Chapter-III
Evaluation of eddy current loss in solid cores under double excitation

3.1 INTRODUCTION

The second chapter dealt with the problem on single excitation in detail. In this chapter double excitation field problem is formulated. A closed form solution is developed for this field problem. The same problem is also solved by Crank-Nicholson numerical method. In addition a graphical solution is also provided for supplementation. Finally an experiment is conducted on two identical toroids.

In electric machines harmonics are due to many reasons like saturation, irregular gap length, type of winding and slots, unbalanced loading etc. Mainly the use of switching devices would inject time-harmonics. Thus, the air gap m.m.f of an electric drive, not only have fundamental component but also harmonics, of course, here time harmonics are only considered. When, the rotor of an induction motor is subjected to such an m.m.f, the magnitude of eddy current loss increases considerably. Induction motors with solid iron rotors are in use. Therefore, it will be useful to know the eddy current distribution in the rotor. Extensive work has been done on single excitation. But, for various reasons, attention has not been given to research on dual excitation, especially in the area of electric machines. Theory of dual excitation is very well applied in the field of Control Engineering. Indeed, it can also be used to analyze eddy current losses in electric machines. For the purpose of evaluating eddy current losses, the input excitation to the rotor can be considered as the addition of fundamental and the harmonic of highest magnitude, neglecting all other harmonics. In other words, the non-sinusoidal excitation can be represented as the sum of two sinusoidal signals of commensurate frequencies i.e. the frequency of one signal is the
multiple of frequency of other signal. There are cases, where the frequencies are incommensurate.

3.2 THEORY

Let

\[ H = H_1 \sin(\omega_1 t + \theta_1) + H_2 \sin(\omega_2 t + \theta_2) \]  

(3.1)

and assume the following

(i) At any layer, the components of \( H \), of angular frequencies \( \omega_1 \) and \( \omega_2 \) alone exist,

(ii) The flux density at any layer consists of above two components of \( H \) only, and

(iii) The input frequencies are treated as incommensurate.

Hence the flux density at any layer is given by

\[ B = B_1 \sin(\omega_1 t + \theta_1) + B_2 \sin(\omega_2 t + \theta_2) \]  

(3.2)

where \( B_1, B_2 \) are fundamental components of output corresponding to the input components \( H_1 \) and \( H_2 \) respectively. The evaluation of \( B_1 \) and \( B_2 \) are discussed in appendices A3-1 & A3-2 for the given non-linearity (B-H curve), both for commensurate and incommensurate frequencies.

Substituting equations (3.1) and (3.2) in (2.3) and equating similar terms, yields the following equations

\[ \frac{d^2 H_1}{dx^2} = H_1 \left( \frac{d\theta_1}{dx} \right)^2 \]  

(3.3a)

\[ \frac{1}{H_1} \frac{d}{dx}\left( H_1 \frac{d\theta_1}{dx} \right) = \frac{\omega_1 B_1}{\rho} \]  

(3.3b)
3.2.1 Solution of Equation (3.3)

The magnetization curve of the material can be approximated by a relay type curve such as shown in fig.2.2 (p18). For this relay type B-H curve, the formulae for $B_1$ and $B_2$ are given in [21], as

$$B_1 = B_m F_1(\lambda) \quad (3.5a)$$

and

$$B_2 = B_m F_2(\lambda) \quad (3.5b)$$

Where $F_1(\lambda)$, $F_2(\lambda)$ are yet to be defined.

The flux density at saturation $B_m$, is the flux density corresponding to the given field strength (H). Hence, it can be read from the actual B-H curve (fig.3.1).

But in this project, the B-H curve is replaced by rational fraction formula as

$$B = \frac{\alpha_0 + \alpha_1 H + \alpha_2 H^3 + \alpha_3 H^4 + \alpha_4 H^5}{1 + \gamma_1 H + \gamma_2 H^3 + \gamma_3 H^5 + \gamma_4 H^7} \quad (3.6)$$

Where, the values of $\alpha_0 = -8.6816 \times 10^{-2}$, $\alpha_1 = 1.0213 \times 10^{-3}$, $\alpha_2 = -1.1625 \times 10^{-10}$, $\alpha_3 = 5.0086 \times 10^{-18}$, $\alpha_4 = -6.5553 \times 10^{-26}$, $\gamma_1 = -6.3403 \times 10^{-5}$, $\gamma_2 = -4.0462 \times 10^{-11}$, $\gamma_3 = 2.0808 \times 10^{-18}$, $\gamma_4 = -2.8879 \times 10^{-26}$ and $H = (H_1 + H_2)/2$. Still for accurate representation, the series should be extended.
Since it is assumed that the two frequencies are incommensurate i.e. the ratio of $(\omega_2/\omega_1)$ is an irrational number. Then, with $\lambda = (H_2/H_1) < 1$, $F_1(\lambda)$, $F_2(\lambda)$ are given by

$$F_1(\lambda) = \frac{8}{\pi^2} E(\lambda)$$  \hspace{1cm} (3.7a)$$

$$F_2(\lambda) = \frac{8 H_1}{\pi^2 \lambda} \left[ E(\lambda) - (1 - \lambda^2) K(\lambda) \right]$$  \hspace{1cm} (3.7b)$$

