CHAPTER II

THERMAL CONVECTION OF MICROPOLAR FLUIDS IN THE PRESENCEOF SUSPENDED PARTICLES IN HYDROMAGNETICS

2.1 INTRODUCTION

Micropolar theory was introduced by Eringen [1966] in order to describe some physical systems which do not satisfy the Navier Stokes equations. These fluids are able to describe the behaviour of colloidal solutions, liquid crystals, animal blood etc.The equations governing the flow of micropolar fluids theory involve a spin vector and a microinertia tensor in addition to velocity vector. A generalization of the theory including thermal effects has been developed by Kazakia and Ariman [1971] and Eringen [1972].

Micropolar fluids stabilities have become an important field of research these days. A particular stability problem is the Rayleigh–Bénard instability in a horizontal thin layer of fluid heated from below. A detailed account of thermal convection in a horizontal thin layer of Newtonian fluid heated from below has been given by Chandrasekhar [1961]. Ahmadi [1976] and Pérez–Garcia et al. [1981] have studied the effects of the microstructures on the thermal convection and have found that in the absence of coupling between thermal and micropolar effects, the principle of exchange of stabilities may not be fulfilled and consequently micropolar fluids introduce oscillatory motions. The existence of oscillatory motions in micropolar fluids has been depicted by Lekkerkerker in liquid crystals [1977, 1978], Bradley in dielectric fluids [1978] and Laidlaw in binary mixture [1979]. In the study of problems of thermal convection, it is frequent practice to simplify the basic equations by introducing an approximation which is attributed to Boussinesq [1903]. In geophysical situations, the fluid is often not pure but contains several suspended particles. Motivation for the study of certain effect of particles immersed in the fluid such as particle heat capacity, particle mass fraction and thermal force is due to the fact that the knowledge concerning fluid–particles mixture is not commensurate with their industrial and scientific importance. Saffman [1962] has considered the stability of laminar flow of a dusty gas. Sharma et al. [1976] have considered the effect of suspended particles on the onset of Bénard convection in hydromagnetics and found that the critical Rayleigh number is reduced because of the heat capacity of particles thereby destabilizing the system. The suspended particles were thus found to destabilize the layer. Palaniswami and Purushotham [1981] have studied the stability of shear flow of stratified fluids with the fine dust and found that the presence of dust particles increases the region of instability. On the other hand, multiphase fluid systems are concerned with the motion of liquid or gas containing immiscible inert identical particles of all multiphase fluid systems observed in nature, blood flow in arteries, flow in rocket tubes, dust-in-gas cooling system to enhance heat transfer processes, movement of inset solid particles in atmosphere, and sand or other particles on sea or ocean beaches are the most common examples of multiphase fluid systems. Sharma and Kumar ([1994], [1995]) have studied the effect of rotation and magnetic field separately, on the thermal convection in micropolar fluids. Sharma and Kumar [2001] have studied the thermal convection in rotating micropolar fluids in hydromagnetics in porous medium.

The problem of hydromagnetics of micropolar fluids has relevance and importance in chemical engineering, bio-mechanics, geophysics and electrically conducting colloidal suspensions. Sharma and Kumar [2002] have studied the stability of micropolar fluids heated from below in the presence of suspended particles (fine dust) and have found that suspended particles number density has destabilizing effect on the convection of micropolar fluids. The present chapter, therefore, deals with the stability of electrically conducting micropolar fluids heated from below in the presence of suspended particles in a uniform vertical magnetic field.

2.2 FORMULATION OF THE PROBLEM AND DISTURBANCE EQUATIONS

Consider an infinite horizontal layer of an incompressible electrically conducting micropolar fluids of thickness d permeated with suspended particles (or fine dust).A uniform vertical magnetic field $\vec{H}(0,0,H)$ pervades the system. This fluid–particles layer is heated from below but convection sets in when the temperature gradient $\left(\beta = \left| {dT}/{dz} \right|\right)$ between the lower and upper boundaries exceeds a certain critical value. The critical temperature gradient depends upon the bulk properties and boundary conditions of the fluid.

Let $\vec{v}$, $\vec{\vartheta}$, $\vec{H}$, p, ρ, T, $\vec{g}$, k_T, c_{pt}, c_v, μ_e, η, $\hat{e}_z$, $\vec{u}$, δ and j_1 denote the velocity, the spin, the magnetic field intensity, the pressure, the density, the temperature, the acceleration due to gravity, the thermal conductivity, the heat capacity of particles, the specific heat at constant volume, the magnetic permeability, the electrical resistivity, the unit vector in $z-$direction, the particle velocity, the coefficient giving account of coupling between spin and heat flux and microinertial constant, respectively. $\varepsilon', \beta'', \gamma'$ are the coefficients of angular viscosity and κ is the dynamic microrotation viscosity. Assume that external couples and heat sources are not present. If N is the number density and mN is the mass of suspended particles per unit volume, $K = 6\pi\mu r'$, r' being the particle radius, is the Stoke's drag coefficient, then the mass, momentum, internal angular momentum, internal energy balance equations using the Boussinesq approximation are

$$\nabla \cdot \vec{v} = 0 , \tag{2.1}$$

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right)\vec{v} = -\frac{1}{\rho_0}\nabla p + \frac{1}{\rho_0}(\mu + \kappa)\nabla^2 \vec{v} + \frac{\kappa}{\rho_0}\nabla \times \vec{\vartheta} - \left(1 + \frac{\delta\rho}{\rho_0}\right) g\,\hat{e}_z + \frac{1}{\rho_0} KN\,(\vec{u} - \vec{v})$$

$$+ \frac{1}{\rho_0}\frac{\mu_e}{4\pi}\left(\nabla \times \vec{H}\right) \times \vec{H} , \tag{2.2}$$

$$\rho_0 j_1 \left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right)\vec{\vartheta} = (\varepsilon' + \beta'')\nabla\left(\nabla . \vec{\vartheta}\right) + \gamma' \nabla^2 \vec{\vartheta} + \kappa \nabla \times \vec{v} - 2\,\kappa\vec{\vartheta} , \tag{2.3}$$

$$\rho_0 c_v \left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right) T + mN\, c_{pt}\left(\frac{\partial}{\partial t} + \vec{u} \cdot \nabla\right) T = k_T \nabla^2 T + \delta\left(\nabla \times \vec{\vartheta}\right) \cdot \nabla T . \tag{2.4}$$

Also the equation of state is given by

$$\rho = \rho_0 [1 - \alpha (T - T_0)] , \tag{2.5}$$

where ρ_0, T_0 are reference density, reference temperature at the lower boundary and α is the coefficient of thermal expansion .

The Maxwell's equations yield

$$\frac{\partial \vec{H}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{H}\right) + \eta \nabla^2 \vec{H} , \tag{2.6}$$

$$\nabla \cdot \vec{H} = 0 . \tag{2.7}$$

Assuming dust particles of uniform size, spherical shape and small relative velocities between the two phases (fluid and particles), the net effect of the particles on the fluid is equivalent to an extra body force term per unit volume $KN\,(\vec{u} - \vec{v})$, as has been taken in equation (2.2). The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. The distance between the particles is assumed to be so large compared with their diameter that interparticle reactions are ignored. The buoyancy force on the particles is also neglected. The equations of motion and continuity for the particles, under these restrictions, are

$$mN\left(\frac{\partial}{\partial t}+\vec{u}\cdot\nabla\right)\vec{u}=KN\left(\vec{v}-\vec{u}\right), \tag{2.8}$$

$$\frac{\partial N}{\partial t}+\nabla\cdot(N\,\vec{u}\,)=0. \tag{2.9}$$

Let us now consider the stability of the system in the usual way by giving small perturbations on the initial state and on seeing the reaction of the perturbations on the system.

The steady state solution is

$\vec{v}=0\,,\vec{u}=0\,,\vec{\vartheta}=0\,,\,N=N_0(\text{constant}),\;T=T_0-\beta\,z,\;\rho=\rho_0(1+\alpha\,\beta\,z),$

$$p=p_0-g\rho_0\left(z+\frac{\alpha\,\beta\,z^2}{2}\right), \tag{2.10}$$

where p_0 is the pressure at $z=0$ and $\beta=\frac{T_0-T_1}{d}$ $(T_0>T_1)$ is the magnitude of uniform temperature gradient.

