Chapter I

Introduction
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Hydrodynamics

All materials show deformation under the action of forces. If the deformation in the material increases continually without limit under the action of shearing forces, however small, the material is called ‘fluid’. This continuous deformation under the action of forces is manifested in the tendency of fluid to flow and this tendency of fluids is called ‘fluidity’.

The science which deals with the properties of fluid in motion is called fluid dynamics. The origin of the science of fluid dynamics is difficult to trace because the excavations of the Indus Valley Civilization and Egyptian ruins show that the principles of flow and resistance to flow were known as long as four thousands years ago. The drainage and irrigation systems of Mohanjo-daro, Egypt and China, the use of siphons and bellows and the construction of windmills and paddle wheels date from ancient times.

Knowledge and understanding of the basic principles and concepts of fluid mechanics are essential in the analysis and design of any system in which fluid is the working medium. The design of virtually all means of transportation requires an application of the principles of fluid mechanics. The circulatory system of the body is essentially a fluid system. It is not surprising then that the design of artificial hearts, heart-lung machines, breathing aids, and other such devices must lie on the basic principles of fluid mechanics.

The types of engineering problems that deal with fluid mechanics would be far too extensive to record. An engineer would employ his knowledge of fluid mechanics in such widely different classes of problems as these: designing turbo- machines that add or extract energy from a fluid system; determining the pumping machinery requirements for an oil field operation; predicting the rate of riverbed erosion around a bridge pier; estimating the size of a smoke cloud downwind of a factory chimney; finding the drag on the fins of a ballistic missile; designing components for a computer that operates on the flow of a fluid rather than on electricity; and extracting power from an ionized gas stream for space propulsion.

The systematic study of fluid mechanics started only after the Euler’s discovery of equations of motion of an inviscid fluid. Earlier, an attempt to describe the effect of fluid motions is due to Newton, who conceived the idea that the fluid consisted of a granulated
structure of discrete particles. After Euler and Newton, some significant contributions to this subject were given by the following scientists. Lagrange gave the concept of velocity potential stream function. The principle of resistance of flow in capillary tubes was given by Poiseuille. The credit for the equations of motion of viscous fluids goes to Navier and Stokes. Reynolds discovered the equations of turbulent motion. Prandtl put forward the boundary layer theory. Later on, many more contributions were given by many famous scientists/mathematicians which include Prandtl, Kutta, Lord Kelvin, Orr, Sommerfeld, Zhukovski and Karman etc. Now-a-days fluid mechanics has become a very vast subject and has given birth to many other subjects like Meteorology, gas dynamics, aerodynamics, non-Newtonian flows, magnetohydrodynamics etc. Classical (or Newtonian) mechanics and continuum hypothesis are going to act as the basis for the study of fluid mechanics.

**Classical Mechanics**

Classical mechanics is the model of the physics of forces acting upon bodies. It is often referred to as Newtonian mechanics after Newton and his laws of motion. We confine ourselves to Newtonian mechanics and shall not evoke the theory of relativity. In other words, we restrict ourselves to those systems where particle velocities are small in comparison to the velocity of light so as to have negligible relativity effects. In this way, we are not concerned with those masses, velocities and temperatures for which Newtonian mechanics does not provide adequate description.

**Continuum Hypothesis**

In fluid mechanics, we make use of continuum theory though we know that matter is composed of atoms and molecules and therefore has necessarily a discrete structure. In normal gases, the masses are concentrated in molecules. These molecules are separated by vacuous regions with linear dimensions much larger than those of the molecules themselves. In liquids and solids, though the average spacing between the molecules and atoms is small, the masses are concentrated in the nuclei of the atoms composing a molecule and are very far from being smeared uniformly over the volume occupied by the liquid. When the fluid is viewed on microscopic scale so as to reveal the individual molecules, the properties of fluids such as composition, velocity and density have violently non-uniform distributions. Since we are generally concerned with the macroscopic behaviour at the mass centres, are smeared out uniformly over a certain
volume surrounding them and treat the matter as continuum. This is called the
"Continuum hypothesis". There is ample evidence that common real fluids, both liquid
and gases move as if they were continuous under normal conditions and even under
considerable departure from normal conditions. The hypothesis is justified when we
consider only those systems in which the characteristic length is much larger than the
mean free path of the fluid molecules. The continuum approach is simpler than the more
rigorous kinematic one. Because our hypothesis has made it possible to give meaning to
terms such as density, pressure, temperature, momentum and angular momentum ‘at a
point’. And, in general, the values of these quantities are continuous functions of
position and time, thus permitting us the use of derivatives and differentials whenever
they are needed.

The foundational axioms of fluid mechanics are the laws of conservation of mass,
conservation of momentum (also known as Newton’s second law) and conservation of
energy. These are based on Classical mechanics and modified in Relativistic mechanics.
The central equations for fluid mechanics are the Navier-Stokes equations which are non­
linear differential equations describing the flow of a fluid whose stress depends linearly
on pressure and velocity. Fundamental equations of a viscous incompressible fluid
motion with constant fluid properties in ordinary Cartesian coordinates
\( \vec{X} = (x^*, y^*, z^*) \) and \( \vec{V} = (u^*, v^*, w^*) \) are

**Equation of State:**

\[ p^* = \rho RT^* \quad (1.1) \]

**Equation of Continuity:**

\[ \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0. \quad (1.2) \]

**Equations of Motion:**

\[ \rho \left( \frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} \right) = -\frac{\partial p^*}{\partial x^*} + F_x^* + \mu \left( \frac{\partial^2 u^*}{\partial x^*^2} + \frac{\partial^2 u^*}{\partial y^*^2} + \frac{\partial^2 u^*}{\partial z^*^2} \right), \]

\[ (1.3) \]

\[ \rho \left( \frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} \right) = -\frac{\partial p^*}{\partial y^*} + F_y^* + \mu \left( \frac{\partial^2 v^*}{\partial x^*^2} + \frac{\partial^2 v^*}{\partial y^*^2} + \frac{\partial^2 v^*}{\partial z^*^2} \right), \]

\[ (1.4) \]
\[
\rho \left( \frac{\partial w^*}{\partial t^*} + u^* \frac{\partial w^*}{\partial x^*} + v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} \right) = -\frac{\partial P^*}{\partial z^*} + F_z^* + \mu \left( \frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}} \right),
\]