Where $E(\lambda)$, $K(\lambda)$ are elliptic integrals of the first and second kind respectively.
It may be noted that the values of $F_1(\lambda)$, $F_2(\lambda)$ of incommensurate frequencies would approach their counterparts of commensurate frequencies for large ratio of $(\omega_2/\omega_1)$[22]. By expanding the elliptic integrals into power series and dividing by $(4/\pi)$ (because of first assumption made in section (3.2)), the equation (3.7) becomes

$$F_1(\lambda) = \left[1 - \frac{\lambda^2}{4} - \frac{3\lambda^4}{64} - \frac{5\lambda^6}{256} - \ldots \right]$$  \hfill (3.8a)

$$F_2(\lambda) = \frac{\lambda}{2} \left[1 + \frac{\lambda^2}{8} + \frac{3\lambda^4}{64} + \frac{25\lambda^6}{1024} + \ldots \right]$$  \hfill (3.8b)

If the ratio, $(H_2/H_1) > 1$, then it can be shown that the above two equations (3.8a) and (3.8b) must be interchanged for $F_1(\lambda)$ and $F_2(\lambda)$ with $\lambda = H_1/H_2$. That is

$$F_1(\lambda) = \frac{\lambda}{2} \left[1 + \frac{\lambda^2}{8} + \frac{3\lambda^4}{64} + \frac{25\lambda^6}{1024} + \ldots \right]$$  \hfill (3.9a)

$$F_2(\lambda) = \left[1 - \frac{\lambda^2}{4} - \frac{3\lambda^4}{64} - \frac{5\lambda^6}{256} - \ldots \right]$$  \hfill (3.9b)

with $\eta = (\omega_2/\omega_1)$, using equation (3.5), the equations (3.3) and (3.4) can be written as,

$$\frac{d^2H_1}{dx^2} = H_1 \left( \frac{d\theta_1}{dx} \right)^2$$  \hfill (3.10a)

$$\frac{1}{H_1} \frac{d}{dx} \left( H_1 \frac{d\theta_1}{dx} \right) = \frac{\omega_2 B_m F_1(\lambda)}{\rho}$$  \hfill (3.10b)

$$\frac{d^2H_2}{dx^2} = H_2 \left( \frac{d\theta_2}{dx} \right)^2$$  \hfill (3.11a)

$$\frac{1}{H_2} \frac{d}{dx} \left( H_2 \frac{d\theta_2}{dx} \right) = \frac{\eta \omega_2 B_m F_2(\lambda)}{\rho}$$  \hfill (3.11b)

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3.2.1(a): \( \lambda = (H_2/H_1) \leq 0.4 \)

Now the equation (3.8) would become

\[
F_1(\lambda) \approx 1 \quad (3.12a)
\]

\[
F_2(\lambda) \approx \frac{\lambda}{2} = \frac{H_2}{2H_1} \quad (3.12b)
\]

Hence, using the above equation (3.12a), equation (3.10) is simplified as

\[
\frac{d^2H_1}{dx^2} = H_1 \left( \frac{d\theta_1}{dx} \right)^2 \quad (3.13a)
\]

\[
\frac{1}{H_1} \frac{d}{dx} \left( H_1^2 \frac{d\theta_1}{dx} \right) = \frac{\omega_1 B_m}{\rho} \quad (3.13b)
\]

The above equations (3.13a) and (3.13b) are similar to that of (2.30a) and (2.30b) respectively. Hence the solution of simultaneous non-linear differential equations (3.13a) and (3.13b) is given by the equation (2.34). That is the solution or the law of variation of \( H_1 \) with 'x' is given here for convenience,

\[
H_1 = a x^2 \quad (3.14)
\]

Where \( a = \omega_1 B_m / (3 \rho \sqrt{2}) \).

