CHAPTER III

THERMAL CONVECTION OF MICROPOLAR FLUIDS IN THE PRESENCE OF SUSPENDED PARTICLES IN HYDROMAGNETICS IN POROUS MEDIUM

3.1 INTRODUCTION

The theoretical and experimental results of the onset of themal instability (Bénard convection) in a fluid layer under varying assumptions of hydromagnetics, has been depicted in a treatise by Chandrasekhar [1961]. Lapwood [1948] has studied the convective flow in porous medium using linearized stability theory. The Rayleigh instability in flow through a porous medium has been considered by Wooding [1960]. The problem of thermal convection in a fluid in porous medium is of importance in geophysics, soil–science, ground–water, hydrology and astrophysics. The developments of geothermal power resources have increased general interest in the study of problems of convection in porous media. Generally, it is accepted that comets consist of a density snowball of a mixture of frozen gases which is in the process of their journey changes from solid to gas and vice–versa. The physical property of comets, meteororites and inter–planetary dust strongly suggests the importance of porosity in the astrophysical context (McDonnel [1978]). Moreover, Saffman and Taylor [1958] have shown that the motion in a Hele–Shaw cell is mathematically analogous to two dimensional flow in porous medium.

When a fluid permeates a porous material, the gross effect is represented by Darcy's law. As a result of this macroscopic law, the usual viscous term in the equations of motion of microscopic fluid is replaced by the resistance term $\left[-\frac{1}{k_1}(\mu+\kappa)\,\vec{q}_2\right]$, where μ and κ are viscosity and dynamic microrotation viscosity respectively, k_1 is the medium permeability and $\vec{q}_2$ is the Darcian (filter) velocity of the fluid. The effect of a magnetic field on the stability of such a flow is of great interest in geophysics e.g. in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like porous medium which can become convectively unable as a result of differential diffusion. Also, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. Sharma and Gupta [1995] have studied the thermal convection in micropolar fluids in porous medium and have found that medium permeability has stabilizing effect on stationary convection and destabilizing effect on the overstable case. Sharma and Kumar [1997] have studied the thermal instability of micropolar fluids in hydromagnetics in porous medium. Sharma and Dutt [2006] have studied the Hall–currents and the thermal convection of micropolar fluids. They have found that Hall–currents have destabilizing effect on the system.

Keeping in mind the importance and relevance of porosity and hydromagnetics in chemical engineering, geophysics and biomechanics, thermal instability of micropolar fluids in the presence of a uniform vertical magnetic field to include the effect of suspended particles (dust particles) in porous medium has been considered in the present chapter.

3.2 FORMULATION OF THE PROBLEM AND DISTURBANCE EQUATIONS

Consider an infinite, horizontal layer of an incompressible electrically conducting micropolar fluid of thickness d permeated with suspended particles (or fine dust) in an isotropic and homogeneous medium of porosity ε and medium permeability k_1. A uniform vertical magnetic field $\vec{H}\,(0,0,H)$ pervades the system. This fluid–particles layer is heated from below but convection sets in when the temperature gradient $\left(\beta = \left| {dT}/{dz} \right|\right)$ between the lower and upper boundaries exceeds a certain critical value. $\vec{q}_2$ and c_s denote, respectively, the filter (seepage) velocity and the heat capacity of solid matrix. Assume that external couples and heat sources are not present. The mass, momentum, internal angular momentum, internal energy balance equations using the Boussinesq approximation are

$$\nabla \cdot \vec{q}_2 = 0\,, \tag{3.1}$$

$$\frac{1}{\varepsilon}\left(\frac{\partial}{\partial t} + \frac{\vec{q}_2}{\varepsilon}\cdot\nabla\right)\vec{q}_2 = -\frac{1}{\rho_0}\nabla p - \frac{1}{\rho_0 k_1}(\mu+\kappa)\vec{q}_2 + \frac{\kappa}{\rho_0}\nabla\times\vec{\vartheta} - \left(1+\frac{\delta\rho}{\rho_0}\right)g\,\hat{e}_z + \frac{1}{\rho_0}\frac{KN}{\varepsilon}(\vec{u}-\vec{q}_2)$$
$$+\frac{1}{\rho_0}\frac{\mu_e}{4\pi}\left(\nabla\times\vec{H}\right)\times\vec{H}\,, \tag{3.2}$$

$$\rho_0 j_1\left(\frac{\partial}{\partial t} + \frac{\vec{q}_2}{\varepsilon}\cdot\nabla\right)\vec{\vartheta} = (\varepsilon' + \beta'')\nabla\left(\nabla\cdot\vec{\vartheta}\right) + \gamma'\,\nabla^2\vec{\vartheta} + \frac{\kappa}{\varepsilon}\nabla\times\vec{q}_2 - 2\,\kappa\vec{\vartheta}\,, \tag{3.3}$$

$$\left[\rho_0 c_v \varepsilon + \rho_s c_s (1-\varepsilon)\right]\frac{\partial T}{\partial t} + \rho_0\, c_v\, \vec{q}_2 \cdot \nabla T = k_T \nabla^2 T + \delta\left(\nabla\times\vec{\vartheta}\right)\cdot\nabla T\,. \tag{3.4}$$

Also the equation of state is given by

$$\rho = \rho_0\left[1 - \alpha\,(T - T_0)\right]\,, \tag{3.5}$$

where ρ_0, T_0 are reference density, reference temperature at the lower boundary and α is the coefficient of thermal expansion coefficient.

The Maxwell's equations yield

$$\varepsilon\frac{\partial\vec{H}}{\partial t} = \nabla\times\left(\vec{q}_2\times\vec{H}\right) + \varepsilon\,\eta\,\nabla^2\vec{H}\,, \tag{3.6}$$

$$\nabla\cdot\vec{H} = 0\,. \tag{3.7}$$

Assuming dust particles of uniform size, spherical shape and small relative velocities between the two phases (fluid and particles), the net effect of the particles on the fluid is equivalent to an extra body force term per unit volume $KN(\vec{u}-\vec{q}_2)$, as has been taken in equation (3.2). The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. The distance between the particles is assumed to be so large compared with their diameter that interparticle reactions are ignored. The buoyancy force on the particles is also neglected. The equations of motion and continuity for the particles, under these restrictions, are

$$mN\left(\frac{\partial}{\partial t}+\frac{\vec{u}}{\varepsilon}\cdot\nabla\right)\vec{u}=KN\left(\vec{q}_2-\vec{u}\right), \tag{3.8}$$

$$\varepsilon\frac{\partial N}{\partial t}+\nabla\cdot(N\vec{u})=0. \tag{3.9}$$

Now we study the stability of the system wherein we give small perturbations on the initial state and on seeing the reaction of the perturbations on the system.

The steady state solution is

$\vec{v}=0$, $\vec{u}=0$, $\vec{\vartheta}=0$, $N=N_0$(constant), $T=T_0-\beta z$, $\rho=\rho_0(1+\alpha\beta z)$,

$$p=p_0-g\rho_0\left(z+\frac{\alpha\beta z^2}{2}\right), \tag{3.10}$$

where p_0 is the pressure at $z=0$ and $\beta=\frac{T_0--T_1}{d}$ $(T_0>T_1)$ is the magnitude of uniform temperature gradient.

Let $\vec{q}_2(u,v,w)$, $\vec{u}(\ell,r,s)$, $\vec{\omega}$, N, δp, $\delta\rho$, θ and $\vec{h}(h_x,h_y,h_z)$ denote, respectively, the perturbations in filter (seepage) velocity $\vec{q}_2(0,0,0)$, particles velocity $\vec{u}(0,0,0)$, spin $\vec{\vartheta}$, particles number density N_0, pressure p, density ρ, temperature T and magnetic field $\vec{H}(0,0,H)$ so that the change in density $\delta\rho$ caused by the perturbation θ

in temperature is given by

$$\delta\rho = -\rho_0 \alpha\, \theta . \tag{3.11}$$

Then equations (3.1)–(3.9) yield the perturbation equations

$$\nabla \cdot \vec{q}_2 = 0, \tag{3.12}$$

$$\frac{\rho_0}{\varepsilon}\left(\frac{\partial}{\partial t} + \frac{\vec{q}_2}{\varepsilon} \cdot \nabla\right)\vec{q}_2 = -\nabla\, \delta p - \frac{1}{k_1}(\mu + \kappa)\vec{q}_2 + \kappa(\nabla \times \vec{\omega}) + g\, \rho_0 \alpha\, \theta\, \hat{e}_z + \frac{KN_0}{\varepsilon}(\vec{u} - \vec{q}_2) + \frac{\mu_e}{4\pi}(\nabla \times \vec{h}) \times \vec{H}\,, \tag{3.13}$$

$$\rho_0 j_1\left(\frac{\partial}{\partial t} + \frac{\vec{q}_2}{\varepsilon} \cdot \nabla\right)\vec{\omega} = (\varepsilon' + \beta'')\nabla\, (\nabla \cdot \vec{\omega}) + \gamma' \nabla^2 \vec{\omega} + \frac{\kappa}{\varepsilon}\nabla \times \vec{v} - 2\,\kappa\vec{\omega}, \tag{3.14}$$

$$[\rho_0\, c_v \varepsilon + \rho_s c_s (1-\varepsilon)]\left(\frac{\partial}{\partial t} + \frac{\vec{q}_2}{\varepsilon} \cdot \nabla\right)\theta = \beta\, (w + h_1 s) + k_T \nabla^2 \theta + \delta\, [\nabla\theta \cdot (\nabla \times \vec{\omega}) - (\nabla \times \vec{\omega})_z \cdot \beta], \tag{3.15}$$

$$\varepsilon \frac{\partial \vec{h}}{\partial t} = \nabla \times \left(\vec{q}_2 \times \vec{h}\right) + \varepsilon\, \eta \nabla^2 \vec{h}\,, \tag{3.16}$$

$$\nabla \cdot \vec{h} = 0, \tag{3.17}$$

$$mN_0\left(\frac{\partial}{\partial t} + \frac{\vec{u}}{\varepsilon} \cdot \nabla\right)\vec{u} = KN_0(\vec{q}_2 - \vec{u}\,), \tag{3.18}$$

$$\varepsilon \frac{\partial M}{\partial t} + \nabla \cdot \vec{u} = 0, \tag{3.19}$$

where $H_1 = 1 + h_1, h_1 = \dfrac{fc_{pt}}{c_v}$, $f = \dfrac{mN_0}{\rho_0}$ and $M = \dfrac{N}{N_0}$.

