CHAPTER V

THERMOSOLUTAL CONVECTION OF MICROPOLAR FLUIDS IN THE PRESENCE OF SUSPENDED PARTICLES

5.1 INTRODUCTION

The heat and solute being two diffusing components, thermosolutal convection is the general term dealing with such phenomena. The buoyancy forces can arise not only from density differences due to variations in temperature, but also from those due to variations in solute concentration. When a stably stratified body of fluid has horizontal temperature and salinity gradients are present, but no horizontal density gradient, it may be unstable to infinitesimal disturbances. These instabilities were first noticed in the context of the mixing of water masses in the oceans. Brakke [1955] explained a double–diffusive instability that occurs when solution of a slowly diffusing protein is layered over a dense solution of more diffusing sucrose. Convection that is dominated by the presence of two components is very common in geophysical systems and has been the subject of many studies since Stern [1960] realized the important implication of an 'oceanographical curiosity' (Stommel, Arons and Blanchard [1956]).The problem of thermosolutal convection (double–diffusive convection) in a layer of fluid heated from below and subjected to a stable solute gradient has been studied by Veronis [1965]. Themosolutal convection problems arise in oceanography, limnology and engineering. Examples of particular interest are provided by ponds built to trap solar heat (Tabor and Matz [1965]) and some Antarctic lakes (Shirtcliffe [1964]). The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions in double–diffusive convection are important (e.g. in lower parts of the Earth's atmosphere, astrophysics and several geophysical situations) are usually far removed from the consideration of a single component fluid and rigid boundaries and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries. Stern [1967] showed that for unbounded bodies of water with uniform horizontal temperature and salinity gradients and where the fluxes were dominated by salt fingers, the fluid was always unstable to motions that took the form of almost horizontal interleaving convection layers. This analysis was extended by Toole and Georgi [1981] with the inclusion of viscous effects. The layered double–diffusive convection in a porous media has been considered by Griffiths [1981].The case of fluids with uniform salinity gradients when the fluxes are driven by other mechanisms has been looked at by McDougall [1985] who assumed that the fluxes were proportional to the salinity difference between the convective layers and independent of the layer thickness and by Holyer [1983], who assumed that the fluxes were driven by molecular diffusivities. In all these cases where an unbounded fluid has uniform horizontal and vertical compositional gradients, the fluid is always unstable and so considerations of marginal stability are inappropriate. A broad view of the subject of double–diffusive convection is given by Brandt and Fernando [1996]. The most intensively studied double–diffusive system is thermosolutal (thermohaline) convection in which the fluid density depends on heat and solute concentrations.

The multidiffusive convection can occur in a fluid subject to a gravitational field when the fluid contains at least two components with different molecular diffusivities each of which affects the density of the fluid. Chemists prefer to use the term "isothermal ternary system", referring to a solvent and two solutes or polymers, or "non–isothermal binary system", thus distinguishing between thermal and other diffusion processes. Huppert and Turner [1981] placed a hot layer of KNO_3 solution under a cold, lighter

$NaNO_3$ layer, taking care to minimize the mixing and to produce a sharp (triple–diffusive) interface. The rapid convective heat transfer through the interface led to the growth of crystals on the bottom, so that the density of the residual lower solution decreased. When it became equal to that in the upper layer, the interface broke down and the layer mixed thoroughly together. This is the behaviour to be expected in a basaltic magma chamber, with nearly equal viscosities of the two layers. Turner [1985] has studied multicomponent convection and has emphasized the very broad range of disciplines in which the effects of multicomponent convection should be taken into account, which lead to dynamic instabilities, often well before a fluid system would become statically unstable. Sharma and Kumar [2000] have studied the thermosolutal convection of micropolar fluids in hydromagnetics and have found that magnetic field and solute parameter have stabilizing effect on the system. Sharma [2001] has studied the thermosolutal convection of micropolar fluids in porous medium and has found that medium permeability has destabilizing effect on the system. Sharma and Sharma [2000] have studied the thermosolutal convection of micropolar fluids in hydromagnetics in porous medium and have found that magnetic field and solute parameter have stabilizing effect on the system while medium permeability has destabilizing effect on the system. Sharma and Dutt [2006] have studied the effect of suspended particles on micropolar fluids on thermosolutal convection of micropolar fluids in porous media. Keeping in mind the importance and relevance of double–diffusive convection with suspended particles (fine dust) in biomechanics, chemical engineering, geophysics and colloidal suspensions industries, the present chapter, therefore, deals with the thermosolutal convection in micropolar fluids in the presence of suspended particles.

5.2 FORMULATION OF THE PROBLEM AND DISTURBANCE EQUATIONS

Consider the stability of an infinite, horizontal layer of an incompressible electrically conducting micropolar fluid of thickness d permeated with suspended particles (or fine dust). This fluid-particles layer is heated and soluted from below but convection sets in when the temperature gradient $\left(\beta = \left| dT/dz \right|\right)$ between the lower and upper boundaries exceeds a certain critical value. This is the Rayleigh–Bénard instability problem in the presence of salinity gradient in micropolar fluids.

Let k'_T, C, δ' denote, respectively, the analogous solute conductivity, the solute concentration and the analogous coupling between spin flux with that solute flux.

The mass, momentum, internal angular momentum, internal energy balance equations using the Boussinesq approximation are

$$\nabla \cdot \vec{v} = 0, \tag{5.1}$$

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\vec{v} = -\frac{1}{\rho_0}\nabla p + \frac{1}{\rho_0}(\mu+\kappa)\nabla^2\vec{v} + \frac{\kappa}{\rho_0}\nabla\times\vec{\vartheta} - \left(1+\frac{\delta\rho}{\rho_0}\right)g\,\hat{e}_z + \frac{1}{\rho_0}KN(\vec{u}-\vec{v}), \tag{5.2}$$

$$\rho_0 j_1\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\vec{\vartheta} = (\varepsilon' + \beta'')\nabla\left(\nabla\cdot\vec{\vartheta}\right) + \gamma'\,\nabla^2\vec{\vartheta} + \kappa\nabla\times\vec{v} - 2\,\kappa\vec{\vartheta}, \tag{5.3}$$

$$\rho_0 c_v\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)T + mN\,c_{pt}\left(\frac{\partial}{\partial t} + \vec{u}\cdot\nabla\right)T = k_T\nabla^2 T + \delta\left(\nabla\times\vec{\vartheta}\right)\cdot\nabla T, \tag{5.4}$$

$$\rho_0 c_v\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)C + mN\,c_{pt}\left(\frac{\partial}{\partial t} + \vec{u}\cdot\nabla\right)C = k'_T\nabla^2 C + \delta'\left(\nabla\times\vec{\vartheta}\right)\cdot\nabla C, \tag{5.5}$$

where the equation of state is given by

$$\rho = \rho_0[1 - \alpha(T - T_0) + \alpha'(C - C_0)], \tag{5.6}$$

where ρ_0, T_0, C_0 are reference density, reference temperature and reference solute concentration at the lower boundary and α, α' are the coefficients of thermal expansion and the analogous solvent coefficient respectively.

