CHAPTER 1

INTRODUCTION

1.1 General Introduction

Viscous flow through porous media has attracted the attention of Scientists and Engineers because of its important applications notably in the flow of oil through porous rock, the extraction of energy from geothermal regions, the filtration of solids from liquids and drug penetration through human skin. The knowledge of flow through porous media is useful in the recovery of crude oil efficiently from the pores of reservoir rocks by displacement with immiscible water. The prime interest in all these problems is to study heat and mass transfer through and past porous media. It is very interesting to study flow past a porous bed, because porous medium is used as insulating material for some type of nuclear reactors in which the appearance of connective flow of interstitial fluid can lead to the failure of insulation. In these situations it is necessary to find the effect of porous layer on the free flow above and below it. Moreover, most of the physiological systems involve movement of fluids in response to mechanical forces. The motility of spermatozoa and ova, propulsion of bacteria and protozoa and blood flow through circulatory systems of mammals are some examples of physiological situations. The characteristic of such mathematical models of the system are under consideration.

Heat transfer in free and mixed convection in vertical channels occurs in many industrial processes and natural phenomena. It has therefore been the subject of many detailed, mostly numerical studies for different flow configurations. Most of the
interest in this subject is due to its applications, for instance, in the design of cooling systems for electronic devices and in the field of solar energy collection.

1.2. Classification of fluids

1.2.1. Newtonian Fluid

If shear stress is linearly proportional to the rate of strain, the fluid is called as a Newtonian fluid. Newtonian behaviour has been observed in all gases, in liquids or solutions of materials of low molecular weight.

The constitutive equation for Newtonian fluid is

\[ \tau = \mu \dot{\gamma} \]

where \( \tau \) is the stress, \( \dot{\gamma} \) is the shear rate and \( \mu \) is the viscosity of the fluid.

1.2.2. Non-Newtonian Fluid

Non-Newtonian fluids generally exhibit a nonlinear relationship between the shear stress and the rate of strain. Foodstuffs (like banana juice, apple juice, chyme), blood, slurries, sperm, intra uterine fluid, etc. behave like non-Newtonian fluids.

In this thesis an attempt is made to study the following non-Newtonian fluids:

(a) Jeffrey Fluid

The Jeffrey model is relatively simpler linear model using time derivatives instead of convected derivatives, for example Oldroyd-B model does; it represents a rheology different from the Newtonian.

The constitutive equation for the Jeffrey fluid is
\[
\tau = \frac{\mu}{1 + \lambda_1} (\dot{\gamma} + \dot{\lambda}_2 \dot{\gamma})
\]

where \(\mu\) is the dynamic viscosity of the fluid, \(\dot{\gamma}\) is the shear rate, \(\lambda_1\) is the ratio of relaxation time to retardation time and \(\lambda_2\) is the retardation time and dots over the quantities denote differentiation with respect to time.

(b) Viscoelastic fluids (Second grade fluids)

Viscoelastic fluids can be modeled by Rivlin – Ericksen constitutive equation

\[
S = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2
\]

where \(S\) is the Cauchy stress tensor, \(p\) is the scalar pressure, \(\mu, \alpha_1\) and \(\alpha_2\) are the material constants, customarily known as the coefficients of viscosity, elasticity and cross - viscosity, respectively. These material constants can be determined from viscometric flows for any real fluid. \(A_1\) and \(A_2\) are Rivlin-Ericksen tensors and they denote, respectively, the rate of strain and acceleration. \(A_1\) and \(A_2\) are defined by

\[
A_1 = \nabla V + (\nabla V)^T
\]

and

\[
A_2 = \frac{dA_1}{dt} + A_1 (\nabla V) + (\nabla V)^T A_1
\]

where \(d / dt\) is the material time derivative and \(\nabla\) gradient operator and \((\ )^T\) transpose operator. The viscoelastic fluids when modeled by Rivlin-Ericksen constitutive equation are termed as second-grade fluids.
(c) Third Order Fluid

Third order fluid behaves like a non-Newtonian fluid. Its viscosity and all other material parameters (non-Newtonian coefficients) are taken constants. These material parameters of third order fluid appropriate to shear thinning.