Using the non–dimensional numbers

$$z = z^* d,\quad \theta = \beta\, d\theta^*,\quad t = \frac{\rho_0 d^2}{\mu} t^*,\quad \vec{q}_2 = \frac{\kappa_T}{d}\vec{q}_2^{\,*}, \nabla = \frac{\nabla^*}{d},$$

$$\vec{u} = \frac{\kappa_T}{d}\vec{u}^*,\quad p = \frac{\mu\kappa_T}{d^2}p^*,\quad \vec{\omega} = \frac{\kappa_T}{d^2}\vec{\omega}^*,\quad \vec{h} = \left(\frac{\mu\kappa_T}{d^2}\right)^{\frac{1}{2}}\vec{h}^* \tag{3.20}$$

and then removing the stars for convenience, the non–dimensional forms of equations

(3.12)–(3.19) become

$$\nabla\cdot\vec{q}_2=0, \tag{3.21}$$

$$\frac{1}{\varepsilon}\left(\frac{\partial}{\partial t}+\frac{\vec{q}_2}{\varepsilon}\cdot\nabla\right)\vec{q}_2=-\nabla\,\delta p-\frac{1}{\bar{k}_1}(1+K_1)\nabla^2\vec{q}_2+K_1\nabla\times\vec{\omega}+\hat{e}_z\,R\theta+\frac{N_2}{\varepsilon}(\vec{u}-\vec{q}_2)+$$

$$\frac{\mu_e}{4\pi}\left(\nabla\times\vec{h}\right)\times\vec{H}\,, \tag{3.22}$$

$$\bar{j}_2\left(\frac{\partial}{\partial t}+\frac{\vec{q}_2}{\varepsilon}\cdot\nabla\right)\vec{\omega}=C_1'\nabla\left(\nabla\cdot\vec{\omega}\right)-C_0'\nabla\times\left(\nabla\times\vec{\omega}\right)+K_1\left(\frac{1}{\varepsilon}\nabla\times\vec{q}_2-2\vec{\omega}\right), \tag{3.23}$$

$$EH_1p_1\left(\frac{\partial}{\partial t}+\frac{\vec{q}_2}{\varepsilon}\cdot\nabla\right)\theta=\beta\left(w+h_1 s\right)+\kappa_T\nabla^2\theta+\bar{\delta}\left[\nabla\theta\cdot(\nabla\times\vec{\omega})-(\nabla\times\vec{\omega})_z\right], \tag{3.24}$$

$$\varepsilon\frac{\partial\vec{h}}{\partial t}=\nabla\times\left(\vec{q}_2\times\vec{h}\right)+\frac{\varepsilon}{p_2}\nabla^2\vec{h}\,, \tag{3.25}$$

$$\nabla\cdot\vec{h}=0, \tag{3.26}$$

$$\left[a\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)+1\right]\vec{u}=\vec{q}_2\,, \tag{3.27}$$

The new dimensionless coefficients are

$$K_1=\frac{\kappa}{\mu},\quad \bar{j}_2=\frac{j}{d^2},\quad \bar{\delta}=\frac{\delta}{\rho_0 c_v d^2},\quad C_0'=\frac{\gamma'}{\mu d^2},\quad C_1'=\frac{\varepsilon'+\beta''+\gamma'}{\mu d^2},$$

$$E=\varepsilon+(1-\varepsilon)\frac{\rho_s c_s}{\rho_0 c_v}\;,\quad \bar{k}_1=\frac{k_1}{d^2},\quad N_2=KN_0\frac{d^2}{\mu},\quad a=\frac{m}{Kd^2}\;\;\frac{\mu}{\rho_0} \tag{3.28}$$

and the dimensionless Rayleigh number R, thermal Prandtl number p_1, magnetic Prandtl number p_2 are

$$R=\frac{g\,\alpha\,\beta\,\rho_0\,d^4}{\mu\,\kappa_T},\;p_1=\frac{\upsilon}{\kappa_T},\;p_2=\frac{\upsilon}{\eta}, \tag{3.29}$$

where $\kappa_T=\dfrac{k_T}{\rho_0 c_v}$ is the thermal diffusivity .

The boundaries are assumed to be free and perfectly heat conducting. Also if the medium adjoining the fluid is electrically non–conducting, then the boundary conditions appropriate to problem are

$$w=\frac{\partial^2 w}{\partial z^2}=\frac{\partial}{\partial z}(\nabla\times\vec{q}_2)_z=0,\left(\nabla\times\vec{h}\right)_z=(\nabla\times\vec{\omega})_z=0,\ \theta=\frac{\partial h_z}{\partial z}=0 \text{ at } z=0 \text{ and } z=d. \quad (3.30)$$

3.3 LINEAR THEORY: DISPERSION RELATION

Since the perturbations applied on the system are assumed to be very small, the products of perturbations, the second and higher order perturbations are negligibly small. Under the linearized theory, second and higher order terms are neglected and only the linear terms are retained. Accordingly, the non–linear terms $(\vec{q}_2\cdot\nabla)\vec{q}_2$, $(\vec{q}_2\cdot\nabla)\theta$, $(\vec{q}_2\cdot\nabla)\vec{\omega}$, $(\vec{q}_2\cdot\nabla)\vec{\omega}$, $\nabla\theta\cdot(\nabla\times\vec{\omega})$ in equations (3.22)–(3.24) are neglected.

Eliminating s between equations (3.24) and (3.27) and applying the curl operator twice to resulting equation and linearizing, we obtain

$$L_2\left[EH_1p_1\frac{\partial}{\partial t}-\nabla^2\right]\theta=\left(a\frac{\partial}{\partial t}+H_1\right)\beta w-L_2\bar{\delta}\Omega_{z1}. \quad (3.31)$$

Eliminating $\vec{u}$ between equations (3.22) and (3.27) and on linearizing, we obtain

$$\varepsilon^{-1}L_1\vec{q}_2=L_2\left[-\nabla\delta p-\frac{1}{\bar{k}_1}(1+K_1)\vec{q}_2+K_1\nabla\times\vec{\omega}+R\theta\,\hat{e}_z+\frac{\mu_e}{4\pi}\left(\nabla\times\vec{h}\right)\times\vec{H}\right], \quad (3.32)$$

where $L_1=a\frac{\partial^2}{\partial t^2}+F\frac{\partial}{\partial t}$, $L_2=a\frac{\partial}{\partial t}+1$ and $F=f+1$.

Applying the curl operator twice to equation (3.22) and taking z–component, we get

$$\varepsilon^{-1}L_1\nabla^2 w=L_2\left[R\nabla_1^2\theta-\frac{1}{\bar{k}_1}(1+K_1)\nabla^2 w+K_1\nabla^2\Omega_{z1}+\frac{\mu_e H}{4\pi}\frac{\partial}{\partial z}\nabla^2 h_z\right], \quad (3.33)$$

where $\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$, $\Omega_{z1}=(\nabla\times\vec{\omega})_z$. (3.34)

Applying the curl operator to equations (3.22), (3.23) and (3.25) taking z –component, we get

$$\varepsilon^{-1} L_2 \frac{\partial}{\partial t}\zeta_z + \varepsilon^{-1} N_2 \zeta_z (L_2 - 1) = -\frac{1}{\bar{k}_1}(1+K_1)\nabla^2 \zeta_z L_2 + \frac{\mu_e H}{4\pi}\frac{\partial \xi_z}{\partial z} L_2, \tag{3.35}$$

$$\bar{j}_2 \frac{\partial \Omega_{z1}}{\partial t} = C_0' \nabla^2 \Omega_{z1} - K_1\left(\frac{1}{\varepsilon}\nabla^2 w + 2\Omega_{z1}\right), \tag{3.36}$$

$$\varepsilon \frac{\partial \xi_z}{\partial t} = H \frac{\partial}{\partial t}\zeta_z + \frac{\varepsilon}{p_2}\nabla^2 \xi_z, \tag{3.37}$$

where $\xi_z = (\nabla \times \vec{h})_z$, $\zeta_z = (\nabla \times \vec{q}_2)_z$ are the z – components of current density and vorticity, respectively. K_1 and C_0' account for coupling between vorticity and spin effects and spin diffusion, respectively.

