Chapter 4

Performance analysis of Cognitive MAC Protocols with Markov Modeling

4.1 Introduction

This chapter develops a Markov model for performance analysis of IEEE 802.11 PSM based cognitive MAC protocols under ideal channel and saturation conditions. This is the first analytic work done for such protocols in multi-channel cognitive environment. The analysis is used to calculate channel capacity, MAC layer throughput and average MAC layer delays. This analysis is helpful in understanding the performance of these MAC protocols in different scenarios. It can also be used to select the design parameters for the protocols based on different user and/or network characteristics. This analysis can be used to improve the protocol designed in chapter 3.

The rest of this chapter is organized as follows. Section 4.2 presents the literature survey for performance analysis of MAC protocols. In section 4.3 the markov model for the performance analysis is developed. Section 4.4 presents the results for channel capacity, MAC layer throughput and average MAC layer delays. Finally, section 4.5 concludes the chapter.

4.2 Literature Survey

First, Bianchi [18] developed an analytic model to compute throughput for the IEEE 802.11 DCF based MAC protocol with the assumption of a finite number of terminals and ideal channel conditions in 2000. Later in 2002, Chatzimisios et al [50] carried out analysis of MAC layer delay observed in IEEE 802.11 DCF. In [51], analytic modeling is provided for IEEE 802.11 DCF based MAC protocols for cognitive radio networks with single spectrum band. This analysis provides results for throughput obtained by secondary users in CRNs. As IEEE 802.11 DCF based MAC protocol does not address the issues related to cognitive networks, this analysis may not be helpful in the design of cognitive MAC protocol.

In 2013, Chen et al [52] presented an analysis of CR-CSMA/CA protocol. This protocol does not provide a solution for the multi-channel hidden terminal problem. Pravati et al [53] presented the analytic model to compute the throughput, average delay and power consumption of IEEE 802.11 IBSS
in PSM under ideal channel and saturation conditions. This work only presented the analysis for single channel case and does not considered the cognitive environment.

4.3 Performance Analysis using Markov Model

As discussed in previous section, Bianchi presented an analytic model for the IEEE 802.11 DCF based MAC protocol. Bianchi’s model cannot be directly applied to the power saving mode (PSM) of IEEE 802.11. Pravati et al [53] presented the analytic model for power saving mode of IEEE 802.11. This model does not consider multi-channel environment. Also, Pravati’s model is not developed for cognitive MAC protocols and hence does not include probabilities related to cognitive environment, like spectrum unavailability. Analytic model for performance analysis of IEEE 802.11 PSM based cognitive multi-channel MAC protocols in not available in existing literature. This section proposes the analytic model for performance analysis of saturation throughput and MAC layer delay with the assumption of ideal channel conditions for IEEE 802.11 PSM based cognitive multi-channel MAC protocols.

4.3.1 System Model

The assumption of a fixed number of cognitive users, each having packets available in MAC buffer, and fixed ATIM window size is considered for the analysis. Single ATIM handshake is allowed for any transmitting cognitive user in an ATIM window. After successful ATIM handshake, a cognitive user can transmit multiple data packets in that beacon interval. All the spectrum bands are assumed to have same bandwidth. The activity of primary users on the spectrum band is modeled as an alternating sequence of ON and OFF periods. ON and OFF denote that the spectrum band is occupied and unoccupied by primary users, respectively. As shown in figure 4.1, ON and OFF periods are exponentially distributed with rate $\alpha$ and $\beta$.

![Activity model of primary users on spectrum band](image)

Figure 4.1: Activity model of primary users on spectrum band
The posterior probabilities for spectrum band being occupied and unoccupied by primary users are calculated using steady state analysis of continuous time markov chains [54].

\[
P(ON) = \frac{\beta}{\alpha + \beta} \quad (4.1)
\]
\[
P(OFF) = \frac{\alpha}{\alpha + \beta} \quad (4.2)
\]

Probability that transition from OFF state to ON state is seen by transmitted packet of the tagged secondary user, \(p_{inf}\), is calculated as:

\[
p_{inf} = P(X_{OFF} < \sigma)
\]
\[
= 1 - e^{-\beta*\sigma}
\]

where \(\sigma\) represents the duration of one time-slot.

### 4.3.2 Markov Model

Let \(s(t)\) be the stochastic process representing the backoff stage at slot time \(t\), \(c(t)\) be the stochastic process representing the backoff time counter at slot time \(t\) and \(l(t)\) be the stochastic process representing the backoff layer for a given station at slot time \(t\). The backoff stage, \(s(t)\), represents the retry limit to transmit an ATIM or data packet within a single beacon interval. The backoff layer, \(l(t)\), represents the number of beacon intervals used to successfully transmit an ATIM packet. IEEE 802.11 wireless LAN MAC & PHY specification [55] does not specify any limit on the number of re-transmission of ATIM packet for inter or intra beacon interval. This work assumes the re-transmission limit as 3 for both inter and intra beacon interval as specified by Jung et al [56]. The ATIM window and data window are modeled as a three state and two state discrete time markov model as shown in figure 4.2 and 4.3, respectively.