The constitute equation for the third grade fluid is

\[ S = A_1 + \alpha_1 A_2 + \beta_1 A_3 + \beta_2 (A_4 A_2 + A_2 A_4) + \beta_3 (t, A_1^2) A_1 \]

\( \mu \) being the coefficient of shear viscosity \( \alpha_1, \alpha_2, \beta_1, \beta_2 \) and \( \beta_3 \) are material constants. The tensors \( A_1, A_2, A_3 \) are respectively given by

\[ A_1 = \nabla V + \nabla V^T \]

\[ A_2 = \frac{dA_1}{dt} + A_1 (\nabla V) + (\nabla V)^T A_1 \]

\[ A_3 = \frac{dA_2}{dt} + A_2 (\nabla V) + (\nabla V)^T A_2 \]

where \( \frac{d}{dt} \) is material derivative, \( V \) is the velocity and \( T \) is the superscript denotes the transpose.

1.3 Literature Survey

Recently, the study of non-Newtonian fluids has attracted much attention because of their practical applications. With the growing importance of non-Newtonian fluids in modern technology and industries, investigations of such fluids are desirable. A number of industrially important fluids including molten plastics,
polymers, pulps, foods and fossil fuels, which may saturate in underground beds are exhibits non-Newtonian behavior. Due to complexity of fluids, several non-Newtonian fluid models have been proposed. In the category of such fluids, second grade fluid is the simplest subclass for which one can hope to gain an analytic solution. Exact analytic solutions for the flows of non-Newtonian fluids are most welcome provided they correspond to physically realistic situations, as they serve a dual purpose. First, they provide a solution to flow that has technical relevance. Second, such solutions can be used as checks against complicated numerical codes that have been developed for much more complex flows. Various studies on the flows of non-Newtonian fluids have been made under different physical aspects. However some recent contributions in the field may be mentioned in Refs. (Fetecau and Fetecau[37], Hayat et al., [43], Chen et al., [24], Fetecau and Fetecau, [38], Tan and Masuoka, [78].

The theory of micropolar fluids introduced by Eringen[34,35] deals with a class of fluids which exhibit certain microscopic effects arising from the local structure and micro-motions of fluid elements. These fluids can support stress moments and body moments and are influenced by the spin-inertia. A subclass of these fluids, introduced by Eringen [36], is the micro polar fluids which exhibit the micro-rotational effects and micro-rotational effects and micro-rotational inertia. The importance of the study of conducting fluid flow influenced by the application of an external magnetic field in many problems of geophysical and astrophysical interest is well-known.

The Problem of convective flow in fluid saturated porous medium has been the subject of several recent papers. Interest in understanding the convective transport
processes in porous material is increasing owing to the development of geothermal energy technology, high performance insulation for building and cold storage, renewed interest in the energy efficient drying processes and many other areas. It is also interest in the nuclear industry, particularly in the evaluation of heat removal from a hypothetical accident in a nuclear reactor and to provide effective insulation. Compressive literature surveys concerning the subject of porous media can be found in the most recent books by Ingham and Pop [45], Nield and Bejan [55], Vafai [83], Pop and Ingham [59], and Bejan and Kraus [16]. Many studies related to non-Newtonian fluids saturated in a porous medium have been carried out. Dharmadhikari and Kale [28] studied experimentally the effect of non-Newtonian fluids in a porous medium.

Heat transfer in free and mixed convection in vertical channels occurs in many industrial processes and natural phenomena. It has therefore been the subject of many detailed, mostly numerical studies for different flow configurations. Most of the interest in this subject is due to its applications, for instance, in the design of cooling systems for electronic devices and in the field of solar energy collection. Some of the related papers on this topic, such as Aung and Worku [9], Cheng et al. Barletta [10,11], Barletta and Zanchini [12], Chamkha [20], El-Din [30], Boulama and Galanis [19], Barletta et al. [13], deal with the evaluation of the temperature and velocity profiles for the vertical parallel-flow fully developed regime. As is well known, heat exchanger technology involves convective flows in vertical channels. In most cases, these flows imply conditions of uniform heating of a channel, which can be modeled either by uniform wall temperature (UWT) or uniform heat flux (UHF) thermal boundary conditions. In all the above studies of free and mixed convection flow in vertical channels are based on the hypothesis that the fluids are Newtonian. However,
because of their fundamental and technological importance, theoretical studies of free, forced and mixed convection flow of non-Newtonian fluids in channels and tubes are very important in several industrial processes. Szeri and Rajagopal [77], have examined the flow of a third grade fluid between heated parallel plates caused by external pressure gradient and obtained similarity solutions of the energy equation, numerically. Chen and Chen [25], investigated the free convection flow along a vertical plate embedded in a porous medium. Rees [70] analyzed the effect of inertia on free convection over a horizontal surface embedded in a porous medium. Nakayama [54] investigated the effect of buoyancy-induced flow over a non-isothermal body of arbitrary shape in a fluid-saturated porous medium. A ray-tracing method for evaluating the radiative heat transfer in a porous medium was examined by Argento [5]. Akyildiz [4] have studied the flow of third grade fluid between heated parallel plates. Chamka et al. [21] have studied the fully developed free connective flow of - micropolar fluid between two vertical parallel plates analytically. Recently, Siddiqui et al. [74] have investigated the flow of a third grade non-Newtonian fluid between two parallel plates separated by a finite gap by using the Adomian decomposition method.