Equation of Energy:

\[
\rho C_p \left( \frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} \right) = \frac{\partial Q^*}{\partial t^*} + k \left( \frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}} \right) + \phi,
\]

where

\[
\phi = 2\mu \left\{ \left( \frac{\partial u^*}{\partial x^*} \right)^2 + \left( \frac{\partial v^*}{\partial y^*} \right)^2 + \left( \frac{\partial w^*}{\partial z^*} \right)^2 \right\}
\]

\[+ \mu \left\{ \left( \frac{\partial v^*}{\partial x^*} + \frac{\partial u^*}{\partial y^*} \right)^2 + \left( \frac{\partial w^*}{\partial y^*} + \frac{\partial v^*}{\partial z^*} \right)^2 + \left( \frac{\partial u^*}{\partial z^*} + \frac{\partial w^*}{\partial x^*} \right)^2 \right\}. \quad (1.7)
\]

Equation of Concentration

\[
\frac{\partial C^*}{\partial t^*} + u^* \frac{\partial C^*}{\partial x^*} + v^* \frac{\partial C^*}{\partial y^*} + w^* \frac{\partial C^*}{\partial z^*} = D \left( \frac{\partial^2 C^*}{\partial x^{*2}} + \frac{\partial^2 C^*}{\partial y^{*2}} + \frac{\partial^2 C^*}{\partial z^{*2}} \right). \quad (1.8)
\]

Many physical situations involve the simultaneous transfer of mass, energy and momentum. The drying of a wet surface by a hot, dry gas is an excellent example in which all these transport phenomena are involved. Energy is transferred to the cooler surface by convection and radiation; mass and its associated enthalpy are transferred back into the moving gas stream. The simultaneous transport processes are more complex, requiring the simultaneous treatment of each transport phenomenon involved.

**Forced and Free Convection**

By forced convection we mean the flow in which velocities arising from the variable density (i.e. due to the force of buoyancy) are negligible in comparison with the velocity of the main or forced flow, whereas in free convection, also known as natural convection, the motion is essentially caused by the effect of gravity on the heated fluid of variable density.
Boussinesq Approximation

For flows satisfying certain conditions, Boussinesq in 1903 suggested that we can neglect the density changes in the fluid, except in the gravity term where \( \rho \) is multiplied by \( g \).

This approximation also treats the properties of the fluid (such as \( \mu, k, C_p \)) as constants.

A formal justification, and the conditions under which the Boussinesq approximation holds, is given in Spiegel and Veronis (1960). The Boussinesq approximation applies if the Mach number of the flow is small, propagation of sound or shock waves is not considered, the vertical scale of the flow is not too large, and the temperature differences in the fluid are small. Then the density can be treated as constant in the continuity, momentum and concentration equations, except in the gravity term. Keeping in mind that the thermodynamic state of the fluid mixture depends on pressure, temperature and concentration, in the limit of small density variations at constant pressure, we can write

\[
\rho \approx \rho_\infty \left[ 1 + \beta (T^* - T^*_\infty) + \beta_c (C^* - C^*_\infty) \right],
\]

where \( \beta = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T^*} \right)_\rho \) is the coefficient of thermal expansion and \( \beta_c = \frac{1}{\rho} \left( \frac{\partial \rho}{\partial C^*} \right)_\rho \) is the coefficient of concentration expansion, \( x^* \)-axis is taken upward. The constant \( \rho_\infty \) is a reference density corresponding to a reference temperature \( T^*_\infty \) and concentration \( C^*_\infty \), which can be taken to be the mean temperature and concentration in the flow or the temperature and concentration at a boundary respectively.

Omitting Coriolis forces, the set of equations corresponding to Boussinesq approximation for the combined heat and mass transfer problem is

\begin{align}
\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} & = 0,
\end{align}

\begin{align}
\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} & = -\frac{1}{\rho} \frac{\partial \rho^*}{\partial x^*} + \nu \left( \frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}} \right) \\
& \quad + g \beta (T^* - T^*_\infty) + g \beta_c (C^* - C^*_\infty),
\end{align}

\begin{align}
\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} & = -\frac{1}{\rho} \frac{\partial \rho^*}{\partial y^*} + \nu \left( \frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}} \right),
\end{align}

\begin{align}
\frac{\partial w^*}{\partial t^*} + u^* \frac{\partial w^*}{\partial x^*} + v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} & = -\frac{1}{\rho} \frac{\partial \rho^*}{\partial z^*} + \nu \left( \frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}} \right),
\end{align}

\begin{align}
\frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} & = \frac{k}{\rho C_p} \left( \frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}} \right) + \frac{\phi}{\rho C_p},
\end{align}
\[
\frac{\partial C^*}{\partial t^*} + u \frac{\partial C^*}{\partial x^*} + v \frac{\partial C^*}{\partial y^*} + w \frac{\partial C^*}{\partial z^*} = D \left( \frac{\partial^2 C^*}{\partial x^*\partial x^*} + \frac{\partial^2 C^*}{\partial y^*\partial y^*} + \frac{\partial^2 C^*}{\partial z^*\partial z^*} \right),
\]

(1.15)

where the viscous dissipative heat \( \phi \) has already been given in equation in (1.6).

**Boundary Layer Theory**

The boundary layer theory finds its applications in the calculation of the skin-friction drag which acts on a body as it is moved through a fluid: For example the drag experienced by a flat plate at zero incidence, the drag of a ship, of an aeroplane wing, aircraft nacelle, or turbine blade. Problems connected with the flow of fluids through the channels formed by the blades of turbo-machines (rotary compressors and turbines) can also be treated with aid of boundary-layer theory. Furthermore, phenomena which occur at the point of maximum lift of an aerofoil and which are associated with stalling can be understood only on the basis of boundary layer theory. Finally, problems of heat and mass transfer between a solid body and a fluid flowing past it also belong to the class of problems in which boundary-layer phenomena plays a decisive part.

