CHAPTER - 2

A PAIR OF SEMICIRCLE THEOREMS IN ROTATORY THERMOHALINE CONVECTION

2.1. Abstract

The investigation presented in this chapter is concerned with the problem of obtaining bounds for the complex growth rate of an arbitrary oscillatory perturbation, neutral or unstable, in the domain of infinitesimal amplitude instability in rotatory thermohaline convection. A pair of semicircle theorems, one each for both Stern's and Veronis' configurations, under the influence of a uniform rotation transverse to the fluid layer, are established in this connection. These results are new and are uniformly valid for all combinations of dynamically free and rigid boundaries.

2.2. Introduction

Pellew and Southwell (1940) have shown that ordinary Bénard convection first appears as an instability governed by the principle of exchange of stabilities.

Stern (1960) has treated the stability of a horizontal layer of fluid which is heated from above and in which the mass concentration of a chemical dissolved is maintained at C_0 at the lower boundary and C_1 at the

---

The contents of this chapter are sent for publication in Communications in Mathematical Physics, 1981.

upper boundary ($C_1 > C_0$). The temperature at the two boundaries is maintained at T_0 and T_1 respectively with $T_1 > T_0$. He has shown that even if the fluid in the undisturbed condition is lighter at the top than at the bottom, instability might still occur in the configuration as exchange of stabilities provided the destabilizing concentration gradient is sufficiently large but compatible with the condition that the total density field is gravitationally stable. The above investigation of Stern is restricted by the assumption that the 'principle of exchange of stabilities' is valid.

Veronis (1965) has treated the configuration in which $C_1 < C_0$ and $T_1 < T_0$ and has shown that even if the fluid in the undisturbed condition is lighter at the top than at the bottom instability might still occur in the configuration as overstability provided the destabilizing temperature gradient is sufficiently large but compatible with the condition that the total density field is gravitationally stable. The above investigation of Veronis is restricted by the assumption that the boundaries are dynamically free.

If a constraint such as rotation, is present in the configuration, instability may occur first as an overstable, i.e. time-dependent, maintained perturbation. This type of instability arises because of steady type of motion may be too restrictive in the sense that it cannot take advantage of sources of potential energy that are available to a time-dependent motion. A further fact, observed both theoretically and experimentally, is that these overstable motions, when they do occur, are generally less efficient in transporting heat and mass and in altering the mean gradients than are steady convective motions. Hence, one can conjecture that, when the determining parameter (usually a Rayleigh number) exceeds the critical eigenvalue, overstable motions will occur first ; as the parameter is increased further, so that steady convective instability can occur, the observed motions will be the latter.

The problem of obtaining bounds for the complex growth rate of an arbitrary oscillatory perturbation, neutral or unstable, in rotatory thermohaline configuration is thus important especially in situations when both the boundaries are not dynamically free so that exact solution in closed form is not available. The present investigation is concerned precisely with this problem and a pair of semi-circle theorems under the influence of a uniform rotation transverse to the fluid layer, are established in this connection. These results are new and are uniformly valid for all combinations of dynamically free and rigid boundaries.

2.3. Mathematical analysis

We can treat the above two configurations by considering the same configuration but with the assumption that gravity is positive downward in one problem and positive upward in the other problem. The relevant governing equations and boundary conditions in nondimensional forms are given by

$$(D^2-a^2)(D^2-a^2-p/\sigma)w = R_1 a^2\theta - R_2 a^2\varphi + TD\zeta, \qquad (2.3.1)$$

$$(D^2-a^2-p)\theta = -w, \qquad (2.3.2)$$

$$(D^2-a^2-p/\tau)\varphi = -\frac{w}{\tau}, \qquad (2.3.3)$$

$$(D^2-a^2-p/\sigma)\zeta = -Dw, \qquad (2.3.4)$$

and $w = 0 = \theta = \varphi = D^2w = D\zeta$ at $z = 0$ and $z = 1$

(both boundaries dynamically free), (2.3.5)

or $w = 0 = \theta = \varphi = Dw = \zeta$ at $z = 0$ and $z = 1$

(both boundaries rigid), (2.3.6)

or $w = 0 = \theta = \varphi = D^2w = D\zeta$ at $z = 1$)
(upper boundary dynamically free),)
and $w = 0 = \theta = \varphi = Dw = \zeta$ at $z = 0$) (2.3.7)
(lower boundary rigid))
without loss of generality.).