The equations of motion and continuity for the particles are

$$mN\left(\frac{\partial}{\partial t}+\vec{u}\cdot\nabla\right)\vec{u}=KN\left(\vec{v}-\vec{u}\right), \tag{5.7}$$

$$\frac{\partial N}{\partial t}+\nabla\cdot\left(N\,\vec{u}\right)=0. \tag{5.8}$$

Now we consider the stability of the system wherein we give small perturbations on the initial state and on seeing the reaction of the perturbations on the system.

The steady state solution of the governing equations (5.1)–(5.8) is given by

$$\vec{v}=0,\ \ \vec{u}=0,\ \ \vec{\vartheta}=0,\ \ N=N_0(\text{constant}),\ \ T=T_0-\beta z,\ \ C=\beta' z-C_0,$$

$$\rho=\rho_0\left(1+\alpha\,\beta\,z-\alpha'\beta' z\right),\ \ p=p_0-g\rho_0\left(z+\frac{\alpha\,\beta\,z^2}{2}-\frac{\alpha'\beta' z^2}{2}\right), \tag{5.9}$$

where p_0 is the pressure at $z=0$ and $\beta=\dfrac{T_0-T_1}{d}$, $\beta'=\dfrac{C_0-C_1}{d}$ are the magnitudes of uniform temperature and concentration gradient, respectively.

Let $\vec{v}(u,v,w)$, $\vec{u}(\ell,r,s)$, $\vec{\omega}, N, \delta p$, $\delta\rho$, θ and γ denote, respectively, the perturbations on fluid velocity $\vec{v}(0,0,0)$, particles velocity $\vec{u}(0,0,0)$, spin $\vec{\vartheta}$, particles number density N_0, pressure p, density ρ, temperature T and solute concentration C, so that the change in density $\delta\rho$ caused by the perturbations θ and γ in temperature and concentration is given by

$$\delta\rho=-\rho_0\left(\alpha\theta-\alpha'\gamma\right). \tag{5.10}$$

Then equations (5.1)–(5.8) yield the perturbation equations

$$\nabla\cdot\vec{v}=0, \tag{5.11}$$

$$\rho_0\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\vec{v}=-\nabla\delta p+(\mu+\kappa)\nabla^2\vec{v}+\kappa(\nabla\times\vec{\omega})+g\rho_0(\alpha\theta-\alpha'\gamma)\hat{e}_z+KN_0(\vec{u}-\vec{v}), \tag{5.12}$$

$$\rho_0 j_1\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\vec{\omega}=(\varepsilon'+\beta'')\nabla(\nabla\cdot\vec{\omega})+\gamma'\nabla^2\vec{\omega}+\kappa\nabla\times\vec{v}-2\kappa\vec{\omega}, \tag{5.13}$$

$$H_1\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\theta=\beta(w+h_1 s)+\kappa_T\nabla^2\theta+\frac{\delta}{\rho_0 c_v}[\nabla\theta\cdot(\nabla\times\vec{\omega})-(\nabla\times\vec{\omega})_z\cdot\beta], \tag{5.14}$$

$$H_1\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\gamma=\beta'(w+h_1 s)+\kappa_T'\nabla^2\gamma+\frac{\delta'}{\rho_0 c_v}[\nabla\gamma\cdot(\nabla\times\vec{\omega})-(\nabla\times\vec{\omega})_z\cdot\beta'], \tag{5.15}$$

$$mN_0\left(\frac{\partial}{\partial t}+\vec{u}\cdot\nabla\right)\cdot\vec{u}=KN_0(\vec{v}-\vec{u}), \tag{5.16}$$

$$\frac{\partial M}{\partial t}+\nabla\cdot\vec{u}=0, \tag{5.17}$$

where $H_1=1+h_1$, $h_1=\frac{fc_{pt}}{c_v}$, $f=\frac{mN_0}{\rho_0}$ and $M=\frac{N}{N_0}$.

Using the non-dimensional numbers

$$z=z^*d,\quad \theta=\beta d\theta^*,\quad \gamma=\beta' d\gamma^*,\quad t=\frac{\rho_0 d^2}{\mu}t^*,\quad \vec{v}=\frac{\kappa_T}{d}\vec{v}^*,$$

$$\nabla=\frac{\nabla^*}{d},\ \vec{u}=\frac{\kappa_T}{d}\vec{u}^*,\ p=\frac{\mu\kappa_T}{d^2}p^*,\ \vec{\omega}=\frac{\kappa_T}{d^2}\vec{\omega}^* \tag{5.18}$$

and then removing the stars for convenience, the non-dimensional forms of equations (5.11)–(5.17) become

$$\nabla\cdot\vec{v}=0, \tag{5.19}$$

$$\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\vec{v}=-\nabla\delta p+(1+K_1)\nabla^2\vec{v}+K_1\nabla\times\vec{\omega}+\hat{e}_z\left(R\theta-\frac{p_1}{q}S\gamma\right)+N_2(\vec{u}-\vec{v}), \tag{5.20}$$

$$j_2\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\vec{\omega}=C_1'\nabla(\nabla\cdot\vec{\omega})-C_0'\nabla\times(\nabla\times\vec{\omega})+K_1(\nabla\times\vec{v}-2\vec{\omega}), \tag{5.21}$$

$$H_1 p_1\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\theta=\beta\left(w+h_1 s\right)+\kappa_T\,\nabla^2\theta+\bar{\delta}\left[\nabla\theta\cdot(\nabla\times\vec{\omega})-(\nabla\times\vec{\omega})_z\right], \qquad (5.22)$$

$$H_1 q\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\gamma=\beta'\left(w+h_1 s\right)+\kappa_T'\,\nabla^2\gamma+\bar{\delta}'\left[\nabla\gamma\cdot(\nabla\times\vec{\omega})-(\nabla\times\vec{\omega})_z\right], \qquad (5.23)$$

$$\left[a\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)+1\right]\vec{u}=\vec{v}\,. \qquad (5.24)$$