The concept of boundary layer theory was first introduced by Prandtl in 1904. The discrepancy between the engineering and mathematics tradition in fluid dynamics was resolved with one stroke in a short paper by Prandtl. In this work, Prandtl gave an outline of his so-called “boundary layer” [Grenzschicht] theory. The concept of the boundary layer made it possible to separate the problem of viscous flow into two interacting components: the first component is a thin layer near the surface of a body, in which the velocity decreases from a finite value \( U \) (the velocity of the outer flow) to zero at the surface of the body itself. The layer thus fulfils the so-called ‘no-slip’ condition, first proposed by Daniel Bernoulli in 1738, which all known (real) fluids and gases satisfy. The second component of the problem is the flow outside the boundary layer, which Prandtl assumed to be inviscid and thus amenable to the Euler equations. The flow of a body in a fluid (e.g. a flying aeroplane) or the flow around an object (e.g. the flow of a river against a pier) can now be seen as the result of an interaction between the two components. The decisive move is to assign a substantial causal role to the boundary layer and to confine the viscous effects to it. Although it is very thin in fluids like water and air, the layer has a considerable effect on the free flow because of its strong velocity gradient. The higher the Reynolds number, the thinner the boundary layer gets. One can
say that modern fluid dynamics arose from a fusion of ideal-flow theory with all its splendid mathematics and the boundary layer concept.

Prandtl's model made it possible for the first time to obtain approximate mathematical solutions for viscous flow. In the case of laminar steady flow, the momentum and energy equations are parabolic and can therefore be calculated much more easily than in the Navier–Stokes case. The number of equations and unknown variables are thus reduced for the viscous part and the applicability of the Euler equations maintained for the outer flow. Prandtl thereby arrived at the following basic differential equations for steady flow in two dimensions:

\[
\rho \left( u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} + \frac{dp^*}{dx^*} \right) = \mu \frac{\partial^2 u^*}{\partial y^*^2}, \tag{1.16}
\]

\[
\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0. \tag{1.17}
\]

**Boundary Conditions:**

\[
\begin{align*}
\text{for } y^* = 0 : & \quad u^* = 0, \quad v^* = 0 \\
\text{for } y^* = \infty : & \quad u^* = U^*(x^*)
\end{align*} \tag{1.18}
\]

The flow over a flat plate is characteristic of external flows in general. This flow is shown in Figure 1.1. The boundary layer thickness is zero at the leading edge and increases with distance along the plate surface. The early portion of the boundary layer is laminar but may be followed by a transition region where the flow changes from laminar to turbulent. This transition region actually consists of burst of turbulence which spread until they intermingle to result in a fully turbulent region.

![Figure 1.1](image-url)
The boundary layer in a decelerating stream has a point of inflection and grows rapidly. The existence of the point of inflection implies a slowing down of the region next to the wall, a consequence of the uphill pressure gradient. Under a strong enough adverse pressure gradient, the flow next to the wall reverses direction, resulting in a region of backward flow as shown in Figure 1.2. Note that $U$ has its maximum value at $B$ and then gets smaller. The reversed flow meets the forward flow at some point $D$ at which the fluid near the surface is transported out into the mainstream. We say that the flow separates from the wall. The separation point $D$ is defined as the boundary between the forward flow and backward flow of the fluid near the wall, where the stress vanishes:

$$\left( \frac{\partial u^*}{\partial y^*} \right)_{wall} = 0. \quad (1.19)$$

**Figure 1.2**: Growth and separation of boundary layer owing to increasing pressure gradient.

Prandtl showed in his paper how to use a standard numerical technique to calculate the drag of water flow along a flat thin plate as caused by friction on the surface of the body. Since many engineering problems require the reduction of drag, e.g. on planes and ships, Prandtl's approach became all the more attractive.

In his paper, Prandtl also explained the phenomenon of flow separation and transition to turbulence in a qualitative way. As a result of surface friction, the boundary layer is put into rotation, separates from the surface and pushes it way through the outer flow. The separation makes a transition from a smooth, laminar state with low drag to a chaotic, turbulent one with higher drag. The problem of reducing drag is thus transformed into the problem of suppressing or delaying flow separation. For the first time, there was hope that the principles governing the problem may after all be found.
Prandtl also presented experimental, albeit only qualitative, evidence for his proposal. He put a cylinder into a flow of water in which mica was suspended in order to visualize vortices and other deformations. The cylinder had a small slit exactly at the point where flow separation occurred. The fact that flow separation disappeared when water was pumped out of the cylinder and the boundary layer thus sucked off was proof for the hypothesis that there is in fact a boundary layer and that its interaction with the outer flow is responsible for the observed phenomena (If the water had not been pumped out, separation of the flow from the cylinder wall would have occurred).

In 1921, Prandtl’s student Karl Pohlhausen showed in his dissertation that in many cases of flow the velocity distribution can be represented by relatively simple polynomials. He used the so-called “integral form” of the boundary layer equations as introduced by Theodore von Kármán the same year. In the early 1950s, S. Kaplun, P.A. Lagerström and M. Van Dyke developed a powerful mathematical technique for deriving asymptotic expansions of Navier–Stokes solutions. This so-called “method of matched asymptotics”, which built upon results attained by K.O. Friedrichs and Prandtl in the 1940s, assumes that inner and outer solutions of the flow are connected — that is, that in their domains they share an overlapping region in which solutions are equal.

It should be mentioned that Prandtl’s boundary layer concept also proved fruitful for the study of heat transfer. Ernst R.G. Eckert hit upon this idea in the 1930s and summed it up in his *Einführung in den Wärme- und Stoffaustausch* of 1949.

**Magnetohydrodynamics**

Fluid dynamics and Electromagnetic theory were developed independently of each other almost up to first half of the twentieth century. A systematic study of the hydrodynamics of a conducting fluid immersed in a magnetic field was started in 1942 by Alfvén, the study is known as “Magnetohydrodynamics (MHD)” or ‘Hydromagnetics’. Thus, Hydromagnetics is the union of two fully developed fields — Electromagnetic theory and dynamics of fluid. Consequently, hydromagnetics or magnetohydrodynamics is the science, which deals with the motion of electrically conducting fluid in the presence of a magnetic field. It is concerned with physical systems specified by the equations that result from the fusion of those of hydrodynamics and electromagnetic theory. It is well known fact that when a conductor moves in a magnetic field, electric currents are induced in it. These currents experience a mechanical force, called the Lorentz force, due to the presence of the magnetic field. This force tends to modify the initial motion of the
conductor. Moreover, a magnetic field which is generated by the induced currents is added on to the applied magnetic field. Thus there is a coupling between the motion of the conductor and electromagnetic field, which is exhibited in a more pronounced form in liquid and gas conductors. This is due to the fact that the molecules composing the liquids and gases enjoy more freedom of movement than those of solid conductors.