In the above equations, z is the vertical coordinate and $0 \le z \le 1$, $D \equiv \frac{d}{dz}$, a^2 is the square of the wave number, $p(= p_r + ip_i)$ is the complex growth rate, σ

is the Prandtl number, R_1 is the usual Rayleigh number, R_2 is the concentration Rayleigh number, T is the Taylor number, τ is the ratio of mass diffusivity to heat diffusivity, w is the vertical velocity, Θ is the temperature, φ is the concentration and ζ is the vertical vorticity.

We prove the following two theorems :

Theorem 1 : (A semicircle theorem for rotatory Veronis' configuration)

For rotatory Veronis' configuration, the complex growth rate of an arbitrary oscillatory perturbation, neutral or unstable, must lie inside a semicircle in the right half of the $p_r p_i$ plane whose centre is origin and radius = $\sqrt{\text{greater of } (T\sigma^2, R_2\sigma)}$ and this result is uniformly valid for all combinations of dynamically free and rigid boundaries.

Proof : Multiplying both sides of (2.3.1) by w^* (the complex conjugate of w) and integrating the resulting equation over the vertical range of z, we obtain

$$\int_0^1 w^*(D^2-a^2)(D^2-a^2-p/\sigma)w\,dz = R_1 a^2 \int_0^1 w^* \Theta\,dz - R_2 a^2 \int_0^1 w^* \varphi\,dz + T \int_0^1 w^* D\zeta\,dz. \qquad (2.3.8)$$

Now, taking the complex conjugate of both sides of (2.3.2), multiplying the resulting equation by Θ throughout and integrating over the vertical range of z, it follows that

$$\int_0^1 w^* \theta \, dz = - \int_0^1 \theta \, (D^2-a^2-p^*) \, \theta^* \, dz \, , \tag{2.3.9}$$

where p^* is the complex conjugate of p and θ^* is the complex conjugate of θ.

Further, since $p_i \neq 0$, we get from (2.3.3) that

$$\varphi = \frac{\tau}{p} (D^2-a^2)\varphi + \frac{w}{p} \, , \tag{2.3.10}$$

which can be written in the form

$$\varphi = \frac{\tau p^*}{|p|^2} (D^2-a^2)\varphi + \frac{p^*}{|p|^2} w. \tag{2.3.11}$$

Multiplying both sides of (2.3.11) by w^* and integrating the resulting equation over the vertical range of z, we obtain

$$\int_0^1 w^* \varphi \, dz = \frac{\tau p^*}{|p|^2} \int_0^1 w^*(D^2-a^2)\varphi \, dz + \frac{p^*}{|p|^2} \int_0^1 |w|^2 \, dz \, . \tag{2.3.12}$$

Again, taking the complex conjugate of both sides of (2.3.3), multiplying the resulting equation by $(D^2-a^2)\varphi$ throughout and integrating over the vertical range of z, it follows that

$$\int_0^1 w^* (D^2-a^2)\varphi \, dz = - \tau \int_0^1 (D^2-a^2)\varphi \, (D^2-a^2-p^*/\tau) \, \varphi^* \, dz \, , \tag{2.3.13}$$

where φ^* is the complex conjugate of φ.

Further differentiating (2.3.4) once with respect to z and using the condition that p_i is non-zero, we get

$$D\zeta = \frac{\sigma p^*}{|p|^2} D(D^2-a^2)\zeta + \frac{\sigma p^*}{|p|^2} D^2 w \, . \tag{2.3.14}$$

Multiplying both sides of (2.3.14) by w^* and integrating the resulting equation over the vertical range of z, we obtain

$$\int_0^1 w^* D\zeta \; dz = \frac{\sigma p^*}{|p|^2} \int_0^1 w^* D(D^2-a^2)\zeta \; dz + \frac{\sigma p^*}{|p|^2} \int_0^1 w^* D^2 w \; dz \, . \tag{2.3.15}$$

Now, integrating by parts once and using the boundary conditions (2.3.5) or (2.3.6) or (2.3.7), we can write

$$\int_0^1 w^* D(D^2-a^2)\zeta \; dz = -\int_0^1 Dw^* (D^2-a^2)\zeta \; dz. \tag{2.3.16}$$

Further, taking the complex conjugate of both sides of (2.3.4), multiplying the resulting equation by $(D^2-a^2)\zeta$ throughout and integrating over the vertical range of z, it follows that

$$\int_0^1 Dw^* (D^2-a^2)\zeta \; dz = -\int_0^1 (D^2-a^2-p^*/\sigma)\zeta^* (D^2-a^2)\zeta \; dz, \tag{2.3.17}$$

where ζ^* is the complex conjugate of ζ .

