Chapter-IV
Double excitation theory-Induction Motor

4.1 INTRODUCTION

In chapter-III, it has been mentioned that the time harmonics will be generated in the supply due to switching devices. When electric motors are given such a supply, the iron losses would increase considerably, especially if the rotor is a solid one. The iron losses in the solid-iron rotor can be estimated by representing the non-linear excitation as the sum of fundamental and the harmonic of highest magnitude. But the test has been performed on single-phase induction motor by feeding the fundamental excitation at 50 Hz and the harmonic excitation at 450 Hz. This type of problem can be called as two-excitation problem or dual excitation problem. One would come across two excitation problem in the feedback control systems, where to stabilize the main signal, a high frequency signal is injected in to the system, of course the frequency separation between the two signals is high. The numerical solution of the two-excitation field problem is achieved by Pseudo-Spectral Method [13]. A computerised graphical method is also developed to find the field distribution.

The theory developed in chapter-III, to find the various quantities like power losses, fluxes etc. has been verified with the practical results of single-phase induction motor with dual excitation. The total losses on the stator side are estimated using the machine design theory [24]. The infinite half-space theory is applied to the actual rotor by modifying the resistivity of the rotor material with the correction factors for curvature and end effects [14]. A correction factor is also incorporated for temperature rise. The interference between the two input signals with regard to the windings is avoided. Since the electrical angle between the windings is 90°.
4.2 DESCRIPTION OF THE PROBLEM

The three-dimensional field in the rotor of poly-phase induction motor can be made two dimensional, if it is assumed that the induced eddy-currents in the rotor are in axial-direction only. The rotor can be viewed as an iron-block, when its curvature is neglected. Such an iron-block is subjected to travelling field on its surface say in y-z plane, it is obvious that, there exists an alternating flux through any section parallel to x-z plane, alternating at slip frequency. Consequently, the evaluation of eddy-current losses in solid-iron rotor of an induction machine can be based on the knowledge of eddy-current distribution in an infinite half-space of iron subjected to pulsating field.

Solid-iron rotor induction motors are in use. Due to many reasons, these motors operate from non-sinusoidal excitation. Such an excitation can be considered to be the sum of fundamental and the harmonic of highest magnitude for the purpose of evaluating eddy-current losses. Therefore, for analysing eddy-current losses in electric machines, the input is taken as the sum of two sinusoidal signals of commensurate frequencies. Therefore, two-excitation field problem is simulated using single-phase induction motor with two windings.

The problem has been formulated in section 2.5. But the boundary conditions are provided in section 3.5 (eqn.(3.38)) Rewrite the equation (2.41) for convenience

\[
\frac{\partial^2 H}{\partial t^2} = S(H) \frac{\partial^2 H}{\partial x^2}
\]

(4.1)

where \( S(H) = \frac{\mu (\gamma + |H|^2)}{\alpha \gamma} \)

The constants are \( \alpha = 2.25 \text{ Tesla} \) and \( \gamma = 787 \text{ A/m} \) for the given magnetization curve of the material as shown in figure 4.1.
The boundary and initial conditions are rewritten here as follows:

\[(i) \quad x = 0, H = H_s = H_{12}\sin(\omega_1 t) + H_{15}\sin(\omega_2 t)\]
\[(ii) \quad x = x_m, \frac{\partial H}{\partial x} = 0 \text{ for all } "T"\]
\[(\text{or}) \quad H = 0 \text{ for large } "x"\]
\[(iii) \text{ Initial values i.e. at } T = 0, H = 0 \text{ for all } "x"\]
4.3 SOLUTION OF THE PROBLEM-PSM

In section 3.2, the analytical solution is developed for the problem. The same problem is also solved numerically by using Crank-Nicholson Method in section 3.5. But in chapter-II, it has been stated that the Pseudo-Spectral Method can be a good substitute for Crank-Nicholson Method. Hence, in this chapter, the dual excitation problem is solved by Spectral method.