Let $\vec{v}\,(u,v,w)$, $\vec{u}\,(\ell,r,s)$, $\vec{\omega},N,\delta p$, $\delta\rho$, θ and $\vec{h}\,(h_x,h_y,h_z)$ denote, respectively, the perturbations in fluid velocity $\vec{v}\,(0,0,0)$, particles velocity $\vec{u}(0,0,0)$, spin $\vec{\vartheta}$, particles number density N_0, pressure p, density ρ, temperature T and magnetic field $\vec{H}(0,0,H)$ so that the change in density $\delta\rho$ caused by the perturbation θ in temperature is given by

$$\delta\rho=-\rho_0\alpha\,\theta\,. \tag{2.11}$$

Then equations (2.1)–(2.9) yield the perturbation equations

$$\nabla\cdot\vec{v}=0\,, \tag{2.12}$$

$$\rho_0\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\vec{v}=-\nabla\delta p+(\mu+\kappa)\nabla^2\vec{v}+\kappa\nabla\times\vec{\omega}+\alpha\,\rho_0\,g\,\theta\,\hat{e}_z+KN_0(\vec{u}-\vec{v})$$

$$+\frac{\mu_e}{4\pi}\left(\nabla\times\vec{h}\right)\times\vec{H}\,, \qquad (2.13)$$

$$\rho_0 j_1\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\vec{\omega}=(\varepsilon'+\beta'')\nabla(\nabla\cdot\vec{\omega})+\gamma'\nabla^2\vec{\omega}+\kappa\nabla\times\vec{v}-2\kappa\vec{\omega}, \qquad (2.14)$$

$$H_1\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\theta=\beta(w+h_1 s)+\kappa_T\nabla^2\theta+\frac{\delta}{\rho_0 c_v}[\nabla\theta\cdot(\nabla\times\vec{\omega})-(\nabla\times\vec{\omega})_z\cdot\beta], \qquad (2.15)$$

$$\frac{\partial\vec{h}}{\partial t}=\nabla\times\left(\vec{v}\times\vec{h}\right)+\eta\nabla^2\vec{h}\,, \qquad (2.16)$$

$$\nabla\cdot\vec{h}=0, \qquad (2.17)$$

$$mN_0\left(\frac{\partial}{\partial t}+\vec{u}\cdot\nabla\right)\vec{u}=KN_0(\vec{v}-\vec{u}), \qquad (2.18)$$

$$\frac{\partial M}{\partial t}+\nabla\cdot u=0\,, \qquad (2.19)$$

where $H_1=1+h_1$, $h_1=\dfrac{f\,c_{pt}}{c_v}$, $f=\dfrac{mN_0}{\rho_0}$ and $M=\dfrac{N}{N_0}$.

Using the non–dimensional numbers

$$z=z^*d,\quad \theta=\beta\,d\theta^*,\quad t=\frac{\rho_0 d^2}{\mu}t^*,\quad \vec{v}=\frac{\kappa_T}{d}\vec{v}^*,\quad \vec{u}=\frac{\kappa_T}{d}\vec{u}^*,$$

$$p=\frac{\mu\kappa_T}{d^2}p^*,\vec{\omega}=\frac{\kappa_T}{d^2}\vec{\omega}^*,\ \vec{h}=\left(\frac{\mu\,\kappa_T}{d^2}\right)^{\frac{1}{2}}\vec{h}^*,\nabla=\frac{\nabla^*}{d} \qquad (2.20)$$

and then removing the stars for convenience, the non–dimensional forms of equations (2.12)–(2.19) become

$$\nabla\cdot\vec{v}=0\,, \qquad (2.21)$$

$$\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\vec{v}=-\nabla\delta p+(1+K_1)\nabla^2\vec{v}+K_1\nabla\times\vec{\omega}+R\theta\,\hat{e}_z+N_2(\vec{u}-\vec{v})+\frac{\mu_e}{4\pi}\left(\nabla\times\vec{h}\right)\times\vec{H}\,, \quad (2.22)$$

$$\bar{j}_2\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\vec{\omega}=C_1'\nabla(\nabla\cdot\vec{\omega})-C_0'\nabla\times(\nabla\times\vec{\omega})+K_1(\nabla\times\vec{v}-2\vec{\omega}), \tag{2.23}$$

$$H_1 p_1\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\theta=\beta(w+h_1 s)+\kappa_T\nabla^2\theta+\bar{\delta}\left[\nabla\theta.(\nabla\times\vec{\omega})-(\nabla\times\vec{\omega})_z\right], \tag{2.24}$$

$$\frac{\partial\vec{h}}{\partial t}=\nabla\times\left(\vec{v}\times\vec{h}\right)+\frac{1}{p_2}\nabla^2\vec{h}, \tag{2.25}$$

$$\nabla\cdot\vec{h}=0, \tag{2.26}$$

$$\left[a\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)+1\right]\vec{u}=\vec{v}, \tag{2.27}$$

where new dimensionless coefficients are

$$K_1=\frac{\kappa}{\mu},\quad \bar{j}_2=\frac{j_1}{d^2},\quad \bar{\delta}=\frac{\delta}{\rho_0\, c_v\, d^2},\quad C_0'=\frac{\gamma'}{\mu\, d^2},\quad C_1'=\frac{\varepsilon'+\beta''+\gamma'}{\mu\, d^2},$$

$$N_2=KN_0\frac{d^2}{\mu},\ a=\frac{m}{Kd^2}\frac{\mu}{\rho_0} \tag{2.28}$$

and the dimensionless Rayleigh number R, thermal Prandtl number p_1, the magnetic Prandtl number p_2 are

$$R=\frac{g\alpha\beta\rho_0 d^4}{\mu\,\kappa_T},\ p_1=\frac{\upsilon}{\kappa_T},\ p_2=\frac{\upsilon}{\eta}, \tag{2.29}$$

where $\kappa_T = \dfrac{k_T}{\rho_0 c_v}$ is the thermal diffusivity.

Let us assume both the boundaries to be free and perfectly heat conducting. The case of two free boundaries, though little artificial is the most appropriate for stellar atmosphere. Since the surfaces are fixed and are maintained at fixed temperature, $w=0=\theta$ at $z=0$ and $z=d$.

The z –component of vorticity vector is given by

$$\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}. \tag{2.30}$$

Further, tangential stresses do not act on free surfaces. The conditions to be satisfied on it are $\tau_{xz} = 0$, $\tau_{yz} = 0$, which yield

$$\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0 \text{ and } \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0. \tag{2.31}$$

Now, as w vanishes for all x and y on the boundaries it follows from the equations (2.31) that

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0 \text{ on free surfaces.} \tag{2.32}$$

Differentiating equation (2.21) with respect to z and using (2.32), we conclude that

$$\frac{\partial^2 w}{\partial z^2} = 0 \text{ on free surfaces.} \tag{2.33}$$

Differentiating (2.30) with respect to z and using (2.32) we conclude that

$$\frac{\partial \zeta}{\partial z} = 0 \text{ on free surfaces.} \tag{2.34}$$

Since the medium adjoining the fluid is perfect conductor,

$$\frac{\partial h_z}{\partial z} = 0. \tag{2.35}$$

Also if the medium adjoining the fluid is electrically non–conducting, then the boundary conditions are

$$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial}{\partial z}(\nabla \times \vec{v})_z = 0, (\nabla \times \vec{h})_z = (\nabla \times \vec{\omega})_z = 0,\ \theta = \frac{\partial h_z}{\partial z} = 0 \text{ at } z = 0 \text{ and } z = d. \tag{2.36}$$

2.3 LINEAR THEORY: DISPERSION RELATION

Since the perturbations applied on the system are assumed to be very small, the products of perturbations, the second and higher order perturbations are negligibly small. Under the linearized theory, second and higher order terms are neglected and only the linear terms are retained. Accordingly, the non–linear terms $(\vec{v} \cdot \nabla)\, \vec{v}$, $(\vec{v} \cdot \nabla)\theta$,

$(\vec{v}\cdot\nabla)\,\vec{\omega}$, $\nabla\theta\cdot(\nabla\times\vec{\omega})$ in equations (2.22)–(2.24) are neglected.