Combining (2.3.8), (2.3.9), (2.3.12), (2.3.13), (2.3.15), (2.3.16) and (2.3.17) we obtain

$$\int_0^1 w^*(D^2-a^2)(D^2-a^2-p/\sigma) w \; dz = R_1 a^2 [-\int_0^1 \Theta (D^2-a^2-p^*)\Theta^* dz] -$$

$$R_2 a^2 [\frac{\tau p^*}{|p|^2} \left\{ -\tau \int_0^1 (D^2-a^2)\varphi . (D^2-a^2-p^*/\tau) \varphi^* \; dz \right\} - \frac{p^*}{|p|^2} \int_0^1 |w|^2 dz] .$$

$$T\left[\frac{\sigma p^*}{|p|^2}\int_0^1 (D^2-a^2-p^*/\sigma)\zeta^*(D^2-a^2)\quad dz + \frac{\sigma p^*}{|p|^2}\int_0^1 w^* D^2 w\, dz\right]. \tag{2.3.18}$$

Integrating by parts a suitable number of times and using the boundary conditions (2.3.5) or (2.3.6) or (2.3.7), we get

$$\left.\begin{aligned}
\int_0^1 w^* D^4 w\, dz &= \int_0^1 |D^2 w|^2\, dz > 0\,, \\
\int_0^1 w^* D^2 w\, dz &= -\int_0^1 |Dw|^2\, dz < 0\,, \\
\int_0^1 \theta\, D^2\theta^*\, dz &= -\int_0^1 |D\theta|^2\, dz < 0\,, \\
\int_0^1 \varphi^* D^2\varphi\, dz &= -\int_0^1 |D\varphi|^2\, dz < 0\,, \\
\int_0^1 \quad D^2\zeta\, dz &= -\int_0^1 |D\zeta|^2\, dz < 0\,.
\end{aligned}\right\} \tag{2.3.19}$$

Making use of (2.3.19), we can write (2.3.18) as

$$\int_0^1 (|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2)\, dz + \frac{p}{\sigma}\int_0^1 (|Dw|^2 + a^2|w|^2)dz =$$

$$R_1 a^2 \int_0^1 (|D\theta|^2 + a^2|\theta|^2 + p^*|\theta|^2)dz - R_2 a^2\Big[-\frac{\tau^2 p^*}{|p|^2}\int_0^1 |(D^2-a^2)\varphi|^2$$

$$dz - \frac{\tau p^{*2}}{|p|^2}\int_0^1 (|D\varphi|^2 + a^2|\varphi|^2)dz + \frac{p^*}{|p|^2}\int_0^1 |w|^2 dz\Big] +$$

$$T\left[\frac{\sigma p^*}{|p|^2}\int_0^1 |(D^2-a^2)\zeta|^2 dz + \frac{p^*}{|p|^2}\int_0^1 (|D\zeta|^2 + a^2|\zeta|^2)dz - \frac{\sigma p^*}{|p|^2}\int_0^1 |Dw|^2 dz.\right. \tag{2.3.20}$$

Equating the imaginary parts of both sides of (2.3.20) we obtain

$$\frac{p_i}{\sigma}\int_0^1(|Dw|^2+a^2|w|^2)dz = -R_1a^2p_i\int_0^1|\Theta|^2dz - \frac{R_2a^2\tau^2p_i}{|p|^2}\int_0^1|(D^2-a^2)\varphi|^2dz - \frac{2R_2a^2\tau p_rp_i}{|p|^2}\int_0^1(|D\varphi|^2+a^2|\varphi|^2)dz + \frac{R_2a^2p_i}{|p|^2}\int_0^1|w|^2dz - \frac{T\sigma p_i}{|p|^2}\int_0^1|(D^2-a^2)\zeta|^2dz - \frac{2Tp_rp_i}{|p|^2}\int_0^1(|D\zeta|^2+a^2|\zeta|^2)dz + \frac{T\sigma p_i}{|p|^2}\int_0^1|Dw|^2\,dz. \quad (2.3.21)$$