Matter is composed of atoms and molecules and has necessarily a discrete structure in MHD, as in fluid dynamics, we shall throughout adopt a continuum picture. We are therefore considering those systems in which characteristic length is much larger than the mean free path. By considering the fluid as a continuous medium, pressure and other quantities vary smoothly from one point to the other and use of derivatives and differentials is allowed whenever they are needed. The hydrodynamic equations are basically non-linear and electrodynamic equations are linear. The coupling of two systems of equations: hydrodynamic and electrodynamic causes the non-linear aspect to be carried over into the resulting MHD equations. The new phenomena, which arise, are interesting. For example, the coupling between longitudinal and transverse fields provides the possibility of energy transfer between the longitudinal and transverse modes of oscillations.

Magnetofluid dynamics (MFD) or Magnetohydrodynamics (MHD) has attracted the attention of large number of scholars due to its diverse applications in engineering, e.g. in MHD generators, ion propulsion, MHD bearings, MHD pumps, MHD boundary layer control of reentry vehicles etc. In geophysics and astrophysics, it is applied to study solar storms and flares, radio propagations through the ionosphere, stellar and solar structures, interplanetary and interstellar matter etc. Yen and Chang (1964) analyzed the effects of wall electrical conductance on the magneto-hydrodynamic Couette flow. Attia and Kotb (1996) studied the MHD flow between two parallel plates with heat transfer.

The Maxwell Equations are

\[
\begin{align*}
\nabla \cdot \vec{D} &= \rho_e, \\
\nabla \times \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \\
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\
\nabla \cdot \vec{B} &= 0
\end{align*}
\]

(1.20)
where $\vec{D}$ is the displacement field, $\rho_e$ is the space charge density, $\vec{H}$ is the magnetic field, $\vec{J}$ is the current density, $\frac{\partial \vec{D}}{\partial t}$ is the displacement current, $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic induction field.

**The Lorentz Transformations**

The Lorentz transformations are listed below. Here $\vec{V}$ is the velocity of the rest frame $S'$ with respect to $S$ and $\perp$ and $\parallel$ indicate the components of the quantities perpendicular to and parallel to the vector $\vec{V}$ respectively.

\[
\begin{align*}
\vec{E}'_\perp &= \beta (\vec{E} + \vec{V} \times \vec{B}), & \vec{E}'_\parallel &= \vec{E}_\parallel \\
\vec{D}'_\perp &= \beta (\vec{D} + \vec{V} \times \vec{H} / c^2) , & \vec{D}'_\parallel &= \vec{D}_\parallel \\
\vec{H}'_\perp &= \beta (\vec{H} - \vec{V} \times \vec{D}) , & \vec{H}'_\parallel &= \vec{H}_\parallel \\
\vec{B}'_\perp &= \beta (\vec{B} - \vec{V} \times \vec{E} / c^2) , & \vec{B}'_\parallel &= \vec{B}_\parallel \\
\vec{J}'_\perp &= \vec{J}_\perp , & \vec{J}'_\parallel &= \vec{J}_\parallel \\
\rho'_e &= \beta (\rho_e - \vec{V} \cdot \vec{J} / c^3) , & \rho'_e &= \rho_e \\
\vec{P}'_\perp &= \beta (\vec{P} - \vec{V} \times \vec{M} / c^2) , & \vec{P}'_\parallel &= \vec{P}_\parallel \\
\vec{M}'_\perp &= \beta (\vec{M} + \vec{V} \times \vec{P}) , & \vec{M}'_\parallel &= \vec{M}_\parallel 
\end{align*}
\]

where $\beta = 1 / \sqrt{1 - v^2 / c^2}$. In the non-relativistic limit, which is the case of interest in MHD, the transformations reduce to:

\[
\begin{align*}
\vec{E}' &= \vec{E} + \vec{V} \times \vec{B} , & \vec{J}' &= \vec{J} - \rho_e \vec{V} \\
\vec{D}' &= \vec{D} + \vec{V} \times \vec{H} / c^2 , & \rho'_e &= \rho_e - \vec{V} \cdot \vec{J} / c^3 \\
\vec{H}' &= \vec{H} - \vec{V} \times \vec{D} , & \vec{P}' &= \vec{P} - \vec{V} \times \vec{M} / c^3 \\
\vec{B}' &= \vec{B} - \vec{V} \times \vec{E} / c^2 , & \vec{M}' &= \vec{M} + \vec{V} \times \vec{P} .
\end{align*}
\]

**The Electromagnetic Body Force**

The electromagnetic body force is derivable from the Coulomb force law: electromagnetic force on a particle of charge $q$ in the local rest frame of the particle is

\[
\vec{F} = q \vec{E}'
\]  

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Now, if we transform to the laboratory frame,

\[ E' = E + V \times B \]  \hspace{1cm} \text{(non-relativistic)} \quad (1.22)

so that \[ F = q \left( E + V \times B \right) \], which is the familiar Lorentz force law. The complete electromagnetic body force in a conducting MHD fluid is

\[ \vec{f}_e = \rho_e \vec{E} + \vec{J} \times \vec{B} \]  \hspace{1cm} (1.23)

**Magnetohydrodynamic (MHD) Assumptions**

The MHD assumptions which are usually made are the following:

1. All velocities are small compared to that of light, so that \( V^2 / c^2 \ll 1 \). This is in keeping with the non-relativistic Newtonian form of equations of motion and allows the \( \sqrt{1 - \left( V^2 / c^2 \right)} \) factor to be taken as unity.

2. The electric field is of order \( \vec{V} \times \vec{B} \), that is, of the order of magnitude of the induced effects. This assumption is equivalent to assuming that the induced magnetic field is much smaller than the externally applied magnetic field. Any \( \vec{E} \) fields involved are induced or of the order of the induced field. We must always write \( \vec{E}' = \vec{E} + \vec{V} \times \vec{B} \) and distinguish between \( \vec{E}' \) and \( \vec{E} \).

3. \( \vec{H} \equiv \vec{H}' \) and \( \vec{B} \equiv \vec{B}' \), since

\[ \vec{B}' = \vec{B} - \frac{\vec{V} \times \vec{E}}{c^2} \equiv \vec{B} - O \left( \frac{V^2 \vec{B}}{c^2} \right). \]

Liquid metals and ionized gases have permeability \( \mu_0 \) so that we write \( \vec{B} = \mu_0 \vec{H} \) in any frame of reference, where \( \mu_0 \) is free space permeability.