The new dimensionless coefficients are

$$K_1=\frac{\kappa}{\mu},\quad \bar{j}_2=\frac{j_1}{d^2},\quad \bar{\delta}=\frac{\delta}{\rho_0\,c_v\,d^2},\quad \bar{\delta}'=\frac{\delta'}{\rho_0\,c_v\,d^2},\quad C_0'=\frac{\gamma'}{\mu\,d^2},$$

$$C_1'=\frac{\varepsilon'+\beta''+\gamma'}{\mu\,d^2},\quad N_2=KN_0\frac{d^2}{\mu},\quad a=\frac{m}{Kd^2}\frac{\mu}{\rho_0} \qquad (5.25)$$

and the dimensionless Rayleigh number R, analogous solute number S, thermal Prandtl number p_1 and the analogous Schmidt number q are

$$R=\frac{g\alpha\beta\rho_0 d^4}{\mu\,\kappa_T},\quad S=\frac{g\alpha'\beta'\rho_0 d^4}{\mu\,\kappa_T'},\quad p_1=\frac{\mu}{\rho_0\,\kappa_T},\quad q=\frac{\mu}{\rho_0\,\kappa_T'}, \qquad (5.26)$$

where $\kappa_T=\dfrac{k_T}{\rho_0 c_v}$ and $\kappa_T'=\dfrac{k_T'}{\rho_0 c_v}$ are the thermal diffusivity and the solute diffusivity, respectively.

The boundaries are considered to be free. The case of two free boundaries is little artificial except in astrophysical situations but it enables us to find analytical solutions. Thus the boundary conditions appropriate to problem are

$$w=0,\frac{\partial^2 w}{\partial z^2}=0,\ (\nabla\times\vec{\omega})_z=0,\ \theta=0,\gamma=0 \text{ at } z=0 \text{ and } z=1. \qquad (5.27)$$

5.3 LINEAR THEORY: DISPERSION RELATION

Since the perturbations applied on the system are assumed to be very small, the products, the second and higher order perturbations are negligibly small.

Under the linearized theory, second and higher order terms are neglected and only the linear terms are retained. Accordingly, the non–linear terms $(\vec{v}\cdot\nabla)\vec{v}$, $(\vec{v}\cdot\nabla)\theta$, $(\vec{v}\cdot\nabla)\vec{\omega}$, $(\vec{v}\cdot\nabla)\gamma$, $\nabla\theta\cdot(\nabla\times\vec{\omega})$ and $\nabla\gamma\cdot(\nabla\times\vec{\omega})$ in equations (5.20)–(5.23) are neglected.

Eliminating s between equations (5.22) and (5.23) with the help of equation (5.24) and applying the curl operator twice to resulting equation and linearizing, we obtain

$$L_2\left[H_1 p_1\frac{\partial}{\partial t}-\nabla^2\right]\theta=\left(a\frac{\partial}{\partial t}+H_1\right)\beta\, w-L_2\bar{\delta}\Omega_{z1}, \tag{5.28}$$

$$L_2\left[H_1 q\frac{\partial}{\partial t}-\nabla^2\right]\gamma=\left(a\frac{\partial}{\partial t}+H_1\right)\beta'\, w-L_2\bar{\delta}'\Omega_{z1}. \tag{5.29}$$

Eliminating $\vec{u}$ between equations (5.20) and (5.24), we obtain

$$L_1\vec{v}=L_2\left[-\nabla\delta p+(1+K_1)\nabla^2\vec{v}+K_1\nabla\times\vec{\omega}+\left(R\theta-\frac{p_1}{q}S\gamma\right)\hat{e}_z\right], \tag{5.30}$$

where $L_1=a\dfrac{\partial^2}{\partial t^2}+F\dfrac{\partial}{\partial t}$, $L_2=a\dfrac{\partial}{\partial t}+1$ and $F=f+1$.

Applying the curl operator to equation (5.21) and taking z–component, we get

$$\bar{j}_2\frac{\partial\Omega_{z1}}{\partial t}=C_0'\nabla^2\Omega_{z1}-K_1\left(\nabla^2 w+2\Omega_{z1}\right), \tag{5.31}$$

where K_1 and C_0' accounts for coupling between vorticity and spin effects and that of spin diffusion, respectively and $\Omega_{z1}=(\nabla\times\vec{\omega})_z$.

Applying the curl operator twice to equation (5.20) and taking z–component, we get

$$L_1\nabla^2 w=L_2\left[R\nabla_1^2\theta-\frac{p_1}{q}S\,\nabla_1^2\gamma+(1+K_1)\nabla^4 w+K_1\nabla^2\Omega_{z1}\right], \tag{5.32}$$

where $\nabla_1^2=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}$, $\nabla^2=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}+\dfrac{\partial^2}{\partial z^2}$. (5.33)

We now analyze an arbitrary perturbation into a complete set of normal modes and then examine the stability of each of these modes individually. For the system of equations (5.28), (5.29), (5.31) and (5.32), the analysis can be made in terms of two dimensional periodic waves of assigned wave-numbers. Thus we ascribe to all quantities describing the perturbation dependence on x, y and t of the form

$$\exp\left[i\left(k_x x+k_y y\right)+nt\right], \tag{5.34}$$

where $k=\left(k_x^2+k_x^2\right)^{1/2}$ is the resultant wave number, k_x and k_y are real constants and n is the stability parameter which can be, complex, in general. The solution of the stability problem requires the specifications of the state for each k. The above considerations allow us to suppose that the perturbations quantities have the form

$$\left[w,\Omega_{z1},\theta,\gamma\right]=\left[W(z),\Omega_2(z),\Theta(z),\Gamma(z)\right]\exp\left(ik_x+ik_y y+nt\right), \tag{5.35}$$

then the equations (5.28), (5.29), (5.31) and (5.32) become

$$(an+1)\left\{H_1 p_1 n-\left(D^2-k^2\right)\right\}\Theta=\left(an+H_1\right)W-(an+1)\,\bar{\delta}\,\Omega_2, \tag{5.36}$$

$$(an+1)\left\{H_1 qn-\left(D^2-k^2\right)\right\}\Gamma=\left(an+H_1\right)W-(an+1)\,\bar{\delta}'\Omega_2, \tag{5.37}$$

$$\left\{\ell_1 n+2A-\left(D^2-k^2\right)\right\}\Omega_2=-A\left(D^2-k^2\right)W, \tag{5.38}$$

$$\left(D^2-k^2\right)\left\{\left(an^2+Fn\right)-(an+1)\left(1+K_1\right)\left(D^2-k^2\right)\right\}W=(an+1)\left\{-Rk^2\Theta+\frac{p_1}{q}S\,k^2\Gamma+K_1\left(D^2-k^2\right)\Omega_2\right\}, \tag{5.39}$$

where $A=\dfrac{K_1}{C_0'}$, $\ell_1=\bar{j}_2\dfrac{A}{K_1}$, $D=\dfrac{d}{dz}$, $\dfrac{\partial}{\partial t}=n$, $L_2=a\dfrac{\partial}{\partial t}+1=an+1$,

$$L_1=a\frac{\partial^2}{\partial t^2}+F\frac{\partial}{\partial t}=an^2+Fn\,. \tag{5.40}$$

The boundary conditions (5.27) transform to

$$W=D^2W=\Omega_2=\Theta=\Gamma=0 \text{ at } z=0,1. \tag{5.41}$$

Using the boundary conditions (5.41), we can show that all the even order derivatives of W must vanish on the boundaries, therefore the proper solution for W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \qquad (5.42)$$

where W_0 is a constant.