Now, since $p_i \neq 0$, we can cancel p_i from both sides of (2.3.21) and write the resulting equation as

$$\int_0^1\frac{(|p|^2-T\sigma^2)}{\sigma|p|^2}|Dw|^2dz + \int_0^1\frac{a^2(|p|^2-R_2\sigma)}{\sigma|p|^2}|w|^2dz + R_1a^2\int_0^1|\Theta|^2dz + \frac{R_2a^2\tau^2}{|p|^2}\int_0^1|(D^2-a^2)\varphi|^2dz + \frac{2R_2a^2\tau p_r}{|p|^2}\int_0^1(|D\varphi|^2+a^2|\varphi|^2)dz + \frac{T\sigma}{|p|^2}\int_0^1|(D^2-a^2)\zeta|^2dz + \frac{2Tp_r}{|p|^2}\int_0^1(|D\zeta|^2+a^2|\zeta|^2)dz = 0. \quad (2.3.22)$$

But, since $p_r \geq 0$, $R_1 > 0$ and $R_2 > 0$, we have from(2.3.22) that

$$|p|^2 < \text{greater of } (T\sigma^2,\ R_2\sigma). \quad (2.3.23)$$

In other words

$$p_r^2 + p_i^2 < \text{greater of } (T\sigma^2,\ R_2\sigma) \quad (2.3.24)$$

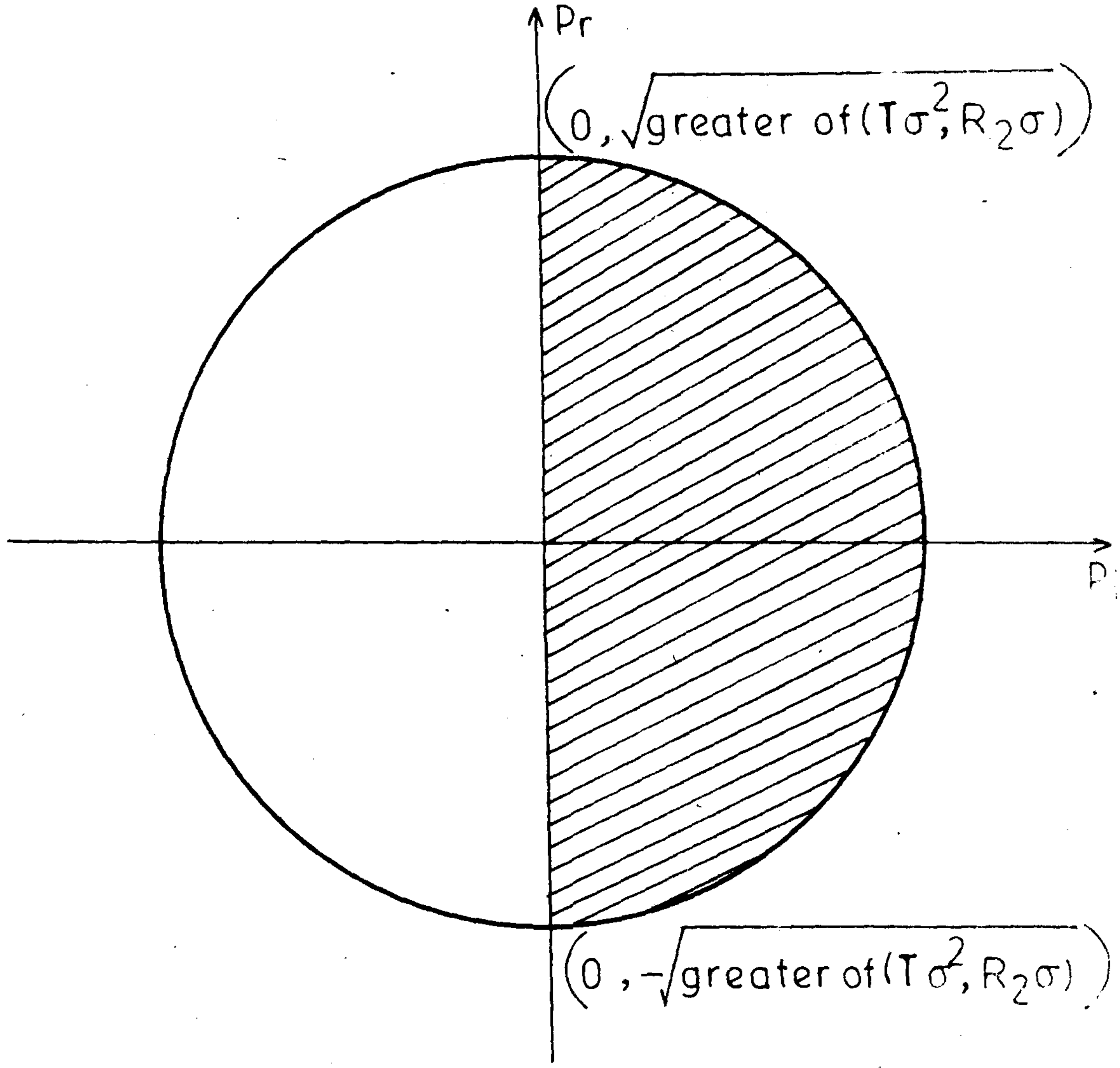

Fig.(c): Shaded area shows the region in which the complex growth rate of an arbitrary oscillatory mode, neutral or unstable, in rotatory Veronis' thermohaline configuration must lie.

and this proves the theorem (see Fig. (c)).

For nonrotatory Veronis configuration, $T = 0$ and (2.3.24) becomes

$$p_r^2 + p_i^2 < R_2\sigma. \qquad (2.3.25)$$

<u>Contributions of Theorem 1</u>

(i) Derives the Pellew and Southwell's (1940) result that for the simple Bénard problem the 'principle of exchange of stabilities' is valid.

(ii) Derives Banerjee's (1972, 1978a, 1978b) result that for the generalized Bénard problem the complex growth rate of an arbitrary oscillatory mode, neutral or unstable, must lie inside a semicircle in the right half of the $p_r p_i$ plane whose centre is origin and radius $= \sqrt{R_2\sigma}$.

(iii) Derives the result (see, for example, Theorem 1 of Chapter 1) of Banerjee et al (1981) in terms of bounds for the complex growth rate of an arbitrary oscillatory mode, neutral or unstable, for all combinations of dynamical free and rigid boundaries for Veronis' (1965) thermohaline configuration.