4. The displacement current \( \frac{\partial \vec{D}}{\partial t} \) is negligible with respect to \( \vec{J} \), the conduction current. This neglect implies, of course, that we are working with conductors and not with dielectrics in which case \( \frac{\partial \vec{D}}{\partial t} \) would have been retained.

5. Ohm’s law is \( \vec{J} = \sigma \vec{E}' = \sigma \left( \vec{E} + \vec{V} \times \vec{B} \right) \), and \( \vec{J}' = \vec{J} \) since \( \rho_e \vec{V} \) is neglected compared to \( \sigma \left( \vec{E} + \vec{V} \times \vec{B} \right) \).

6. The space charge \( \rho'_e \) is zero, but \( \rho_e \neq 0 \). We must write \( \nabla \cdot \vec{D} = \rho_e \) and not set \( \nabla \cdot \vec{D} \) to zero. Since \( \rho_e \) is not usually known. \( \nabla \cdot \vec{D} = \rho_e \) is not a useful equation. It
7. The force \( \rho_e \vec{E} \) is negligible compared to \( \vec{J} \times \vec{B} \). The electric stress and energy, proportional to \( \vec{E} \cdot \vec{D} \), is negligible compared to \( \vec{H} \cdot \vec{B} \).

8. For high conductivity, \( \sigma \to \infty \), Ohm’s law indicates that the finite \( \vec{J}, \vec{E} \) = 0 and \( \vec{E} = -\vec{V} \times \vec{B} \). The current is then determined by \( \nabla \times \vec{H} = \vec{J} \) and not by Ohm’s law. Here the prime designated quantities measured in the rest frame.

The basic equations of MHD can be written as the Maxwell equations, Ohm’s law, the equation of continuity, the equation of motion with the \( \vec{J} \times \vec{B} \) body force, and the energy equation with Joule heating. In addition, we must use the non-relativistic Lorentz transformations. The equation of state for a perfect gas can still be used under the MHD approximations. We can list these equations as:

**Maxwell’s Equation:**

\[
\begin{align*}
\nabla \times \vec{H} &= \vec{J}, \\
\nabla \cdot \vec{B} &= 0, \\
\n\nabla \cdot \vec{E} &= -\frac{\partial \vec{B}}{\partial t}, \\
\n\nabla \cdot \vec{J} &= 0.
\end{align*}
\]

**Ohm’s Law:**

\[
\vec{J} = \sigma (\vec{E} + \vec{V} \times \vec{B}).
\]

**Equation of Continuity:**

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0
\]

**Equation of Motion:**

\[
\rho \left[ \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} \right] = -\nabla p^* + \nabla \cdot \vec{V} + \vec{J} \times \vec{B} - \rho \nabla \psi,
\]

where \( \tau \) is the mechanical stress tensor and \( \psi \) is the gravitational potential.

**Equation of Energy:**

\[
\rho C_p \frac{DT^*}{Dt^*} = k \nabla^2 T^* + \phi + \frac{J^2}{\sigma} - p^* \nabla \cdot \vec{V}.
\]

**Equation of State:**

\[
p^* = \rho RT^*.
\]
Rarefied Gas Dynamics

Rarefied gas dynamics is concerned with flows at such low density that the molecular mean free path is not negligible. Under these conditions, the gas no longer behaves as a continuum. Important modifications in aerodynamic and heat transfer characteristics occur which are ascribable to the basic molecular structure of the gas.

This branch of gas dynamics has been the subject of many investigations since the time of Maxwell. Most of the early studies were related to very low speed flows, and usually to, “internal” geometries – pipes, ducts, orifices, and the like – in connection with vacuum problems. The results of this classical research are to be found in the standard texts on kinetic theory. Since World War-II, however, a revival of interest in the field has occurred due to applications to very high-altitude, high speed flight.

It is convenient to subdivide rarefied gas dynamics into four different flow regimes. These are called “free molecular flow”, “near-free-molecular flow”, “transition flow” and “slip-flow”, corresponding, respectively, to extremely rarefied, highly rarefied, moderately rarefied and only slightly rarefied gas flows. This subdivision is desirable because the four flow regimes exhibit quite different phenomena and the basic theoretical approaches are entirely different. Since “rarefied” is a relative term, the demarcation of these four subdivisions is not characterized by absolute pressure or gas density levels, but rather in terms of the ratio of the mean free path $\lambda$ to some dimension $L'$ characteristic of the flow field. The ratio $\lambda / L'$ is called the Kudsen number, $Kn$. Free-molecular flow corresponds to very large Kudsen number, $Kn > 10$; slip flow corresponds to Kudsen numbers in the range $0.01 < Kn < 0.1$; while the transitional regimes lies in between, with $0.10 < Kn < 10$.

Slip Flow

Slip flow is the part of rarefied gas dynamics corresponding to only slight rarefaction and for which the departure from continuum gas behaviour is slight. It takes its name from the phenomenon of slip, i.e., the gas immediately adjacent to a solid surface may possess a finite velocity with respect to it. There is also a temperature of the gas next to a surface which will in general differ from the surface temperature. These are boundary condition effects.

First order velocity slip and temperature jump boundary conditions (neglecting the thermal creep terms) at the surface are:
\[ u^* = \frac{2 - f_1 \chi \frac{\partial u^*}{\partial y^*}}{f_1} = L_1^* \frac{\partial u^*}{\partial y^*} \]
\[ T^* - T_0^* = \frac{2 - f_2}{f_1} \frac{2\gamma \frac{\partial T^*}{\partial y^*}}{\gamma + 1 \text{Pr} \frac{\partial T^*}{\partial y^*}} = L_2^* \frac{\partial T^*}{\partial y^*} \]

Here \( f_1 \) is the tangential momentum accommodation coefficient, \( f_2 \) is the thermal accommodation coefficient, \( \gamma \) is the specific heat ratio, \( C_p / C_v \), \( \text{Pr} \) is the Prandtl number, and \( \lambda = \mu \left( \frac{\pi}{(2p^* \rho)} \right)^{\frac{1}{2}} \) is the mean free path and is a constant for an incompressible fluid. Consequently \( L_1^* \) and \( L_2^* \) can be taken as constants.