3.2.2 Solution of equation (3.4)

Making use of equation (3.12b), equation (3.11) is written as

\[
\frac{d^2H_2}{dx^2} = H_2 \left( \frac{d\theta_2}{dx} \right)^2 \quad (3.15a)
\]

\[
\frac{1}{H_2} \frac{d}{dx} \left( H_2^2 \frac{d\theta_2}{dx} \right) = \frac{\eta \omega_1 B_m H_2}{\rho 2H_1} \quad (3.15b)
\]
Assume a solution of the form

\[ H_2 = ax^b \]  

(3.16)

and substituting in 3.15(a), gives

\[ \beta(\beta - 1)ax^{b-2} = ax^b \left( \frac{d\theta_2}{dx} \right)^2 \]

(or)

\[ \frac{d\theta_2}{dx} = \frac{\sqrt{\beta(\beta - 1)}}{x} \]  

(3.17)

Now substituting (3.17) for \( \frac{d\theta_2}{dx} \) in (3.15b), yields

\[ \frac{1}{ax^b} \frac{d}{dx} \left[ a^2x^{2b} \sqrt{\beta(\beta - 1)} \right] = \frac{\eta \omega \beta m H_2}{2\rho H_1} \]

After differentiation and substitution of (3.14) for \( H_1 \), the result is

\[ \frac{\sqrt{2}}{3} (2\beta - 1)\sqrt{\beta(\beta - 1)} = \eta \]  

(3.18)

It may be noted here, that the unknown \( \beta \) depends only on frequency ratio \( \eta \). By inspection it is clear that \( \beta > 2 \), when \( \eta > 2 \) and

\[ \frac{H_2}{H_1} = x^{b-2} \rightarrow 0 \text{ as } x \rightarrow 0 \]  

(3.19)

The inference from the above equation (3.19) is that the high frequency signal attenuates faster than low frequency signal.

For the given value of \( \eta > 2 \), the \( \beta \) is found from equation (3.18) and substituted in (3.16) so as to obtain solution for \( H_2 \).
3.2.2(a): \( \lambda = (H_1 / H_2) \leq 0.4 \)

Now the equation (3.9) reduces to

\[
F_1(\lambda) \approx \frac{\lambda}{2} = \frac{H_1}{2H_2} \tag{3.20a}
\]

\[
F_2(\lambda) \approx 1 \tag{3.20b}
\]

Consequently, the solution of \( H_1 \) would be the solution of \( H_2 \) and vice-versa. Also note that the equation (3.18) would become

\[
\frac{\sqrt{2}}{3} (2\beta - 1)\sqrt{\beta(\beta - 1)} = \frac{1}{\eta} \tag{3.21}
\]

Further the statement of (3.19) holds good. It is concluded that the derived equations pertaining to signal \( H_1 \) would belong to signal \( H_2 \) and vice-versa, after replacing \( H_{1S} \) and \( \omega_1 \) by \( H_{2S} \) and \( \omega_2 \) in all the relevant equations of the next section (3.3). Of course the parameter \( \beta \) should be from equation (3.21).

3.3 DETERMINATION OF LOSSES

3.3.1 Computation of eddy current loss, flux, power factor, etc., due to \( H_1 \)

The loss per unit surface area, flux per unit length etc., are given by the equations (2.35) through (2.38). They are as follows:

Loss per unit surface area is

\[
P_1 = \sqrt{\frac{\rho \omega B_m}{3\sqrt{2}}} (H_{1S})^3 \text{ watts/m}^2 \tag{3.22}
\]

Flux per unit length of perimeter is

\[
\phi_{1S} = \sqrt{\frac{\sqrt{2}\rho B_m}{\omega_1}} H_{1S} \text{ webers/m} \tag{3.23}
\]
Power factor at the surface is

\[ \sin \psi_{15} = \sqrt{2/3} \]  \hspace{1cm} (3.24)

Depth of penetration is

\[ X_1 = \sqrt{\frac{3\sqrt{2\rho}}{\omega_1 B_m} H_{15}} \text{ meters} \]  \hspace{1cm} (3.25)

3.3.2 Computation of eddy current loss, flux, power factor, etc., due to \( H_2 \)

Knowing \( H_2 = a \times 10^\beta \), the following formulae are obtained, using the equations (2.27), (2.28), (2.29) and (3.16)

\[ P_2 = \frac{\beta}{2\sqrt{(2\beta - 1)\sqrt{\beta (\beta - 1)}}} \left( \frac{\rho \omega_2 B_m}{2 H_{15}} \right) [H_{25}]^2 \]  \hspace{1cm} (3.26)

\[ \phi_{25} = \left( \frac{\beta}{(\beta - 1)} \right)^{1/2} \frac{\rho B_m}{2\omega_2 H_{15}} H_{25} \]  \hspace{1cm} (3.27)

\[ \sin \psi_{25} = \frac{\beta}{\sqrt{(2\beta - 1)}} \]  \hspace{1cm} (3.28)

\[ X_2 = \left( \frac{3\sqrt{2\rho}}{\omega_2 B_m H_{25}} \right)^{1/4} \]  \hspace{1cm} (3.29)