4.3.1 Implicit Pseudo Spectral Method

In section 2.6, Pseudo Spectral Method or Chebyshev collocation method is discussed elaborately. The implicit time-stepping scheme to equation (4.1) is given by equation (2.67) i.e.,

\[
\left( H_0 \right)^{n+1} = \text{INV} \left( \begin{array}{c} \text{D} \\ \text{(N+1)X(N+1)} \end{array} \right)^{n+1} \left( H_0 \right)
\]

It may be noted that to find the values of H at (n+1) time-step the derivatives are evaluated at (n+1) time-step itself. The boundary conditions of equation (4.2) are implemented by changing the entries of first-row and last-row of coefficient matrix 'D' by referring section (2.61). Then the equation (4.3) is solved for finding the field distribution. For the surface excitation of \( H_{1s} = 11518 \text{ A/m} \) at \( \omega_1 = 314.2 \text{ rad/sec} \); \( H_{2s} = 2303 \text{ A/m} \) at \( \omega_2 = 2827.8 \text{ rad/sec} \), the field distribution at various layers is shown in figure 4.2.
4.3.2 Evaluation of eddy current loss

Once the field distribution is obtained the iron loss can be calculated using the procedure enclosed in section 3.5. In this chapter the effect of harmonics on iron losses is also considered.

For this purpose, along with the fundamental components, various harmonic components of field are determined at various layers, consequently, the current densities are evaluated using the formula give by equation (3.41) as

\[ J_{j,k} = \frac{H_j^{(k)} - H_j^{(k+1)}}{\Delta x} \] (4.4)

For each value of \( j = 1, 2; \) fix \( h = 1, 3, 5, \) and run \( k = 0 \) to \( N. \)

The figure 4.3 shows the profile of fundamental and harmonic current densities, when the surface excitations are \( H_{15} = 11518 \) A/m, at \( \omega_1 = 314.2 \) rad/sec; and \( H_{25} = 2303 \) A/m, at \( \omega_2 = 2827.8 \) rad/sec.

From the figure 4.3, it is understood that the high frequency signal attenuates faster than low frequency signal confirming the validity of equation (2.38) with respect to frequency. Moreover, the attenuation is nonlinear with depth.

Fourier series is employed to separate the fundamental components at each layer from the resultant wave. Therefore, it has been assumed that the two frequency signals are commensurate at all layers. For the incommensurate signals at the surface, it is difficult to separate the fundamental components in interior of the material. In fact it will be very interesting to find the ways to separate the fundamental components of incommensurate signals from the distorted resultant field.
4.3.3 Estimation of Surface Field Strengths and applied voltages

For evaluating eddy current-losses by the numerical method and compare with that of experimental values, the surface excitation must be the same as that of rotor. The procedure follows to estimate field strength at the rotor surface for the given the stator current.

If \( D \) is the diameter of rotor, then the number of conductors on the rotor is \( \pi D \).

So the ratio of transformation is

\[
K_T = \frac{2 \zeta N}{\pi D}
\]  

(4.5)

Where \( N \) is the effective number of turns of each stator winding.
If the two phases have unequal number of turns, then

\[ \zeta = \frac{(N_1 + N_2)}{N_1} \quad \text{for the first winding} \]

\[ \zeta = \frac{(N_1 + N_2)}{N_2} \quad \text{for the second winding} \]

Let \( H_s \) be the magnetising force at the surface of iron in Amp/m. Then the equivalent r.m.s current of the rotor referred to the stator is

\[ I_r = \frac{H_s}{\sqrt{2} K_T} \quad \text{(or)} \]

\[ H_s = \sqrt{2} K_T I_r \quad (4.6) \]

Since, it has been observed practically that the magnetising current drawn by solid iron rotor induction motor is as high as 30%. If one assumes the angles between voltage \& \( I_1 \) and voltage \& \( I_\mu \) are 36° and 82° respectively, the rotor component \( I_r \) will be 76% of stator current. Hence

\[ H_s = 0.76 \sqrt{2} K_T I_s \quad (4.7) \]

Where \( I_s \) is the stator current.