When the gas is only slightly rarefied, results agreeing with the observed physical phenomena can be obtained by solving the usual Navier-Stokes equations together with modified boundary conditions for a velocity slip and temperature jump at the surface. This scheme of theoretical investigation of the so-called "slip-flow regime" is particularly suitable for studying the effects of gas rarefaction on any classical viscous flow problem. Sharma (2005) studied fluctuating thermal and mass diffusion on unsteady free convection flow past a vertical plate in slip-flow regime. Recently Mansour et al. (2007) extended it to the MHD micropolar fluid embedded in porous medium. The effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel has been investigated by Mehmood and Ali (2007).

**Non-Dimensional Parameters**

Several independent non-dimensional parameters that commonly enter fluid flow problems in fluid mechanics, magnetofluid mechanics and slip flow regime are listed and briefly discussed below:

**Reynolds Number** \((Re)\)

The Reynolds number is defined as

\[ Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{UL^*}{\nu}. \]

It is the most important dimensionless number in fluid dynamics providing a criterion for dynamic similarity. The Reynolds number is used for determining whether a flow is laminar or turbulent.
Froude Number \((Fr)\)

The Froude number is defined as
\[
Fr = \left[ \frac{\text{Inertia force}}{\text{Gravity force}} \right]^{\frac{1}{2}} = \frac{U}{\sqrt{gL^*}}.
\]

Equality of Froude number is a requirement for the dynamical similarity of flows with a free surface, in which gravity forces are dynamically significant.

Mach Number \((Ma)\)

The Mach number is defined as
\[
Ma = \left[ \frac{\text{Inertia force}}{\text{Compressibility force}} \right]^{\frac{1}{2}} = \frac{U}{c}
\]
where \(c\) is the speed of the sound. It is a measure of the compressibility of the fluid.

The Ratio of Specific Heats \((\gamma)\)

The ratio of specific heat at constant pressure \(C_p\) to that at constant volume \(C_v\) is usually designated as \(\gamma\), therefore
\[
\gamma = \frac{C_p}{C_v}.
\]
It is a measure of the relative complexity of the gas molecules.

Eckert Number \((Ec)\)

The Eckert number is defined as
\[
Ec = \frac{U^2}{C_vT^*}.
\]
In compressible fluids it determines the relative rise in temperature of the fluid due to adiabatic compression.

Grashoff Number \((Gr)\)

The Grashoff number usually occurs in free convection problems. This gives the relative importance of buoyancy force to the viscous forces. This number is defined as
\[
Gr = \frac{gL^3(T_0^* - T_\infty^*)}{U^3T_\infty^*}.
\]

Modified Grashoff Number \((Gm)\)

The modified Grashoff number usually occurs in free convection problems, when the effect of mass transfer is also considered. This number is defined as
Prandtl Number \((Pr)\)

It is a measure of the relative importance of heat conduction and viscosity of the fluid. The Prandtl number, like the viscosity and thermal conductivity, is a material property and it thus varies from fluid to fluid. Usually Prandtl number is large when thermal conductivity is small and viscosity is large and small when viscosity is small and thermal conductivity is large. It is defined as

\[
Pr = \frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\nu}{a} = \frac{\mu C_p}{k}.
\]

Nusselt Number \((Nu)\)

The dimensionless coefficient of heat transfer which is generally known as the Nusselt number, is defined as

\[
Nu = -\frac{L^*}{(T_0^* - T_w^*)} \left( \frac{\partial T^*}{\partial y^*} \right)_{y^*=0}.
\]

Sherwood Number \((Sh)\)

The dimensionless coefficient of mass transfer which is generally known as the Sherwood number, is defined as

\[
Sh = -\frac{L^*}{(C_0^* - C_w^*)} \left( \frac{\partial C^*}{\partial y^*} \right)_{y^*=0}.
\]

Magnetic Reynolds Number \((Rm)\)

The magnetic Reynolds number is defined as

\[
Rm = UL^* \sigma \mu = \frac{UL^*}{\eta^*},
\]

which is a measure of the ratio of magnetic convection to magnetic diffusion. If \(Rm \ll 1\), it can be shown that the induced magnetic field is small compared to the applied magnetic field.

Hartmann Number \((M)\)

The Hartmann number is defined as

\[
M = \sqrt{\frac{\sigma B_0^2 L^*}{\mu}},
\]

which is a measure of the ratio of the magnetic body force to the viscous force.
Magnetic Prandtl Number \((Pm)\)

The magnetic Prandtl number is defined as

\[ Pm = \frac{v}{\eta}, \]

which is a measure of the ratio of vorticity diffusion to magnetic diffusion.

Schmidt Number \((Sc)\)

This number is the ratio of momentum diffusivity to molecular diffusivity. It is defined as

\[ Sc = \frac{\text{Momentum diffusivity}}{\text{Molecular diffusivity}} = \frac{\nu}{D}. \]

The Schmidt number plays a role in convective mass transfer analogous to that of Prandtl number in convective heat transfer.

Rarefaction Parameter \((h)\)

The relative importance of effects due to the Rarefaction of a fluid may be indicated by a comparison of the magnitude of the mean free molecular path in the fluid with some significant body dimension. It is defined as

\[ h = \frac{v_0 L^*}{v}, \]

where \(v_0\) is the suction velocity and \(L^*\) is the characteristic dimension in the flow field.

Kundsen Number \((Kn)\)

The Kundsen number can be defined as

\[ Kn = \frac{\text{Mean free path of a gas}}{\text{characteristic length}} = \frac{\lambda}{L^*} = 1.255 \sqrt{\gamma} \frac{Ma}{Re}, \]

where mean free path of a gas \(\lambda\) may be taken as follows:

\[ \lambda = 1.255 \sqrt{\gamma} \left( \frac{v}{c} \right). \]

We usually take Kundsen number very small so that the gas may be considered as a continuum.

Solution of Fluid Dynamic (FD) and Magnetofluid Dynamic (MFD) Equations

The fluid dynamic and Magnetofluid dynamic equations can be solved by different methods like Laplace transform method, series method, numerical techniques and perturbation techniques etc. To obtain the solutions of some physical problems, we have
used perturbation technique in this thesis. Problems in non-steady boundary layers involve an essentially steady flow on which there is superimposed a small non-steady perturbation. If it is assumed that the perturbation is small compared with the steady basic flow, it is possible to split the equations into non-linear equation for the steady part.