Eliminating s between equations (2.24) and (2.27) and applying the curl operator twice to resulting equation and linearizing, we obtain

$$L_2\left[H_1 p_1 \frac{\partial}{\partial t}-\nabla^2\right]\theta=\left(a\frac{\partial}{\partial t}+H_1\right)\beta\, w-L_2\bar{\delta}\Omega_{z1}\,. \tag{2.37}$$

Eliminating $\vec{u}$ between (2.22) and (2.27) and on linearizing, we obtain

$$L_1\,\vec{v}=L_2\left[-\nabla\delta p+(1+K_1)\nabla^2\vec{v}+K_1\nabla\times\vec{\omega}+R\theta\,\hat{e}_z+\frac{\mu_e}{4\pi}\left(\nabla\times\vec{h}\right)\times\vec{H}\right], \tag{2.38}$$

where $L_1=a\dfrac{\partial^2}{\partial t^2}+F\dfrac{\partial}{\partial t}$, $L_2=a\dfrac{\partial}{\partial t}+1$ and $F=f+1$.

Applying the curl operator twice to equation (2.22) and taking z–component, we get

$$L_1\nabla^2 w=L_2\left[R\,\nabla_1^2\theta+(1+K_1)\nabla^4 w+K_1\nabla^2\Omega_{z1}+\frac{\mu_e H}{4\pi}\frac{\partial}{\partial z}\nabla^2 h_z\right], \tag{2.39}$$

$$\text{where}\quad \nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},\ \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2},\ \Omega_{z1}=(\nabla\times\vec{\omega})_z. \tag{2.40}$$

Applying the curl operator to equations (2.22), (2.23) and (2.25), taking $z-$ component, we get

$$L_2\frac{\partial}{\partial t}\zeta_z+n_1\zeta_z(L_2-1)=(1+K_1)\nabla^2\zeta_z L_2+\frac{\mu_e H}{4\pi}\frac{\partial \xi_z}{\partial z}L_2, \tag{2.41}$$

$$\bar{j}_2\frac{\partial\Omega_{z1}}{\partial t}=C_0'\nabla^2\Omega_{z1}-K_1\left(\nabla^2 w+2\Omega_{z1}\right), \tag{2.42}$$

$$\frac{\partial\xi_z}{\partial t}=H\frac{\partial}{\partial t}\zeta_z+\frac{1}{p_2}\nabla^2\xi_z, \tag{2.43}$$

where $\xi_z=\left(\nabla\times\vec{h}\right)_z$, $\zeta_z=(\nabla\times\vec{v})_z$ are the $z-$components of current density and vorticity, respectively. K_1 and C_0' account for coupling between vorticity and spin effects and spin diffusion, respectively.

Taking the z-component of equation (2.25), we get

$$\frac{\partial h_z}{\partial t} = H\frac{\partial w}{\partial z} + \frac{1}{p_2}\nabla^2 h_z . \tag{2.44}$$

We now analyze an arbitrary perturbation into a complete set of normal modes and then examine the stability of each of these modes individually. For the system of equations (2.37), (2.39) and (2.41)–(2.44), the analysis can be made in terms of two dimensional periodic waves of assigned wave-numbers. Thus we ascribe to all quantities describing the perturbation a dependence on x, y and t of the form

$$\exp\left[i(k_x x + k_y y) + nt \right], \tag{2.45}$$

where $k = \left(k_x^2 + k_x^2\right)^{1/2}$ is the resultant wave-number, k_x and k_y are real constants and n is the stability parameter which can be, complex, in general. The solution of the stability problem requires the specifications of the state for each k.The above considerations allow us to suppose that the perturbation quantities have the form

$$[w, \Omega_{z1}, \zeta_z, \xi_z, \theta, h_z] = [W(z), \Omega_2(z), Z(z), G(z), \Theta(z), B(z)]\exp\left(ik_x x + ik_y y + nt\right), \tag{2.46}$$

then the equations (2.37), (2.39) and (2.41)–(2.44) become

$$(an+1)\{H_1 p_1 n - (D^2 - k^2)\}\Theta = (an + H_1)W - (an+1)\bar{\delta}\,\Omega_2, \tag{2.47}$$

$$(D^2 - k^2)\{(an^2 + Fn) - (an+1)(1+K_1)(D^2 - k^2)\}\, W = \{(an+1) - Rk^2\Theta + K_1\,(D^2 - k^2)\Omega_2 + \frac{\mu_e H}{4\pi}(D^2 - k^2)DB\}, \tag{2.48}$$

$$\{(an^2 + Fn) - (an+1)(D^2 - k^2)(1+K_1)\}Z = \frac{\mu_e H}{4\pi}(an+1)DG, \tag{2.49}$$

$$\{\ell_1 n + 2A - (D^2 - k^2)\}\Omega_2 = -A(D^2 - k^2)W, \tag{2.50}$$

$$\left\{n - \frac{1}{p_2}(D^2 - k^2)\right\}G = H\, DZ, \tag{2.51}$$

$$\left\{n - \frac{1}{p_2}(D^2 - k^2)\right\} B = H\ DW\ , \tag{2.52}$$

where $A = \frac{K_1}{C_0'}$, $\ell_1 = \bar{j}_2 \frac{A}{K_1}$, $D = \frac{d}{dz}$, $\frac{\partial}{\partial t} = n$, $L_2 = a\frac{\partial}{\partial t} + 1 = an + 1$,

$$L_1 = a\frac{\partial^2}{\partial t^2} + F\frac{\partial}{\partial t} = an^2 + Fn\,. \tag{2.53}$$

Eliminating Θ, Z, B, Ω_2 from equations (2.47)–(2.52), we get

$$\begin{aligned}&(D^2 - k^2)\ \{(an^2 + Fn) - (an+1)(1+K_1)(D^2 - k^2)\}\ \{H_1 p_1 n - (D^2 - k^2)\}\\ &\{\ell_1 n + 2A - (D^2 - k^2)\}\ \{\ell_1 n + 2A - (D^2 - k^2)\}\ W = -Rk^2\ \{\ell_1 n + 2A - (D^2 - k^2)\}\\ &\left\{n - \frac{1}{p_2}(D^2 - k^2)\right\}(an + H_1)W - Rk^2\left\{n - \frac{1}{p_2}(D^2 - k^2)\right\}(an+1)\,\delta A\,(D^2 - k^2)W\\ &- AK_1(D^2 - k^2)^2(an+1)\{H_1 p_1 n - (D^2 - k^2)\}\left\{n - \frac{1}{p_2}(D^2 - k^2)\right\}W + \frac{H^2}{4\pi}(D^2 - k^2)\\ &\{H_1 p_1 n - (D^2 - k^2)\}(an+1)\{\ell_1 n + 2A - (D^2 - k^2)\}D^2 W.\end{aligned} \tag{2.54}$$

The boundary conditions (2.36) transform to

$$W = 0,\ D^2W = 0,\ DZ = 0,\ G = 0,\ \Omega_2 = 0,\ \Theta = 0,\ DB = 0 \text{ at } z = 0 \text{ and } 1. \tag{2.55}$$

Using boundary equations (2.55), equations (2.47)–(2.52) give

$$D^2\Theta = 0, D^2\Omega_2 = 0, D^3Z = 0, D^3G = 0, D^3B = 0\,, \tag{2.56}$$

where $D^3 = \frac{d^3}{dz^3}, D^2 = \frac{d^2}{dz^2}$.

Differentiating equation (2.48) twice with respect to z and using the boundary conditions (2.56), it can be shown that $D^4W = 0$, where $D^4 = \frac{d^4}{dz^4}$. It can be shown from equations (2.47)–(2.52) and boundary conditions (2.55), (2.56) that all even order derivatives of W vanish on the boundaries .The proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \tag{2.57}$$

where W_0 is a constant. Substituting equation (2.57) in equation (2.54) and putting $b = \pi^2 + k^2$, we obtain

$$Rk^2\left\{n+\frac{b}{p_2}\right\}\{(an+H_1)(\ell_1 n+2A+b)-(an+1)\bar{\delta}Ab\}=b\{(an^2+Fn)+(an+1)(1+K_1)b\}$$
$$(H_1p_1n+b)(\ell_1 n+2A+b)\left\{n+\frac{b}{p_2}\right\}-K_1Ab^2(an+1)(H_1p_1n+b)\left(n+\frac{b}{p_2}\right)+$$
$$\frac{H^2\pi}{4}(H_1p_1nb+b^2)(an+1)(\ell_1 n+2A+b). \tag{2.58}$$

