# A SYNOPSIS

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Department of Mathematics,
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*Title*

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## ONSET OF INSTABILITY IN SOME MAGNETO FLUID DYNAMICAL SYSTEMS

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By

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# S Y N O P S I S

The present dissertation is constituted of five chapters and is devoted to a mathematical investigation of the problem of onset of linear instability in some mageneto fluid dynamical systems with primary emphasis on the development and elucidation of integral inequalities that play an important role in the characterization of marginal states of such systems.

Chapter I is introductory in nature and concises the essential results in the respective fields of magnetohydrodynamic simple Bénard convection, magnetohydrodynamic thermohaline convection of the Veronis type and magnetohydrodynamic thermohaline convection of the Stern type which concern the present dissertation . It is followed by a critical analysis of the above results. The analysis reveals some gaps in the literature of the subject matter; these are proposed to be filled up. The analysis also propounds chief motivations that form the basis of the present work. The chapter ends with a brief account of the results that we have derived and their significance in the context.

The contents of Chapter II are along lines suggested by Chandrasekhar (1952) in his celebrated paper wherein he predicted that in an appropriate parameter range the 'principle of exchange of stabilities' is valid in the problem of magnetohydrodynamic simple Bénard convection and that the total kinetic energy associated with a disturbance exceeds its total magnetic energy.An integral inequality of fundamental importance is developed in this chapter for quite general boundary conditions on the velocity and magnetic field in an appropriate parameter range by making use of the equation of magnetic induction only and it is shown that this result together with the result of Banerjee et al (1985) on the 'principle of exchange of stabilities' for the problem , comprehensively establishes Chandrasekhar's prediction that remained unresolved till recently.

The contents of chapter III extends the considerations of chapter II to a more general class of problems wherein two diffusive mechanisms, instead of a single one as in chapter II, on account of causes that constitute the density are simultaneously in operation. The specific problem chosen for the purpose here is that of magnetohydrodynamic thermohaline convection of the Veronis (1965) type, and an integral inequality of fundamental importance is developed for quite general boundary conditions on the velocity, concentration and magnetic field in an appropriate parameter range by making use of the equations of magnetic induction and concentration diffusion only. It is established that this result together with the result of Gupta et al(1986) on the 'principle of exchange of stablilities' for the problem proves that at the marginal state the total kinetic energy associated with a disturbance exceeds the sum of its total magnetic and concentration energy in the above parameter range.

The contents of chapter IV are exactly on the lines of chapter III except that here we investigate the problem of magnetohydrodynamic thermohaline convection of the Stern (1960) type which in essence represents a physical configuration that is gravitationally opposite to that treated in chapter III. An integral inequality of fundamental importance is developed in this chapter for quite general boundary conditions on velocity, temperature and magenetic field in an appropriate parameter range by making use of the equations of magnetic induction and heat diffusion only. It is established that this result together with the result of Gupta et al (1986) on the 'principle of exchange of stabilities' for the problem proves that at the marginal state the total kinetic energy associated with a disturbance exceeds the sum of its total magnetic and thermal energy in the above parameter range.

The contents of chapter V are primarily motivated by the famous $\pi^2$ Q-law that Chandrasekhar (1952,1961) derived for the asymptotic dependence on Q of the critical Rayleigh number in the problem of magnetohydrodynamic simple Bénard convection when both the bounding surfaces are free and the 'principle of exchange of stabilities' is valid. Chandrasekhar predicted from the results of his numerical calculations that the same power law with the same constants of proportionality would hold for more general bounding surfaces also but no mathematical proof to this effect has appeared in the literature as yet. In the present chapter we mathematically examine the validity of the above prediction of Chandrasekhar for more general bounding surfaces.

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