Taking the z –component of equation (3.25), we get

$$\varepsilon \frac{\partial h_z}{\partial t} = H \frac{\partial w}{\partial z} + \frac{\varepsilon}{p_2}\nabla^2 h_z. \tag{3.38}$$

We now analyze an arbitrary perturbation into a complete set of normal modes and then examine the stability of each of these modes individually. For the system of equations (3.31), (3.33) and (3.35)–(3.38), the analysis can be made in terms of two dimensional periodic waves of assigned wave-numbers. Thus we ascribe to all quantities describing the perturbation a dependence on x, y and t of the form

$$\exp\left[i\left(k_x x + k_y y\right) + nt \right], \tag{3.39}$$

where $k = \left(k_x^2 + k_x^2\right)^{1/2}$ is the resultant wave-number, k_x and k_y are real constants and n is the stability parameter which can be, complex, in general. The solution of the stability problem requires the specifications of the state for each k. The above considerations allow us to suppose that the perturbation quantities have the form

$$[w,\Omega_{z1},\zeta_z,\xi_z,\theta,h_z]=[W(z),\Omega_2(z),Z(z),G(z),\Theta(z),B(z)]\exp\left(ik_xx+ik_yy+nt\right), \tag{3.40}$$

then the equations (3.31), (3.33) and (3.35)–(3.38) become

$$(an+1)\left\{EH_1p_1n-\left(D^2-k^2\right)\right\}\Theta=(an+H_1)W-(an+1)\bar{\delta}\,\Omega_2, \tag{3.41}$$

$$\left(D^2-k^2\right)\left\{\left(an^2+Fn\right)+\frac{1}{\bar{k}_1}(an+1)(1+K_1)\left(D^2-k^2\right)\right\}W=(an+1)\Big\{-Rk^2\Theta+K_1\left(D^2-k^2\right)\Omega_2$$

$$+\frac{\mu_eH}{4\pi}\left(D^2-k^2\right)DB\Big\}, \tag{3.42}$$

$$\left\{\varepsilon^{-1}\left(an^2+Fn\right)+(an+1)\frac{1}{\bar{k}_1}(1+K_1)\right\}Z=\frac{\mu_eH}{4\pi}(an+1)DG, \tag{3.43}$$

$$\left\{\ell_1n+2A-\left(D^2-k^2\right)\right\}\Omega_2=-A\varepsilon^{-1}\left(D^2-k^2\right)W, \tag{3.44}$$

$$\left\{n-\frac{1}{p_2}\left(D^2-k^2\right)\right\}G=\varepsilon^{-1}H\,DZ, \tag{3.45}$$

$$\left\{n-\frac{1}{p_2}\left(D^2-k^2\right)\right\}B=\varepsilon^{-1}H\,DW, \tag{3.46}$$

where $A=\frac{K_1}{C_0'}$, $\ell_1=\bar{j}_2\frac{A}{K_1}$, $D=\frac{d}{dz}$, $L_2=a\frac{\partial}{\partial t}+1=an+1$,

$$L_1=a\frac{\partial^2}{\partial t^2}+F\frac{\partial}{\partial t}=an^2+Fn. \tag{3.47}$$

Eliminating Θ, Z, B, Ω_2 from equations (3.41)–(3.46), we get

$$\left(D^2-k^2\right)\left\{\varepsilon^{-1}\left(an^2+Fn\right)+\frac{1}{\bar{k}_1}(an+1)(1+K_1)\left(D^2-k^2\right)\right\}\left\{EH_1p_1n-\left(D^2-k^2\right)\right\}$$

$$\left\{\ell_1n+2A-\left(D^2-k^2\right)\right\}\left\{n-\frac{1}{p_2}\left(D^2-k^2\right)\right\}W=-Rk^2\left\{\ell_1n+2A-\left(D^2-k^2\right)\right\}$$

$$\left\{n-\frac{1}{p_2}\left(D^2-k^2\right)\right\}(an+H_1)W-Rk^2\left\{n-\frac{1}{p_2}\left(D^2-k^2\right)\right\}(an+1)\bar{\delta}A\,\varepsilon^{-1}\left(D^2-k^2\right)W$$

$$-\varepsilon^{-1}AK_1\left(D^2-k^2\right)^2(an+1)\left\{EH_1p_1n-\left(D^2-k^2\right)\right\}\left\{n-\frac{1}{p_2}\left(D^2-k^2\right)\right\}W+\frac{H^2}{4\pi}\left(D^2-k^2\right)$$

$$\{EH_1p_1n-(D^2-k^2)\}\,(an+1)\,\{\ell_1n+2A-(D^2-k^2)\}\varepsilon^{-1}D^2W\,. \tag{3.48}$$

The boundary conditions (3.30) transform to

$$W=0,\ D^2W=0,\ DZ=0,\ G=0,\ \Omega_2=0,\ \Theta=0,\ DB=0 \ \text{ at } \ z=0 \text{ and } 1. \tag{3.49}$$

Using boundary conditions (3.49), equations (3.41)–(3.46) transform to

$$D^2\Theta=0,\ D^2\Omega_2=0,\ D^3Z=0,\ D^3G=0,\ D^3B=0, \tag{3.50}$$

where $\quad D^3=\dfrac{d^3}{dz^3}, D^2=\dfrac{d^2}{dz^2}.$

Differentiating (3.42) twice with respect to z and using boundary conditions (3.50), it can be shown that $D^4W=0$, where $D^4=\dfrac{d^4}{dz^4}$. It can be shown from equations (3.41)–(3.46) and boundary conditions (3.49), (3.50) that all even order derivatives of W vanish on the boundaries .The proper solution of W belonging to the lowest mode is

$$W=W_0\sin\pi z\,, \tag{3.51}$$

where W_0 is a constant. Substituting equation (3.51) in equation (3.48) and putting $b=\pi^2+k^2$, we obtain

$$Rk^2\left\{n+\frac{b}{p_2}\right\}\{(an+H_1)(\ell_1n+2A+b)-(an+1)\varepsilon^{-1}\bar{\delta}Ab\}=b\left\{\varepsilon^{-1}(an^2+Fn)+\frac{1}{\bar{k}_1}(an+1)(1+K_1)\right\}$$
$$(EH_1p_1n+b)(\ell_1n+2A+b)\left\{n+\frac{b}{p_2}\right\}-\varepsilon^{-1}K_1Ab^2(an+1)(EH_1p_1n+b)\left\{n+\frac{b}{p_2}\right\}+$$
$$\frac{H^2\pi}{4}(EH_1p_1nb+b^2)(an+1)\varepsilon^{-1}(\ell_1n+2A+b). \tag{3.52}$$

3.4 THE CASE OF OSCILLATORY MODES

Equating the imaginary parts of equation (3.52), we have

$$n_i\left[n_i^4EabH_1p_1\ell_1\varepsilon^{-1}+n_i^2\left(-2Aab^2\varepsilon^{-1}-ab^3\varepsilon^{-1}-2EH_1p_1A\frac{ab^2}{p_2}\varepsilon^{-1}-2EH_1p_1A\frac{ab^3}{p_2}\varepsilon^{-1}-\frac{a^2b^3}{p_2}\ell_1\varepsilon^{-1}\right.\right.$$
$$-EFH_1p_1b\,\varepsilon^{-1}-EFH_1p_1b^2\varepsilon^{-1}-F\,b^2\ell_1\varepsilon^{-1}-EH_1p_1\ell_1\varepsilon^{-1}\frac{b^2}{p_2}+\frac{1}{\bar{k}_1}\left\{-2AEH_1p_1ab-EH_1p_1ab^2-\frac{ab^2\ell_1}{p_2}\right.$$

$$- EH_1 p_1 \ell_1 \frac{ab^2}{p_2} - 2AEH_1 p_1 abK_1 - EH_1 p_1 K_1 b^2 - ab^2 \ell_1 K_1 - EH_1 p_1 \ell_1 bK_1 + \frac{b^3 a}{p_2} K_1 \Bigg\} + Rk^2 a \ell_1 +$$

$$2A \frac{b^5}{p_2} F\varepsilon^{-1} + \frac{b^4}{p_2} F\varepsilon^{-1} \Bigg) + \frac{ab^4}{p_2} + \frac{2b^3}{p_2} \left(Aa + \frac{1}{\bar{k}_1} \{AaK_1 + AEH_1 p_1 K_1\} \right) + \frac{b^2}{p_2} \Bigg(\frac{1}{\bar{k}_1} 2Ab^3 EH_1 p_1 K_1$$

$$+ Rk^2 \varepsilon^{-1} a \bar{\delta} A - Rk^2 a \Big) + b^2 \frac{1}{\bar{k}_1} 2AK_1 + b \left(Rk^2 \left\{ -H_1 + \bar{\delta} A \varepsilon^{-1} - \frac{2A}{p_2} a - \frac{1}{p_2} H_1 \ell_1 \right\} \right) - 2Rk^2 AH_1 \Bigg] = 0. \quad (3.53)$$

Equation (3.53) yields that either $n_i = 0$ or $n_i \neq 0$, which means that the modes are either non–oscillatory or oscillatory. In the absence of suspended particles number density, magnetic field intensity and medium permeability, equation (3.53) reduces to

$$n_i \left(2A\, b^2 K_1 + R\, k^2 \bar{\delta} A\, b \right) = 0 \quad (3.54)$$

and term within the brackets is definitely positive, which implies that $n_i = 0$. Therefore, the oscillatory modes are not allowed and principal of exchange of stabilities is satisfied for porous medium in the absence of suspended particles and magnetic field. The presence of the suspended particles number density, the magnetic field intensity and medium permeability bring oscillatory modes (as n_i may not be zero) which were non–existent in their absence.

3.5 THE CASE OF OVERSTABILITY

We now discuss the possibility of whether the instability may occur as overstability. Put $n = in_i$, it being remembered that n may be complex. Since for overstability, we wish to determine critical Rayleigh number for the onset of overstability via a state of pure oscillations, it suffices to find conditions for which equation (3.52) will admit of solution with n_i real.