2.4 THE CASE OF OSCILLATORY MODES

Equating the imaginary parts of equation (2.58), we have

$$n_i\Bigg[n_i^4(abH_1p_1\ell_1-abn)+n_i^2\Bigg(-2AH_1p_1\frac{ab^2}{p_2}-H_1p_1\frac{ab^3}{p_2}-\ell_1n_i^2\frac{ab^3}{p_2}-2AFb-FH_1p_1b^2-F\ell_1b^2$$
$$-FH_1p_1\ell_1\frac{b^2}{p_2}-H_1p_1K_1b^3-ab^3\ell_1K_1-aH_1p_1\ell_1K_1\frac{b^3}{p_2}-2AH_1p_1ab^2-H_1p_1ab^4-ab^3\ell_1-2AH_1p_1aK_1b^2$$
$$-H_1p_1\ell_1b^2-H_1p_1\ell_1\frac{ab^3}{p_2}+Rk^2a\ell_1\Bigg)+\frac{ab^5}{p_2}(K_1+1)+\frac{b^4}{p_2}(aK_1+2Aa+\ell_1+H_1p_1+F)+\frac{b^3}{p_2}(H_1p_12A+2AF)$$
$$+\frac{b^2}{p_2}(-Rk^2a+Rk^2a\bar{\delta}A)+\frac{b}{p_2}(-2Rk^2aA-Rk^2H_1\ell_1)+b(-Rk^2H_1-\bar{\delta}A)+b^4$$
$$+2Ab^3-2Rk^2H_1A\Bigg]=0. \tag{2.59}$$

Equation (2.59) yields that either $n_i = 0$ or $n_i \neq 0$, which means that the modes are either non–oscillatory or oscillatory. In the absence of suspended particles number density, magnetic field intensity and magnetic permeability equation (2.59) reduces to

$$n_i\left(b^4\ell_1+Rk^2\bar{\delta}Ab\right)=0 \tag{2.60}$$

and term within the brackets is definitely positive, which implies that $n_i = 0$.Therefore, the oscillatory modes are not allowed and principal of exchange of stabilities is satisfied in the absence of suspended particles and magnetic field. The presence of the suspended particles and the magnetic field, bring oscillatory modes (as n_i may not be zero) which were non–existent in their absence.

2.5 THE CASE OF OVERSTABILITY

We now discuss the possibility whether instability may arise as oscillations of increasing amplitude i.e. as overstability. Put $n = in_i$, it being remembered that n may be complex. Since for overstability, we wish to determine critical Rayleigh number for the onset of overstability via a state of pure oscillations, it suffices to find conditions for which equation (2.58) will admit of solution with n_i real.

Substituting $n = in_i$ in equation (2.58), the real and imaginary parts of equation (2.58), yield

$$Rk^2\left[\frac{b^2}{p_2}\{2H_1A + b(1-\bar{\delta}A)\} - n_i^2\left\{b\ell_1\left(1+\frac{a}{p_2}\right) + \{2aAH_1 + b(1-\bar{\delta}A)\}\right\}\right] =$$
$$n_i^4\left[H_1p_1\ell_1b^2\{1+a(1+K_1)\} + ab\left\{b\ell_1\left(1+\frac{H_1p_1}{p_2}\right)\right\} + H_1p_1(2aA+F\ell_1)\right] - n_i^2\left[\left\{(2A+b)\left(1+\frac{H_1p_1}{p_2}\right) + \frac{H_1b\ell_1}{p_2}\right\}\right.$$
$$\{1+a(1+K_1)\}b^3 + b^3(2aA+F\ell_1)\left\{H_1p_1(1+K_1) + \frac{a}{p_2}\right\} + b^4a\ell_1(1+K_1)\left(1+\frac{H_1p_1}{p_2}\right) - K_1Ab^3$$
$$\left.\left\{p_1H_1 + a\left(1+\frac{p_1H_1}{p_2}\right)\right\}\right] + \frac{\pi H^2 b}{4}\left[-n_i^2\{b\ell_1(H_1p_1+a) + H_1p_1(2A+b)\} + (2A+b)b^2\right]$$
$$+\left[\frac{1}{p_2}(1+K_1)b^6 + \frac{A}{p_2}(2+K_1)b^5\right] \tag{2.61}$$

and

$$Rk^2\left[-a\ell_1n_i^3 + 2A\,n_iH_1 + n_iH_1b - n_i\bar{\delta}Ab + \frac{2A\,b}{p_2}an_i + \frac{b^2}{p_2}an_i + \frac{b}{p_2}H_1\ell_1n_i - \frac{1}{p_2}an_i\bar{\delta}Ab^2\right]$$
$$= abH_1p_1\ell_1n_i^5 - 2Aab^2n_i^4 - abn_i^5 - 2AH_1p_1n_i^3\frac{ab^2}{p_2} - H_1p_1n_i^3\frac{ab^3}{p_2} - \ell_1n_i^3\frac{ab^3}{p_2} - 2AFn_i^3b$$
$$-Fn_i^3H_1p_1b^2 - Fn_i^3\ell_1b^2 - FH_1p_1\ell_1n_i^3\frac{b^2}{p_2} + 2AFn_i\frac{b^3}{p_2} + Fn_i\frac{b^4}{p_2} - H_1p_1n_i^3K_1b^3 - an_i^3b^3\ell_1K_1$$

$$- aH_1 p_1 \ell_1 n_i^{\,3} K_1 \frac{b^3}{p_2} + n_i a K_1 \frac{b^4}{p_2} + a n_i K_1 \frac{b^5}{p_2} - 2H_1 p_1 n_i^{\,3} a b^2 A - H_1 p_1 n_i^{\,3} a b^4 - a b^3 \ell_1 n_i^{\,3} - H_1 p_1 \ell_1$$

$$n_i^{\,3} \frac{ab^3}{p_2} + 2Aan_i \frac{b^4}{p_2} + \frac{b^5}{p_2} an_i - 2AH_1 p_1 n_i^{\,3} aK_1 b^2 - H_1 p_1 \ell_1 n_i^{\,3} b^2 + 2Ab^3 n_i + b^4 n_i + \frac{b^4}{p_2}\ell_1 n_i$$

$$+ 2AH_1 p_1 n_i \frac{b^3}{p_2} + H_1 p_1 n_i \frac{b^4}{p_2}. \tag{2.62}$$

Eliminating R between equations (2.61) and (2.62), we get

$$n_i^{\,6}\left[- a^2 \ell_1^2 \{1 + H_1 p_1 (1 + K_1)\} b^2 + ab\ell_1 H_1 p_1 \left(H_1 \ell_1 - ab\bar{\delta}A - F\ell_1\right) - b^3 a \ell_1 H_1 \left(\frac{1}{p_2} + (1 + K_1)\right)\right]$$

$$+ n_i^4 \left[b^5 \left\{ H_1 p_1 a^2 \left(1 - \bar{\delta}A\right) + H_1 p_1 \ell_1 \bar{\delta}Aa \frac{1}{p_2}(1 + K_1) + \frac{p_1}{p_2}\bar{\delta}A(H_1 - 1) \right\}\right.$$

$$+ b^4 \left\{ 2H_1 p_1 a^2 (1 + K_1) A - a^2 \bar{\delta}A \ell_1 (1 + K_1) - a^2 H_1^{\,2} p_1 (H_1 - 1)\frac{p_1}{p_2} \right\}$$

$$+ b^3 \left\{ H_1 p_1 Fa\left(1 - \bar{\delta}A\right) + H_1 p_1 \ell_1^2 \frac{a^2}{p_2^2}(H_1 - 1) - H_1 p_1 \ell_1 K_1 a\left(2 - \bar{\delta}A\right) + \frac{1}{p_2^2} H_1 p_1 a \ell_1^2 (F - aK_1) \right\}$$

$$+ b^2 \left\{ \frac{-2a}{p_2} AF\ell_1 (H_1 - 1) - \frac{H^2 \pi}{4} a\ell_1 \left(a\ell_1 - \frac{p_1}{p_2} + p_1 \bar{\delta}A \right) - 2Aa\ell_1 H_1 (H_1 - 1) \right\}$$

$$\left. + b \left\{ - 2A^2 a H_1^{\,2} p_1 (H_1 - 1) - \frac{\pi H^2}{4} a\ell_1 2A \left(a\ell_1 - \frac{p_1}{p_2} a\ell_1 + p_1 \bar{\delta}A \right) \right\}\right]$$

$$+ n_i^{\,2} \left[b^7 \left\{ - H_1 p_1 \frac{a}{p_2^2}\left(1 - \bar{\delta}A\right) - \bar{\delta}A \frac{a}{p_2^2}(1 + K_1) \right\} + b^6 \left\{ \frac{a^2}{p_2} \bar{\delta}A(1 + K_1) - H_1 p_1 \ell_1 \frac{1}{p_2^2}\left(2 - \bar{\delta}A\right) \right.\right.$$