For different values of stator currents \( I_1 \) and \( I_2 \) of two windings, the corresponding values of \( H_{1s} \) and \( H_{2s} \) are found using equation (4.7). Subsequently, the flux components \( \phi_1 \) and \( \phi_2 \) are determined using the equations (3.23) and (3.27) respectively. Thus the induced emfs are calculated using the following equations:

\[ E_1 = 4.44 f_1 N_1 (2 L \phi_1) \quad (4.8a) \]

\[ E_2 = 4.44 f_2 N_2 (2 L \phi_2) \quad (4.8b) \]

Where \( N_1 \) and \( N_2 \) are effective number of turns of respective stator windings.
Now the applied voltages are given by

\[ V_1 = E_1 + I_1 (R_1 + jX_1) \]  
\[ V_2 = E_2 + I_2 (R_2 + jX_2) \]

(4.9a)  
(4.9b)

### 4.4 GRAPHICAL SOLUTION

In section 2.4.1, graphical method is discussed for single excitation problem. Whereas in section 3.4, geometrical construction is provided for double excitation problem, with the condition \(1 \leq (\omega_2/\omega_1) \leq 2\). In this section, a similar graphical construction will be explained for \((\omega_2/\omega_1) > 2\). The graphical construction starts with the assumption of values for \(H\) and \(\phi\) at zeroth-layer. Then the values of \(H\) and \(\phi\) at first-layer are found vectorially. Having calculated the values of \(H\) and \(\phi\) at first-layer, the values at second-layer are evaluated, and so on. The above statements can be written in the mathematical form as,

\[
H^{(i)}_{1'} = H^{(0)}_{1'} + j(\omega/\rho)\phi^{(0)}_{1'} \Delta x \\
H^{(i)}_{2'} = H^{(0)}_{2'} + j(\omega/\rho)\phi^{(0)}_{2'} \Delta x \\
\phi^{(i)}_{1'} = \phi^{(0)}_{1'} + B^{(0)}_{1'} \Delta x \\
\phi^{(i)}_{2'} = \phi^{(0)}_{2'} + B^{(0)}_{2'} \Delta x \\
\ldots \\
H^{(k-1)}_{1'} = H^{(k)}_{1'} + j(\omega/\rho)\phi^{(k)}_{1'} \Delta x \\
H^{(k-1)}_{2'} = H^{(k)}_{2'} + j(\omega/\rho)\phi^{(k)}_{2'} \Delta x \\
\phi^{(k-1)}_{1'} = \phi^{(k)}_{1'} + B^{(k)}_{1'} \Delta x \\
\phi^{(k-1)}_{2'} = \phi^{(k)}_{2'} + B^{(k)}_{2'} \Delta x
\]  

(4.10)
but the ratio (co/\( d_0 \)) should be 9.0 since the construction is made for \( n = 9.0 \).

are helpful, especially to find small power losses for any values of \( p \), \( B \), and \( n \).

The results of normalized graphical method are given in Table. These results

\[ n = \frac{1.0 + 0.5}{1.0} \times \frac{3}{2} \]

wapped and by deriving the scale factors given by equations (3.33) to (3.36).

p = 1.0 \times 10^{-1}, \ D' \ M' \ x = 5 \times 1.0 \times m. \ However, \ the \ loci \ can \ be \ obtained \ for \ any \ values \ of

The results shown in the table are corresponding to \( m = 1.1, n = 1.0, B = 1.1 \).

4.4.2 Normalization of Graphical Construction

Procedure described above. The results are tabulated in the table.

of \( H \) and \( \phi \) are computed at various layers for a local depth of 1.0. Then computing the

these initial values, the computed series of graphical construction is started and the values

values namely \( \phi \) and \( H' \) are respectively found first. Then, with

Then by using the equations (3.23), (3.24), and (3.25) the other starting

\[ H = \frac{3}{2} \]

given below.

(3.18) \( H = 0' \phi \) \( \chi = 0' \phi \) \( \omega = 0' \phi \) \( n = 0' \phi \) \( m = 1.1 \), \( D' \ M' \ x = 5 \times 10^{-1} \times m. \)

4.4.1 Choice of Initial Values

taken care of while using equation (3.5).

