Chapter-V
Performance evaluation of two-phase induction motor with solid iron rotor under unbalanced load conditions

5.1 INTRODUCTION

Two-phase induction motor operates under unbalanced load conditions, when it is used as servomotor. It is interesting to evaluate the performance of such a motor with solid iron rotor. The analysis developed, based on 1-D model of the rotor. Under unbalanced conditions the rotor is subjected to two-sinusoidally distributed, oppositely rotating magnetic fields at the surface. Their speeds relative to the rotor being determined by the slip (s) and pole pitch (τ). For the 1-D model, the rotor can be considered as an infinite half-space of iron subjected to two pulsating magnetic fields at the surface. So the problem is to determine the forward and backward rotor sequence fluxes, the equivalent circuit and hence the performance.

5.2 THEORY

Consider, the machine operating on unbalanced set of voltages applied to the stator phases. This set can be resolved into two balanced sets of positive and negative sequence voltages. The positive sequence voltages produce a field that travel in the same direction of the rotation of rotor. The relative velocity of this forward field with the velocity of rotor is (sωrτ)/π. Whereas, the negative sequence voltages produces a backward travelling field which runs in the opposite direction at a velocity (2-s)ωrτ/π with respect to the rotor.

If the φf and φb are the total forward and backward fluxes per pole, then the fluxes entering the rotor surface will appear as alternating fluxes of amplitudes φf/2 and φb/2 at frequencies sf and (2-s)f respectively. However, as we move along the
periphery, the phase shift between these flux waves at different sections can be seen to vary uniformly taking all possible values. The proof of this is as follows;

Consider the instant of time at which the axes of the forward and backward travelling flux waves coincide. Let this instant, reckoned as $t=0$ and it is happened in section AA in space as shown in figure 5.1.

![Diagram](image)

**Fig. 5.1:** Forward and Backward Traveling Fields

For the direction shown, as the forward field moves by a distance $y_1$ towards the right, the backward field moves by a distance $y_1(2-s)/s$ towards the left. Taking the
direction of flux from left to right as positive, the flux in the y-direction at AA due to forward field increases negatively with time, while that due to the backward field increases positively. Hence, at section AA, the two components of fluxes, in the y-direction can be expressed as

\[\phi_r = -\phi_r \sin(s\omega_o t) = -\phi_r \sin(\omega_r t) \quad (5.1a)\]

\[\phi_b = \phi_b \sin[(2-s)\omega_o t] = \phi_b \sin(\omega_b t) \quad (5.1b)\]

Where \(\omega_o\) is operating frequency

Consider next, a section at an arbitrary distance \(y\) from the section AA. The two-components of fluxes at this section are

\[\phi_r = -\phi_r \sin(\omega_r t - \pi y/\tau) \quad (5.2a)\]

\[\phi_b = \phi_b \sin(\omega_b t + \pi y/\tau) \quad (5.2b)\]

Substituting \((\omega_r t - \pi y/\tau) = \omega_r t'\) and putting \(\omega_b / \omega_r = (2-s) / s = \gamma\), equation (5.2) is written,

\[\phi_r = -\phi_r \sin(\omega_r t) \quad (5.3a)\]

\[\phi_b = \phi_b \sin[\gamma \omega_r t + (\gamma+1)(\pi y/\tau)] \quad (5.3b)\]

Hence the total flux in the y-direction can be expressed as

\[\phi = -\phi_r \sin(\omega_r t) + \phi_b \sin(\gamma \omega_r t + \alpha) \quad (5.4)\]

Where \(\alpha = (\gamma+1)(\pi y/\tau) = 2 \pi y/\tau\).

Thus it can be seen from the equation (5.4) that, as one moves along the periphery, the phase shift between the two components of fluxes at different sections
varies uniformly assuming all possible values. In fact, it changes by $2\pi$ over the distance $y=\pi$.