Substituting $n = in_i$ in equation (3.52), the real and imaginary parts of equation (3.52), yield

$$Rk^2\left[\frac{b^2}{p_2}\{2H_1A+b(1-\varepsilon^{-1}\bar{\delta}A)\}-n_i^2\left\{b\ell_1\left(1+\frac{a}{p_2}\right)+a\{2AH_1E+b(1-\varepsilon^{-1}\bar{\delta}A)\}\right\}\right]=$$

$$n_i^4\left[EH_1p_1\ell_1b^2\varepsilon^{-1}a\frac{(1+K_1)}{\bar{k}_1}+\varepsilon^{-1}ab\left\{b\ell_1\left(1+\frac{EH_1p_1}{p_2}\right)\right\}+EH_1p_1\varepsilon^{-1}(2aA+F\ell_1)\right]$$
$$-n_i^2\left[\left[\left\{(2A+b)\varepsilon^{-1}\left(1+\frac{EH_1p_1}{p_2}\right)+\frac{EH_1p_1\ell_1}{p_2}\right\}\left\{2aAEH_1\frac{(1+K_1)}{\bar{k}_1}b^3+b^3\varepsilon^{-1}(2aA+F\ell_1)\right\}\right.\right.$$
$$\left\{EH_1p_1\frac{(1+K_1)}{\bar{k}_1}\varepsilon^{-1}+\frac{\varepsilon^{-1}aE}{p_2}\right\}+b^4a\ell_1E\frac{(1+K_1)}{\bar{k}_1}\left(1+\frac{EH_1p_1}{p_2}\right)-K_1A\varepsilon^{-1}b^3$$
$$\left.\left.\left\{Ep_1H_1+a\varepsilon^{-1}\left(1+\frac{Ep_1H_1}{p_2}\right)\right\}\right]\right]+\frac{H^2\pi}{4}\varepsilon^{-1}\left[-n_i^2\{b\ell_1(EH_1p_1+a)+EH_1p_1\varepsilon^{-1}(2A+b)\}\right]$$
$$+\left[\frac{1}{p_2}\left\{\frac{(1+K_1)}{\bar{k}_1}-\varepsilon^{-1}K_1A\right\}b^5+\frac{b^4}{p_2}\left\{2A\frac{(1+K_1)}{\bar{k}_1}\right\}\right] \tag{3.55}$$

and

$$Rk^2\left[-a\ell_1n_i^3+2AH_1n_i+H_1bn_i-n_i\bar{\delta}Ab\varepsilon^{-1}+\frac{2Ab}{p_2}an_i+\frac{b^2}{p_2}an_i+\frac{b}{p_2}H_1\ell_1n_i-\frac{b^2}{p_2}\varepsilon^{-1}an_i\bar{\delta}A\right]=$$
$$EabH_1p_1\ell_1n_i^5\varepsilon^{-1}-2Aab^2n_i^3\varepsilon^{-1}-ab^3n_i^3\varepsilon^{-1}-2EH_1p_1n_i^3A\frac{ab^2}{p_2}\varepsilon^{-1}-2EH_1p_1n_i^3A\frac{ab^3}{p_2}\varepsilon^{-1}-\frac{a^2b^3}{p_2}$$
$$\ell_1n_i^3\varepsilon^{-1}-EFH_1p_1n_i^3b\varepsilon^{-1}-EFn_i^3H_1p_1b^2\varepsilon^{-1}-Fb^2\ell_1n_i^3\varepsilon^{-1}-EH_1p_1\ell_1n_i^3\varepsilon^{-1}\frac{b^2}{p_2}+2A\frac{b^5}{p_2}$$
$$Fn_i\varepsilon^{-1}+\frac{b^4}{p_2}Fn_i\varepsilon^{-1}+\frac{1}{\bar{k}_1}\left[-2AEH_1p_1abn_i^3-EH_1p_1n_i^3ab^2-\frac{an_i^3b^2\ell_1}{p_2}-EH_1p_1\ell_1n_i^3\frac{ab^2}{p_2}\right.$$
$$+2Aan_i\frac{b^3}{p_2}+\frac{ab^4}{p_2}n_iK_1-2AEH_1p_1n_i^3abK_1-EH_1p_1n_i^3K_1b^2-ab^2\ell_1n_i^3K_1+2Aan_i\frac{b^3}{p_2}K_1$$
$$+\frac{ab^4}{p_2}n_iK_1-EH_1p_1n_i^3\ell_1b+2Ab^2n_i+b^3n_i+2AEH_1p_1n_i\frac{b^2}{p_2}+EH_1p_1n_i\frac{b^3}{p_2}+\frac{b^3}{p_2}\ell_1n_iK_1$$
$$\left.-EH_1p_1n_i^3\ell_1bK_1+2Ab^2n_iK_1+2AEH_1p_1n_i\frac{b^2}{p_2}K_1+2AEH_1p_1n_i\frac{b^3}{p_2}K_1+\frac{b^3}{p_2}n_iK_1\right]. \tag{3.56}$$

Eliminating R between equations (3.55) and (3.56), we get

$$n_i^6\left[b^2\left\{\varepsilon^{-1}a^2H_1^2\ell_1^2\frac{(1+K_1)}{\bar{k}_1}\right\}+b\left\{\frac{\varepsilon^{-1}aEH_1p_1}{\bar{k}_1}(EH_1\ell_1-ab\varepsilon^{-1}\bar{\delta}A-F\ell_1)-\varepsilon^{-1}b^3a\ell_1H_1E\left(\frac{E}{p_2}+\frac{(1+K_1)}{\bar{k}_1}\right)\right\}\right]$$
$$+n_i^4\left[b^5\left\{EH_1p_1a^2(1-\varepsilon^{-1}\bar{\delta}A)+H_1p_1\ell_1\bar{\delta}A\frac{E}{p_2}\frac{1+K_1}{\bar{k}_1}+\frac{Ep_1}{p_2}\bar{\delta}A(H_1-1)\right\}+b^4\left\{2EH_1p_1a^2A\frac{(1+K_1)}{\bar{k}_1}\right.\right.$$

$$-\frac{Ep_1}{p_2}a^2p_1H_1^2(H_1-1)\Big\}+b^3\Big\{\frac{1}{p_2^2}EH_1p_1aF(1-\varepsilon^{-1}\bar{\delta}A)+EH_1p_1\ell_1^2a^2\frac{1}{p_2^2}(H_1-1)-EH_1p_1\ell_1K_1a$$

$$(2-\varepsilon^{-1}\bar{\delta}A)+\frac{E}{p_2^2}H_1p_1a\ell_1^2(1-\varepsilon^{-1}\bar{\delta}A)\Big\}+b^2\Big\{\frac{-aE^2}{p_2}2AF\ell_1(H_1-1)-\frac{H^2\pi}{4}\varepsilon^{-1}\ell_1a\Big(a\ell_1-\frac{Ep_1}{p_2}a\ell_1+p_1\varepsilon^{-1}\bar{\delta}A\Big)$$

$$-2Aa\ell_1H_1E^2\frac{(1+K_1)}{\bar{k}_1}+2AE^2\varepsilon^{-1}(2-K_1Ep_1)\Big\}+b\Big\{\frac{1}{p_2^2}-2A^2E^2aH_1p_1(H_1-F)-\frac{2H^2\pi}{4}\varepsilon^{-1}a\ell_1A$$

$$\Big(a\ell_1-\frac{Ep_1}{p_2}+p_1\varepsilon^{-1}\bar{\delta}A\Big)\Big\}\Big]+n_i^2\Big[b^7\Big\{-E^2H_1p_1\frac{a}{p_2^2}(1-\varepsilon^{-1}\bar{\delta}A)-\varepsilon^{-1}\bar{\delta}A\frac{aE}{p_2^2}\frac{(1+K_1)}{\bar{k}_1}\Big\}+b^6\Big\{\frac{E^2a^2}{p_2^2}\varepsilon^{-1}\bar{\delta}A$$

$$\frac{(1+K_1)}{\bar{k}_1}-EH_1p_1\ell_1\frac{1}{p_2^2}(2-\varepsilon^{-1}\bar{\delta}A)-\frac{2Ep_1}{p_2}H_1^2aA\frac{(1+K_1)}{\bar{k}_1}-\frac{H^2\pi}{4}E^2\frac{H_1p_1}{p_2}(2-\varepsilon^{-1}\bar{\delta}A)-EH_1p_1\bar{\delta}A$$

$$\frac{1}{p_2^2}\varepsilon^{-1}(F-aK_1)-\bar{\delta}A\varepsilon^{-1}H_1\frac{1}{p_2}\ell_1\frac{(1+K_1)}{\bar{k}_1}\Big\}+b^5\Big\{-\frac{E^2H_1p_1}{p_2^2}a\ell_1(H_1-1)+\varepsilon^{-1}H_1p_1\frac{a^2}{p_2^2}E^2\frac{(1+K_1)}{\bar{k}_1}$$

$$+\bar{\delta}A\frac{\varepsilon^{-1}a}{p_2}\frac{(1+K_1)}{\bar{k}_1}-\frac{E^2\ell_1^2}{p_2^2}(2aA+F\ell_1)-2A^2\varepsilon^{-1}\frac{\bar{\delta}E}{p_2}(H_1-F)+E^2H_1ap_1\frac{1}{\bar{k}_1p_2^2}(F-aK_1)+\frac{\varepsilon^{-1}K_1aA}{p_2^2}$$

$$(E\ell_1+H_1p_1\varepsilon^{-1}\bar{\delta}A)+\frac{4AaE^2}{p_2^2}\Big(1+H_1p_1E\frac{(1+K_1)}{\bar{k}_1}-EK_1p_1\Big)+2\Big\{1+EH_1p_1\frac{(1+K_1)}{\bar{k}_1}\Big\}\{1-\varepsilon^{-1}\bar{\delta}A\}\Big\}$$

$$+b^4\Big\{-E^2H_1^2\frac{p_1}{p_2^2}(F\bar{\delta}\varepsilon^{-1}-\varepsilon^{-1}\bar{\delta}-aK_1)+\frac{EH_1p_1\ell_1}{p_2}(EH_1p_1+\varepsilon^{-1}F\ell_1)-\frac{H^2\pi}{4}\varepsilon^{-1}\Big(\varepsilon^{-1}a^2\ell_1-\frac{Ep_1}{p_2}a\ell_1\Big)$$