Eliminating Θ, Γ, Ω_2 from equations (5.36)–(5.39) and substituting above solution, we obtain

$$\{b\,(an^2 + Fn) + b^2(an+1)(1+K_1)\}\{H_1p_1n + b\}\{H_1qn + b\}\{\ell_1 n + 2A + b\} = \{Rk^2(an + H_1)$$

$$(\ell_1 n + 2A + b) - Rk^2(an+1)\,Ab\bar{\delta}\}\;\{H_1qn + b\} - \left\{\frac{p_1}{q} S\,k^2(an + H_1)(\ell_1 n + 2A + b) - \right.$$

$$\left.\frac{p_1}{q} S\,k^2(an+1)\,A\,b\bar{\delta}'\right\}\{H_1p_1n + b\} + (an+1)\{H_1p_1n + b\}\;\{H_1qn + b\}K_1Ab^2, \qquad (5.43)$$

where $b = \pi^2 + k^2$.

5.4 THE CASE OF OSCILLATORY MODES

Equating the imaginary parts of equation (5.43), we have

$$n_i\Big[n_i^4 ab^2H_1^2p_1q\ell_1 + n_i^2\{-2AH_1qab^2 - H_1qab^3 - H_1Fp_1\ell qb^2 - \frac{p_1}{q}Sk^2H_1^2K_1p_1qAb^2a - 2aAH_1^2p_1qb^2$$

$$-H_1^2p_1qb^2\ell_1 - aH_1^2p_1qb^3 - aH_1\ell_1qb^3 - 2AaH_1^2p_1K_1qb^2 - aH_1^2p_1qb^3K_1 - aH_1\ell_1qb^3K_1$$

$$-H_1^2p_1qb^2\ell_1K_1 - 2AH_1p_1ab^2 - H_1p_1ab^3 - ab^3\ell_1 - H_1p_1\ell_1Fb^2 - aH_1p_1n_i^2b^3K_1 + H_1p_1\ell_1ab^3$$

$$+Rk^2a\ell_1H_1q\} + b^5\{aK_1 + a\} + b^4\{2A + 2AaK_1 + H_1p_1 + \ell_1 - 2AH_1^2Fbp_1 - H_1^2Fp_1qb^2 + K_1H_1p_1$$

$$+K_1\ell_1 - \frac{p_1}{q}Sk^2aK_1A + H_1q + K_1H_1q + F\Big\} + b^3\Big\{2aH_1p_1\ell_1 - \frac{p_1}{q}Sk^2H_1p_1K_1A + 2AH_1q$$

$$+2AH_1p_1 + 2AH_1K_1q + 2AF\} + b^2\Big\{-\frac{p_1}{q}Sk^2H_1qK_1A - Rk^2a + Rk^2\bar{\delta}A\Big\} + b\Big\{-\frac{p_1}{q}Sk^2a\bar{\delta}'A$$

$$+\frac{p_1}{q}Sk^2a - Rk^2\left(H_1^2q + 2Aa + H_1\ell_1 - H_1q\bar{\delta}A\right)\Big\} + \frac{p_1}{q}Sk^2\{2Aa + H_1\ell_1\} - Rk^2\,2aH_1^2q\Big] = 0. \qquad (5.44)$$

Equation (5.44) yields that either $n_i = 0$ or $n_i \neq 0$, which means that the modes are either non–oscillatory or oscillatory. In the absence of suspended particles and solute parameter (5.44) reduces to

$$n_i \left[b^4\{2A + \ell_1(1+K_1)\} + Rk^2\bar{\delta}Ab^2\right] = 0 \tag{5.45}$$

and term within in the brackets is definitely positive, which implies that $n_i = 0$. Therefore, the oscillatory modes are not allowed and principal of exchange of stabilities is satisfied in the absence of suspended particles number density and solute parameter. The presence of the suspended particles number density and the solute parameter, bring oscillatory modes (as n_i may not be zero) which were non–existent in their absence.

5.5 THE CASE OF OVERSTABILITY

We now discuss the possibility of whether the instability may occur as overstability. Put $n = in_i$, it being remembered that n may be complex. Since for overstability, we wish to determine critical Rayleigh number for the onset of overstability, it suffices to find conditions for which (5.43) will admit of solution with n_i real.

Substituting $n = in_i$ in equation (5.43), the real and imaginary parts of equation (5.43), yield

$$Rk^2 = \frac{p_1}{q} S k^2 \Big[-n_i^2\{-H_1 p_1(1 - H_1 K_1 Aqb^2) + (\ell_1 - H_1 Aqb^3 K_1)\} + A\{-\bar{\delta}' b + (1+K_1)ab^3 + H(2A+b)\}\Big]$$

$$+ n_i^4\Big[b^2\{aH_1^2 p_1 q + aH_1\ell_1(q+p_1) + a^2 H_1 qp_1\ell_1 + aH_1^2\ell_1 qp_1(1+K_1)\} + b\{2AH_1^2 qap_1 + p_1\ell_1 F\}\Big]$$

$$- n_i^2\Big[2FH_1 qA + 2Aa\{H_1 q(1+K_1) + \ell_1 + H_1 p_1 K_1\} + H_1^2 p_1 q(1+K_1)$$

$$+ H_1\{(1+K_1)(\ell_1 q + \ell_1 p_1) + p_1(F+2A)\} + F\ell_1 + b^4\{aH_1(q+p_1)(1+K_1) + a\ell_1(1+K_1)\}$$

$$+ b^2\{2AH_1 F(p_1 + 2q) + 2H_1^2 p_1(aq + AK_1 q) + a\}\Big] + b^5(1+K_1) + b^4(2+K_1)A$$