$$\left. - 2H_1^2 \frac{p_1}{p_2} aA(1 + K_1) - \frac{H^2 \pi}{4} H_1 p_1 \frac{1}{p_2}\left(2 - \bar{\delta}A\right) - H_1 p_1 \bar{\delta}A \frac{1}{p_2}(F - aK_1) \right\} + b^5 \left\{ - H_1 p_1 \ell_1 \frac{a}{p_2}(H_1 - 1) \right.$$

$$+ H_1 p_1 a^2 \frac{1}{p_2^2}(1 + K_1) + \frac{a\bar{\delta}A}{p_2}(1 + K_1) - \frac{\ell_1^2}{p_2^2}(2Aa + F\ell_1) - 2A^2 \frac{\bar{\delta}}{p_2}(H_1 - F) + H_1 p_1 \frac{a}{p_2^2}(F - aK_1)$$

$$\left. + \frac{K_1 A a}{p_2^2}\left(\ell_1 + H_1 p_1 \bar{\delta}A\right) + \frac{2Aa}{p_2^2}\{2 + 2H_1 p_1 (1 + K_1) - K_1 p_1\} + 2\{1 + H_1 p_1 (1 + K_1)\}\left(1 - \bar{\delta}A\right) \right\}$$

$$+ b^4 \left\{ H_1 p_1 - \frac{1}{p_2}\left(F\bar{\delta} - \bar{\delta} - aK_1\right) + \frac{H_1 p_1 \ell_1}{p_2^2}(F\ell_1 + H_1 p_1) - \frac{H^2 \pi}{4}\left(a^2 \ell_1 - \frac{p_1}{p_2} a\ell_1 + p_1 \bar{\delta}A \right) \right.$$

$$+ \frac{\ell_1^2}{p_2^2}(H_1 - F) - \frac{H^2 \pi}{4}\left(\left\{ F\left(1 - \frac{p_1}{p_2}\right) - \frac{H_1 p_1 \ell_1}{p_2} \right\}\left(1 - \bar{\delta}A\right) + \frac{F\ell_1}{p_2}\left(H_1 p_1 + \bar{\delta}A\right) \right) + 4A^2 \{1 + H_1 p_1 (1 + K_1)\}$$

$$\left. + \frac{2A^2 F}{p_2^2}\{2 + 2H_1 p_1 (1 + K_1) - K_1 p_1\} \right\}$$

$$+ b^3 \left\{ \frac{1}{p_2} 2H_1^2 p_1 A\ell_1 F(H_1 - 1) + \frac{H^2 \pi}{4}\left[H_1 p_1 \left(2 - \bar{\delta}A\right)\left(1 - \frac{H_1 p_1}{p_2}\right)\right] - \frac{H^2 \pi}{4}\left[F\left(1 - \frac{H_1 p_1}{p_2}\right)\frac{H_1 p_1 \ell_1}{p_2} \right] \right.$$

$$+2H_1p_1A\frac{1}{p_2}\left(F\bar{\delta}-\bar{\delta}-aK_1\right)-\frac{K_1a\ell_1}{p_2}(H_1-1)+\frac{H_1\ell_1}{p_2^2}\left(\ell_1+H_1p_1\bar{\delta}A\right)+2H_1^2p_1F\frac{1}{p_2}A\left(1-\bar{\delta}A\right)\Big\}$$

$$+b^2\left\{\frac{H^2\pi}{4}H_1p_1\ell_1\frac{a}{p_2}\left(2-\bar{\delta}A\right)+\frac{H^2\pi}{4}p_1H_1\ell_1\left(1-\bar{\delta}A\right)+H_1p_1\frac{1}{p_2}\left(2aA+F\ell_1\right)-\frac{\pi H^2}{4}H_1p_1\ell_1F\left(1-\frac{p_1}{p_2}\right)\right\}$$

$$+b\left\{\frac{2H^2\pi}{4}H_1p_1\ell_1FA\left(1-\frac{p_1}{p_2}\right)\right\}\Bigg]+b^8\left[\frac{1}{p_2^2}\left\{1+H_1p_1(1+K_1)\left(2-\bar{\delta}A\right)-F\ell_1(1+K_1)\bar{\delta}Aa\right\}\right]$$

$$+b^7\left\{\frac{4A}{p_2^2}\{1+H_1p_1(1+K_1)\}\left(1-\bar{\delta}A\right)+2aA\{1+H_1p_1(1+K_1)\}\right\}+b^6\left[\frac{2A^2F}{p_2^2}\{1+H_1p_1(1+K_1)\}\left(1-\bar{\delta}A\right)\right.$$

$$\left.+\frac{1}{p_2^2}H_1p_1(H_1-1)+\frac{H^2\pi}{4}\left\{\left(\frac{H_1p_1}{p_2}-1\right)\left(2-\bar{\delta}A\right)-\frac{H_1p_1\ell_1}{p_2}\bar{\delta}AF\right\}\right]+b^5\left[4H_1p_1A^2\frac{1}{p_2^2}\left(2-\bar{\delta}A\right)\right.$$

$$\left.+\frac{2}{p_2^2}AH_1(F-aK_1)+\frac{H^2\pi}{4}\left\{2AF\left(\frac{H_1p_1}{p_2}-1\right)\left(2-\bar{\delta}A\right)-\frac{H_1p_1F}{p_2}\bar{\delta}AF\right\}\right]+b^4\left[\frac{-4}{p_2}A^2H_1(1+K_1)\right.$$

$$\left.+\frac{H^2\pi}{4}\left\{\left(1-\bar{\delta}A\right)\left(\frac{H_1p_1}{p_2}-1\right)+2A^2\frac{a}{p_2^2}K_1(H_1-1)\right\}\right]+b^3\left[\frac{-H^2\pi}{2}\frac{Aa}{p_2}(H_1-1)+\frac{H_1a}{p_2}(1+K_1)\right]$$

$$+b^2\left[\frac{H^2\pi}{4}\left\{\left(2-\bar{\delta}A\right)\frac{\ell_1H_1}{p_2}\right\}-\frac{H^2\pi}{4}\frac{H_1a}{p_2}\right]=0. \tag{2.63}$$

It is evident from the equation (2.63) that overstable modes will not be present for all values of parameters. For example, in the absence of coupling between spin and heat flux $(\bar{\delta}=0)$, magnetic field $(\vec{H}=0)$ and in the absence of suspended particles $(a=0=f=h_1)$, equation (2.63) allows only $n_i=0$ and so overstable solution will not take place if $K_1p_1<2$.

Thus for stationary convection i.e. $n_i=0$ and in the presence of coupling between spin and heat fluxes $(\bar{\delta}\neq 0)$, equation (2.61) reduces to

$$R=\frac{b^4(1+K_1)+Ab^3(2+K_1)+\frac{H^2\pi}{4}(2A+b)\,bp_2}{k^2\left\{H_1 2A+b\left(1-\bar{\delta}A\right)\right\}}. \tag{2.64}$$

In the absence of magnetic field intensity $(\vec{H}=0)$, equation (2.65) reduces to

$$R=\frac{b^4(1+K_1)+Ab^3(2+K_1)}{k^2\left\{2H_1A+b\left(1-\bar{\delta}A\right)\right\}}, \tag{2.65}$$

a result in good agreement with Sharma and Kumar [2002].

In the absence of suspended particles $(a=0=f=h_1)$ and coupling between spin and heat fluxes $(\bar{\delta}=0)$, equation (2.65) further reduces to

$$R=\frac{(1+K_1)b^4+b^3A(2+K_1)}{k^2(2A+b)}, \tag{2.66}$$

a result derived by Pérez-Garcia and Rubi [1982].

For Newtonian viscous fluids i.e. $\bar{\delta}=0=\vec{H}=K_1=C_0'$ and $a=0=f=h_1$, equation (2.66) further reduces to

$$R=\frac{b^3}{k^2}, \tag{2.67}$$

which agrees with earlier result Chandrasekhar [1961].

2.6 RESULTS AND DISCUSSION

Equation (2.63) has been examined numerically using the Newton–Raphson method through the software Fortran 90. We have plotted the variation of Rayleigh number with respect to wave-number using equation (2.61) satisfying equation (2.63) for overstable case and equation (2.65) for stationary case, for the fixed permissible values of the dimensionless parameters $K_1=1,\ A=0.5, \bar{\delta}=1,\ \ell_1=1,\ \ p_1=5,\ \ p_2=1,\ a=10, F=1.005$ and $H_1=1.01$.