3.4 Graphical Solution of the Problem

The closed form solutions derived for \( H_1 \) and \( H_2 \) given by equations (3.14) and (3.16) are valid for \( \eta > 2 \). However, a graphical solution is suggested for \( 2 \geq \eta \geq 1 \) is as follows:

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Consider the equation

\[ \lambda_c F_1(\lambda_c) = \eta F_2(\lambda_c) \]  

(3.30)

Where \( \lambda_c = (H_2 / H_1) < 1 \).

It may be noted that, in the above equation (3.30) the range of \( \eta \) is \( 2 \geq \eta \geq 1 \), for \( 0 \leq \lambda_c \leq 1 \). Moreover, for this range of \( \eta \), \( H_1 \) and \( H_2 \rightarrow 0 \) as \( x \rightarrow 0 \), but the ratio of starting values \( (H_2^{(0)} / H_1^{(0)}) \rightarrow \lambda_c \). Where the superscript 'o' within brackets is an indication of starting value.

In this graphical construction, the values of \( H_1 \) and \( \phi_1 \) at \( x = x^{(0)} \) are assumed to be known. Then, the problem is of determining graphically the values of

\[ \begin{align*}
H_1 + \Delta H_1 \\
\phi_1 + \Delta \phi_1 \\
H_2 + \Delta H_2 \\
\phi_2 + \Delta \phi_2
\end{align*} \]  

(3.31)

at an adjacent layer, i.e at \( x^{(0)} + \Delta x \).

Where \( \Delta H_1 = j \phi_1 (\omega_1 \Delta x / \rho) \), \( \Delta H_2 = j \phi_2 (\omega_2 \Delta x / \rho) \),

\[ \Delta \phi_1 = B_1 \Delta x, \; \Delta \phi_2 = B_2 \Delta x \]

The values \( B_1 \) and \( B_2 \) are given by (3.5).

also \[ \Delta H^{(n)}_1 \perp to \phi^{(n)}_1 \; \text{ (n = 1,2,...)} \]

(3.32)

\[ \Delta H^{(n)}_2 \perp to \phi^{(n)}_2 \; \text{ (n = 1,2,...)} \]

\[ \Delta \phi^{(n)}_1 \parallel to H^{(n)}_1 \; \text{ (n = 1,2,...)} \]

\[ \Delta \phi^{(n)}_2 \parallel to H^{(n)}_2 \; \text{ (n = 1,2,...)} \]

With the above information the geometrical construction can be made for \( \phi_1 \) and \( H_1 \),

which will be same as that shown in fig. 2.3 (p19). Similarly, the loci of \( \phi_2 \) and \( H_2 \) can be drawn.
3.4.1 Procedure

(i) Select a value for $\eta$ say 1.2,

(ii) Determine $\lambda_c$ using (3.30), so that RHS equal to LHS,

(iii) Choose the starting values as $H_1^{(0)} = 1$, $(H_2^{(0)}/H_1^{(0)}) = \lambda^{(0)}$, with $\omega_1 = 1$, $B_m = 1$ Tesla, and $\rho = 10^{-8}$ $\Omega \cdot$ m,

(iv) Using the equations derived in section 3.3, of course with $\beta = 2$, got the other starting values, namely;

$$X^{(0)} = \sqrt{5}(2) \times 10^{-4}$$

$$\phi_1^{(0)} = 2^{1/2} \times 10^{-4} \frac{\omega b}{\eta}$$

$$\phi_2^{(0)} = 2^{1/2} \sqrt{\frac{1}{2\eta}} \lambda^{(0)} \times 10^{-4} \frac{\omega b}{\eta}$$

$$\psi_1^{(0)} = \psi_2^{(0)} = \sin^{-1}(\sqrt{2/3})$$

(v) Fix the surface ratio $\lambda_s = (H_{2x}/H_{1x})$,

(vi) Choose $\lambda^{(0)} = \lambda_c$, if $\lambda_s = \lambda_c$,

$$\lambda^{(0)} = \lambda_c + \varepsilon \lambda_c$$, if $\lambda_s > \lambda_c$,

$$\lambda^{(0)} = \lambda_c - \varepsilon \lambda_c$$, if $\lambda_s < \lambda_c$,

Where $\varepsilon$ is fractional value.