$$\Big[\{2A\,bH_1 + b^2(H_1 - \bar{\delta}A)\} - n_i^2\{H_1 q(2A + b - \bar{\delta}Ab)a + \ell_1(H_1^2 q + ab)\}\Big]^{-1} \tag{5.46}$$

and

$$Rk^2\left[-a\ell_1 H_1 q\,n_i^3+2aH_1^2 q\,n_i+H_1^2 b\,q\,n_i+2Ab\,a\,n_i+ab^2 n_i+H_1\ell_1 b\,n_i-a\bar{\delta}Ab^2 n_i-H_1 q\bar{\delta}Abn_i\right]$$

$$+\frac{p_1}{q}Sk^2\left[a\bar{\delta}'Ab\,n_i-2Aa\,n_i-ab\,n_i-H_1\ell_1 n_i-H_1^2K_1 p_1 q\,Ab^2 a\,n_i^3+aK_1Ab^4 n_i+H_1p_1K_1Ab^3 n_i\right.$$

$$\left.+H_1 q\,K_1 Ab^2 n_i\right]=ab^2H_1^2p_1q\,\ell_1 n_i^5-2AH_1q\,ab^2n_i^3-H_1q\,ab^3n_i^3-2AH_1^2Fb\,p_1n_i^3-H_1^2F\,p_1q\,n_i^3b^2$$

$$-H_1F\,p_1\ell_1 q\,n_i^3b^2-2aAH_1^2\,p_1\,q\,n_i^3b^2-aH_1^2p_1q\,n_i^2b^3-aH_1\ell_1 q\,n_i^3b^3-2AaH_1^2p_1K_1q\,n_i^3\,b^2$$

$$-H_1^2\,p_1q\,n_i^3\,b^2\,\ell_1+2AH_1q\,n_i\,b^3+H_1q\,n_i\,b^4-H_1^2p_1q\,n_i^3\,b^2\,\ell_1K_1-aH_1^2\,p_1\,q\,n_i^3\,b^3K_1-aH_1\ell_1q\,n_i^3\,b^3K_1$$

$$-2AH_1\,n_i\,qb^3+K_1H_1\,q\,n_i\,b^4-2AH_1p_1a\,n_i^3\,b^2-H_1\,p_1\,a\,n_i^3\,b^3-a\,n_i^3\,b^3\,\ell_1-H_1\,p_1\,\ell_1F\,n_i^3\,b^2+F\,n_i\,b^4$$

$$+2AF\,n_i\,b^3-H_1\,p_1\ell_1\,a\,n_i^3\,b^3+2A\,n_i\,b^4K_1+a\,n_ib^5-aH_1p_1\,n_i^3\,b^3+2AaK_1b^4n_i+aK_1b^5n_i$$

$$+2\,aH_1p_1\ell_1b^3n_i+H_1p_1b^4n_i+b^4n_i\,\ell_1+2A\,H_1p_1b^3n_i+K_1H_1p_1b^4n_i+K_1\,\ell_1\,n_i\,b^4. \qquad (5.47)$$

Eliminating R between equations (5.46) and (5.47), we get

$$n_i^6\left[a^2H_1^2p_1\ell_1^2q^2(1+K_1)+a^2b^2H_1^2p_1q^2\left(1-\bar{\delta}A\right)+a^2b^2H_1^2\ell_1^2p_1q(H_1-1)\right]$$

$$+n_i^4\left[H_1^3b^2ap_1q^2(H_1-F)+a^3b^2H_1qp_1\ell_1\left(1-\bar{\delta}A\right)+a^2b^2H_1^2\ell_1p_1q\left(1-\bar{\delta}A\right)+H_1^4b^2q^2p_1\ell_1(F-aK_1)\right.$$

$$+a^2q^2\ell_1H_1^2p_1b^2\left(1-\bar{\delta}A\right)+a^2b^4H_1^2p_1\ell_1q\left(1-\bar{\delta}A\right)-H_1^2p_1q\ell_1ab^2(-aK_1+F)+a^2b^4H_1^2p_1q\left(1-\bar{\delta}A\right)$$

$$+a^2H_1^2\ell_1\bar{\delta}Ab^4q(p_1-q)-H_1^2q^2ab^3\bar{\delta}A(2Aa+F\ell_1)+H_1^2p_1q\bar{\delta}Aab^3\ell_1(1-Fq)-p_1H_1^2q^2\bar{\delta}Aa^2b^3(F-aK_1)$$

$$+a\ell_1^2H_1^2q^2(1+K_1)-ab^2\ell_1H_1^2p_1q(H_1-1)+H_1^3Fp_1q^2ab^3\left(1-\bar{\delta}A\right)+4A^2H_1^3q^2bp_1a(H_1-F)$$

$$-ab\ell_1^2FH_1(p_1-q)-\ell_1^2H_1^2q^2b^3(2aA+F\ell_1)-H_1^2q^2a^2b^4\left(1-\bar{\delta}A\right)+b^4a^2H_1^3q^2\ell_1p_1\bar{\delta}A(F-aK_1)$$

$$-H_1^2\ell_1^2ab(q-p_1)+2Aa^2H_1^3p_1K_1q^2b^3\left(\bar{\delta}A-1\right)+2AH_1^3q^2p_1\ell_1b^2q^2(H_1-F)-a^2H_1^3q^2p_1b^3-a^2b^2\ell_1^2$$

$$-a^2H_1p_1\ell_1^2b^2-a^2b^2\ell_1^2H_1(q-p_1K_1)-4A^2H_1^2q^2a^2b^2(1+H_1p_1)-a\ell_1H_1^3p_1q^2\left(b^3-q\right)$$

$$+H_1qp_1a^2b(\ell_1-H_1K_1b)-a^2b^2\ell_1H_1p_1(\bar{\delta}A-q)+\frac{p_1}{q}Sk^2H_1^3K_1p_1q^2Ab^2a^2(b+2A)-H_1^3p_1ab^3q(\ell_1+aq)$$

$$+\frac{p_1}{q}Sk^2H_1^4K_1q^2Ab^2a(H_1-b\bar{\delta}A)+\frac{p_1}{q}Sk^2aH_1^2qAb^3K_1(-\ell_1q+p_1a)+\frac{p_1}{q}Sk^2a\ell_1H_1q(a\ell_1-H_1p_1)$$

$$\left.+\frac{p_1}{q}Sk^2a\ell_1H_1^3p_1K_1Aq^2b^2\right]+n_i^2\left[b^6\left\{a^2H_1qp_1(1-\bar{\delta}A)+a^2H_1p_1K_1q(1-\bar{\delta}A)+\bar{\delta}a^2\ell_1q(2-K_1p_1)\right.\right.$$