As only one data window is used for data transfer related to each successful ATIM packet transmission, \(l(t)\) is not needed to model data window. If OFF period is observed in spectrum sensing slot, then the channel is considered as available for data transmission. In figure 4.2 and 4.3, \(p_a\) and \(p_d\) represent the collision probabilities in ATIM window and data window. These probabilities are fixed for specified network size. \(q_a\) and \(q_d\) represent the probabilities that ATIM window or data window ends when secondary user is transmitting its packet. \(W_i\) represents the size of contention window at the \(i^{th}\) backoff stage. \(W_i = 2^iW_0\), where \(W_0\) represents the minimum contention window size given by \(W_0 = CW_{min} + 1\). In data transmission modeling, \((m + 1)\) represents the Station Short Retry Count (SSRC). According to IEEE 802.11 wireless LAN MAC & PHY specification value of SSRC is considered as 7. If current OFF period finishes during the transmission of data packet with probability \(p_{inf}\), then transmission is considered to be unsuccessful because of primary user activity. In the presented
model, $p'_a$, $p''_a$, $p'_d$ and $p''_d$ are given by:

\[
\begin{align*}
  p'_a &= (1 - q_a)(1 - p_a) \\
  p''_a &= p_a(1 - q_a) \\
  p'_d &= (1 - q_d)(1 - p_d)(1 - p_{inf}) \\
  p''_d &= (1 - q_d)p_{inf} + p_d - p_dp_{inf}
\end{align*}
\]  

(4.3)

The following notations are used in presenting the Markov model:

\[
\begin{align*}
  P\{(i_1, j_1, k_1)'|(i_0, j_0, k_0)\}' &= P\{s(t + 1) = i_1, c(t + 1) = j_1, l(t + 1) = k_1 \\
  &\quad |s(t) = i_0, c(t) = j_0, l(t) = k_0\} \\
  P\{(i_1, j_1)''|(i_0, j_0)''\}' &= P\{s(t + 1) = i_1, c(t + 1) = j_1 |s(t) = i_0, c(t) = j_0\}
\end{align*}
\]  

(4.4)
The non zero one-step transition probabilities of the Markov chain in figures 4.2 and 4.3 and their inferences are listed below:

- $P\{(i, j, k)'|(i, j + 1, k)'\} = (1 - q_a)$, for $i \in [0, 2], j \in [0, w_0 - 1], k \in [0, 2]$
  This equation indicates that the ATIM frame backoff counter decrements with probability $(1 - q_a)$ within the ATIM window.

- $P\{(0, j, k - 1)'|(i, j_0, k)'\} = \frac{2e}{w_0}$, for $i \in [0, 2], j \in [0, w_0 - 1], k \in [1, 2], j_0 \in [0, w_i - 1]$
  This equation indicates that at any backoff stage and for any backoff counter value if the ATIM window ends, the protocol tries to re-transmit the ATIM frame with backoff stage 0 in the next ATIM window.

- $P\{(0, j, 2)'|(i, j_0, 0)'\} = \frac{2e}{w_0}$, for $i \in [0, 2], j \in [0, w_0 - 1], j_0 \in [0, w_i - 1]$
  This equation indicates the third unsuccessful transmission of an ATIM frame in one ATIM window. This means that the data cannot be sent in this beacon interval.

- $P\{(0, j, 2)'|(i, 0, k)'\} = (1 - p_a)(1 - q_a)$, for $i \in [0, 2], j \in [0, w_0 - 1], k \in [0, 2]$
  This equation indicates a successful transmission of an ATIM frame.

- $P\{(0, j, 2)'|(2, 0, 0)'\} = \frac{1}{w_0}$, for $j \in [0, w_0 - 1]$
  This equation indicates that at either an ATIM frame is transmitted successfully in the third beacon interval or the ATIM frame is discarded due to maximum trials reached.
\[ P\{(0, j, k - 1)|\{(2, 0, k)\}\} = \frac{p_a(1 - q_a)}{w_0}, \text{ for } j \in [0, w_0 - 1], k \in [1, 2] \]

This equation indicates that there is a collision at the last try within an ATIM window.

\[ P\{(i + 1, j, k)|\{(i, 0, k)\}\} = \frac{p_a(1 - q_a)}{w_i}, \text{ for } i \in [0, 1], j \in [0, w_{i+1} - 1], k \in [0, 2] \]

This equation indicates that after an unsuccessful transmission of an ATIM frame, the node increases the backoff stage for transmission of same ATIM frame and selects the backoff time counter uniformly.

\[ P\{(i, j)|\{(i, j + 1)\}\} = 1 - q_d, \text{ for } i \in [0, m], j \in [0, w_i - 1] \]

This equation indicates that within the data window, the data frame backoff counter decrements with probability \((1 - q_d)\).

\[ P\{(0, j)|\{(i, j)\}\} = \frac{p_a}{w_0}, \text{ for } i \in [0, m], j \in [0, w_0 - 1] \]

This equation indicates that at any backoff stage and for any backoff counter value if the data window ends, the protocol drops the data frame.

\[ P\{(0, j)|\{(i, 0)\}\} = \frac{p_d}{w_0}, \text{ for } i \in [0, m], j \in [0, w_0 - 1] \]

This equation indicates the successful transmission of a data frame.

\[ P\{(i + 1, j)|\{(i, 0)\}\} = \frac{p_d}{w_i}, \text{ for } i \in [0, m - 1], j \in [0, w_i - 1] \]

This equation indicates that the station increases the backoff stage and chooses the backoff counter uniformly after an unsuccessful transmission of a data frame within the data window.

\[ P\{(0, j)|\{(m, 0)\}\} = \frac{q_a + p_d}{w_0}, \text{ for } j \in [0, w_0 - 1] \]

This equation indicates the unsuccessful transmission of a data frame at the last backoff stage.