(vii) Knowing the values of $H_1$, $H_2$, $\phi_1$ and $\phi_2$ at $x^{(0)}$, find the values of $H_1, H_2, \phi_1$ and $\phi_2$ at an adjacent layer $x^{(0)} + \Delta x$, using the equation (3.31). In the same manner vectorially find out the values of $H$, $\phi$ and $P.F$ at subsequent layers.

(viii) Find the power losses $P_1$ and $P_2$ employing Poynting theorem (eqn. (3.39))
3.4.2 Normalisation of graphical construction

The graphical construction is carried out for $\omega_1 = 1$, $\eta_0 = 1.2$, $B_m = 1$ Tesla, $\rho = 1 \times 10^{-4}$ $\Omega$-m, and $\Delta x = 5 \times 10^{-4}$ m. The results are tabulated in the table.

However, using normalization technique, results can be obtained for any values of $\omega_1$, $B_m$ and $\rho$ by adapting the following scale factors:

Scale factor for $H_1$ and $H_2$ is

$$k(H) = \frac{(\omega_1 B_m) / \rho_n}{(\omega_1 B_m) / \rho}$$

(3.33)

Scale factor for $\phi_1$ and $\phi_2$ is

$$k(\phi) = \frac{B_m}{B_m}$$

(3.34)

Scale factor for $P_1$ and $P_2$ is

$$k(P) = \frac{\omega_{in}}{\omega_1} \times k(H) k(\phi)$$

(3.35)

Scale factor for $x$ is

$$k(x) = \sqrt{\frac{H_{in}}{k(H) H_1}}$$

(3.36)

Where $H_{in}, \omega_{in}, B_m$ and $\rho_n$ are the new values of $H_1, \omega_1, B_m$ and $\rho$ respectively.
### TABLE:

**Results of Computerized graphical construction**

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<th>H2 A/m</th>
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<th>(\phi_2) mWb/(cm²)</th>
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<th>(\sin\psi_2)</th>
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<tr>
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<td>0.850</td>
<td>5.4244</td>
<td>20.2676</td>
<td></td>
</tr>
</tbody>
</table>

#### 3.4.3 Use of table

The outcome of normalized graphical construction is provided in the table. These results are useful to find out power losses for any data. But \(\eta\) should be 1.20. Because graphical construction is carried out for \(\eta=1.20\). To illustrate,

(i) Let \(H_{15} = \text{36500 A/m, } H_{25} = \text{51500 A/m, } \omega_1 = \text{261.8 rad/sec, } \omega_2 = \text{314.16 rad/sec},\) \(B_m = 2\) Tesla, \(\rho = 15.1 \times 10^{-8} \Omega \cdot \text{m}.

(ii) The scale factor for \(H\) using equation (3.33) is,

\[ k(H) = 34.68 \]
\( k(H) = 34.68 \)

(iii) The field strengths of 13th row, when they are multiplied by \( k(H) \) are equal to the given values of \( H_{15} \) and \( H_{25} \) respectively.

(iv) The scale factor for \( P \) is found using equation (3.35),

\[ k(P) = 18158 \]

(v) Now multiplying the power losses of 13th row by \( k(P) \) gives,

\[ P_1 = 29172 \text{ watts/m}^2 \quad P_2 = 69910 \text{ watts/m}^2 \]

(vi) The scale factor for \( \phi \) from equation (3.34) is \( K(\phi) = 2.0 \).

The curves of field strength, power loss, flux etc. of each signal are drawn after multiplying the table values by the above scale factors. The curves are shown in fig3.2, so that one can have freedom in choosing either \( H_{15} \) or \( H_{25} \).

![Graph](attachment:image.png)
3.5 NUMERICAL SOLUTION OF THE PROBLEM - CNM

Let the excitation at the surface of a material be given by

\[ H_s = H_{15} \sin(\omega_1 t) + H_{25} \sin(\omega_2 t) \]  

(3.37)

and the boundary conditions and initial condition are

\[
\begin{align*}
(1) \ & \text{at } x = 0, \ H = H_s \\
(11) \ & \text{at } x = x_w, \ \frac{\partial H}{\partial x} = 0 \quad \text{for all } "T" \\
(111) \ & \text{at } T = 0, \ H = 0 \quad \text{for all } "x"
\end{align*}
\]

(3.38)

By employing the Crank-Nicholson Method described in section 2.5, the field distribution for the surface values of \( H_{15} = 7070 \) A/m at \( \omega_1 = 2618 \) rad/sec and \( H_{25} = 1414 \) A/m at \( \omega_2 = 3142 \) rad/sec is obtained and shown in figure 3.3. Having found the field distribution the fundamental components of two input signals are calculated at each layer. Subsequently the power losses are determined using the following eqns,

\[ P_1 = 0.5 \rho J_1 \sin \psi_1 \]  