$$\left.+\bar{\delta}AqH_1p_1a^2(1+K_1)\right\}+b^5\left\{H_1a^2p_1q(\bar{\delta}A-2)+\bar{\delta}Aa^2\ell_1q(-H_1+1)+aq(\ell_1+p_1\bar{\delta}A)+H_1a(F-aAK_1)q^2\right.$$

$$\left.+\frac{p_1}{q}Sk^2H_1aK_1\ell_1q(\bar{\delta}A-1)+aqK_1(H_1-F)-2AH_1a^2p_1q(1+K_1)\right\}$$

$$+b^4\left\{a^2(1-\bar{\delta}A)+H_1q\ell_1a(1+K_1)+H_1^2Fp_1\ell_1q(1-\bar{\delta}A)+a^2H_1p_1(1+K_1)\bar{\delta}A+H_1p_1q(H_1-F)\right.$$

$$+H_1^3p_1q^2\bar{\delta}Aa^2(H_1-F)+2AH_1^3\ell_1q(1+K_1)+H_1^2p_1\ell_1q(2-\bar{\delta}A)+\frac{p_1}{q}Sk^2H_1^2qK_1a(1-\bar{\delta}A)$$

$$-\frac{p_1}{q}Sk^2aA\ell_1H_1^2q(1+K_1)+H_1^2\ell_1qa^2(2-\bar{\delta}A)-4A\ell_1q(H_1-F)+H_1^2q(K_1+2H_1)-\bar{\delta}A(1+K_1)$$

$$\left.+4A^2H_1q(1+K_1)+\ell_1^2H_1^2q(1+K_1)\right\}+b^3\left\{2AH_1p_1q(H_1-F)+2AH_1^2F\ell_1q(H_1-\bar{\delta}A)+2aAH_1p_1(1+K_1)\right.$$

$$+H_1^3p_1K_1q(1-\bar{\delta}A)+aA\ell_1(F\bar{\delta}-\bar{\delta}-aK_1)+2A(\bar{\delta}A-2)+2A^2aH_1(H_1-F)-H_1p_1a(1-\bar{\delta}A)(F-aAK_1)$$

$$+2AH_1^3q^2(H_1-F)+H_1q(1-\bar{\delta}A)+2a^2A(\bar{\delta}A-2)+2A^2aH_1(H_1-F)-H_1p_1a(1-\bar{\delta}A)(F-aAK_1)$$

$$+2aH_1^3q^2(H_1-F)+H_1q(1-\bar{\delta}A)+4A^2H_1^3p_1K_1a(H_1-F)+aH_1p_1(H_1-\bar{\delta}A)$$

$$\left.-\frac{p_1}{q}Sk^2H_1p_1K_1q^2A(aAK_1+2H_1)-aH_1p_1(1-\bar{\delta}A)(F-aAK_1)\right\}+b^2\left\{-4a^2A^2\{1+H_1p_1(1+K_1)\}\right.$$

$$+\bar{\delta}Aab^2H_1\ell_1q(1+K_1)+\bar{\delta}Aab^2H_1p_1\ell_1(1+K_1)+4A^2\bar{\delta}H_1qa^2b^2(1+K_1)+H_1p_1\ell_1ab^2(\bar{\delta}A-1)-H_1\ell_1qab^2(1+K_1)$$ ^)

$$-AH_1p_1\bar{\delta}(2aA+F\ell_1)+2aAH_1p_1(aAK_1+2H_1)-\ell_1^2H_1^2p_1(1+K_1)-2aAFH_1p_1(2-\bar{\delta}A)-FH_1\ell_1^2$$

$$\left.+aK_1H_1p_1(H_1-1)-\frac{p_1}{q}Sk^2H_1qa^2b^2(\bar{\delta}A-\bar{\delta}'A)-\frac{p_1}{q}Sk^2H_1p_1q^2K_1(2-\bar{\delta}A-\bar{\delta}'A)+H_1p_1\bar{\delta}Ab^2(H_1-F)\right\}$$

$$\left.\left.+b\left\{2AH_1^2qa(H_1-F)+4aA^2H_1p_1(H_1-F)+4A^2H_1^3p_1q\ell_1(H_1-F)+\frac{p_1}{q}Sk^2H_1^3p_1qb(H_1-1)\right\}\right\}\right]$$

$$+b^7[(1-H_1)a-aH_1(1+K_1)]+b^6\left[H_1^2q(1+K_1)+H_1p_1(1-\bar{\delta}A)q-H_1p_1(H_1-\bar{\delta}A)q\right.$$

$$\left.+\frac{p_1}{q}Sk^2aK_1(1-\bar{\delta}A)+H_1p_1(H_1-F)\right]+b^5\left[\frac{p_1}{q}Sk^2H_1p_1K_1(H_1-\bar{\delta}A)+4A^2H_1q(H_1-F)\right.$$

$$+H_1^2 p_1 K_1 A.\frac{p_1}{q}Sk^2\left(\bar{\delta}A-1\right)+a\{(1-\bar{\delta}A)+K_1(1-H_1)\}\Bigg]+b^4\left[\frac{p_1}{q}Sk^2 H_1 q K_1 A(H_1-\bar{\delta}A)\right.$$

$$+4A^2H_1q(H_1-F)+4A^2H_1^2q(1-\bar{\delta}A)+(1+K_1)\{4aA(1-H_1)-H_1p_1(H_1-\bar{\delta}A)+A\ell_1\bar{\delta}\}-aAK_1(1-H_1)\Big]$$

$$+b^3\left[\frac{2p_1}{q}Sk^2A^2K_1ab^3(2-\bar{\delta}A-\bar{\delta}'A)+a\bar{\delta}Ab^3(\bar{\delta}'A-1)+2A^2a\{(2+K_1)(1-H_1)\}-4A^2H_1p_1b^3q(H_1-F)\right.$$

$$-\bar{\delta}A^2\{2F+H_1p_1(2+K_1)\}\Big]+b^2\left[\frac{p_1}{q}Sk^2H_1\ell_1(\bar{\delta}A-\bar{\delta}'A)-2A^2H_1^2p_1(1+K_1)+\frac{p_1}{q}Sk^2(1-\bar{\delta}A)H_1q\right.$$

$$\left.+\frac{p_1}{q}Sk^2\bar{\delta}'AH_1q(H_1-1)\right]+b\left[\frac{p_1}{q}Sk^2H_1^2q\left(1-\bar{\delta}A\right)+\frac{p_1}{q}Sk^2H_1^2q(1-\bar{\delta}'A)\right]=0. \qquad (5.48)$$

It is evident from the equation (5.48) that oscillatory modes will not be present for all values of parameters. For example, in the absence of coupling between spin and heat flux $(\bar{\delta}=0)$ and that of between spin and analogous solute flux $(\bar{\delta}'=0)$, solute parameter $(S=0)$ and in the absence of suspended particles $(a=0=f=h_1)$, equation (5.48) allows only $n_i=0$ and so overstable solution will not take place if $K_1p_1<2$.