### 4.3.3 Model Analysis

Let \( b'_{i,j,k} = \lim_{t \to \infty} P\{s(t) = i, c(t) = j, \{t\} = k\} \) and \( b''_{i,j} = \lim_{t \to \infty} P\{s(t) = i, c(t) = j\} \) represents the steady state distribution of Markov models for ATIM window and data window, respectively.

The steady state equations follow the rules given in equation 4.5 and 4.6.

\[ b'_{i,j,k} = \frac{p''}{w_i} \sum_{l=0}^{w_i-(j+1)} (1 - q_a)^l \cdot b'_{i-1,0,k} \text{ for } i \in [1, 2] \text{ & } j \in [0, w_i - 1] \text{ & } k \in [0, 2] \]

\[ b'_{i,0,k} = (p_a')^i \left( \prod_{n=1}^{i} \frac{\sum_{l=0}^{w_n-1} (1 - q_a)^l}{w_n} \right) \cdot b'_{0,0,k} \text{ for } i \in [1, 2] \text{ & } j \in [0, w_i - 1] \]  \(4.5\)

\[ b''_{i,j} = \frac{p_d''}{w_i} \sum_{l=0}^{w_i-(j+1)} (1 - q_d)^l \cdot b''_{i-1,0} \text{ for } i \in [1, m] \text{ & } j \in [0, w_i - 1] \]

\[ b''_{i,0} = (p_d')^i \left( \prod_{n=1}^{i} \frac{\sum_{l=0}^{w_n-1} (1 - q_d)^l}{w_n} \right) \cdot b'_{0,0} \text{ for } i \in [1, m] \text{ & } j \in [0, w_i - 1] \]  \(4.6\)
Based on the above rules, steady state equation for all the states shown in Markov model are given by equations 4.7 and 4.8.

\[
b'_{i,j,k} = \begin{cases} 
\frac{1}{w_0} \left[ p'_d \sum_{i=0}^{2} \sum_{k=0}^{2} b'_{i,0,k} + q_a \sum_{i=0}^{2} b'_{i,0,0} + p''_a \sum_{i=0}^{2} \sum_{k=0}^{2} b''_{i,0,0} \right] & \text{for } i = 0, j = w_0 - 1, k = 2 \\
\frac{1}{w_0} \left[ p''_d b'_{2,0,k+1} + q_a \sum_{j=0}^{w_0 - 1} \sum_{i=0}^{2} w_i \sum_{j=0}^{1} b'_{i,j,k+1} \right] & \text{for } i = 0, j = w_0 - 1, k = [0, 1] \\
\frac{1}{w_0} \left[ p''_d b'_{2,0,k+1} + q_a \sum_{j=0}^{w_0 - 1} \sum_{i=0}^{2} w_i \sum_{j=0}^{1} b'_{i,j,k+1} \right] & \text{for } i = 0, j = w_0 - 1, k = [0, 1] \\
\frac{1}{w_i} \sum_{j=0}^{(j+1)} (1 - q_a)^j \sum_{i=0}^{w_i - 1} b''_{i-1,0,k} & \text{for } i = [1, 2], j = [0, w_i - 1], k = [0, 2] 
\end{cases}
\] (4.7)

\[
b''_{i,j} = \begin{cases} 
\frac{1}{w_0} \left[ q_d \sum_{i=0}^{m} \sum_{j=0}^{w_i - 1} b''_{i,j} + p'_d \sum_{i=0}^{m} \sum_{j=0}^{w_i - 1} b''_{i,0} + (1 - q_d)b''_{m,0} \right] & \text{for } i = 0, j = w_0 - 1 \\
1 - \sum_{i=0}^{m} \sum_{j=0}^{w_i - 1} b''_{i,j} & \text{for } i = [1, m], j = [0, w_i - 1] 
\end{cases}
\] (4.8)

As sum of all the state probabilities should be one, normalization equation for the model is shown in equation 4.9.

\[
\sum_{i=0}^{2} \sum_{j=0}^{2} \sum_{k=0}^{2} b'_{i,j,k} = 1 \\
\sum_{i=0}^{m} \sum_{j=0}^{w_i - 1} b''_{i,j} = 1
\] (4.9)

Let \( \tau_a \) is the probability that a station transmits ATIM packet in randomly chosen slot. As transmission of ATIM packet occurs when backoff time counter is zero, \( \tau_a \) is given by equation 4.10.

\[
\tau_a = \sum_{i=0}^{2} \sum_{k=0}^{2} b'_{i,0,k}
\] (4.10)

However, \( \tau_a \) depends on the conditional collision probability in ATIM window. The relation between \( \tau_a \) and \( p_a \) is given by:

\[
p_a = 1 - (1 - \tau_a)^{N-1}
\] (4.11)

where \( N \) is the number of cognitive users. Values of \( \tau_a \) and \( p_a \) can be calculated by solving equations 4.7 - 4.11.

