xi' + 2\alpha^2 \xi_n \xi' + \alpha^4 \xi_n \xi' \right) dz + i\alpha \sum_{n=0}^{N} a_n \int_{-1}^{1} \left(D^2u_b \xi_n \xi' + \alpha^2 u_b \xi_n \xi' - u_b \xi_n \xi' \right) dz
\]

\[
- i\alpha Al \sum_{n=0}^{N} b_n \left(\zeta_n \zeta' + \alpha^2 \zeta_n \zeta' \right) dz + \frac{\sigma_p^2}{\text{Re}} \sum_{n=0}^{N} a_n \int_{-1}^{1} \left(\xi_n \xi' + \alpha^2 \xi_n \xi' \right) dz \ ,
\]

\[
= -i\alpha c \sum_{n=0}^{N} a_n \int_{-1}^{1} \left(\xi_n \xi' + \alpha^2 \xi_n \xi' \right) dz
\]
\[ i\alpha \sum_{n=0}^{N} a_n \int_{-1}^{1} \xi_n \zeta_m dz - \frac{1}{Re Pm} \sum_{n=0}^{N} b_n \int_{-1}^{1} \left( \xi_n \zeta_m \zeta_m + \zeta_n \zeta_m \right) dz - \]
\[ i\alpha \sum_{n=0}^{N} b_n \int_{-1}^{1} \zeta_n \zeta_m \zeta_m dz = -i\alpha c \sum_{n=0}^{N} b_n \int_{-1}^{1} \zeta_n \zeta_m \zeta_m dz \]  

in which the primed quantities denote differentiation with respect to \( z \).

The above equations form the following system of linear algebraic equations
\[ \tilde{A}X = c \tilde{B}X, \]

where \( \tilde{A} \) and \( \tilde{B} \) are the complex matrices, \( c \) is the eigenvalue and \( X \) is the eigenvector. For fixed values of \( \sigma_p, Pm \) and \( Al \), the values of \( c \) which ensure a non-trivial solution of Eq. (5.5.4) are obtained as the eigenvalues. The critical wave speed \( c_c \) and the corresponding critical Reynolds number \( Re_c \) and the critical wave number \( \alpha_c \) are determined for various values \( \sigma_p, Pm \) and \( Al \) following the procedure explained in Shankar et al. (2014). In the calculations, instead of the magnetic Reynolds number we used the magnetic Prandtl number \( Pm = \frac{Rm}{Re} \), which is directly proportional to the electrical conductivity.

### 5.6 Results and discussion

The numerical results are presented with the main objective of investigating the flow of fluid in the porous medium. The resulting eigenvalue problem of Orr-Sommerfeld type is solved numerically using Galerkin method with Legendre polynomials as trial functions. The parameters involved in the present study are the Alfvén number \( Al \), magnetic Prandtl number \( Pm \), Reynolds number \( Re \) and porous parameter \( \sigma_p \).

The convergence of the numerical method is tested for different sets of parametric values by varying the order of base polynomial \( N \) and results are shown in table 5.1. From the table it is observed that the results are found to remain consistent and accuracy improved up to four digits by taking 51 terms of the approximation in Galerkin method. Hence for further studies we have taken \( N=50 \) and ratio of effective to fluid viscosity \( \Lambda = 1 \). For code validation, we had to compare with published results for porous channel flow (Makinde 2009) in the absence of magnetic field,
the results so obtained for different values of porous parameter are compared in Table 5.2 and the results are found to be in good agreement. Also, when \( Al = 0, \Lambda = 1 \) and \( \sigma_p = 0 \), the Eqs. (5.4.7) and (5.4.8) reduces to Orr-Sommerfeld equation. For this nonporous domain case, it is seen that \( Re_c=5772.291851 \) and \( \alpha_c=1.020 \), which are in excellent agreement with those of Orszag (1974).

The magnetic Reynolds number and Alfven number are found to have no influence on the basic flow. Nonetheless, the porous parameter influences the same. Fig 5.2 show the influence of \( \sigma_p \) on the basic velocity \( U_b \), this figure indicate that velocity profiles are symmetric about \( z = 0 \) with maximum value along the centerline and minimum at the wall. From this figure it is also observed as porous parameter, \( \sigma_p \) increases from 1, 4, 8 to 10 the basic velocity profiles flattens out due to a gradual decrease in the porous medium permeability.