To determine the rotor eddy-current losses, consider a span along $y$-axis ($y=\pi$) over which the phase angle has assumed all values from 0 to $2\pi$. Divide this span into infinitesimal strips, parallel to the $x$-$z$ plane, each strip characterized by a definite phase relationship between the two components of fluxes. The total losses in the span is equal to the sum of the losses occurred in all such strips. Since the amplitudes of the two components of fluxes remains the same in all strips, the effect of phase angle should be summed up, as it varies uniformly from 0 to $2\pi$. In other words, the average loss per unit length in this region involves an averaging over the effects produced by the phase angle, when it varies uniformly from 0 to $2\pi$. This average loss can be seen to be same as the average loss that would have been occurred, if the two components of fluxes have the same phase relationship in all strips at any instant, but this phase continuously varying with time and uniformly taking all the values from 0 to $2\pi$. Now it may be thought of such a distribution of flux that would have been produced by two alternating magnetic fields at frequencies $f$ and $(2-s)f$ acting simultaneously at the surface, directed along the $y$-axis, the phase angle between them being uniformly varied over all possible values.

Therefore, under unbalanced operating conditions, the rotor can be represented by an infinite half-space of material excited at the surface by two alternating magnetic fields of different magnitudes and frequencies along the $y$-axis. Hence, the two-frequency excitation theory developed in chapter-III can be very well applied. So the two fields $H_{15}$ and $H_{25}$ at the surface as mentioned in chapter-III will be designated as forward and backward fields $H_f$ and $H_b$ respectively.
The flux components $\Phi_f$ and $\Phi_b$ corresponding to $H_f$ and $H_b$ are given by the equations (3.23) and (3.27) respectively as

$$\Phi_f = \sqrt{\frac{\gamma}{2}} \frac{pB_m}{s\omega_e} H_f \text{ webers}$$

$$\Phi_b = \left( \frac{\beta}{\beta - 1} \right) \frac{1}{2} \sqrt{\frac{\gamma}{2}} \frac{pB_m}{s\omega_e} H_b \text{ webers}$$

Where $$\frac{\sqrt{2}}{3} (2\beta - 1) \sqrt{\beta(\beta - 1)} = \gamma = \frac{\omega_b}{\omega_e}$$

The rotor power factors are given by the equations (3.24) and (3.28) are

$$\cos\psi_f = \sqrt{(2/3)}$$

$$\cos\psi_b = \frac{\beta}{\sqrt{(2\beta - 1)}}$$

The above equations (5 5), (5 6) & (5 7) are valid for $\gamma > 2$

For $\gamma = [(2 - s)/s] < 2$, however, the graphical method provided in section 3.4, is exploited to determine $\Phi_f$ and $\Phi_b$ for the given values of $H_f$ and $H_b$ at surface of the rotor. The graphical construction also yields phase shifts $\Psi_f$ and $\Psi_b$ of $H_f$ and $H_b$ with $\Phi_f$ and $\Phi_b$ respectively. Consequently, the rotor power factors $\cos\Psi_f$ and $\cos\Psi_b$ can be found. In the graphical construction, it is achieved by trial and error that the given set $H_f$ and $H_b$ would co-exist.
Having found the forward and backward sequence fluxes \( \Phi_r \) and \( \Phi_b \), the equations of performance of 2-phase induction motor under unbalanced load conditions with solid iron rotor are written as follows:

(i) Air-gap sequence voltages \( E_f \) and \( E_b \) are given by

\[
E_f = \sqrt{2} \omega_o N L \phi_f
\]

\[
E_b = \sqrt{2} \omega_o N L \phi_b
\]

Where \( N \) is effective number of turns/phase.

(ii) Equivalent rotor currents referred to the stator \( I_{rf} \) and \( I_{rb} \) are given by the equation (4.6) as

\[
I_{rf} = \frac{\pi D H_r}{4 \sqrt{2} N}
\]

\[
I_{rb} = \frac{\pi D H_b}{4 \sqrt{2} N}
\]

Where \( D \) is the diameter of rotor and \( N \) is effective number of turns of each phase.

(iii) Stator forward and backward sequence currents \( I_{sf} \) and \( I_{sb} \) are given by

\[
I_{sf} = I_{rf} + \frac{E_f}{Z_m}
\]

\[
I_{sb} = I_{rb} + \frac{E_b}{Z_m}
\]

Where \( Z_m \) is the magnetizing branch impedance.

(iv) Stator forward and backward sequence voltages \( V_f \) and \( V_b \) are given by

\[
V_f = E_f + I_{sf} Z_r
\]

\[
V_b = E_b + I_{sb} Z_r
\]
Where $Z_s$ is the stator impedance per phase.