Any layer only fundamental components are existing. Therefore, this point has been

at various layers. If may be noticed that in the graphical method, it is assumed that at

corresponding to (3.23), (3.24), etc., \( \phi \) and \( H' \) are from equation (3.5),
Let $\omega_1 = 314.2$ rad/sec, $\omega_2 = 2827.8$ rad/sec, $\rho = 16.58 \times 10^{-4}$ $\Omega \cdot m$, $B_m = 2.0$ T. The scale factor for $H$, $\phi$ and $P$ are 37.88, 2.0 and 23789 respectively, from the equations (3.33), (3.34) and (3.35). After multiplying the values of the table by these scale factors, the curves of field of strengths, power losses, fluxes etc., are drawn and shown in fig. 4.4.

### TABLE:

Results of computerized graphical construction

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<tr>
<th>$X \times 10^{-3}$ m</th>
<th>$H_1 $ A/m</th>
<th>$H_2 $ A/m</th>
<th>$H_3 $ A/m</th>
<th>$\Phi_1$ mWb</th>
<th>$\Phi_2$ mWb</th>
<th>$\cos \varphi_1$</th>
<th>$\cos \varphi_2$</th>
<th>$P_1$ W/m$^2$</th>
<th>$P_2$ W/m$^2$</th>
</tr>
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<td>0.005</td>
<td>0.118</td>
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<td>0.761</td>
<td>0.0000</td>
<td>0.0000</td>
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<td>7.06</td>
<td>11</td>
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<td>0.036</td>
<td>0.407</td>
<td>0.003</td>
<td>0.816</td>
<td>0.761</td>
<td>0.0020</td>
<td>0.0000</td>
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<td>12.06</td>
<td>34</td>
<td>2</td>
<td>0.087</td>
<td>0.695</td>
<td>0.013</td>
<td>0.816</td>
<td>0.760</td>
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<td>10</td>
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<td>0.330</td>
<td>0.816</td>
<td>0.759</td>
<td>0.0275</td>
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<td>0.660</td>
<td>0.817</td>
<td>0.758</td>
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<td>0.818</td>
<td>0.756</td>
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<td>0.454</td>
<td>1.822</td>
<td>0.190</td>
<td>0.820</td>
<td>0.753</td>
<td>0.1806</td>
<td>0.0708</td>
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<tr>
<td>37.06</td>
<td>321</td>
<td>190</td>
<td>0.591</td>
<td>2.083</td>
<td>0.292</td>
<td>0.824</td>
<td>0.749</td>
<td>0.2764</td>
<td>0.1877</td>
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<tr>
<td>42.06</td>
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<td>311</td>
<td>0.753</td>
<td>2.322</td>
<td>0.433</td>
<td>0.830</td>
<td>0.744</td>
<td>0.3983</td>
<td>0.4522</td>
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<td>0.840</td>
<td>0.736</td>
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<td>0.848</td>
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<td>0.776</td>
<td>1.1207</td>
<td>7.4793</td>
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<td>2.063</td>
<td>3.008</td>
<td>1.692</td>
<td>0.891</td>
<td>0.785</td>
<td>1.3423</td>
<td>12.3577</td>
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<td>1137</td>
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<td>3.065</td>
<td>1.981</td>
<td>0.902</td>
<td>0.792</td>
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<td>19.2042</td>
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<td>1277</td>
<td>3478</td>
<td>2.723</td>
<td>3.112</td>
<td>2.270</td>
<td>0.911</td>
<td>0.798</td>
<td>1.8123</td>
<td>28.3720</td>
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<td>3.150</td>
<td>2.560</td>
<td>0.919</td>
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<td>3.183</td>
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<td>3.737</td>
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<td>4.412</td>
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<td>0.808</td>
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4.5 EXPERIMENTAL PROCEDURE, RESULTS AND DISCUSSION

An experiment was conducted on a single-phase induction motor after removing the capacitor from the starting winding and replacing the conventional rotor by a solid-iron rotor with no air-gap. The specifications of the motor are given in the appendix A4-1. The starting winding was connected to the 50 Hz supply. Where as, the running winding was excited by 450 Hz signal. Magnetic interference is avoided since the electrical angle between the two windings is 90°. The practical eddy-current losses in the material (rotor) at these frequencies are found-out from the measured input powers. It may be noted that the iron and stray losses in the stator, including
yoke, are estimated approximately as 15% of the total (input) losses, taking into account the total amount of iron. The details of calculations are included in the appendix A4-2. The simulated and measured iron losses are shown in fig.4.5.