(3.39a)

\[ P_2 = 0.5 \rho J_2 \sin \psi_2 \]  

(3.39b)

where \( J_{1,2}^{(k)} = \frac{H_{1,2}^{(k)} - H_{1,2}^{(k+1)}}{\Delta x} \) \( j = 1, 2, \ k = 0 \)

\[ \sin \psi_{1s} = 0.8165 \{\text{using equation (3.24)}\} \]

\[ \sin \psi_{2s} = 0.9165 \{\text{using equation (3.28)}\} \]
Alternatively,

\[ P_1 = \sum_{k=0}^{N} 0.5 \omega_1 B_1 \Delta X H_{1k} \sin \psi_{1k} \]  
(3.40a)

\[ P_2 = \sum_{k=0}^{N} 0.5 \omega_2 B_2 \Delta X H_{2k} \sin \psi_{2k} \]  
(3.40b)

Where \( B_1, B_2 \) can be evaluated using the equation (3.5) and \( \sin \psi_{1k} = \sin \psi_{2k} = 0.8165 \)

\[ \text{Fig. 3.3: Field profile} \]
3.5.1 Evaluation of harmonics

Along with fundamental components, various harmonics are determined at different layers. Consequently, harmonic current densities are evaluated using the equation,

$$ J_{jh} = \frac{H_{hj}^{(k)} - H_{hj}^{(k+1)}}{\Delta x} $$

(3.41)

For each \( j = 1, 2; \) fix \( h = 3, 5, 7, \ldots \) and run \( k \) from 0 to \( N \).

Where subscript ‘\( j \)’ refers to the input signal number and ‘\( h \)’ indicates the fundamental and harmonics depending upon its value. The superscript ‘\( k \)’ pointing the layer number.

But the figure 3.4 shows only the current densities of frequencies same as those of input signals.

![Figure 3.4: Variation of current densities with depth](image-url)
3.6 EXPERIMENTAL PROCEDURE AND RESULTS

An experiment was conducted on two identical toroids, the details of which are provided in the appendix (A3-3). The primary windings are connected in parallel, whereas the secondary windings are so connected in series such that the emfs induced in them due to primary Amp-Turns (AT) will cancel each other. Therefore, the interference caused by the primary AT is neutralised at all excitations. The flux produced by the secondary AT would induce an emf in each of the primary windings and hence circulate current in the internal circuit, since the primaries are connected in parallel. To swamp-out this circulating current, two equal high value resistors are inserted, one in each branch, so that the AT measured on each side would be corresponding to the respective excitations and loading effect on secondaries is made negligible.

The primaries, which are connected in parallel, are excited by 48 Hz supply. Whereas the series combination of the secondaries are excited by 16 Hz signal. The low frequency signal is generated by A.C generator. Power losses on both sides are measured, keeping the magnitude of low frequency signal constant and varying the magnitude of the high frequency signal. Two wattmeters were placed on the primary side, one in each of the parallel branch. The average of these two meters readings is taken for further calculations. The iron losses are found out from the measured input powers. The results of analytical solution and the simulated values of the Crank-Nicholson Method are compared with the experimental power losses. From the figure 3.5, it is known that the theoretical values of power losses are close to that of practical values.
3.7 CONCLUSIONS

An analytical method is devised to evaluate eddy current loss in ferromagnetic cores, when they are subjected to two sinusoidal signals of different frequencies, under saturated conditions. To verify the validity of the proposed method an experiment was conducted on two identical toroids. It is observed that the calculated power loss of analytical method and simulated results of numerical method are agreeing with the experimental values of power loss. To complete the project, a graphical solution is also suggested.
A3: APPENDIX

A3-1 Evaluation of fundamental components of output using M-functions

(a) Commensurate frequencies

The magnetisation curve of any machine is nonlinear in nature. When this nonlinearity is subjected to two sinusoidal signals of commensurate frequencies, the output wave is analysed by Fourier series, to find the response of the element.

Consider an input

\[ H = H_1 \sin(\omega_1 t) + H_2 \sin(\omega_2 t) \]  \hspace{1cm} \text{(A3.1)}

The resulting response is given by

\[ B = f(H) \] \hspace{1cm} \text{(A3.2)}

This output will contain fundamental components of frequencies \( \omega_1 \) and \( \omega_2 \), their harmonics \( m\omega_1 \) and \( n\omega_2 \) and combination frequencies \( (m\omega_1 \pm n\omega_2) \), where 'm' and 'n' are integers.