The essential idea is that the problem has a small parameter (say $\varepsilon$) in either the governing equations or in the boundary conditions. In a flow at high Reynolds number, the small parameter is $\varepsilon = \frac{1}{Re}$, in a creeping flow $\varepsilon = Re$, and in flow around an airfoil $\varepsilon$ is the ratio of thickness to chord length. The solutions of these problems can frequently be written in terms of a series involving the small parameter, the higher order terms acting as a perturbation on the lower order terms. These methods are called perturbation techniques. A well known example is that for which the external stream has the form

$$U(x,t) = \bar{U}(x) + \varepsilon U_1(x,t) + \cdots,$$

(1.31)

where $\varepsilon$ denotes a very small number. The most important special case when the external perturbation is purely harmonic, was studied by Lighthill (1954). The same type of linearization can be employed when the perturbation at the wall is represented by the expression

$$T(x,t) = \bar{T}(x) + \varepsilon T_1(x,t)$$

(1.32)

or when the wall itself performs small, non-steady, perturbing motions (oscillating bodies).

In such cases we start with the assumption that the solutions for the dynamics as well as for the thermal boundary layer are of the following forms:

$$
\begin{align*}
u(x,y,t) &= u_0(x,y) + \varepsilon u_1(x,y,t) + \varepsilon^2 u_2(x,y,t) + \cdots \\
T(x,y,t) &= T_0(x,y) + \varepsilon T_1(x,y,t) + \varepsilon^2 T_2(x,y,t) + \cdots
\end{align*}
$$

(1.33)

The postulated forms from equations are introduced into the governing equations of the flow and the resulting terms are ordered with respect to the powers of $\varepsilon$. From the requirement that the differential expressions, which multiply each power of $\varepsilon$ must vanish separately, we obtain a cascade of differential equations. All the sets of differential equations except those of order zero will be linear and can be solved. The method can also be applied to the calculation of periodic boundary layers.
Flow Through Porous Media

The increasing demands for oil, water and food produced in an environmentally sound manner have placed emphasis on the manner of their production, a major part of which is concerned with flow through porous medium. Flow through porous media is also of interest in chemical engineering (adsorption, filtration, flow in packed columns), in petroleum engineering, in hydrology, in soil physics, in biophysics and in geophysics.

A medium which is a solid body containing pores is called a porous medium. Extremely small void spaces in a solid are called ‘molecular interstices’ and very large ones are called ‘caverns’. Pores are void spaces intermediate in size between caverns and molecular interstices. Flow of fluid is possible only if at least part of pore space is interconnected. The interconnected part of the pore system is called ‘effective pore space’ of the porous medium. A porous medium is a solid with holes in it. The manner in which the holes are imbedded, how they are interconnected and the description of their location, shape and interconnection characterize the porous medium. Accordingly, we have different classes of porous media. Porous medium are classified as unconsolidated or consolidated and as ordered or random. Examples of unconsolidated media are beach sand, glass beads, catalyst pellets, soil, gravel etc. Examples of consolidated media are most of the naturally occurring rocks such as sandstones, limestone and so forth. In addition concrete, cement, bricks, paper, cloth etc., are man-made consolidated media. Ordered porous media are regular packing of various types of materials such as spheres, column packing etc. Random media are media without any particular correlating factor.

When we consider flow in a porous medium we have to take into consideration some additional complexities, which are principally due to the interactions between the fluids and the porous material. When a fluid permeates through a porous medium, we can not follow analytically the actual path of an individual fluid particle because of the fluid-rock boundary conditions, which must be considered. Thus in a porous medium one generally considers the fluid motion in terms of volume or ensemble average of the motion of individual fluid elements over regions of space. This was usually done by famous Darcy’s (1856) law, as a result of this the viscous term in the equations of fluid motion will be replaced by the resistance term \(-\frac{\mu}{k_1} \bar{q}\), where \(\mu\) is the viscosity of the fluid, \(k_1\) the permeability of the medium and \(\bar{q}\) the seepage velocity of the fluid.

The experimental work [Wallace et al. (1969)] on the flow of mercury in porous medium, in the presence of a transverse magnetic field, has revealed that the behaviour of
fluids in petroleum reservoir rock depends to a large extent on the properties of rock. Techniques of course study that yield new or additional information on the characteristics of the rock, would contribute to a better understanding of the petroleum reservoir performance.

A macroscopic equation which describes incompressible creeping flow of a Newtonian fluid of viscosity \( \mu \) through a macroscopically homogeneous and isotropic porous medium of permeability \( k_i \) is the well-known Darcy's equation.

\[
-\frac{\mu}{k_i} \bar{q} = \nabla p^*, \quad (1.34)
\]

where \( p^* \) is the interstitially average pressure within the porous medium and \( \bar{q} \) is the filter velocity (or Darcian velocity).

For an incompressible Newtonian fluid and creeping flow conditions for a non-porous medium

\[
\mu \nabla^2 \bar{u} = \nabla p^*, \quad (1.35)
\]

where \( \bar{u} \) is the actual fluid velocity and \( p^* \) is the pressure. \( \bar{u} \) and \( \bar{q} \) are connected by the relation

\[
\bar{u} = \frac{\bar{q}}{\epsilon}, \quad (1.36)
\]

\( \epsilon \) is the medium porosity.

There is mounting evidence, both theoretical and experimental, which suggests that the Darcy's equation will sometimes provide an unsatisfactory description of the hydrodynamic conditions, particularly, near boundaries of a porous medium. Slattery (1969) has demonstrated

\[
-\frac{\mu}{k_i} \bar{q} + \frac{\mu}{\epsilon} \nabla^2 \bar{q} = \nabla p^*, \quad (1.37)
\]

as the governing equation for incompressible, creeping flow of a Newtonian fluid within an isotropic and homogeneous porous medium. This equation was originally proposed by Brinkman (1947, 1949) and so is known as the Brinkman equation.

The most important argument in the favour of the Darcy's equation and against Brinkman equation is that the latter is an immensely more difficult equation to solve. Secondly, for porous medium of relatively low permeability (low porosity) the first term of Brinkman's equation dominates the second term. Also, outside a zone next to the
boundaries of the porous medium, the contribution of the term $\frac{\mu}{\varepsilon} \nabla^2 \mathbf{q}$ becomes insignificant. The thickness of the boundary zone is actually quite small; hence, the Brinkman equation effectively reduces to the Darcy’s equation within the biggest portion of the porous medium outside the thin boundary zones.

**Governing Equations in Porous Media**

The porous material containing the fluid is in fact a non-homogeneous medium, but for the sake of analysis, it may be possible to replace it with a homogeneous liquid which has dynamical properties equal to the local average of the original non-homogeneous continuum. Then one can study the flow of a hypothetical homogeneous fluid under the action of the properly averaged external forces. Then a complicated problem of the flow through a porous medium changes to the flow problem of a homogeneous fluid with some additional resistance.