(iv) Shows the compatibility of the exact calculations known to date (see, for example, Veronis 1965, Banerjee 1972) with the semicircle restriction of <u>Theorem 1</u>.

(v) Shows that the smaller is the value of $R_2\sigma$ in Veronis' (1965) thermohaline configuration, the smaller is the growth rate of an arbitrary oscillatory perturbation, neutral or unstable.

(vi) *Derives Banerjee's and Kalthia's (1971) result on* the complex growth rate of an arbitrary oscillatory perturbation in the Rayleigh-Taylor configuration of a Boussinesq fluid of constant coefficient of viscosity.

(vii) Derives a new result in terms of bounds for the complex growth rate of an arbitrary oscillatory mode, neutral or unstable, for all combinations of dynamically free and rigid boundaries for rotatory Veronis' (1965) thermohaline configuration.

(viii) Shows the compatibility of the numerical calculations of Chandrasekhar (1953) and Chandrasekhar and Elbert (1955), on the complex growth rate of an arbitrary neutral oscillatory mode in the rotatory simple Bénard problem, with the semicircle restriction of Theorem 1.

(ix) Shows that the smaller the values of $T\sigma^2$ and $R_2\sigma$ in rotatory Veronis' (1965) thermohaline configuration, the smaller is the growth rate of an arbitrary oscillatory perturbation, neutral or unstable.

Theorem 2 : (A semicircle theorem for rotatory Stern's configuration)

For rotatory Stern's configuration, the complex growth rate of an arbitrary oscillatory perturbation, neutral or unstable, must lie inside a semicircle in the right half of the $p_r p_i$ plane whose centre is the origin and radius = $\sqrt{\text{greater of } (T\sigma^2, -R_1\sigma)}$ and this result is uniformly valid for all combinations of dynamically free and rigid boundaries.