$$+\frac{\ell_1^2\varepsilon^{-1}}{p_2^2}(H_1-F)-\frac{H^2\pi}{4}\varepsilon^{-1}\Big\{F\Big\{\Big(1-\frac{Ep_1}{p_2}\Big)-\frac{EH_1p_1\ell_1}{p_2}\Big\}(1-\varepsilon^{-1}\bar{\delta}A)+\frac{F\ell_1E}{p_2}(EH_1p_1+\varepsilon^{-1}\bar{\delta}A)\Big\}+4A^2$$

$$\Big(1+EH_1p_1\frac{(1+K_1)}{\bar{k}_1}\Big)+\frac{2A^2FE^2}{p_2^2}(1-\varepsilon^{-1}\bar{\delta}A)\Big\}+b^3\Big\{\frac{EH_1p_1}{p_2^2}\ell_1F(H_1-1)+\frac{H^2\pi}{4}\varepsilon^{-1}\{EH_1p_1(2-\varepsilon^{-1}\bar{\delta}A)\}$$

$$\Big(1-\frac{EH_1p_1}{p_2}\Big)-\frac{H^2\pi}{4}\varepsilon^{-1}\Big(\Big\{F\ell_1\Big(1-\frac{EH_1p_1}{p_2}\Big)-\frac{EH_1p_1\ell_1}{p_2}\Big\}(2-\varepsilon^{-1}\bar{\delta}A)\Big)-\frac{(1+K_1)}{\bar{k}_1}\frac{a\ell_1}{p_2}(H_1-\ell_1)+\frac{H_1\ell_1E^2}{p_2^2}$$

$$(\ell_1+\varepsilon^{-1}H_1p_1\bar{\delta}A)+H_1^2p_1F\frac{1}{p_2}2A(1-\varepsilon^{-1}\bar{\delta}A)\Big\}+b^2\Big\{\frac{\varepsilon^{-1}H^2\pi}{4}\frac{EH_1p_1\ell_1}{p_2}(2-\varepsilon^{-1}\bar{\delta}A)+\frac{H^2\pi}{4}\varepsilon^{-1}\ell_1$$

$$\Big\{\ell_1+Ep_1\Big(\frac{\ell_1}{p_2}-\bar{\delta}A\varepsilon^{-1}\Big)\Big\}+\frac{H^2\pi}{4}H_1p_1\ell_1(1-\varepsilon^{-1}\bar{\delta}A)+\frac{H_1p_1E}{p_2}\frac{(1+K_1)}{\bar{k}_1}\Big\}+b\Big\{\varepsilon^{-1}\frac{H^2\pi}{2}EH_1p_1\ell_1FA$$

$$\Big(1-\frac{Ep_1}{p_2}\Big)\Big\}\Big]+b^8\Big\{\frac{E^2}{p_2}\Big(1+H_1p_1\frac{(1+K_1)}{\bar{k}_1}(2-\varepsilon^{-1}\bar{\delta}A)-F\ell_1\varepsilon^{-1}\frac{(1+K_1)}{\bar{k}_1}\bar{\delta}Aa\Big)\Big\}$$

$$+b^7\Big[\frac{2E^2}{p_2^2}\Big\{1+EH_1p_1\frac{(1+K_1)}{\bar{k}_1}-EK_1p_1\Big\}A(1-\varepsilon^{-1}\bar{\delta}A)+2Aa\varepsilon^{-1}\Big\{1+EH_1p_1\frac{(1+K_1)}{\bar{k}_1}\Big\}\Big]$$

$$+b^6\Big[\frac{2A^2FE^2}{p_2^2}\Big\{1+H_1p_1\frac{(1+K_1)}{\bar{k}_1}(1-\varepsilon^{-1}\bar{\delta}A)+\frac{E}{p_2^2}H_1p_1(H_1-1)\Big\}+\frac{H^2\pi}{4}\varepsilon^{-1}\Big\{\Big(\frac{EH_1p_1}{p_2}-1\Big)$$

$$(2-\varepsilon^{-1}\bar{\delta}A)-\frac{EH_1p_1F}{p_2}\varepsilon^{-1}\bar{\delta}A\ell_1\Big\}\Big]+b^5\Big[\frac{2A^2\varepsilon^{-1}H_1}{p_2^2}(2-K_1Ep_1)+\frac{2E^2}{p_2^2}AH_1\varepsilon^{-1}(F-aK_1)+$$

$$\frac{\varepsilon^{-1}H^2\pi}{4}\left\{2AF\left(\frac{EH_1p_1}{p_2}-1\right)\left(2-\varepsilon^{-1}\bar{\delta}A\right)-\frac{EH_1p_1\ell_1}{p_2}\varepsilon^{-1}\bar{\delta}AF\right\}\Bigg]+b^4\left[-\frac{2EH_1}{p_2}A^2\frac{(1+K_1)}{\bar{k}_1}+\frac{H^2\pi}{4}\varepsilon^{-1}\right.$$

$$\left.\left\{\left(1-\varepsilon^{-1}\bar{\delta}A\right)\left(\frac{EH_1p_1}{p_2}-1\right)+\frac{2A^2Ea}{p_2^2}K_1(H_1-1)\right\}\right]+b^3\left[-\varepsilon^{-1}\frac{H^2\pi}{2}A\frac{a}{p_2}(H_1-1)+\frac{EH_1a}{p_2}\frac{(1+K_1)}{\bar{k}_1}\right]$$

$$+b^2\left[\frac{H^2\pi}{4}\varepsilon^{-1}\left(2-\varepsilon^{-1}\bar{\delta}A\right)\frac{E\ell_1H_1}{p_2}-\varepsilon^{-1}A^2H^2\pi\frac{aEH_1}{p_2}(H_1-1)\right]=0. \qquad (3.57)$$

It is evident from the equation (3.57) that overstable modes will not be present for all values of parameters. For example, in the absence of coupling between spin and heat flux $(\bar{\delta}=0)$, magnetic field $(\vec{H}=0)$, $\bar{k}_1\to\infty$ and in the absence of suspended particles $(a=0=f=h_1)$, equation (3.57) allows only $n_i=0$ and so overstable solution will not take place if $EK_1p_1<2$.

Thus for stationary convection i.e. $n_i=0$ and in the presence of coupling between spin and heat fluxes $(\bar{\delta}\neq 0)$, equation (3.54) reduces to

$$R=\frac{b^3\left[\frac{(1+K_1)}{\bar{k}_1}-\varepsilon^{-1}K_1A\right]+2b^2A\frac{(1+K_1)}{\bar{k}_1}+\frac{H^2\pi}{4}(2A+b)\varepsilon^{-1}bp_2}{k^2\left\{2H_1A+b\left(1-\varepsilon^{-1}\bar{\delta}A\right)\right\}}. \qquad (3.58)$$

In the absence of magnetic field intensity $(\vec{H}=0)$ and suspended particles $(a=0=f=h_1)$ equation (3.57) further reduces to

$$R=\frac{b^3\left[\frac{(1+K_1)}{\bar{k}_1}-\varepsilon^{-1}K_1A\right]+2A\frac{(1+K_1)}{\bar{k}_1}b^2}{k^2\left\{2A+b\left(1-\varepsilon^{-1}\bar{\delta}A\right)\right\}}, \qquad (3.59)$$

a result in good agreement with Sharma and Gupta [1995].

3.6 RESULTS AND DISCUSSION

Equation (3.57) has been examined numerically using the Newton–Raphson method through the software Fortran 90. We have plotted the variation of Rayleigh number with respect to wave-number using equation (3.55) satisfying (3.57) for overstable case and equation (3.58) for stationary case, for the fixed permissible values of the dimensionless parameters $K_1 = 1$, $A = 0.5$, $\bar{\delta} = 1, \ell_1 = 1$, $p_1 = 5, p_2 = 1$, $a = 10, F = 1.005$, $H_1 = 1.01, \varepsilon = 0.5$, $E = 0.9$, $\bar{k}_1 = 2$.

Figures (3.1)–(3.3) correspond to three values of magnetic field intensity H = 70, 100 and 120 Gauss, respectively. The graphs show that Rayleigh number increases with increase in magnetic field intensity depicting thereby the stabilizing effect of magnetic field intensity. Moreover, the magnetic field introduces the oscillatory modes in the system.

Figures (3.4)–(3.6) correspond to three values of medium permeability $\bar{k}_1 = 5$, 10 and 30. The graphs show that the Rayleigh number for the stationary convection and for the case of overstability decreases with the increase in medium permeability depicting thereby destabilizing effect of medium permeability.

Figures (3.7)–(3.9) correspond to three values of micropolar coefficient $\kappa = 0.5$, 0.7 and 1.0, respectively, accounting for dynamic microrotation viscosity. The graphs show that the Rayleigh number for the stationary convection and for the case of overstability decreases with the increase in micropolar coefficient κ implying thereby the destabilizing effect of dynamic microrotation viscosity.

Figures (3.10)–(3.12) correspond to three values of micropolar coefficient γ' =1.0, 1.2 and 1.4, respectively. The graphs show that the Rayleigh number for the stationary convection and for the case of overstability decreases with the increase in micropolar coefficient γ' implying thereby the destabilizing effect of coefficient of angular viscosity, therefore micropolar coefficients have destabilizing effects on the system. Thus there is a competition between the large enough stabilizing effect of magnetic intensity and the destabilizing effect of the micropolar coefficients. The presence of coupling between thermal and micropolar effects, magnetic field and suspended particles number density may bring overstability in the system. It is also noted from the figures (3.3), (3.4), (3.7) and (3.10) that the Rayleigh number for overstability is always less than the Rayleigh number for stationary convection, for a fixed wave-number. However, the reverse may also occur for large wave-numbers, as has been depicted in figures (3.1), (3.2), (3.5), (3.6), (3.8), (3.9), (3.11) and (3.12).