The calculated applied voltages are compared with that of measured values as shown in figure 4.6.

**Fig.4.5 : Iron Loss curves**
For generating the high frequency signal, oscillator and high capacity (80 Watts) power amplifier are used. It is taken care that the two signals starts at the same time or in other-words that the phase-shift is adjusted to zero. But, the phase-shift does not have much effect on the power losses. The reason is that, the frequency separation between the two signals is more. Hence the average power losses are almost independent of phase-shift.

The infinite half-space theory is adapted for electrical machines, by modifying the specific resistance of rotor material so as to incorporate the correction factor for
curvature and end effects [14]. Finally, a suitable correction factor is also incorporated into resistivity to account for temperature rise. The theory is provided in appendix A4-3. The effective resistivity is $1.06 \times 1.77 \times 1.2 \times 16.58 \times 10^{-8}$ Ω-m.

4.6 CONCLUSIONS

A new classical numerical method called Pseudo-Spectral method is successfully implemented to find the field distribution in the solid-iron rotor of induction motor, when it is subjected to two-frequency excitation. It is found that the simulated power losses in the rotor are agreeing with the experimental values. A graphical method is also presented to evaluate the field distribution in the specimen.

A4: APPENDIX

A4-1: Details of single-phase induction motor:

- **Rated output power**: 1.0 kW
- **Input voltage (V)**: 220 volts
- **Effective number of turns of (starting) winding A, ($N_1$)**: 334
- **D.C resistance of winding A, ($R_1$)**: 8.0 Ohms
- **Inductance of winding A, ($L_1$)**: 0.045 Henrys
- **Effective number of turns of (running) winding B, ($N_2$)**: 286
- **D.C resistance of winding B, ($L_2$)**: 2.5 Ohms
- **Inductance of winding B, ($L_2$)**: 0.026 Henrys
- **Diameter of rotor (D)**: 0.106 m
- **Length of rotor (L)**: 0.109 m
- **Specific resistance of the rotor material ($\rho_o$)**: $16.58 \times 10^{-8}$ Ω-m
- **B-H curve of rotor material**: Figure 4.1(p72)
- **Length of the air-gap**: Zero
- **End rings**: Nil
**A4-2: Procedure to find total losses in the stator**

The losses in the stator are

(a) Copper losses in the windings and

(b) Iron loss in the material

(a) Evaluation of copper loss: Knowing the resistance of each winding, copper losses at any load can be calculated.

(b) Evaluation of iron loss: These losses can be further classified as stator teeth and core losses. The calculations of iron losses are based on the total weight and the flux density in the material. The values given in the brackets immediately after the formulae are referred to the specific motor, whose details are given in the appendix A4-1.

(i) Stator teeth loss

The total weight of all stator teeth is given by

\[ w_{st} = S_s \cdot W_{ss} \cdot d_{ss} \cdot L_{sc} \cdot g, \]

\[ (w_{st} = 36 \times 5 \times 10^{-3} \times 2 \times 10^{-2} \times 0.109 \times 7.65 \times 10^3 = 3 \text{ Kgs}) \]

Where \( S_s \) - number of stator slots

\( W_{ss}, d_{ss} - \) width and depth of each stator slot, meters

\( L_{sc} - \) length of stator core, meters

\( g - \) specific weight of iron, kgs/m^3

The iron loss in the stator teeth \( (P_{it}) = a \cdot B_m^2 \cdot w_{st} \)

\[ (P_{it} = 6.5 \times 1.5^2 \times 3 = 43.87 \text{ watts}) \]

Where \( 'a' \) is a constant. Its value is 6.5 for teeth and 4.7 for core [24].
(ii) Stator core loss

The iron losses in the stator core are also estimated in the same manner as that of stator teeth loss.