Figures (2.1)–(2.3) correspond to three values of the magnetic field intensity $H=70$, 100 and 120 Gauss, respectively. Figure (2.4) corresponds to the absence of magnetic field intensity i.e. $H=0$. The graphs show that Rayleigh number increases with increase in magnetic field intensity for a fixed wave-number depicting thereby the stabilizing effect of magnetic field intensity. Moreover, the magnetic field introduces the oscillatory modes in the system.

Figures (2.5)–(2.7) correspond to three values of micropolar coefficient $\kappa = 0.5, 0.7$ and 1.0, respectively, accounting for dynamic microrotation viscosity. The graphs show that the Rayleigh number for the stationary convection and for the case of overstability decreases with the increase in micropolar coefficient κ for a fixed wave-number implying thereby the destabilizing effect of dynamic microrotation viscosity.

Figures (2.8)–(2.10) correspond to three values of micropolar coefficient γ' =1.0, 1.2 and 1.4, respectively, accounting for coefficient of angular viscosity. The graphs show that the Rayleigh number for the stationary convection and for the case of overstability decreases with the increase in micropolar coefficient γ' for a fixed wave-number implying thereby the destabilizing effect of coefficient of angular viscosity. Thus there is a competition between the large enough stabilizing effect of magnetic field intensity and the destabilizing effect of the micropolar coefficients. The presence of coupling between thermal and micropolar effects, magnetic field and suspended particles number density may bring overstability in the system. It is also noted from the figures (2.3)–(2.10) that the Rayleigh number for overstability is also less than the Rayleigh number for stationary convection, for a fixed wave-number. However, the reverse may also occur for large wave-numbers, as has been depicted in figures (2.1) and (2.2) for $H = 70, 100$ Gauss, respectively.

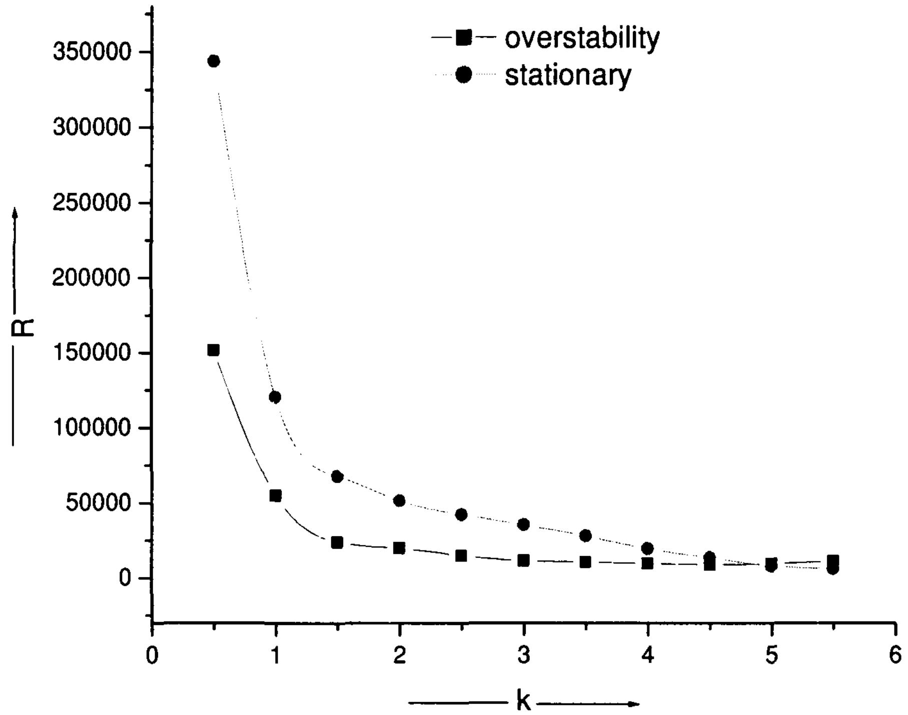

Figure 2.1: The variation of Rayleigh number (R) with wave number(k) for $A = 0.5$, $p_1 = 5$, $p_2 = 1$, $F = 1.005, H_1 = 1.01, a = 10, K_1 = 1$, $\ell_1 = 1$ and $H = 70$Gauss.

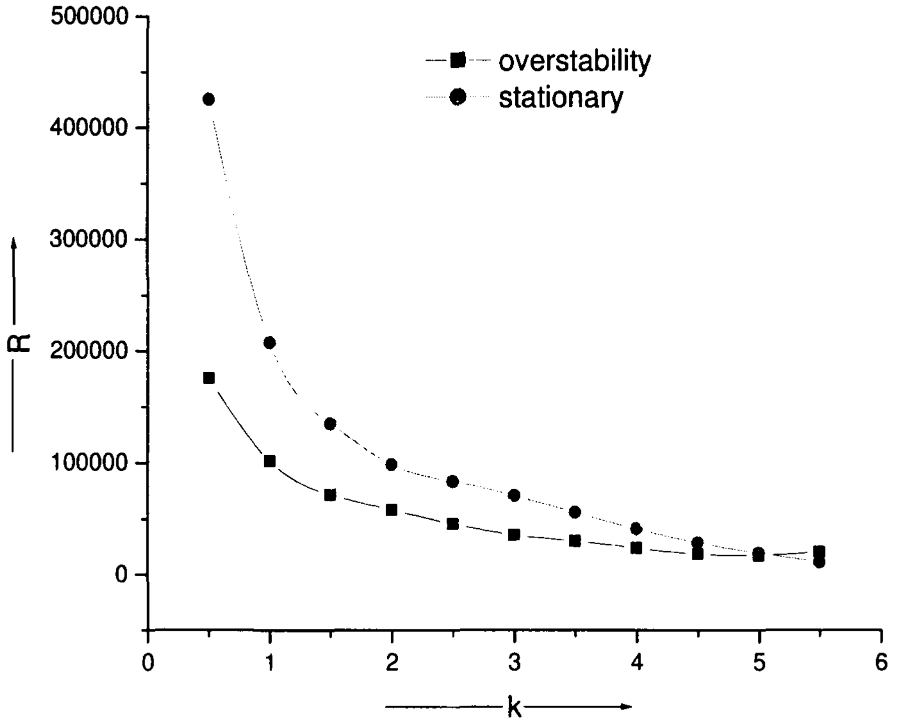

Figure 2.2: The variation of Rayleigh number (R) with wave number(k) for $A = 0.5$, $p_1 = 5$, $p_2 = 1$, $F = 1.005, H_1 = 1.01, a = 10, K_1 = 1$, $\ell_1 = 1$ and $H = 100$ Gauss.

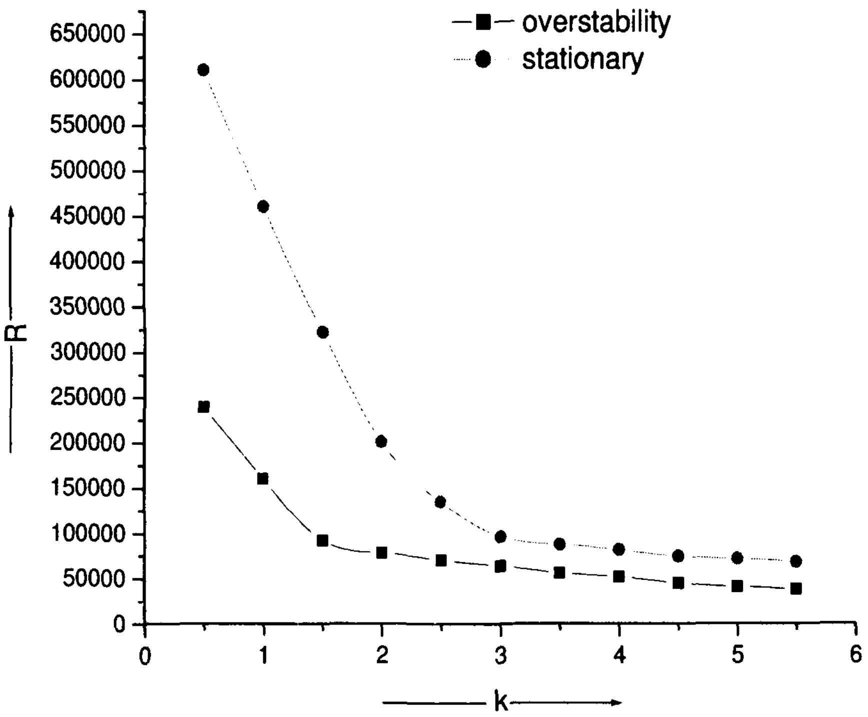

Figure 2.3: The variation of Rayleigh number (R) with wave number(k) for $A = 0.5$, $p_1 = 5$, $p_2 = 1$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $K_1 = 1$, $\ell_1 = 1$ and $H = 120$Gauss.