Similarly probability that a station transmits data packet in randomly chosen slot, \( \tau_d \), is given by:

\[
\tau_d = \sum_{i=0}^{m} b''_{i,0}
\] (4.12)
The relation between $\tau_d$ and $p_d$ is given by:

$$p_d = 1 - (1 - \tau_d)^N \times p_{as}^{N-1}$$  \hspace{1cm} (4.13)

where $p_{as}$ is the probability of successful transmission of ATIM packet in randomly chosen slot, given by equation 4.14.

$$p_{as} = \frac{N \times \tau_a \times (1-\tau_a)^{N-1}}{1-(1-\tau_a)^N}$$  \hspace{1cm} (4.14)

$p_{as}$ is calculated by the probability that exactly one ATIM packet is transmitted on condition that at least one cognitive user transmit the ATIM packet. Values of $\tau_d$ and $p_d$ can be calculated by solving equations 4.8 - 4.14.

### 4.3.4 Throughput Calculations

Let $S$ be the normalized system throughput, defined as a fraction of time the system is busy for successful transmission of data. To compute $S$, let us analyze the possibilities of what can happen to a considered time slot. Any considered time slot could be idle, could have collision or could have a successful transmission. Let $P_{idle}^d$, $P_{coll}^d$ and $P_{suc}^d$ represents the probabilities of randomly chosen slot in the data window is idle, leads to collision and leads to successful transmission, respectively. As the actual duration between two transmitting slots varies with the events occurred, the average length of data slot for successful transmission, $\sigma_{avg}^d$, is calculated with equation 4.15.

$$\sigma_{avg}^d = P_{idle}^d \sigma + P_{suc}^d T_s^d + P_{coll}^d T_c^d$$  \hspace{1cm} (4.15)

where $T_s^d$ and $T_c^d$ are average time the medium is sensed busy due to successful transmission and due to collision of data packet, respectively. $T_s^d$, $T_c^d$, $P_{idle}^d$, $P_{coll}^d$ and $P_{suc}^d$ can be expressed as:

$$T_s^d = T_{DIFS} + 3 \times T_{SIFS} + T_{RTS} + T_{CTS} + H + E[P] + T_{ACK} + \sigma$$  \hspace{1cm} (4.16)

$$T_c^d = T_{DIFS} + T_{RTS} + \sigma$$  \hspace{1cm} (4.17)

$$P_{idle}^d = (1 - P_{tr}^d)$$  \hspace{1cm} (4.18)

$$P_{suc}^d = P_{ds}^d P_{tr}^d$$  \hspace{1cm} (4.19)

$$P_{coll}^d = (1 - P_{ds}^d) P_{tr}^d$$  \hspace{1cm} (4.20)

where $T_{DIFS}$, $T_{SIFS}$ represents the DCF Inter-frame Space (DIFS) and Short Inter-frame Space (SIFS) duration. $T_{RTS}$, $T_{CTS}$ and $T_{ACK}$ represents the time required to transmit RTS, CTS and acknowledgement packets. $H$ is the size of packet header calculated as $H = PHYHeader + MACHeader\cdot E[P]$.
is the size of data payload. It has been assumed for the analysis that all the data packets have same size. \( P_{tr}^d \) is the probability that there is at least one data packet transmitted in the considered slot. Probability of successful data packet transmission is given by \( p_{ds} \). Similar to \( p_{as} \), \( p_{ds} \) is calculated by the probability that exactly one data packet is transmitted on condition that at least one cognitive user transmit the data packet. Expressions of \( P_{tr}^d \) and \( p_{ds} \) are given by equations 4.21 and 4.22.

\[
P_{tr}^d = 1 - (1 - \tau_d)^{N_p_{as}} \tag{4.21}
\]

\[
p_{ds} = \frac{N \cdot p_{as} \cdot \tau_d \cdot (1 - \tau_d)^{N_p_{as} - 1}}{P_{tr}^d} \tag{4.22}
\]

Saturation throughput for data window, \( S_{Data} \), can be calculated as:

\[
S_{Data} = \frac{\text{Payload information transmitted in a slot}}{\text{Average length of the slot}} = \frac{p_{ds} \cdot P_{tr}^d \cdot E[P]}{\sigma_{avg}^2} \tag{4.23}
\]

Based on the saturation throughput for data window, normalized saturation throughput for one channel in cognitive networks is given by equation 4.24.

\[
S = S_{Data} \cdot \frac{T_{BI} - T_{ATIM-Window}}{T_{BI}} \cdot P(OFF) \tag{4.24}
\]

where \( T_{BI} \) and \( T_{ATIM-Window} \) represents the duration of beacon interval and ATIM window, respectively. \( P(OFF) \) is used to consider the duration in which channel is free for cognitive user.