(v) Terminal voltages $V_1$ and $V_2$ are given by

$$V_1 = V_f + V_b$$ (5.12a)

$$V_2 = j(V_f - V_b)$$ (5.12b)

(vi) Stator phase currents $I_1$ and $I_2$ are given by

$$I_1 = I_{rf} + I_{sb}$$ (5.13a)

$$I_2 = j(I_{rf} - I_{sb})$$ (5.13b)

(vii) Torque developed 'T' is given by

$$T = 2(E_f I_{rf} \cos\psi_r - E_b I_{sb} \cos\psi_b)/2\pi n_s$$ (5.14)

Where $n_s$ is the synchronous speed in r.p.s.

It may be noted that the analysis so far assumed $H_f$ and $H_b$ are known priorily. But in practice, the problem is one of determining $H_f$ and $H_b$ for the given values of $V_1$ and $V_2$. So an iterative procedure must be used. The steps involved are illustrated to find the performance of 2-phase induction motor, whose specifications are given in the appendix with end rings.

### 5.3 PROCEDURE TO EVALUATE PERFORMANCE

(i) Let $S = 0.3$, with the applied voltages as $V_1 = 230$ and $V_2 = 200$ Volts

(ii) Find the effective resistivity of the rotor material using the emperical formulae provided in the appendix A4-3 (p87),

- Correction factor for curvature $K_1 = 1.06$
- Correction factor for end effects $K_2 = 1.0(1+0.49S)$
Correction factor for temperature rise $K_3 = 1.2$

Effective resistivity $\rho = K_1 K_2 K_3 \rho = 21.09 \times 10^{-8} \Omega\cdot m$.

(iii) To start with assume an arbitrary values for $H_r$ and $H_b$ as

$H_r = 2000 \text{ A/m}$

$H_b = 500 \text{ A/m}$

(iv) The limiting value of flux density $B_m$ corresponding to $H = \sqrt{H_r H_b}$, from the B-H curve (Fig. 4.1, p72) is

$B_m = 1.25 \text{ Tesla}$.

(v) The ratio $\gamma$ is given by

$\gamma = \left( \frac{\omega_b}{\omega_r} \right) = (2-S)/S = 5.66$

Corresponding to this ratio, the value of $\beta = 2.977$ is from equation (5.6).

(vi) Now, find the forward and backward flux components $\Phi_r$ and $\Phi_b$ for the given values of $H_r$ and $H_b$ using the equations 5.5(a) and 5.5(b) respectively,

$\Phi_r = 3.0 \text{ mwb}$

$\Phi_b = 0.20 \text{ mwb}$

(vii) The air-gap sequence voltages from the equations (5.8a) and (5.8b) are

$E_r = 56 \text{ Volts}$

$E_b = 3.86 \text{ Volts}$

(viii) Rotor currents referred to the stator side are given by the equations (5.9a) and (5.9b) are

$I_{r'} = 0.31 \text{ A}$

$I_{b'} = 0.077 \text{ A}$

(ix) The equations (5.7a) and (5.7b) gives the positive and negative sequence rotor powers factors as

$\text{Cos} \psi_r = 0.8165$

$\text{Cos} \psi_b = 0.7751$

(x) Hence

$I_{r'} = 0.31(-36.26)^\circ \text{ A}$ with reference to $E_r$

$I_{b'} = 0.077(-43.53)^\circ \text{ A}$ with reference to $E_b$
(xi) The stator sequence currents using the equations (5.10a) and (5.10b) are
\[ I_a = 1.42(-72^\circ) \text{ A} \quad I_b = 0.15(-61^\circ) \text{ A} \]

(xii) The stator sequence voltages using the equations (5.11a) and (5.11b) are
\[ V_f = 90.42 \text{ Volts} \quad V_b = 7.48 \text{ Volts} \]

(xiii) The stator terminal voltage are from the equations (5.12a) and (5.12b) are
\[ V_1 = 98 \text{ volts} \quad V_2 = 83 \text{ volts} \]

The calculated values of stator terminal voltages \( V_1 \) and \( V_2 \) are not agreeing with the applied voltage. Hence the new values for \( H_f \) and \( H_b \) should be chosen to recalculate \( V_1 \) and \( V_2 \). If one assumes linear variation, then the new values of \( H \) would be given in terms of old values as

\[ H_f(\text{new}) = H_f(\text{old}) \frac{V_1 + V_2}{V_1^* + V_2^*} \]
\[ H_b(\text{new}) = H_b(\text{old}) \frac{V_1 + V_2}{V_1^* + V_2^*} \]

Where \( V_1^* \), \( V_2^* \) are the calculated values of \( V_1 \) and \( V_2 \) respectively. Now repeat the procedure until the calculated values of terminal voltages tally with the applied voltages.