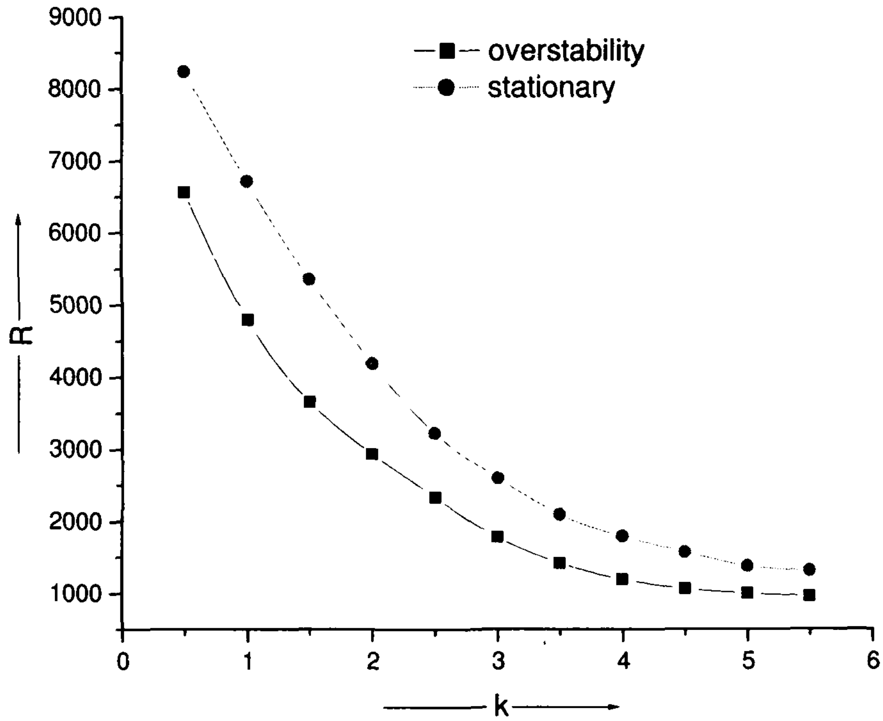

Figure 2.4: The variation of Rayleigh number (R) with wave number(k) for $A = 0.5$, $p_1 = 5$, $p_2 = 1$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $K_1 = 1$, $\ell_1 = 1$ and $H = 0$ Gauss.

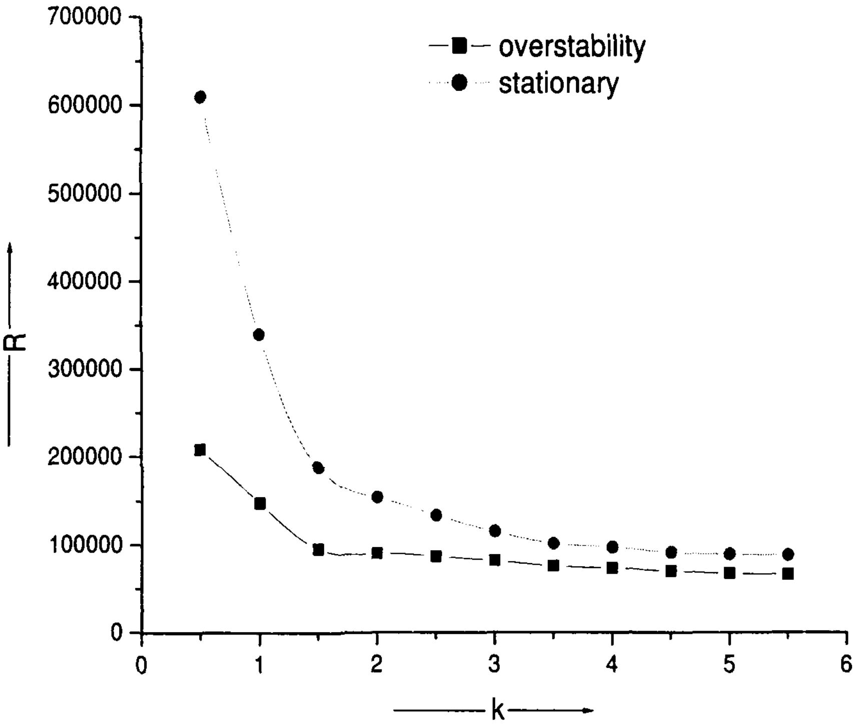

Figure 2.5: The variation of Rayleigh number (R) with wave number (k) for $H=120$, $\bar{\delta}=1$, $p_1=5$, $p_2=1$, $F=1.005$, $H_1=1.01$, $a=10$, $K_1=1$, $\ell_1=1$ and $\kappa=0.5$.

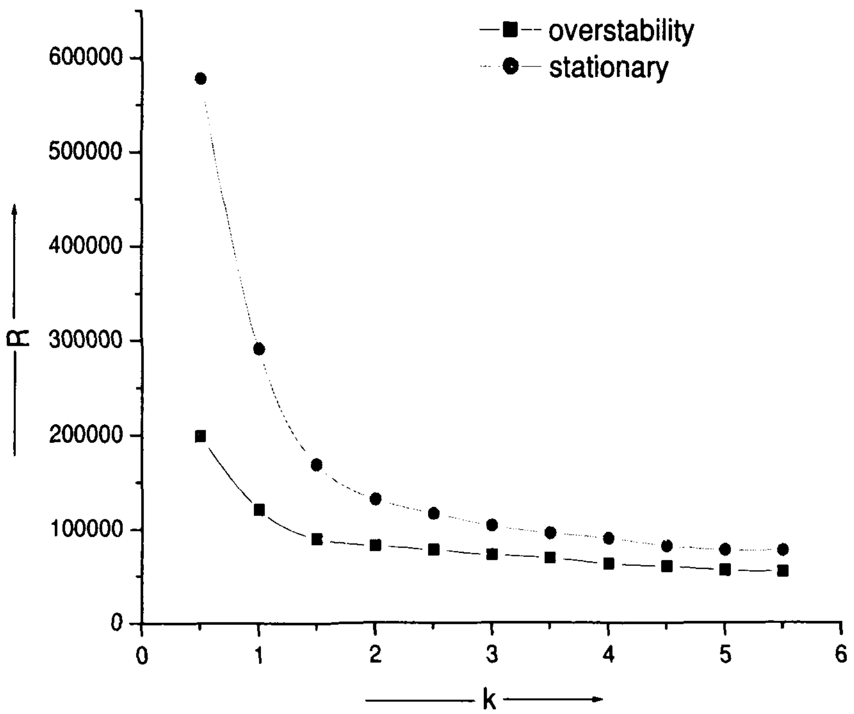

Figure 2.6: The variation of Rayleigh number (R) with wave number (k) for $H=120$, $\bar{\delta}=1$, $p_1=5$, $p_2=1$, $F=1.005$, $H_1=1.01$, $a=10$, $K_1=1$, $\ell_1=1$ and $\kappa=0.7$.