In order to determine these components, the output wave should be analysed by Fourier method over a period of time \( T \), during which both signals complete an integral number of cycles. These components appear as certain harmonics of the wave being analysed.

Thus, fundamental \( \omega_1 \) and \( \omega_2 \), harmonics \( m\omega_1 \), \( n\omega_2 \) and combination \( (m\omega_1 \pm n\omega_2) \) frequency components in the output are given by

\[ B_{mp} = \frac{2}{T} \int_0^T [f(H, \sin(\omega_1 t) + H_2 \sin(\omega_2 t))] \sin(\omega_1 t) \, dt \] \hspace{1cm} \text{(A3.3a)}

\[ B_{mq} = \frac{2}{T} \int_0^T [f(H, \sin(\omega_1 t) + H_2 \sin(\omega_2 t))] \cos(\omega_1 t) \, dt \] \hspace{1cm} \text{(A3.3b)}

Where the subscripts \( p \& q \) stands for in-phase and quadrature components.
Similarly, the other equations for harmonics and combination frequency components can be written.

(b) Incommensurate frequencies

The method described above is straightforward for the case, when the ratio of frequencies is a rational number. But one encounters an apparent difficulty, when this ratio is irrational. Since for incommensurate frequencies, there will be no finite period of time, however large, during which the two signals will have an integral number of cycles. However, this difficulty is overcome by using M-functions [23];

\[ B_{1\omega} = \frac{1}{\pi} \int_{0}^{2\pi} M(h_1, H_2) \sin(\omega_1 t) \, d\omega_1 t \]  

(A3.4)

Where \( h_1 = H_1 \sin(\omega_1 t) \)

\[ M = \frac{1}{2\pi} \int_{0}^{2\pi} f(h_1 + H_2 \sin(\omega_2 t)) \, d\omega_2 t \]  

(A3.5)

So the Dual Input Describing Function is given by

\[ \text{DIDF} = \frac{1}{H_1 \pi} \int_{0}^{2\pi} M(h_1, H_2) \sin(\omega_1 t) \, d\omega_1 t \]  

(A3.6)

The equation (A3.4) gives the fundamental component of the output of nonlinear characteristics \( M(h_1, H_2) \), when its input \( h_1 \) varies sinusoidally with time. In other words, in the presence of given amplitude \( H_2 \) of the second signal, the original nonlinearity behaves with respect to the signal \( h_1 \), as if it is modified to \( B(h_1) = M(h_1, H_2) \). The evaluation of the equation (A3.4) involves two stages. In the first stage the altered characteristics are obtained. Whereas in the second stage the Dual Sinusoidal Input characteristics are found from the Altered characteristics.
(c) Determination of Altered Characteristics

Based on equation (A3.5), for a given B-H curve, the altered characteristics can be obtained as follows. Suppose the low frequency signal $H_1 \sin (\omega_1 t) = h_{1a}$, the operating point on the B-H curve is at point $Y_1$ and the corresponding output is $M_1 Y_1$ as shown in figure 3.6. As the high frequency signal $H_2 \sin(\omega_2 t)$ varies, the operating point moves on the B-H curve accordingly.

![Diagram of B-H curve and operating points](image)

*Fig. 3.6: Construction of Altered characteristics*
But for one complete cycle of $h_2$, the average output will be $M_1D_1$, which is different from $M_1Y_1$, because of the unsymmetrical nature of the B-H curve about the point $Y_1$. In a similar manner, the other points $D_2, D_3, \ldots, D_N$ etc., representing the average outputs can be obtained, when the signal $H_1 \sin(\omega_1 t)$ takes the values $h_{1b}$, $h_{1a}$, $h_{1c}$, etc., with $H_2$ being kept fixed. Thus the curve $OD_1D_2\ldots$ is called as Altered or Modified characteristics. For different values of $H_2$, Modified characteristics are drawn and shown in fig. 3.7 for the given B-H curve (fig.3.1,p46).

Fig. 3.7: Altered characteristics
(d) Dual Sinusoidal Input (DSI) characteristics

Based on the equation (A3.4), the fundamental component $B_{1F}$ of output corresponding to $H_1$, is the fundamental component of Altered characteristics ($B_1 / h_1$) obtained for constant values of $H_2$, when $h_1$ varies sinusoidally.

The curves $B_{1F} V_5 H_1$ for different values of $H_2$ are called DSI characteristics.

The DSI curves corresponding to the altered characteristics (figure 3.7) are shown in figure 3.8.