As cognitive networks supports simultaneous transmissions on different channels, the capacity and throughput of the system will increase in multi-channel environment. Let us assume that there are total \( M' \) channels available for cognitive network. The number of simultaneous transmissions can be calculated as:

\[
M = \min(\text{MaxRes}, M') \tag{4.25}
\]

where \( \text{MaxRes} \) is the maximum number of ATIM handshakes possible in ATIM window. To calculate \( \text{MaxRes} \), we first calculate the average length of ATIM slot for successful transmission, \( \sigma_{avg}^a \), as shown in equation 4.26.

\[
\sigma_{avg}^a = (1 - P_{tr}^a)\sigma + P_{tr}^a p_{as} T_s^a + P_{tr}^a (1 - p_{as}) T_c^a \tag{4.26}
\]

where \( P_{tr}^a \) is the probability that at least one ATIM packet is transmitted in the considered slot. \( T_s^a \) and \( T_c^a \) are average time the medium is sensed busy due to successful transmission and due to collision of
ATIM packet. Expressions for $P_a$, $T_s$ and $T_c$ are given in equations 4.27 to 4.29.

$$P_a = 1 - (1 - \tau_d)^N$$  \hspace{1cm} (4.27)

$$T_s = T_{DIFS} + T_{ATIM} + T_{SIFS} + T_{ATIM-ACK}$$  \hspace{1cm} (4.28)

$$T_c = T_{DIFS} + T_{ATIM}$$  \hspace{1cm} (4.29)

where $T_{ATIM}$ and $T_{ATIM-ACK}$ represents the time required to transmit ATIM packet and ATIM-ACK packet, respectively.

Once average length of slot for successful transmission is calculated, the maximum number of ATIM handshakes possible in ATIM window can be calculated as:

$$MaxRes = \frac{T_{ATIM-Window}}{\sigma_{avg}^a}$$  \hspace{1cm} (4.30)

Probability of $M$ successful transmissions of ATIM packets in ATIM window, $p_{as}^{(M)}$, assuming that there are $M$ simultaneous transmissions possible is given by:

$$p_{as}^{(M)} = \left(\frac{MaxRes}{M}\right)(p_{as})^{M}(1 - p_{as})^{MaxRes-M}$$  \hspace{1cm} (4.31)

System saturation throughput can be calculated by adding the throughput with 1 to $M$ simultaneous transmissions. Expression for calculating system saturation throughput $S^{(M')}$ and system capacity $C^{(M')}$ are given by equations 4.32 and 4.33.

$$S^{(M')} = \sum_{i=1}^{M} S \cdot p_{as}^{(i)} \cdot i \cdot Bandwidth$$  \hspace{1cm} (4.32)

$$C^{(M')} = S \cdot M \cdot Bandwidth$$  \hspace{1cm} (4.33)

### 4.3.5 Delay Calculations

MAC layer delay is a random value as shared opportunistic medium is used with random access mechanism in cognitive networks. Hence, a detailed analysis of MAC layer delay is required. MAC layer packet could be successfully transmitted or deleted due to maximum retry limit. In both the cases MAC layer notifies the successful transmission or deletion of MAC packet to upper layers. Thus, the MAC layer delay is defined as the time interval from the instant packet received by MAC layer to the instant notification about the final status of the packet sent. In this subsection, MAC layer delay calculations are presented in three parts. First part present the calculations for ATIM window, second part presents the calculations for data window and third part presents the calculations for average MAC layer delays.
4.3.5.1 Delay Calculations in ATIM Window

The probability that a ATIM packet is successfully transmitted in backoff stage \( i \) and backoff window \( k \) is given by:

\[
P_{\text{succ}}^a(i, k) = P'_a \times \tau^a(i, k) \quad (4.34)
\]

where \( \tau^a(i, k) \) represents the probability that a station tries to transmit ATIM packet in backoff stage \( i \) and backoff window \( k \). \( \tau^a(i, k) \) can be calculated with:

\[
\tau^a(i, k) = \begin{cases} 
(p''_a)^i & \text{if } k = 2 \\
((p''_a)^3 + q_a)(p''_a)^i & \text{if } k = 1 \\
(p''_a)^{3*2} + 2q_a(p''_a)^3 + q_a^2(p''_a)^i & \text{if } k = 0
\end{cases} \quad (4.35)
\]

Delay experienced by ATIM packet when it is transmitted in \( k^{th} \) ATIM window can be given by:

\[
D^a(k) = k \times T_{BI} + T_{ATIM-Window} \quad (4.36)
\]

Probability that a ATIM packet is dropped due to maximum retry limit reached, \( P_{\text{drop}}^a \), can be calculated with the probability that ATIM packet is not transmitted successfully in any of the backoff stage of any backoff window.

\[
P_{\text{drop}}^a = 1 - \sum_{i=0}^{2} \sum_{k=0}^{2} P_{\text{succ}}^a(i, k) \quad (4.37)
\]

Conditional probability that the backoff process of a ATIM packet transmission ends at backoff stage \( i \) and backoff window \( k \) given that the ATIM packet is transmitted successfully, \( P'_{\text{succ}}^a(i, k) \), can be calculated with:

\[
P'_{\text{succ}}^a(i, k) = \frac{P_{\text{succ}}^a(i, k)}{1 - P_{\text{drop}}^a} \quad (4.38)
\]

Let \( \overline{D^a_{\text{succ}}} \) represents the average delay experienced by successfully transmitted ATIM packet considering all transmissions and \( \overline{D'_{\text{succ}}} \) represents the average delay experienced by successfully transmitted ATIM packet considering only the successful transmissions. \( \overline{D^a_{\text{succ}}} \) and \( \overline{D'_{\text{succ}}} \) can be calculated by using equation 4.34, 4.36 and 4.38.