The presence of suspended particles and coupling between spin and heat flux, and that of between spin and analogous solute fluxes bring overstability in the system.

For stationary convection i.e. $n_i=0$ and in the presence of coupling between spin and heat fluxes $(\bar{\delta}\neq 0)$ and that of between spin and solute fluxes $(\bar{\delta}'\neq 0)$, equation (5.46) reduces to

$$R=\frac{\frac{p_1}{q}Sk^2\left[A\{-\bar{\delta}'b+(1+K_1)ab^3+H_1(2A+b)\}\right]+b^4(2+K_1)A+b^5(1+K_1)}{k^2\{2AbH_1+b^2(1-\bar{\delta}A)\}}. \qquad (5.49)$$

In the absence of stable solute parameters $(S=0,\bar{\delta}'=0)$, equation (5.49) reduces to

$$R=\frac{(1+K_1)\,b^4+A(2+K_1)\,b^3}{k^2\{2AH_1+b(1-\bar{\delta}A)\}}, \qquad (5.50)$$

a result in good agreement with that of Sharma and Kumar [2002].

In the absence of suspended particles $(a = 0 = h_1 = f)$ and coupling between spin and heat fluxes $(\bar{\delta} = 0)$, equation (5.49) further reduces to

$$R = \frac{(1+K_1)b^4 + A(2+K_1)b^3}{k^2(2A+b)}, \tag{5.51}$$

a result derived by Pérez–Garcia and Rubi [1982].

For Newtonian viscous fluid, i.e. $\bar{\delta} = 0 = \bar{\delta}'$, $K_1 = 0$ and in the absence of solute parameter $(S = 0)$, equation (5.51) further reduces to

$$R = \frac{b^3}{k^2}, \tag{5.52}$$

which agrees with earlier result (Chandrasekhar [1961]).

5.6 RESULTS AND DISCUSSION

Equation (5.48) has been examined numerically using the Newton–Raphson method through the software Fortran 90. We have plotted the variation of Rayleigh number with respect to wave-number using equation (5.46) satisfying equation (5.48) for overstable case and equation (5.50) for stationary case, for the fixed permissible values of the dimensionless parameters $A = 0.5, \bar{\delta} = 1$, $\bar{\delta}' = 1.5$, $p_1 = 5$, $q =$ 0.035, $F =$ 1.005, $H_1 =$ 1.01, $a =$ 10, $K_1 =$ 1 and $\ell_1 = 1$.

Figures (5.1)–(5.4) correspond to three different values of the solute parameter i.e. $S =$ 30, 20, 10 and in the absence of solute parameter, respectively. It is evident from the graphs that Rayleigh number increases with the increase in stable solute parameter even in the presence of suspended particles (fine dust) number density depicting the stabilizing effect of solute parameter. Moreover, the solute parameter and the suspended particles introduce oscillatory modes in the system. It is also noted from figures that Rayleigh number for overstability is always less than the Rayleigh number for stationary convection, for a fixed wave-number.

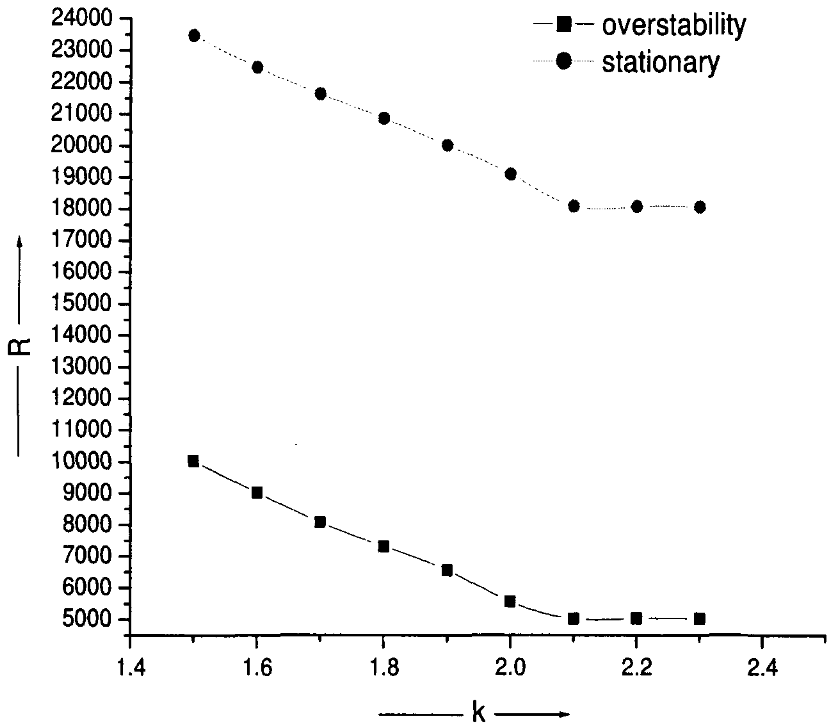

Figure 5.1: The variation of Rayleigh number (R) with wave number (k) for $A = 0.5$, $\bar{\delta} = 1$, $\bar{\delta}' = 1.5$, $p_1 = 3, q = 0.035, F = 1.005, H_1 = 1.01, a = 10, K_1 = 1$, $\ell_1 = 1$ and $S = 30$.