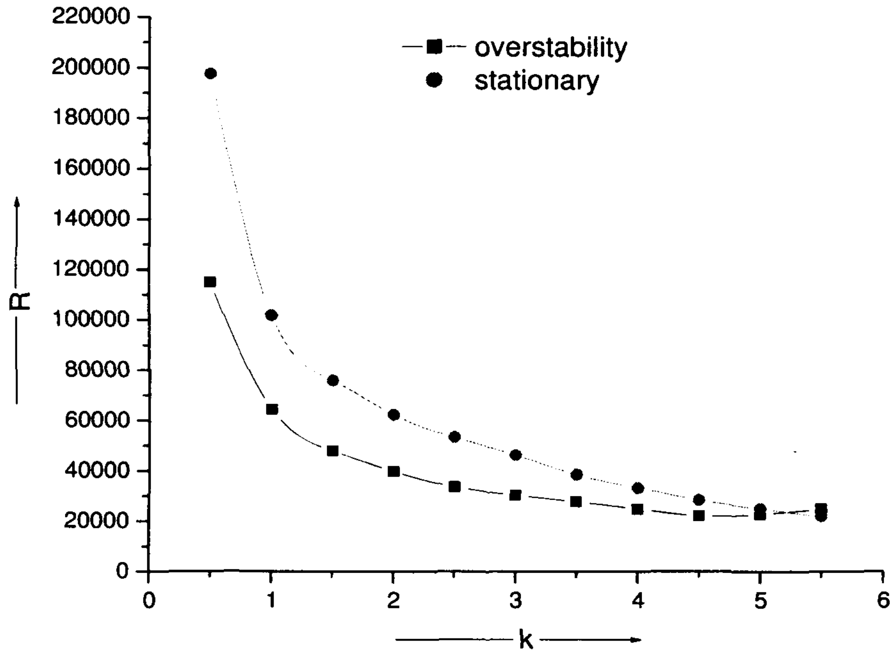

Figure 3.1: The variation of Rayleigh number(R) with wave number(k) for $A = 0.5$, $\bar{\delta} = 1, \varepsilon = 0.5, E = 0.9, \bar{k}_1 = 2, p_1 = 5,, p_2 = 1, F = 1.005, H_1 = 1.01, a = 10, K_1 = 1$, $\ell_1 = 1$ and $H = 70$ Gauss.

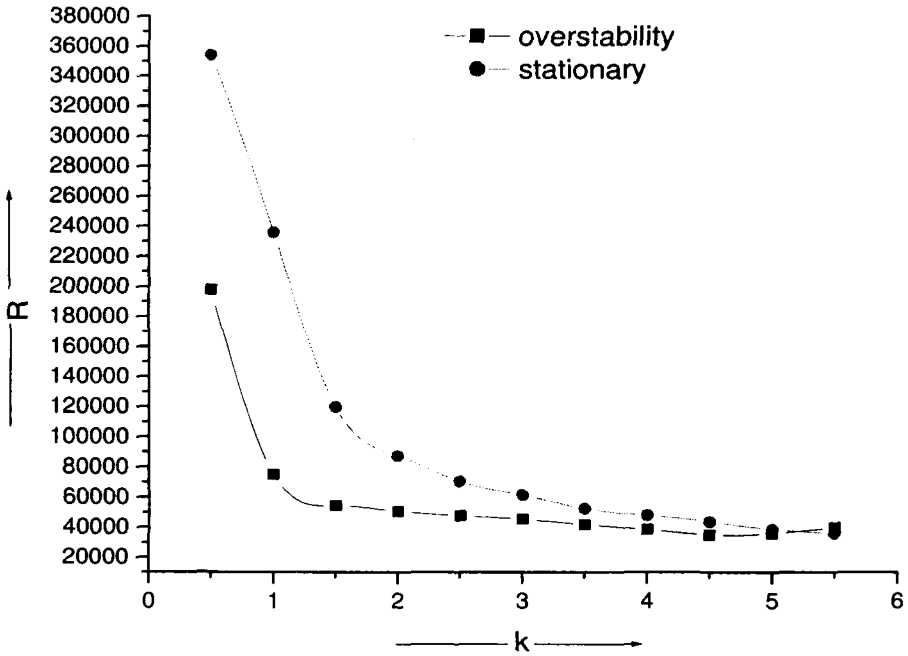

Figure 3.2: The variation of Rayleigh number(R) with wave number $(k)$for $A = 0.5$, $\bar{\delta} = 1, \varepsilon = 0.5, E = 0.9, \bar{k}_1 = 2, p_1 = 5,, p_2 = 1, F = 1.005, H_1 = 1.01, a = 10, K_1 = 1$, $\ell_1 = 1$ and $H = 100$ Gauss.

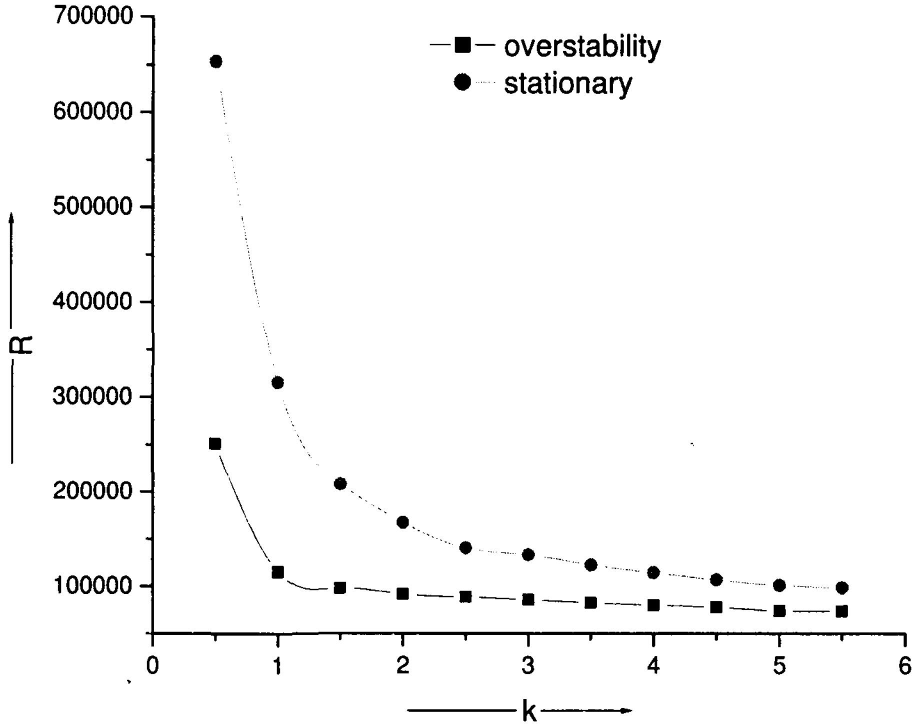

Figure 3.3: The variation of Rayleigh number(R) with wave number (k) for $A = 0.5$, $\bar{\delta} = 1$, $\varepsilon = 0.5$, $E = 0.9$, $\bar{k}_1 = 2$, $p_1 = 5$, $p_2 = 1$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $K_1 = 1$, $\ell_1 = 1$ and $H = 120$ Gauss.

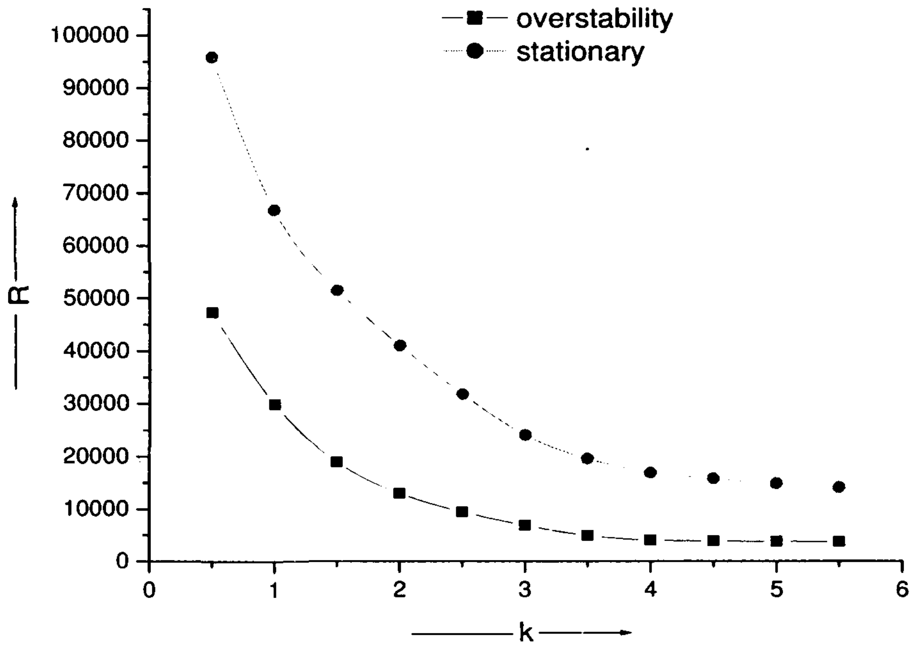

Figure 3.4: The variation of Rayleigh number(R) with wave number (k) for $A = 0.5$, $\bar{\delta} = 1$, $\varepsilon = 0.5$, $E = 0.9$, $p_1 = 5$, $p_2 = 1$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $K_1 = 1$, $\ell_1 = 1$, $H = 100$ Gauss and $\bar{k}_1 = 5$.