Proof : Let
$$\left.\begin{aligned} R_1 &= -\hat{R}_1, \\ R_2 &= -\hat{R}_2. \end{aligned}\right) \qquad (2.3.26)$$
so that $\hat{R}_1 > 0$ and $\hat{R}_2 > 0$.

Equations (2.3.1), (2.3.2), (2.3.3) and (2.3.4) thus respectively become

$$(D^2-a^2)(D^2-a^2-p/\sigma)w = \hat{R}_2 a^2 \varphi - \hat{R}_1 a^2 \theta + TD\zeta , \qquad (2.3.27)$$

$$(D^2-a^2-p)\,\theta = -w , \qquad (2.3.28)$$

$$(D^2-a^2-p/\tau)\varphi = -\frac{w}{\tau} , \qquad (2.3.29)$$

$$(D^2-a^2-p/\sigma)\zeta = -Dw . \qquad (2.3.30)$$

Multiplying both sides of (2.3.27) by w^* and integrating the resulting equation over the vertical range of z, we obtain

$$\int_0^1 w^*(D^2-a^2)(D^2-a^2-p/\sigma)w\,dz = \hat{R}_2 a^2 \int_0^1 w^* \varphi\,dz -$$

$$\hat{R}_1 a^2 \int_0^1 w^* \theta\,dz + T\int_0^1 w^* D\zeta\,. \qquad (2.3.31)$$

Now, taking the complex conjugate of both sides of (2.3.29), multiplying the resulting equation by φ throughout and integrating over the vertical range of z, it follows that

$$\int_0^1 w^* \varphi\,dz = -\tau \int_0^1 \varphi(D^2-a^2-p^*/\tau)\,\varphi^*\,dz. \qquad (2.3.32)$$

Further, since $p_i \neq 0$, we get from (2.3.28) that

$$\theta = \frac{1}{p}(D^2-a^2)\,\theta + \frac{w}{p}, \qquad (2.3.33)$$

which can be written in the form

$$\theta = \frac{p^*}{|p|^2}(D^2-a^2)\theta + \frac{p^*}{|p|^2}\,w. \qquad (2.3.34)$$

Multiplying both sides of (2.3.34) by w^* and integrating the resulting equation over the vertical range of z, we obtain

$$\int_0^1 w^* \theta\,dz = \frac{p^*}{|p|^2}\int_0^1 w^*(D^2-a^2)\theta\,dz + \frac{p^*}{|p|^2}\int_0^1 |w|^2\,dz. \qquad (2.3.35)$$

Again, taking the complex conjugate of both sides of (2.3.28), multiplying the resulting equation by $(D^2-a^2)\theta$ throughout and integrating over the vertical range of z, it follows that

$$\int_0^1 w^*(D^2-a^2)\theta\, dz = -\int_0^1 (D^2-a^2)\theta.(D^2-a^2-p^*)\theta^*\, dz. \qquad (2.3.36)$$

Now, combining (2.3.31), (2.3.32), (2.3.35), (2.3.36), (2.3.15), (2.3.16) and (2.3.17), we obtain

$$\int_0^1 w^*(D^2-a^2)(D^2-a^2-p/\sigma)w\, dz = \hat{R}_2 a^2[-\tau\int_0^1 \varphi(D^2-a^2-p^*/\tau)\varphi^* dz] -$$

$$\hat{R}_1 a^2[\frac{p^*}{|p|^2}\left\{-\int_0^1 (D^2-a^2)\theta.(D^2-a^2-p^*)\theta^* dz\right\} + \frac{p^*}{|p|^2}\int_0^1 |w|^2 dz] +$$

$$T[\frac{\sigma p^*}{|p|^2}\int_0^1 (D^2-a^2-p^*/\sigma)\zeta^*(D^2-a^2)\zeta\, dz + \frac{\sigma p^*}{|p|^2}\int_0^1 w^* D^2 w\, dz]. \qquad (2.3.37)$$