\[
\overline{D^a_{\text{succ}}} = \sum_{i=0}^{2} \sum_{k=0}^{2} P_{\text{succ}}^a(i, k) \times D^a(k) \quad (4.39)
\]

\[
\overline{D'_{\text{succ}}} = \sum_{i=0}^{2} \sum_{k=0}^{2} P'_{\text{succ}}^a(i, k) \times D^a(k) \quad (4.40)
\]
Probability that a ATIM packet is dropped due to maximum retry limit reached in backoff stage $i$ and backoff window $k$, $P^a_{\text{drop}}(i, k)$, is given by:

$$P^a_{\text{drop}}(i, k) = p^a \cdot \tau^a(i, k) \quad (4.41)$$

Conditional probability that the backoff process of a ATIM packet transmission ends at backoff stage $i$ and backoff window $k$ given that the ATIM packet is dropped due to maximum retry limit reached, $P^a'_{\text{drop}}(i, k)$, can be calculated with:

$$P^a'_{\text{drop}}(i, k) = \frac{P^a_{\text{drop}}(i, k)}{P^a_{\text{drop}}} = \tau^a(i, k) \quad (4.42)$$

Let $\overline{D^a_{\text{drop}}}$ represents the average delay experienced by ATIM packet when it is dropped due to maximum retry limit reached considering all the packets and $\overline{D^a'_{\text{drop}}}$ represents the average delay experienced by ATIM packet when it is dropped due to maximum retry limit reached considering only the dropped packets. $\overline{D^a_{\text{drop}}}$ and $\overline{D^a'_{\text{drop}}}$ can be calculated using equation 4.36, 4.41 and 4.42.

$$\overline{D^a_{\text{drop}}} = \sum_{k=0}^{2} \sum_{i=0}^{2} P^a_{\text{drop}}(i, k) \cdot D^a(k) \quad (4.43)$$

$$\overline{D^a'_{\text{drop}}} = \sum_{k=0}^{2} \sum_{i=0}^{2} P^a'_{\text{drop}}(i, k) \cdot D^a(k) \quad (4.44)$$

### 4.3.5.2 Delay Calculations in Data Window

Let $B_i$ is the value of backoff counter generated at stage $i$. The pdf of $B_i$ follows a uniform distribution of range $[0, CW_i]$. Let $B(v)$ is the sum of backoff slots generated from stage 0 to $v$, where $v \in [0, m]$. Therefore,

$$B(v) = \sum_{i=0}^{v} B_i \quad (4.45)$$

The pmf of $B(v)$, given by $Pr(B(i) = v)$, is a convolution of the pmfs of all $B_i$’s with $i \in [0, v]$. It can be noted that $B(v)$ is in the range of $[0, B(v)_{\text{max}}]$, where $B(v)_{\text{max}} = \sum_{i=0}^{v} CW_i$.  

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Probability that a data packet is successfully transmitted in backoff stage $i$, $P_{succ}^d(i)$, is given by:

$$P_{succ}^d(i) = (P_d')^i \times P_d \times P(OFF) \tag{4.46}$$

Let $P_{succ}^d(i, b)$ denote the probability that a data packet is successfully transmitted in backoff stage $i$ where the sum of backoff values up to stage $i$ is $b$. $P_{succ}^d(i, b)$ is given by:

$$P_{succ}^d(i, b) = P_{succ}^d(i) \times Pr(B(i) = b) \tag{4.47}$$

Probability that a data packet is dropped due to maximum retry limit reached, $P_{drop}^d$, can be calculated by the probability that data packet is not transmitted successfully in any of the backoff stage. Thus, $P_{drop}^d$ is given by:

$$P_{drop}^d = 1 - \sum_{i=0}^{m} P_{succ}^d(i) \tag{4.48}$$

Let $P_{succ}^{d'}(i, b)$ denotes the probability that the backoff process of a packet transmission ends at backoff stage $i$ where the sum of backoff values upto stage $i$ is $b$, given that the data packet is transmitted successfully. Expression for $P_{succ}^{d'}(i, b)$ is given by:

$$P_{succ}^{d'}(i, b) = \frac{P_{succ}^d(i, b)}{1 - P_{drop}^d} \tag{4.49}$$

Delay experienced by data packet when it is successfully transmitted in backoff stage $i$ where the sum of backoff values up to stage $i$ is $b$, $D_{succ}^d(i, b)$, can be calculated as:

$$D_{succ}^d(i, b) = b \times \sigma_{avg}^d + i \times T_c^d + T_s^d \tag{4.50}$$

Expressions for $\sigma_{avg}^d$, $T_c^d$ and $T_s^d$ are given in subsection 4.3.4.