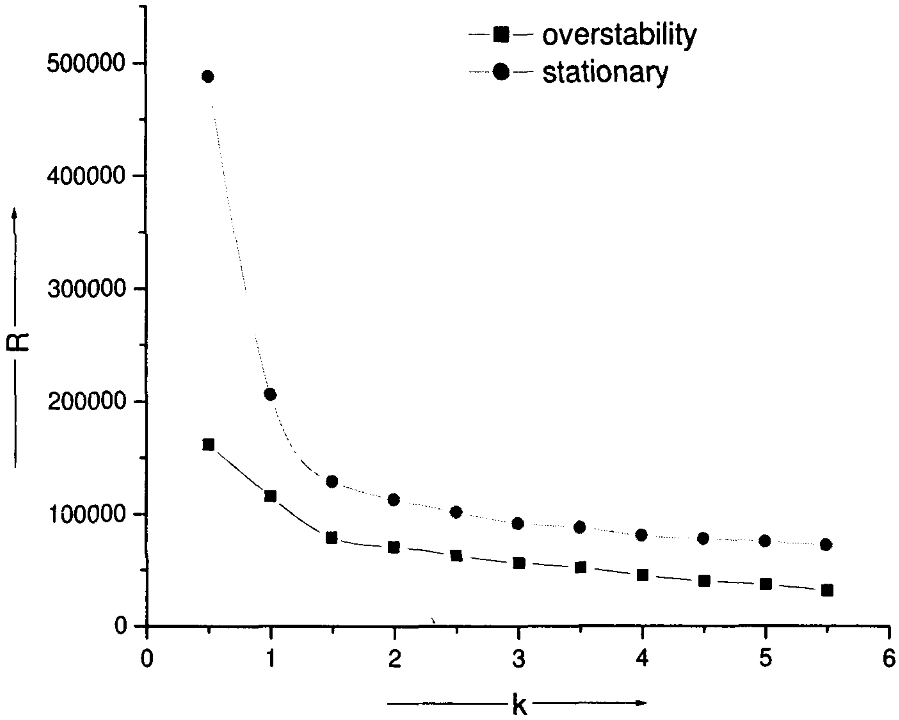

Figure 2.7: The variation of Rayleigh number (R) with wave number (k) for $H = 120$, $\bar{\delta} = 1$, $p_1 = 5$, $p_2 = 1$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $K_1 = 1$, $\ell_1 = 1$ and $\kappa = 1.0$.

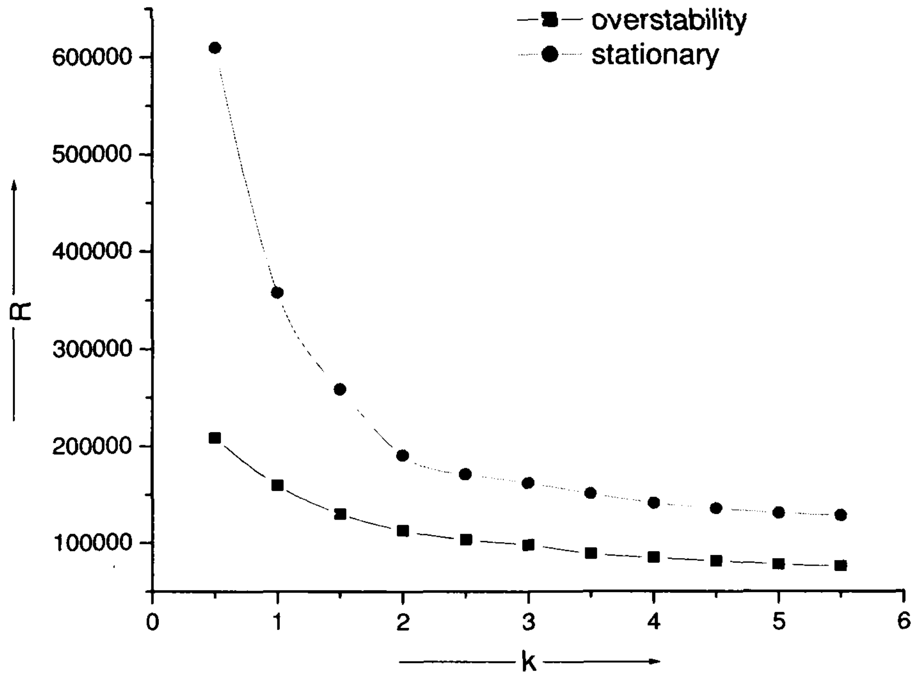

Figure 2.8: The variation of Rayleigh number (R) with wave number (k) for $H = 120$, $\bar{\delta} = 1$, $p_1 = 5$, $p_2 = 1$, $F = 1.005$, $H_1 = 1.01$, $a = 10$, $A = 0.5$, $\ell_1 = 1$ and $\gamma' = 1.0$.

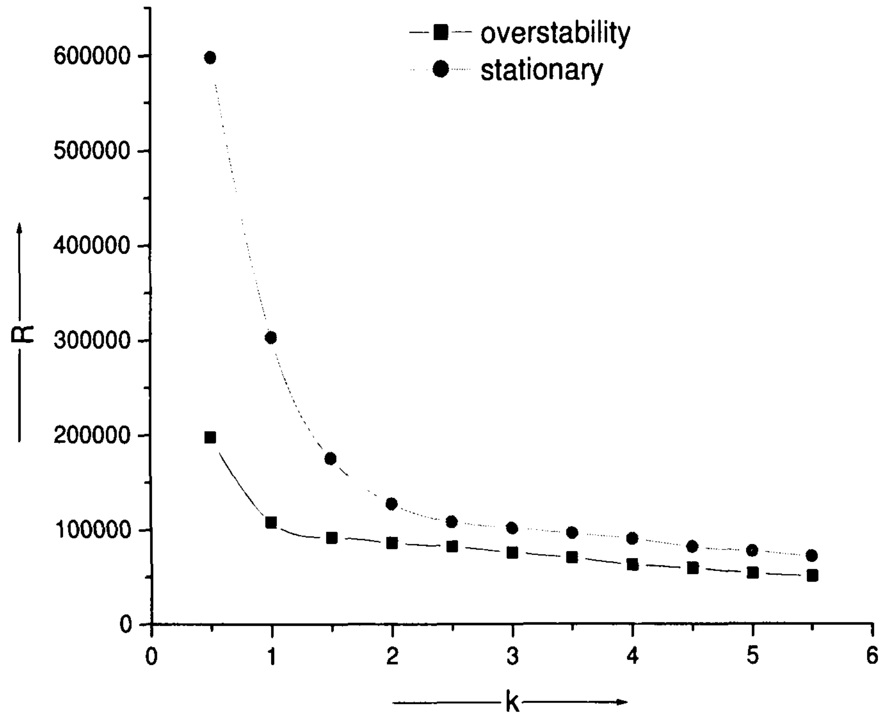

Figure 2.9: The variation of Rayleigh number (R) with wave number (k) for $H=120,\ \bar{\delta}=1,\ p_1=5,\ p_2=1,\ F=1.005, H_1=1.01, a=10,\ A=0.5,\ \ell_1=1$ and $\gamma'=1.2$.

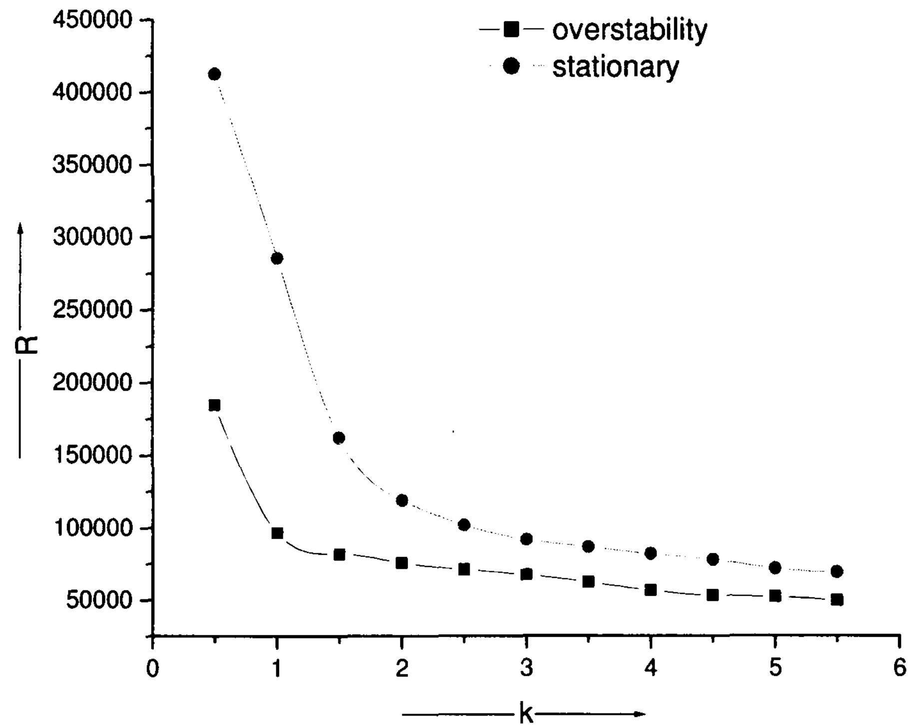

Figure 2.10: The variation of Rayleigh number (R) with wave number (k) for $H=120,\ \bar{\delta}=1,\ p_1=5,\ p_2=1,\ F=1.005, H_1=1.01, a=10,\ A=0.5, \ell_1=1$ and $\gamma'=1.4$.