The final results after eight iterations are as follows:
\[ I_1 = 3.36 (3.7) \text{ Amps} \quad I_2 = 3.25 (3.05) \text{ Amps} \]
\[ P_1 = 352 (340) \text{ watts} \quad P_2 = 235 (210) \text{ watts} \]
\[ T = 2.2 (2.35) \text{ N-m} \]
\[ V_1 = V_1^* = 230 \text{ Volts} \quad V_2 = V_2^* = 200 \text{ Volts} \]

It may be noted that the values given in the brackets are experimental results.
5.4 EXPERIMENTAL RESULTS AND DISCUSSION

A two-phase induction motor is wound and provided with solid iron rotor having copper end-rings. The details of this motor are given in the appendix. The equivalent circuit of the motor is obtained by conducting suitable tests. The load test is also conducted on the same motor under unbalanced voltages. The simulated results obtained by the dual excitation theory along with the equivalent circuit of the machine are compared with the experimental values. The performance curves are shown in figures 5.2 and 5.3. From these figures, it is clear that the theoretical values are closer to practical values. The difficulty faced with the solid-iron rotor is that, during the process of machining the material became hard. Hence, the motor was drawing current more than full-load value even on no-load. To overcome this problem, the material is made to undergo annealing. It is practically observed that the speed fall is drastic with load. This is due to heavy losses in the solid iron rotor.

![Graph showing input phase currents](image-url)

Legend:
A: Ph1 Practical
B: Ph1 Theoretical
C: Ph2 Theoretical
D: Ph2. Practical

Fig. 5.2: Input Phase Currents
5.5 Conclusions

It has been proved that under unbalanced operating conditions, the rotor of two-phase induction motors can be represented by an infinite half-space of material excited at the surface by two alternating magnetic fields of different magnitudes and frequencies. Hence the two excitation theory developed in Section 3.3 is employed to determine the flux components namely forward and backward waves, subsequently, the air-gap sequence voltages are found. Ultimately, the rotor terminal voltages and currents are calculated making use of the proximities of equivalent circuit. An illustrative example is provided in Section 5.3. A two-phase induction motor is wound with solid iron rotor having copper end-winding and load test is conducted. The simulated results of load current and torque are compared with experimental values. It is learnt from the ref [14] that solid iron rotor induction motor could produce more torque when it is provided copper
5.5 CONCLUSIONS

The dual excitation theory is employed to find the forward and backward sequence fluxes and hence the equivalent circuit of two-phase induction motor with solid iron rotor under unbalanced voltages conditions. For this purpose, a two-phase motor is wound and provided with solid iron rotor having copper end-rings and load test is conducted. The simulated results are compared with that of experimental values. It is observed that solid iron rotor induction motor with copper end-rings produces more torque than the one without end-rings.
A5: APPENDIX

Specifications of 2-phase induction motor:

- Rated power : 1.0 Kw
- Rated voltage and frequency : 230V, 50 Hz
- Number of phases : 2
- Number of poles : 6
- Nature of operation : Constant rated voltage
- Winding factor : 0.88
- Effective no. of turns per phase in series (N) : 380.16
- Stator impedance per phase : 9.3+j22.5 Ω
- Magnetising branch resistance $R_m$ : 306 Ω
- Magnetizing branch reactance $X_m$ : 47.7 Ω
- Rotor diameter (D) : 0.106 m
- Active rotor length (L) : 0.110 m
- End-ring material : Copper
- End-ring dimensions : 2cm×1cm
- $\rho$ of the material : 16.58×10^{-8} Ω-m
- Correction factor for curvature : 1.06
- Correction factor for temperature rise : 1.2
- Correction for end effects : (1+0.49S)
- Effective $\rho$ : 21.09(1+0.49S)×10^{-8} Ω-m