Let $\overline{D_{succ}^d}$ represents the average delay experienced by successfully transmitted data packet considering all the transmissions and $\overline{D'_{succ}^d}$ represents the average delay experienced by successfully transmitted data packet considering only the successful transmissions. $\overline{D_{succ}^d}$ and $\overline{D'_{succ}^d}$ can be calculate by using equation 4.47, 4.49 and 4.50.

$$\overline{D_{succ}^d} = \sum_{i=0}^{m} \sum_{b=0}^{B(i1)_{max}} P_{succ}^d(i, b) \times D_{succ}^d(i, b) \tag{4.51}$$

$$\overline{D'_{succ}^d} = \sum_{i=0}^{m} \sum_{b=0}^{B(i1)_{max}} P_{succ}^{d'}(i, b) \times D_{succ}^d(i, b) \tag{4.52}$$
Probability that a data packet is dropped due to maximum retry limit reached where the sum of backoff values till it is dropped is \( b \), \( P^{d}_{drop}(b) \), can be calculated as:

\[
P^{d}_{drop}(b) = P^{d}_{drop} \times Pr(B(i) = b)
\]  
(4.53)

Conditional probability that the backoff process of a data packet transmission ends given that the data packet is dropped due to maximum retry limit reached, \( P^{d}_{drop} \), is given by:

\[
P^{d}_{drop} = \frac{P^{d}_{drop}(b)}{P^{d}_{drop}} = Pr(B(i) = b)
\]  
(4.54)

The delay experienced by data packet when it is dropped due to maximum retry limit reached where the sum of backoff values till it is dropped is \( b \), \( D^{d}_{drop}(b) \), can be calculated as:

\[
D^{d}_{drop}(b) = b \times \sigma^{d}_{avg} + (m + 1) \times T^{d}_{c}
\]  
(4.55)

Let \( D^{d}_{drop} \) represents the average delay experienced by dropped data packet considering all the packets and \( D^{d}_{drop} \) represents the average delay experienced by dropped data packet considering only the dropped packets. \( D^{d}_{drop} \) and \( D^{d}_{drop} \) can be calculated with equation 4.53, 4.54 and 4.55.

\[
\overline{D^{d}_{drop}} = \sum_{b=0}^{B(i)_{max}} D^{d}_{drop}(b) \times P^{d}_{drop}
\]  
(4.56)

\[
\overline{D^{d}_{drop}} = \sum_{b=0}^{B(i)_{max}} D^{d}_{drop}(b) \times P^{d}_{drop}
\]  
(4.57)

4.3.5.3 Combined MAC Layer Delays

Let \( \overline{D^{suc}_{ucc}} \) and \( \overline{D^{drop}} \) represents the average MAC layer delays experienced in ATIM window and data window for successfully transmitted and dropped data packet, respectively. \( \overline{D^{suc}_{ucc}} \) and \( \overline{D^{drop}} \) can be calculated as:

\[
\overline{D^{suc}_{ucc}} = \overline{D^{a}_{suc}_{ucc}} + \overline{D^{d}_{suc}_{ucc}}
\]  
(4.58)

\[
\overline{D^{drop}} = \overline{D^{a}_{drop}} + \overline{D^{d}_{drop}}
\]  
(4.59)

Let \( \overline{D^{notify}} \) represents the average MAC layer delay experienced by packets either dropped or successfully transmitted. Note that we should use the probabilities of ATIM/data packets transmitted successfully or dropped rather than the conditional probabilities while calculating \( \overline{D^{notify}} \). Thus, \( \overline{D^{notify}} \)
can be calculated as:

\[
D_{notfy} = \sum_{k=0}^{2} \sum_{i=0}^{2} P_{succ}^a(i, k) \ast D^a(k) \\
+ \sum_{i=0}^{m} \sum_{b=0}^{B(i1)_{max}} P_{succ}^d(i, b) \ast D_{succ}^d(i, b) \\
+ \sum_{k=0}^{2} \sum_{i=0}^{2} P_{drop}^a(i, k) \ast D^a(k) \\
+ \sum_{b=0}^{B(i1)_{max}} D_{drop}^d(b) \ast P_{drop}^d
\]  \hspace{1cm} (4.60)

Equation 4.60 can be simplified to equation 4.61.

\[
D_{notfy} = (1 - P_{drop}^d)D_{succ}^a + P_{drop}^d D_{drop}^a + (1 - P_{drop}^d)D_{succ}^d + P_{drop}^d D_{drop}^d
\]  \hspace{1cm} (4.61)

### 4.4 Analysis of throughput, capacity and delay

This section presents the performance analysis of CCM-MAC. Table 4.1 lists the values of the parameters used in the calculations.