Making use of (2.3.19), we can write (2.3.37) as

$$\int_0^1 (|D^2w|^2+2a^2|Dw|^2 + a^4|w|^2)\, dz + \frac{p}{\sigma}\int_0^1 (|Dw|^2+a^2|w|^2)\, dz =$$

$$\hat{R}_2 a^2\tau\int_0^1 (|D\varphi|^2+ a^2|\varphi|^2 + \frac{p^*}{\tau}|\varphi|^2)dz - \hat{R}_1 a^2[\frac{p^*}{|p|^2}\ -\int_0^1 |(D^2-a^2)\theta|^2 dz -$$

$$p^*\int_0^1 (|D\theta|^2+a^2|\theta|^2)dz + \frac{p^*}{|p|^2}\int_0^1 |w|^2 dz] + T[\frac{\sigma p^*}{|p|^2}\left\{\int_0^1 |(D^2-a^2)\zeta|^2 dz +\right.$$

$$\left.\frac{p^*}{\sigma}\int_0^1 (|D\zeta|^2+a^2|\zeta|^2)dz\right\} - \frac{\sigma p^*}{|p|^2}\int_0^1 |Dw|^2\, dz]. \qquad (2.3.38)$$

Equating the imaginary parts of both sides of (2.3.38), we obtain

$$\frac{p_i}{\sigma}\int_0^1 (|Dw|^2+a^2|w|^2)dz = -\hat{R}_2 a^2 p_i\int_0^1 |\varphi|^2 dz -$$

$$\frac{\hat{R}_1 a^2 p_i}{|p|^2}\int_0^1 |(D^2-a^2)\theta|^2\, dz - \frac{2\hat{R}_1 a^2 p_r p_i}{|p|^2}\int_0^1 (|D\theta|^2+a^2|\theta|^2)dz +$$