![DSI curves](image)

**Legend:**

- A: $H_2=0$ A/m
- B: $H_2=2000$
- C: $H_2=4000$
- D: $H_2=6000$
- E: $H_2=8000$
- F: $H_2=10000$

**Fig.3.8:** DSI curves
However, for the purpose of computing eddy current loss, the B-H curve has been approximated by relay type nonlinearity, so as to use readily available formulae to compute output of each of the input component.

A3-2 Evaluation of fundamental components of output using power series [21]

For accurate evaluation of $B_{1F}$ and $B_{2F}$, represent the B-H curve by Frohlich equation as

$$B = \frac{\alpha |H|}{\gamma + |H|} = \frac{\alpha [H_1 \sin(\omega t) + H_2 \sin(\omega t)]}{\gamma + H_1 \sin(\omega t) + H_2 \sin(\omega t)}$$  \hspace{1cm} (A3.7)

The describing function of the signal $H_1$, when $H_2=0$, is

$$N_1(H_1) = \frac{4\alpha}{\pi H_1} \frac{\sin(\omega t)}{\gamma + H_1 \sin(\omega t)} \sin(\omega t) \, d\omega t$$

After integration,

$$N_1(H_1) = \frac{4\alpha}{\pi H_1} - \frac{4\alpha \gamma}{\pi H_1^2} + ...$$  \hspace{1cm} (A3.8)

The describing function of signal $H_1$ in the presence of signal $H_2$, is given by the power series expansion technique, as follows

$$N_1(H_1, H_2) = \sum_{p=0}^{\infty} \left( \frac{H_2^p}{2^{2p} (p!)^2} \right) V_p(H_1)$$  \hspace{1cm} (A3.9)

Where $V_p(H_1)$ is computed recursively as

$$V_{p+1}(H_1) = \frac{d^2}{dH_1^2} V_p(H_1) + \frac{3}{H_1} \frac{d}{dH_1} V_p(H_1)$$

with $V_0(H_1) = N_1(H_1)$. 

67
In a similar way, we can obtain describing function of signal $H_2$, when $H_1$ presents. The series is

$$N_2(H_1, H_2) = \sum_{p=0}^{\infty} \left( \frac{H_2^{2p}}{2^{2p} (p!)(p+1)!} \right) W_p(H_1)$$  \hspace{1cm} (A3.10)

where

$$W_{p+1}(H_1) = \frac{d^2}{dH_1^2} W_p(H_1) + \frac{1}{H_1} \frac{d}{dH_1} W_p(H_1)$$

and

$$W_0(H_1) = N_1(H_1) + \frac{H_1}{2} \frac{d}{dH_1} N_1(H_1)$$

The truncated series of equation (A3.9) is

$$N_1(H_1, H_2) = \frac{4\alpha}{\pi H_1} - \frac{2\alpha \gamma}{H_1^3} - \frac{\alpha}{\pi} \left( \frac{H_2}{H_1^2} \right)$$  \hspace{1cm} (A3.11)

Similarly, the truncated series of equation (A3.10) is

$$N_2(H_1, H_2) = \frac{2\alpha}{\pi H_1} + \frac{\alpha}{4 \pi} \left( \frac{H_2}{H_1^3} \right)$$  \hspace{1cm} (A3.12)

The fundamental component of output, corresponding to the fundamental component of the input $H_1$ is

$$B_{11} = N_1(H_1, H_1)H_1 = \frac{4\alpha}{\pi} - \frac{2\alpha \gamma}{H_1} - \frac{\alpha}{\pi} \left( \frac{H_2}{H_1} \right)^2$$  \hspace{1cm} (A3.13)

Similarly,

$$B_{21} = N_2(H_1, H_1)H_1 = \frac{2\alpha}{\pi} \left( \frac{H_2}{H_1} \right) + \frac{\alpha}{4 \pi} \left( \frac{H_2}{H_1} \right)^3$$  \hspace{1cm} (A3.14)
The results of power series are meaningful only when $H_1 >> \gamma$. The above equations (A3.13) and (A3.14) for $B_{1F}$ and $B_{2F}$ are valid for $(H_2/H_1) < 1$. If $(H_2/H_1) > 1$, then the same equations must be interchanged for $B_{1F}$ and $B_{2F}$ along with replacement of $H_1$ by $H_2$ and vice-versa.

A3-3 The details of each toroid

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<th>Material</th>
<th>Mild steel</th>
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<td>Resistivity</td>
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<td>B-H Curve</td>
<td>Fig. 3.1(p 46)</td>
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<tr>
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</tr>
<tr>
<td>D.C resistance of each winding</td>
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</tr>
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</table>