The depth of stator core is given by

\[
d_{sc} = \frac{D_o - D - 2d_{ss}}{2}
\]

\[
(\text{d}_{sc} = \frac{0.179 - 0.109 - 2 \times 0.02}{2} = 0.015 \text{ m})
\]

Where \( D, D_0 \) are inner and outer diameters of stator.

The mean diameter of stator core is given by

\[
D_{msc} = D_0 - d_{sc}
\]

\[
(D_{msc} = 0.179 - 0.015 = 0.164 \text{ m})
\]

Hence the weight of the stator core is,

\[
w_{sc} = \pi \cdot D_{msc} \cdot d_{sc} \cdot L_{sc} \cdot g
\]

\[
(w_{sc} = \pi \cdot 0.164 \cdot 0.015 \cdot 7.65 \times 10^3 = 6.44 \text{ kgs})
\]

The iron loss in the stator core \( (P_{isc}) = a \cdot B_m^2 \cdot w_{sc} \)

\[
(P_{isc} = 4.7 \cdot 1^2 \cdot 6.44 = 30.26 \text{ watts})
\]

The total iron loss in the stator \( (P_t) = P_{int} + P_{isc} \)

\[
(P_t = 43.87 + 30.26 \equiv 75 \text{ watts})
\]

The iron losses and stray losses in stator & yoke are taken as two-times \( P_t \).

So

\[
P = 2 \cdot P_t
\]

\[
(P = 2 \cdot 75 = 150 \text{ watts})
\]
Therefore, for this machine the total iron losses on the stator side, including stray losses are 15% of total input power.

A4.3: The correction factors for curvature and end effects [14]

(a) Correction for curvature

When an infinite half-space theory is applied to a cylindrical co-ordinate system, it becomes necessary to make an allowance for the curvature to confirm the physical fact that actual current is reduced. This reduction in-turn depends upon the depth of penetration. The reduction factor given in the reference is \((D - 2x_1/3)/D\).

Where \(D\) is the diameter of the rotor, and \(x_1\) is the depth of penetration of signal which goes deeper than another signal. In this case it is the depth of penetration of low frequency signal. For the given surface excitation of the rotor, the stator current is reduced by the above factor i.e \((D - 2x_1/3)/D\). In effect, the ratio of transformation is increased by the same factor. Hence, an allowance for the curvature can be made by increasing the specific resistance by a factor,

\[
K_1 = \frac{D^2}{\left(D - \frac{2x_1}{3}\right)^2}
\]

\[
K_1 = \frac{0.106^2}{\left(0.106 - \frac{2 \times 0.0041}{3}\right)^2} = 1.06
\]

Where the value of \(x_1\) is determined using the equation (3.25) for the maximum value of \(H_{ls}\)
(b) Correction for end effects

In the infinite half-space analysis, it has been assumed that all the currents in the rotor flow axially. In other words, the end effects have been ignored. It is obvious that the end effects would depend on the physical dimensions of the rotor, the type of end-rings used, no end-rings used and the rotor frequency. An empirical correction for these effects is to modify the specific resistance of the rotor by a factor

\[ K_2 = K \left[ 1 + \frac{SL}{4\tau} \right] \]

Where \( K = 1 \), for the rotor with copper end rings,

\[ K = (1+\tau/L), \text{ for the rotor with steel end rings.} \]

\[ K = 1.77(1+0.49S) \text{ with no end-rings,} \]

\( S, L \text{ and } \tau \text{ are slip, rotor active length and pole pitch respectively.} \)

\( K_2 = K \) (Since \( S = 0 \), the slip does not come into picture as there is no revolving magnetic field)

So \( K_2 = 1.77 \)

(c) Correction for temperature rise

To consider the effect of increase in temperature on the resistivity of the material, an appropriate multiplication factor is assumed as \( K_3 = 1.2 \).

To summarize, the effective correction factor is \( K = K_1K_2K_3 = 1.06*1.77*1.2 = 2.25 \)

Therefore, the modified resistivity is \( \rho = \rho_0 \times K = 16.58 \times 10^{-4} \times 2.25 = 37.33 \times 10^{-4} \text{ Ohm-m.} \)