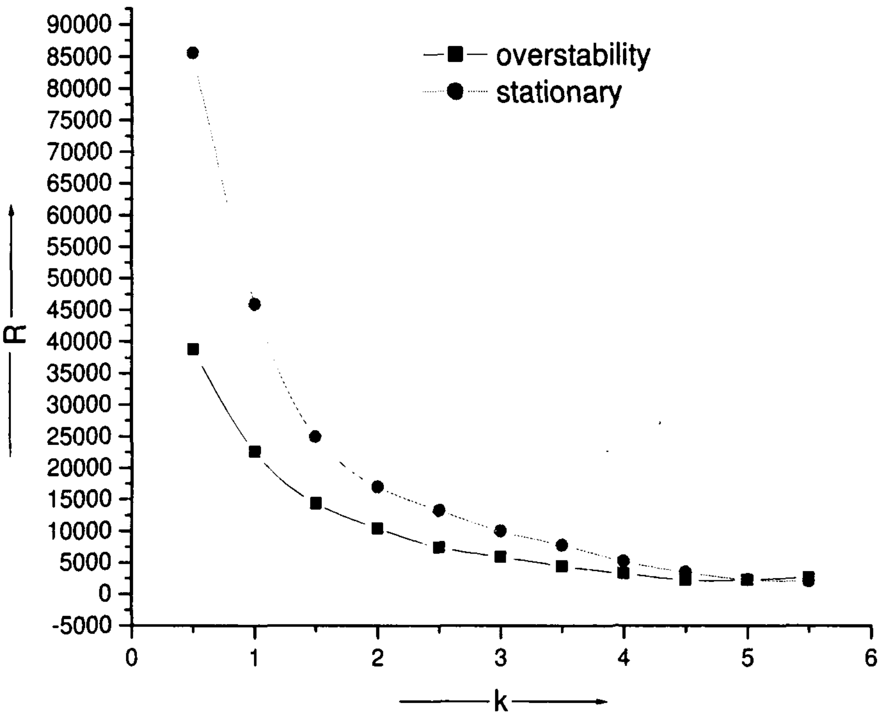

Figure 3.5: The variation of Rayleigh number(R) with wave number $(k)$for $A = 0.5$, $\bar{\delta} = 1, \varepsilon = 0.5, E = 0.9$, $p_1 = 5$, $p_2 = 1$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $K_1 = 1$, $\ell_1 = 1$, $H = 100$ Gauss and $\bar{k}_1 = 10$.

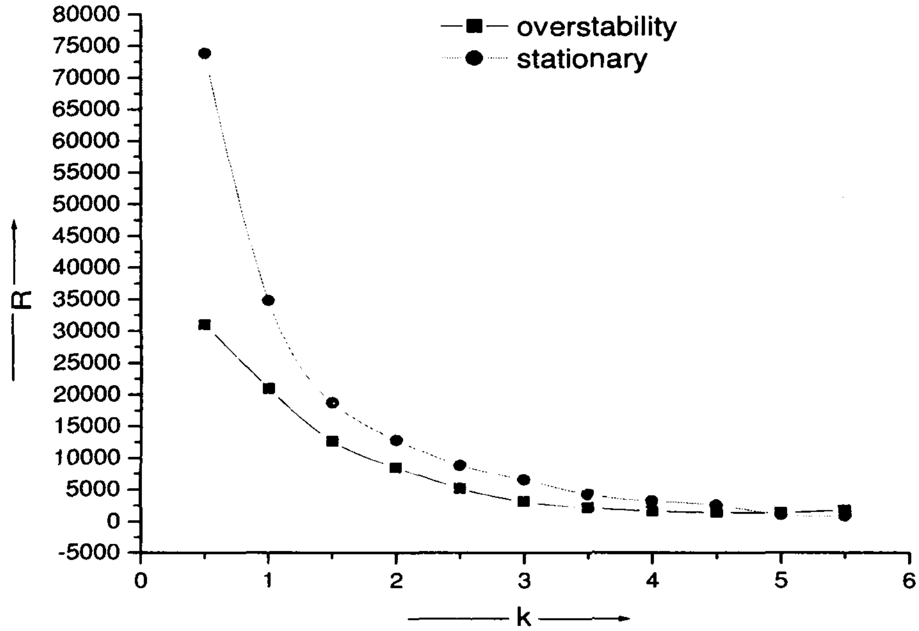

Figure 3.6: The variation of Rayleigh number (R) with wave number $(k)$for $A = 0.5$, $\bar{\delta} = 1, \varepsilon = 0.5, E = 0.9$, $p_1 = 5$, $p_2 = 1$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $K_1 = 1$, $\ell_1 = 1$, $H = 100$ Gauss and $\bar{k}_1 = 30$.

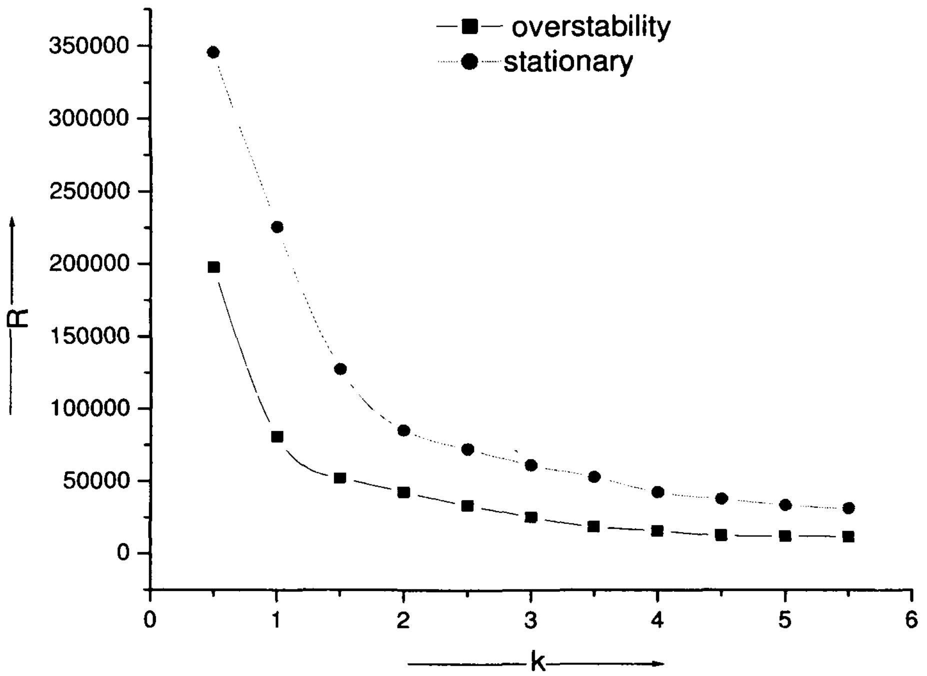

Figure 3.7: The variation of Rayleigh number(R) with wave number(k) for $\bar{\delta}=1, \varepsilon=0.5, E=0.9, p_1=5, p_2=1, F=1.005, H_1=1.01, a=10, K_1=1, \ell_1=1,$ $H=100$ Gauss, $\bar{k}_1=2$ and $\kappa=0.5$.

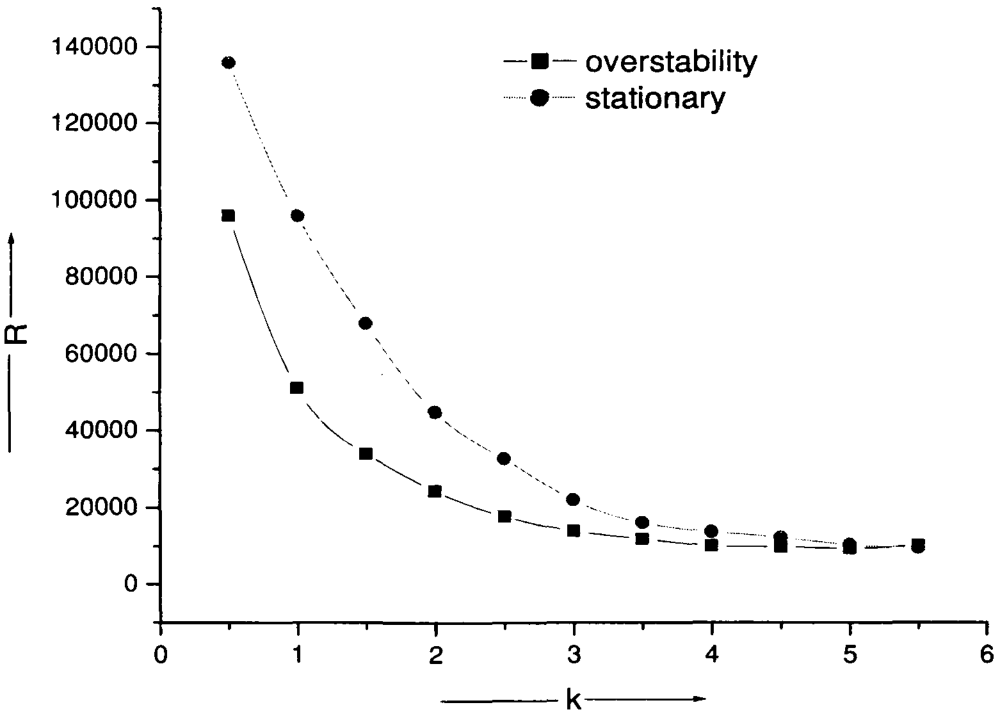

Figure 3.8: The variation of Rayleigh number(R) with wave number(k) for $\bar{\delta}=1, \varepsilon=0.5, E=0.9, p_1=5, p_2=1, F=1.005, H_1=1.01, a=10, K_1=1, \ell_1=1,$ $H=100$ Gauss, $\bar{k}_1=2$ and $\kappa=0.7$.