#### 4.4.1 Model Verification

This subsection validates the proposed Markov model with the simulation results. The model results depends upon \( q_a \) and \( q_d \). Here \( q_a \) is the probability of reaching to the end of the ATIM window when a secondary user is transmitting its packet. This probability depends on network size and ATIM window size. Similarly, \( q_d \) is the probability of reaching to the end of the data window when a secondary user is transmitting its packet. \( q_d \) depends on the number of active sessions in data window and the size of data window. The number of active sessions in data window is equal to \( N \ast p_{as} \). Thus, for fixed size data window \( q_d = c \ast N \ast p_{as} \). For selecting the value of \( q_a \) and \( c \), theoretical results with various values of parameters are matched with the CCM-MAC NS2 simulation results with DSSS physical layer. Other parameters are selected from IEEE 802.11 wireless LAN MAC & PHY specification to model OFDM PHY layer specification. The proposed mathematical model is validated against simulation using NS2 as shown in figure 4.4. For simulation ATIM window size and number of channels are taken as 20ms and 10.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Payload</td>
<td>1024 bytes</td>
</tr>
<tr>
<td>RTS packet length</td>
<td>20 bytes</td>
</tr>
<tr>
<td>CTS packet length</td>
<td>14 bytes</td>
</tr>
<tr>
<td>ACK packet length</td>
<td>14 bytes</td>
</tr>
<tr>
<td>ATIM packet length</td>
<td>20 bytes</td>
</tr>
<tr>
<td>ATIM-ACK packet length</td>
<td>14 bytes</td>
</tr>
<tr>
<td>Time slot $\sigma$</td>
<td>$9 \mu s$</td>
</tr>
<tr>
<td>Beacon Interval ($T_{BI}$)</td>
<td>100 ms</td>
</tr>
<tr>
<td>SIFS ($T_{SIFS}$)</td>
<td>$16 \mu s$</td>
</tr>
<tr>
<td>DIFS ($T_{DIFS}$)</td>
<td>$T_{SIFS} + (2 * \sigma) = 34 \mu s$</td>
</tr>
<tr>
<td>Air Propagation Time</td>
<td>$1 \mu s$</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>1 Mbps</td>
</tr>
<tr>
<td>$CW_{min}$</td>
<td>15</td>
</tr>
<tr>
<td>$q_a$</td>
<td>0.002</td>
</tr>
<tr>
<td>$c$</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Table 4.1: Parameters Used

4.4.2 Markov Model Analysis

4.4.2.1 Effect of number of secondary users & number of channels in system

For this analyses, $\alpha$ and $\beta$ are taken as 0.5, i.e. half of the time the channel is occupied by primary user. The size of ATIM Window, $T_{ATIM-Window}$, is taken as 20 ms. Results for multi-channel system capacity and throughput are shown in figure 4.5 and 4.6. It is clear from figure 4.5 that the capacity of system is not impacted by the number of cognitive users. It also shows that the capacity of system increases with the increase in the number of channels. Figure 4.6 shows that the throughput of system reduces with the increase in the number of users. This reduction is due to the increase of collision probability within the system. Figure 4.7 shows the MAC layer delay experienced by data packets. The average MAC layer delay includes transmission time and waiting time in MAC layer queue. If a node does not have a successful ATIM transmission then the data packet has to wait for the next beacon
interval to reserve a slot. As $D_{\text{disco}}$ only considers successful transmissions, the increase in the number of users does not have significant impact on its value. Probability of successful ATIM transmission decreases with the increase in user density. Packets are dropped if maximum retry limit is reached.
Figure 4.6: Effect of number of users and number of channels on Throughput

Figure 4.7: Effect of number of users and number of channels on Delay

Thus, $D_{\text{drop}}$ increases much faster with the increase in number of users. $D_{\text{notify}}$ considers packets that are being dropped or successfully transmitted. It also increases with the increase in the number of users.
4.4.2.2 Effect of ATIM Window Size

For analyzing the effect of ATIM window size, the number of cognitive users is considered as 50. Relation between system capacity and throughput with ATIM window size is shown in figure 4.8 and 4.9, respectively. It can be noted that if the size of ATIM window is too low then the number of successful ATIM handshakes will be less than the number of channels available for cognitive users. In such situations, capacity and throughput of system will be less. But if ATIM window size increases more than certain value then spectrum bandwidth is wasted. Thus, very large ATIM window size also limits the capacity and throughput performance. It can be observed that the size of ATIM window affects the performance of protocol and should be selected carefully. Figures also show that the required value of ATIM window size increases with the increase in the number of channels. Thus, the number of channels available for cognitive users should be considered while selecting the size of ATIM window for multi-channel system.

4.4.2.3 Effect of PU Activity

Figure 4.10, 4.11 and 4.12 shows the relation between primary user activity model with system capacity, throughput and average MAC delay. For this analysis, ATIM window size is considered as 20 ms and the number of cognitive users is considered as 50. As discussed in section 4.3, primary user
activity is modeled as an alternating sequence of ON and OFF periods with parameters $\alpha$ and $\beta$. With the increase in $\alpha$, availability of channel for cognitive user increases and hence system capacity and throughput increases. Also with the increase in $\beta$, primary user activity on system increases and hence system capacity and throughput decreases. It can also be seen from figure 4.12 that average MAC layer
Throughput of the system (Mb/s) with $M = 1$

$M = 10$

$M = 20$

$M = 50$

Figure 4.11: Effect of PU Activity on Throughput

Average MAC Layer Delay ($D_{notify}$)

Figure 4.12: Effect of PU Activity on Average MAC Layer Delay

delay decreases with the increase in $\alpha$ and increases with the increase in $\beta$. This analysis can be used to design and analyze systems with different primary user characteristics.
4.5 Conclusion

This chapter develops a discrete time Markov chain model for IEEE 802.11 PSM based cognitive MAC protocols. The theoretical analysis of system capacity, achievable throughput and MAC layer delays are presented. The analysis helps in understanding the performance of IEEE 802.11 PSM based cognitive MAC protocols in different scenarios. Presented results show that it is advisable to have flexible ATIM window size for fixed beacon interval system in cognitive networks, where ATIM window size should vary with the variation in available channels and number of users. Thus, this analysis can be used in designing better MAC layer protocol with dynamic ATIM window size.