The neutral stability curves \( c_i(Re, \alpha) = 0 \) are can be seen in figures 5.3(a), (b) and (c) for various values of the parameters \( \sigma_p, Pm \) and \( Al \) respectively. In all these figures, the portions below each neutral stability curve is corresponds to stable region represented by \( c_i(Re, \alpha) < 0 \) and the portion above neutral stability curves that is \( c_i(Re, \alpha) > 0 \) represents unstable region. Also it is observed that the marginal stability curves exhibit different minima’s for different values of the parameters \( \sigma_p, Pm \) and \( Al \). From Fig. 5.3(a), it is clear that the unstable modes begin to exist for varying values of \( \sigma_p \). We observe that an increase in \( \sigma_p \) leads to an increase in the critical Reynolds number and a slight increase in the critical wave number. This means that the stable region in \( Re - \alpha \) plane increases as \( \sigma_p \) increases and thus they have stabilizing effect on the fluid flow due to decrease in the permeability of the porous medium. In Fig. 5.3(b) the stable region rapidly increases with decreasing the magnetic Prandtl number and a sudden stabilization of this flow can be observed. In Fig.5.3(c) an increase in the value of Alfven number \( Al \) for some values of the parameters can lead to increase in the critical Reynolds numbers can be seen and hence it has a stabilizing effect on the fluid flow.
Figure 5.4(a) shows variation of critical Reynolds number with $\sigma_p$, for different values of magnetic Prandtl number, when the Alfven number $A\ell = 0.05$. For $Pm = 0.001$, there is linear increase in the curve and as the curve progresses; it becomes steeper for certain value of $\sigma_p$, further it is observed that the curve decreases with increase in $\sigma_p$. The same trend is observed for $Pm = 0.01$, except that the curve decreases with $\sigma_p$, after reaching the maximum value of the $Re_c$. From these two curves it is observed that the critical Reynolds number is maximum for $Pm = 0.001$ followed by $Pm = 0.01$. The critical Reynolds number increases by small amount with increase in $\sigma_p$ for $Pm = 0.1$. For $Pm = 1$ and $Pm = 1.5$ the curve follows the same trend as that of the previous but with a lesser slope. Therefore increase in the value of $\sigma_p$ is to increases the critical Reynolds number and hence the flow becomes more and more stable. Fig. 5.4(b) shows the plot of critical wave number $\alpha_c$ as a function of the porous parameter $\sigma_p$ for different values of the magnetic Prandtl number $Pm$. These curves shows the effect of $Pm$ on the values of $\alpha_c$ and $\sigma_p$. From this figure it is clear that as $Pm$ increases leads to an significant increase in the values of $\alpha_c$. Fig. 5.4(c) depicts the variation of $c_c$ versus $\sigma_p$ for different values of the parameter $Pm$ is considered. From these curves it is observed that the critical wave speed $c_c$ decreases as the magnetic Prandtl number $Pm$ increases.

Fig. 5.5(a) illustrates the variation of critical Reynolds number with $\sigma_p$, for different values of Alfven number, when the magnetic Prandtl number $Pm = 0.01$ is fixed. For $A\ell = 0.05$, non linear increase in the curve, is observed and further, the Reynolds number decreases by small amount as $\sigma_p$ increases. A sharp increase in the curve is observed for $A\ell = 0.005$, which is almost linear, as the curve progresses; it becomes steeper for certain value of $\sigma_p$, further it decreases with increase in $\sigma_p$. For $A\ell = 0.0005$, the curve follows the same trend as that of the previous curve, but the peak Reynolds number is lesser than that for the curve for $A\ell = 0.05$ and $A\ell = 0.005$. Dissipation was shown to have a considerable effect on the stability of this flow. Fig. 5.5(b) shows the variation of critical wave number $\alpha_c$ with respect to the porous parameter $\sigma_p$, for different values of Alfven number $A\ell$. Three different values of $A\ell$ (=0.0005, 0.005 and
0.05) are taken for observation point of view. The critical wave number corresponding to \( Al = 0.0005 \) and 0.005 exhibit a decreasing trend initially with \( \sigma_p \) but increases with further increase of the same and finally becomes independent of the parameter \( Al \) with increasing \( \sigma_p \) can be seen. Although for \( Al = 0.05 \) the critical value of the wave number is independent initially with respect to \( \sigma_p \) but increases with further increase of \( \sigma_p \). Fig. 5.5(c) shows the plot critical wave speed versus porous parameter \( \sigma_p \) for fixed values of \( Al \) (=0.0005 and 0.05). It is observed that the value of \( c_c \) increases for \( Al = 0.05 \) with increasing the value of \( \sigma_p \) (= 0 to 4) but decreases with further increasing \( \sigma_p \) (= 6 to 8) and again increases with increasing the same. For \( Al = 0.0005 \), the value of \( c_c \) decays initially with increase of \( \sigma_p \) (=0 to 4) and gradually increases with increasing \( \sigma_p \).

![Diagram](image)

**Fig. 5.1:** Physical configuration

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Fig. 5.2: Basic flow profile with increasing values of $\sigma_p$. 
Fig. 5.3: Neutral stability curves.
Fig. 5.4: Variation of (a) critical Reynolds number $Re_c$, (b) critical wave number $\alpha_c$ and (c) critical wave speed $c_v$ with the porous parameter $\sigma_p$ for various values of magnetic Prandtl number $Pm$ when $Al=0.05$. 
Fig. 5.5: Variation of (a) critical Reynolds number $R_{c}$, (b) critical wave number $\alpha_{c}$ and (c) critical wave speed $c_{c}$ with the porous parameter $\sigma_{p}$ for various values of Alfvén number $Al$ when $Pm = 0.01$. 
| N  | \( A_l = 0.05 \) | \( P_m = 0.1 \) | \( \sigma_r = 4 \) | \( Re = 10000 \) | \( \alpha = 8 \) | \( c_r \)  | \( c_i \)  | \( A_l = 0.05 \) | \( P_m = 0.001 \) | \( \sigma_r = 10 \) | \( Re = 30000 \) | \( \alpha = 10 \) | \( c_r \)  | \( c_i \) \\ 
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Table 5.1: Convergence of the Galerkin method.

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<th>Present work</th>
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Table 5.2: Comparison of the eigenvalue of the most unstable mode when \( Re=20000 \), \( \alpha = 1 \), \( A=2 \).