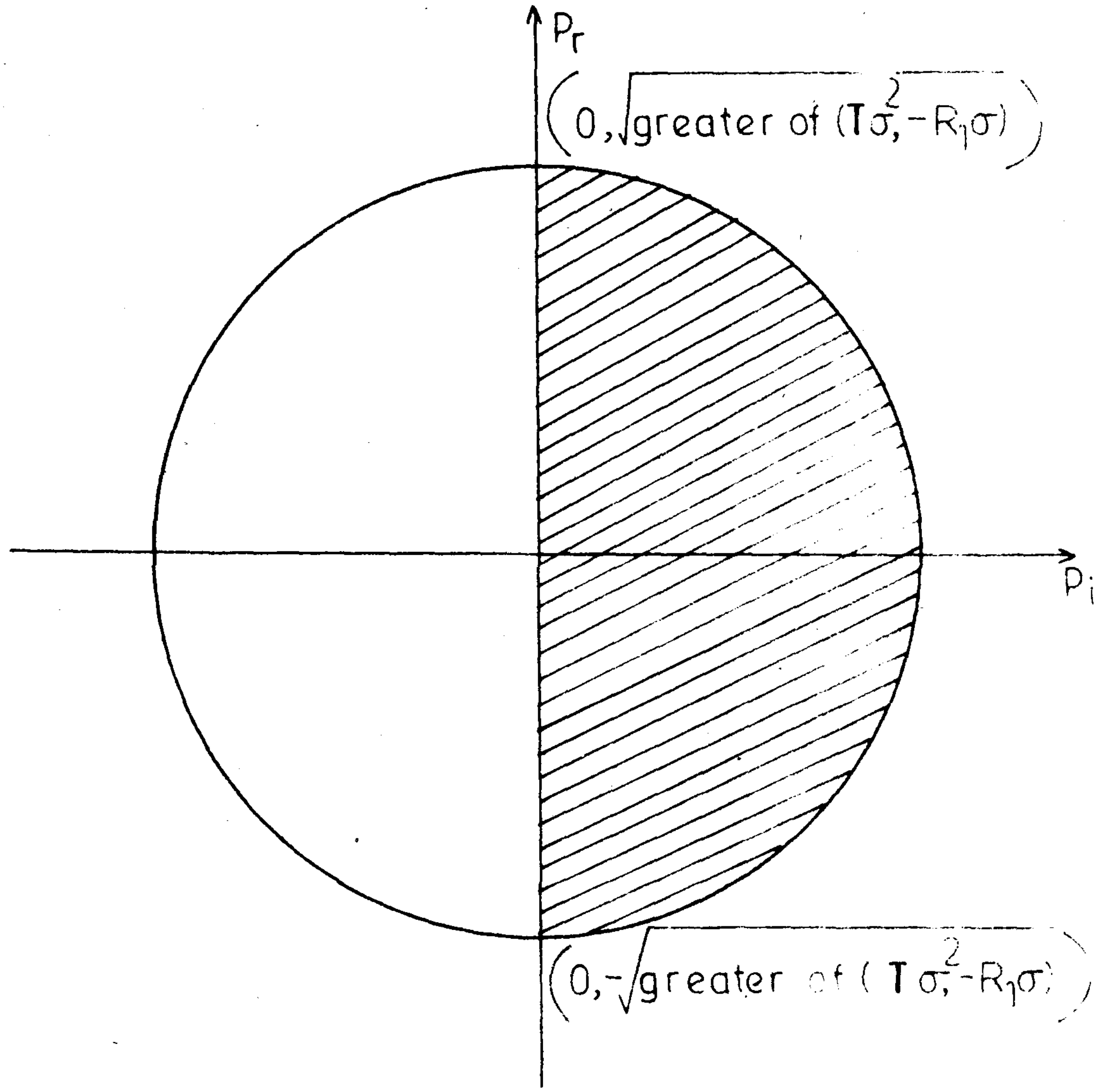

Fig.(d):-Shaded area shows the region in which the complex growth rate of an arbitrary oscillatory mode, neutral or unstable, in rotatory Stern's thermohaline configuration must lie.

$$\frac{\hat{R}_1 a^2 p_i}{|p|^2} \int_0^1 |w|^2 \, dz - \frac{T\sigma p_i}{|p|^2} \int_0^1 |(D^2-a^2)\zeta|^2 dz - \frac{2Tp_r p_i}{|p|^2} \times$$

$$\int_0^1 (|D\zeta|^2 + a^2|\zeta|^2) dz + \frac{T\sigma p_i}{|p|^2} \int_0^1 |Dw|^2 \, dz. \qquad (2.3.39)$$

Now, since $p_i \neq 0$, we can cancel p_i from both sides of (2.3.39) and write the resulting equation as

$$\int_0^1 \frac{(|p|^2 - T\sigma^2)}{\sigma |p|^2} |Dw|^2 dz + \int_0^1 \frac{(|p|^2 - \hat{R}_1 \sigma)}{\sigma |p|^2} |w|^2 dz + \hat{R}_2 a^2 \int_0^1 |\varphi|^2 dz +$$

$$\frac{\hat{R}_1 a^2}{|p|^2} \int_0^1 |(D^2-a^2)\Theta|^2 \, dz + \frac{T\sigma}{|p|^2} \int_0^1 |(D^2-a^2)\zeta|^2 \, dz +$$

$$\frac{2Tp_r}{|p|^2} \int_0^1 (|D\zeta|^2 + a^2|\zeta|^2) dz = 0. \qquad (2.3.40)$$

But, since $p_r \geq 0$, $\hat{R}_1 > 0$ and $\hat{R}_2 > 0$, we have from (2.3.40) that

$$|p|^2 < \text{greater of } (T\sigma^2, +\hat{R}_1\sigma). \qquad (2.3.41)$$

In other words, since $R_1 = -\hat{R}_1$, we obtain from (2.3.41) that

$$p_r^2 + p_i^2 < \text{greater of } (T\sigma^2, -R_1\sigma), \qquad (2.3.42)$$

and this proves the theorem (see Fig.(d)).

For nonrotatory Stern's configuration, $T = 0$, and (2.3.42) becomes

$$p_r^2 + p_i^2 < -R_1\sigma. \qquad (2.3.43)$$

Contributions of Theorem 2

(i) Derives the result (see for example Theorem 2 of Chapter 1) of Banerjee et al (1981) in terms of bounds for the complex growth rate of an arbitrary oscillatory mode, neutral or unstable, for all combinations of dynamically free and rigid boundaries for Stern's (1960) thermohaline configuration.

(ii) Shows that the smaller is the value of $-R_1\sigma$ in Stern's (1960) thermohaline configuration, the smaller is the growth rate of an arbitrary oscillatory perturbation, neutral or unstable.

(iii) Derives a new result in terms of bounds for the complex growth rate of an arbitrary oscillatory mode, neutral or unstable, for all combinations of dynamically free and rigid boundaries for rotatory Stern's (1960) thermohaline configuration.

(iv) Shows that the smaller the values of $T\sigma^2$ and $-R_1\sigma$ in rotatory Stern's (1960) thermohaline configuration, the smaller in the growth rate of an arbitrary oscillatory perturbation, neutral or unstable.