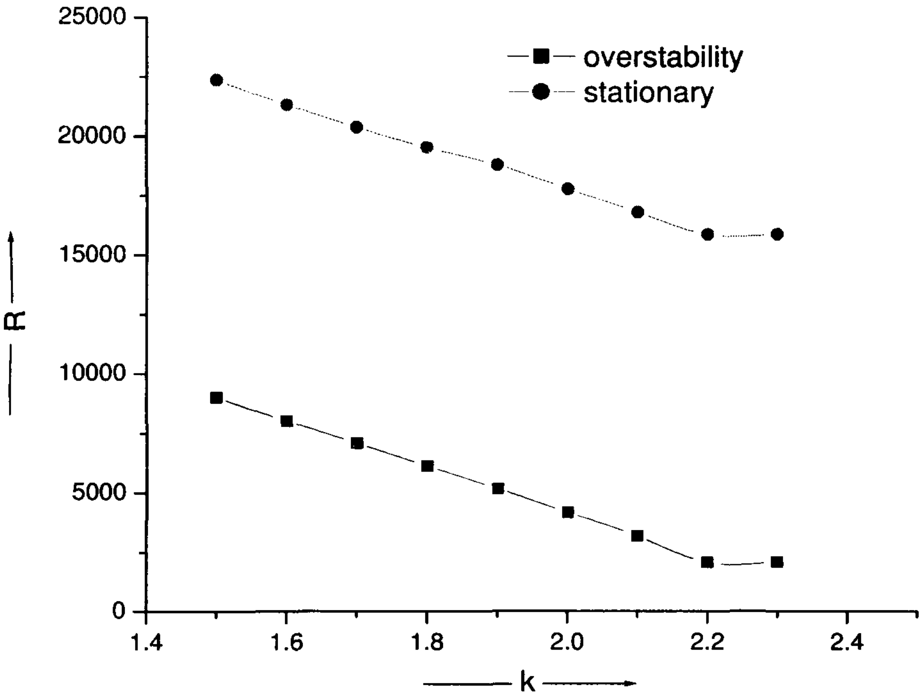

Figure 5.2: The variation of Rayleigh number (R) with wave number (k) for $A = 0.5$, $\bar{\delta} = 1, \bar{\delta}' = 1.5$, $p_1 = 3, q = 0.035, F = 1.005, H_1 = 1.01, a = 10, K_1 = 1$, $\ell_1 = 1$ and $S = 20$.

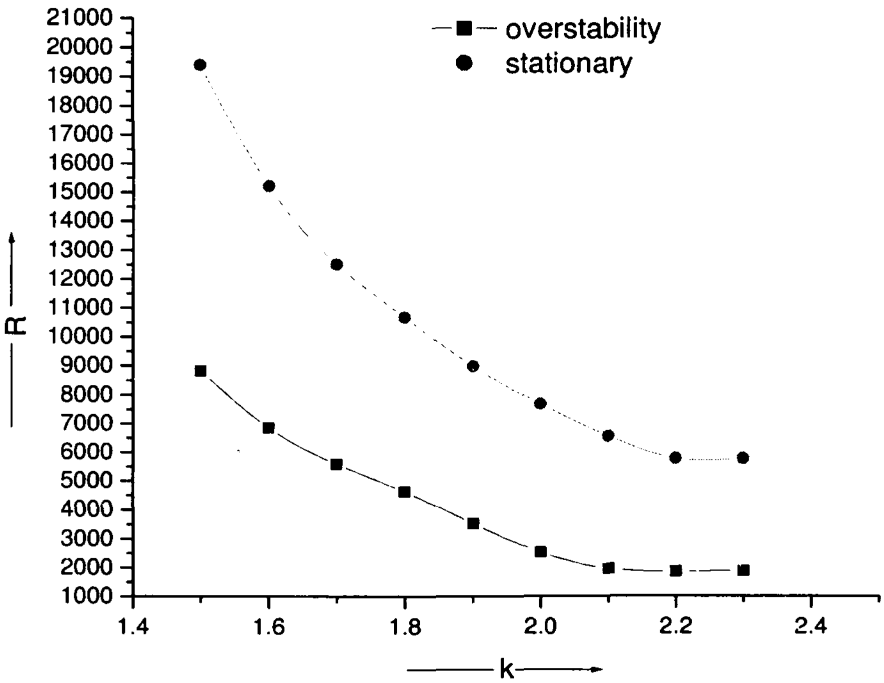

Figure 5.3: The variation of Rayleigh number (R) with wave number (k) for $A = 0.5$, $\bar{\delta} = 1, \overline{\delta'} = 1.5$, $p_1 = 3, q = 0.035, F = 1.005, H_1 = 1.01, a = 10, K_1 = 1$, $\ell_1 = 1$ and $S = 10$.

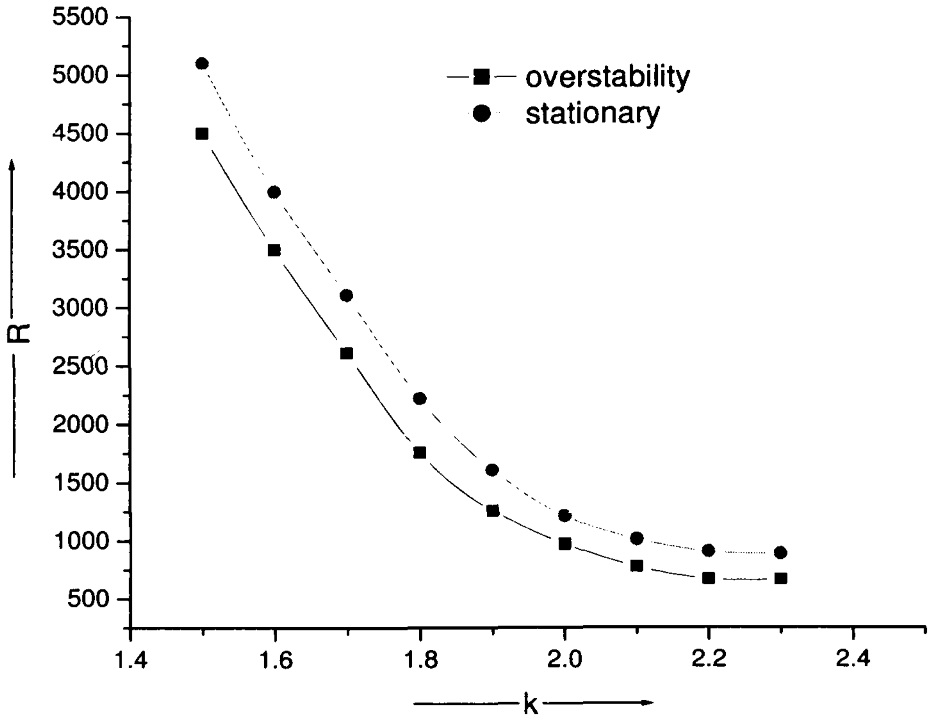

Figure 5.4: The variation of Rayleigh number (R) with wave number (k) for $A = 0.5$, $\bar{\delta} = 1, \overline{\delta'} = 1.5$, $p_1 = 3, q = 0.035, F = 1.005, H_1 = 1.01, a = 10, K_1 = 1$, $\ell_1 = 1$ and $S = 0$.