In classical unsteady heat transfer problems, the basic equations are derived from Fourier’s law of heat conduction, which results in a parabolic equation for the temperature field and an infinite speed of heat propagation, thus violating the principle of causality. Maxwell [50] derived the generalization of Fourier’s heat law for the dynamical theory of gases. Maxwell’s heat flux equation contains a term proportional to the time derivative of the heat flux vector multiplied by a constant relaxation time $\tau$ had a very small magnitude in Maxwell’s work, he took it to be zero. In justification he remarked, “The first term of this equation may be neglected,
as the rate of conduction will rapidly establish itself”. Ackerman et al. [3] established the second sound in solid helium, which gave a finite speed of propagation of thermal waves. Puri and Kythe [63] have studied the influence of generalized law of heat conduction, using the Maxwell- Cattaneo-Fox (MCF) model, on Stokes’ first and second problems for Rivlin-Ericksen fluids with non-classical heat conduction. Kythe and Puri [47] studied the unsteady MHD free-convection flows on a porous plate with time-dependent heating in a rotating medium. Puri and Kythe [61] have studied an unsteady flow problem which deals with non-classical heat conduction effects and the structure of waves in the Stokes’ second problem. Based on these, Ibrahem et al. [44] have investigated the thermal effects in Stokes’ second problem for unsteady micropolar fluid flow.

The motion of a viscous fluid caused by the sinusoidal oscillation of a flat plate is termed as Stokes’ second problem by Schliching [72]. Initially, both the plate and fluid are assumed to be at rest. At time $t = 0+$, the plate suddenly starts oscillating with the velocity $U_0 e^{i\omega t}$. The study of the flow of a viscous fluid over an oscillating plate is not only of fundamental theoretical interest but it also occurs in many applied problems such as acoustic streaming around an oscillating body, an unsteady boundary layer with fluctuations etc (Tokuda,[80]). Penton [58] has presented a closed-form to the transient component of the solution for the flow of a viscous fluid due to an oscillating plate. Puri and Kythe [64] have discussed an unsteady flow problem which deals with non-classical heat conduction effects and the structure of waves in Stokes’ second problem. Erdogan [33] analyzed the unsteady flow of viscous fluid due to an oscillating plane wall by using Laplace transform technique. Vajravelu and Rivera [84] discussed the hydromagnetic flow at an oscillating plate.

Much work has been published on the flow of fluid over an oscillating plate for
different constitutive models (Erdogan, [32]; Zeng and Weinbaum, [86]; Puri and Kythe, [64]; Asghar et al., [7] Ibrahem et al., [44].

The study of oscillatory flow of a viscous fluid in cylindrical tubes has received the attention of many researchers as they play a significant role in understanding the important physiological problem, namely the blood flow in arteriosclerotic blood vessel. Womersley [85] have investigated the oscillating flow of thin walled elastic tube. Detailed measurements of the oscillating velocity profiles were made by Linford and Ryan [48]. Unsteady and oscillatory flow of viscous fluids in locally constricted, rigid, axisymmetric tubes at low Reynolds number has been studied by Ramachandra Rao and Devanathan [68], Hall[41] and Schneck and Ostrach [73]. Haldar [40] have considered the oscillatory flow of a blood through an artery with a mild constriction. Several other workers, Misra and Singh [53], Ogulu and Alabraba [56], Tay and Ogulu [79] and Elshahed [31], to mention but a few, have in one way or the other modeled and studied the flow of blood through a rigid tube under the influence of pulsatile pressure gradient.

In view of these, we studied the non-Newtonian fluid flows through porous medium in different geometries with heat transfer.