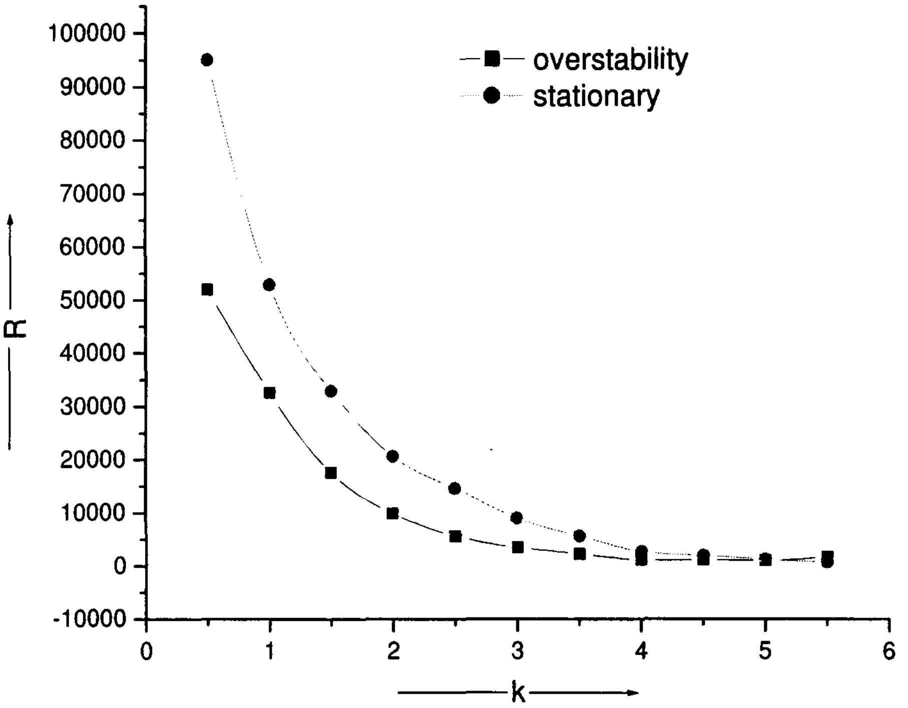

Figure 3.9: The variation of Rayleigh number(R) with wave number (k) for $\bar{\delta}=1, \varepsilon=0.5, E=0.9, p_1=5, p_2=1, F=1.005, H_1=1.01, a=10, K_1=1, \ell_1=1$, $H=100$ Gauss, $\bar{k}_1=2$ and $\kappa=1.0$.

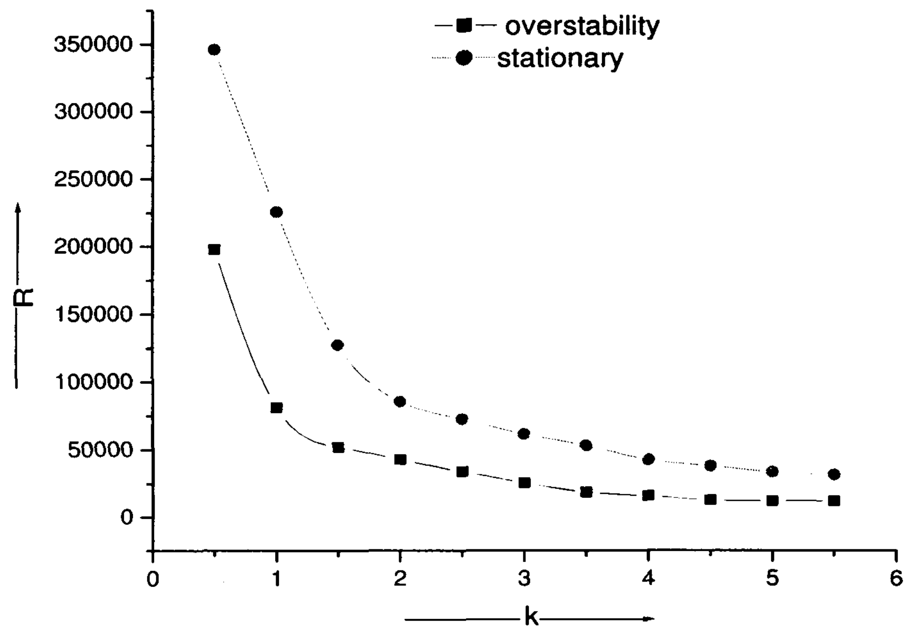

Figure 3.10: The variation of Rayleigh number(R) with wave number(k) for $\bar{\delta}=1, \varepsilon=0.5, E=0.9, p_1=5, p_2=1, F=1.005, H_1=1.01, a=10, \ell_1=1$, $A=0.5$, $H=100$ Gauss, $\bar{k}_1=2$ and $\gamma'=1.0$.

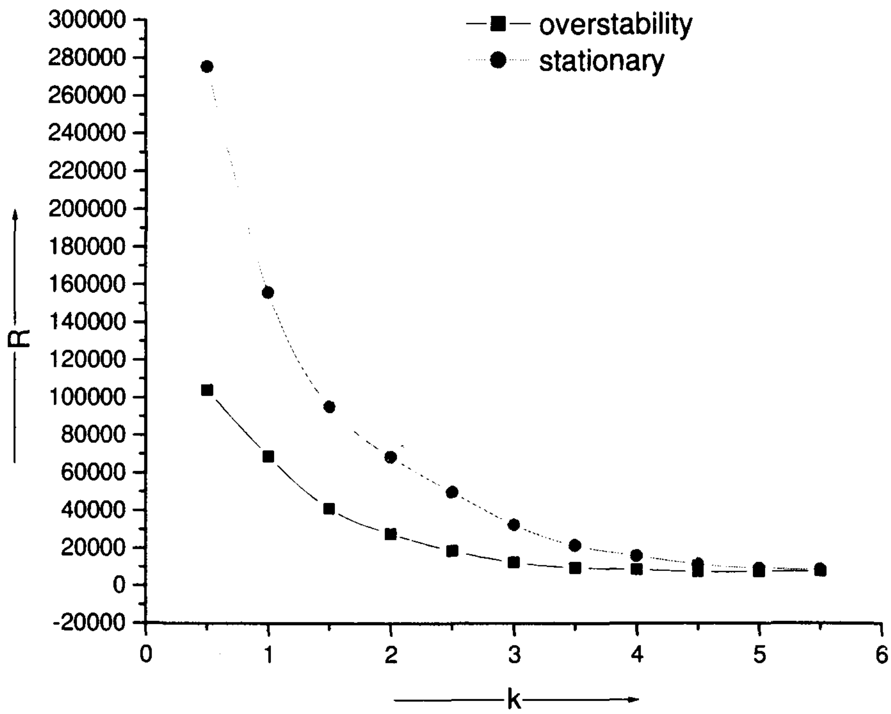

Figure 3.11: The variation of Rayleigh number(R) with wave number(k) for $\bar{\delta}=1, \varepsilon=0.5, E=0.9, p_1=5, p_2=1, F=1.005, H_1=1.01, a=10, \ell_1=1, A=0.5,$ $H=100$ Gauss, $\bar{k}_1=2$ and $\gamma'=1.2$.

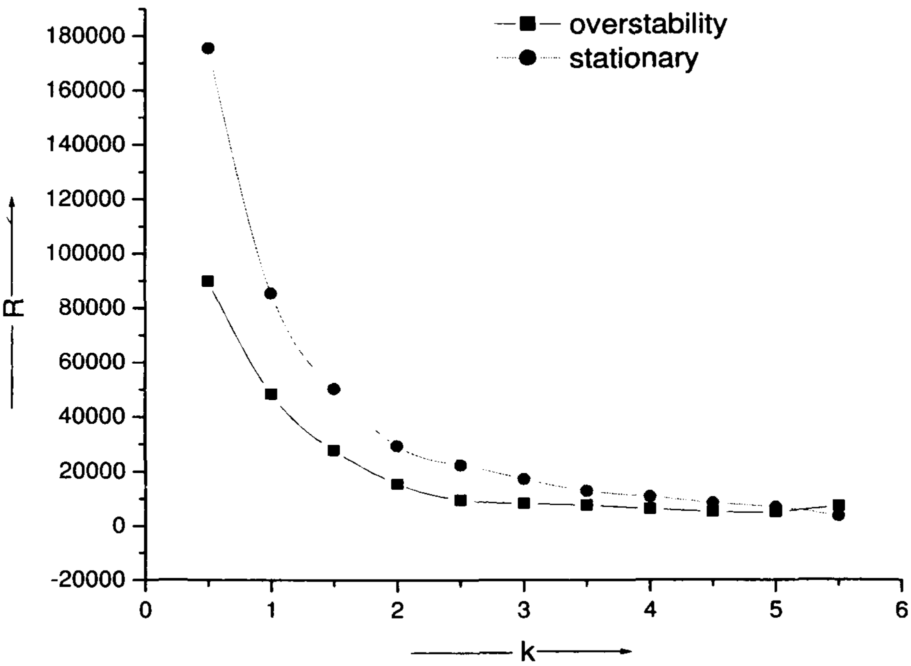

Figure 3.12: The variation of Rayleigh number(R) with wave number (k) for $\bar{\delta}=1, \varepsilon=0.5$ $E=0.9, p_1=5, p_2=1, F=1.005, H_1=1.01, a=10, \ell_1=1, A=0.5,$ $H=100$ Gauss, $\bar{k}_1=2$ and $\gamma'=1.4$.