Muskat (1946) discussed flow of a homogeneous fluid through various types of porous medium following classical Darcy’s law, which states that the seepage velocity of the fluid is proportional to the pressure gradient. This law fails to explain the phenomena occurring in highly porous medium.

Following the method of local averages, given by Eringen in 1964, Ahmadi and Manvi (1971) analytically derived general equations of motion for the flow of a viscous incompressible fluid through rigid porous medium. These equations are:

\[
\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0,
\]

\[
\frac{Du^*}{Dt^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \nabla^2 u^* - \frac{\nu u^*}{K^*},
\]

\[
\frac{Dv^*}{Dt^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \nabla^2 v^* - \frac{\nu v^*}{K^*},
\]

\[
\frac{Dw^*}{Dt^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*} + \nu \nabla^2 w^* - \frac{\nu w^*}{K^*},
\]

\[
\rho C_p \frac{DT^*}{Dt^*} = k \nabla^2 T^* + \phi,
\]

where
\[ \phi = 2\mu \left\{ \left( \frac{\partial u^*}{\partial x^*} \right)^2 + \left( \frac{\partial v^*}{\partial y^*} \right)^2 + \left( \frac{\partial w^*}{\partial z^*} \right)^2 \right\} 
+ \mu \left\{ \left( \frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*} \right)^2 + \left( \frac{\partial v^*}{\partial z^*} + \frac{\partial w^*}{\partial y^*} \right)^2 + \left( \frac{\partial w^*}{\partial x^*} + \frac{\partial u^*}{\partial z^*} \right)^2 \right\} \] (1.43)

These equations were later used by Gulab Ram and Mishra (1977), Vershney and Singh (2005) to study some flows through porous media. These equations were also used by Raptis (1983), Raptis and Perdikis (1985). Recently Thakur et al. (2006) studied exact solutions of steady orthogonal plane MHD flows through porous media. More recently Singh and Kumar (2009a) investigated an exact solution of an oscillatory MHD flow through a porous medium bounded by rotating porous channel in the presence of Hall current.

Various Effects on Fluid Flow Problems

Radiation Effects

Transmission of heat takes place via conduction, convection and radiation. Heat transfer by radiation is explained on the basis of radiant energy that is emitted by the bodies. Thermal radiation is an electromagnetic phenomenon. The radiation between wave lengths of 0.1 and 100 microns is treated as thermal radiation. All electromagnetic radiations, when absorbed by a system, produce thermal energy, that is, a heating effect. This radiant energy gives the measure of the absorptive and emissive power of a body. The absorptive power of a body is defined as the ratio of the amount of radiant energy (heat) absorbed per second by the surface to the amount of total energy (heat) incident on the surface at the same time. Similarly, emissive power is defined as the ratio of the amount of heat radiations emitted by unit area of a surface in one second to the amount of heat radiated by a perfectly black body per unit area in one second under classical conditions.

Kirchhoff's Law of Radiation

Kirchhoff's law states that the ratio of the emissive power and the absorptive power for radiation of a particular wavelength and at a particular temperature is constant for all bodies. This ratio is also equal to the emissive power of a perfectly black body at the temperature i.e.
\[
\frac{E_i}{1} = \frac{Q}{d_i} \quad \text{and} \quad \frac{e_i}{a_i} = E_i = \text{constant},
\]
where \(e_i\), \(a_i\), and \(Q\) are emissive power, absorptive power and quantity of heat radiation incident on the surface respectively.

**Planck’s Law of Radiation**

Planck introduced the quantum concept in 1900 and with it the idea that radiation is emitted not in a continuous energy state but in discrete amounts or quanta. The intensity of radiation emitted by a black body, derived by Planck, is

\[
I_{b,i} = \frac{2c^2h'\ell^{-5}}{\exp \left( \frac{ch'}{\kappa \ell T^*} \right) - 1},
\]

Where \(I_{b,i}\) is the intensity of radiation from a black body between wavelengths \(\ell\) and \(\ell + d\ell\), \(c\) is the speed of light, \(h'\) is Planck’s constant, \(\kappa\) is the Boltzmann constant and \(T^*\) is the temperature. The total emissive power between wavelengths \(\ell\) and \(\ell + d\ell\) is then

\[
E_{b,i} = \frac{2\pi c^2h'\ell^{-5}}{\exp \left( \frac{ch'}{\kappa \ell T^*} \right) - 1}.
\]

**Stefan-Boltzmann’s Law**

It states that the total amount of energy radiated per second per unit area of a perfect black body is directly proportional to the fourth power of the absolute temperature of the surface of the body i.e.

\[
E \propto T^{*4} \quad \text{or} \quad E = \sigma^* T^{*4},
\]

where \(\sigma^*\) is called the Stefan’s constant. Its value is

\[
5.67 \times 10^{-8} \ \text{ergs s}^{-1} \ \text{cm}^{-2} \ \text{K}^{-4} \quad \text{in C.G.S. system} \quad \text{or} \quad \begin{aligned}
5.67 \times 10^{-8} \ \text{J} \ \text{s}^{-1} \ \text{m}^{-2} \ \text{K}^{-4} & \quad \text{or} \quad 5.67 \times 10^{-8} \ \text{W} \ \text{m}^{-2} \ \text{K}^{-4} \quad \text{in S.I.}
\end{aligned}
\]

**Gray-Gas Approximation**

If the absorption coefficient \(a_i\) of a surface is independent of the wavelength \(\ell\), then the surface is called gray. Actually, we use some mean value of the absorption coefficient for the overall flow phenomena. The local radiant for the case of an optically thin gray gas is expressed by
where $a^*$ is the mean absorption coefficient and $\sigma^*$ is Stefan-Boltzmann constant.

We assume that the temperature differences within the flow are sufficiently small such that $T''''$ may be expressed as a linear function of the temperature. This is accomplished by expanding $T''''$ in a Taylor series about $T_\infty$ and neglecting higher-order terms, thus

$$T'''' \equiv 4T_\infty T' - 3T''''.$$  \hfill (1.49)

Radiation effects may be very significant in some convection problems, particularly in the case of gases and where high temperature exists in either gas or solid surface. In free convection problems with gases particularly, where the convective coefficients may be small, radiation heat transfer may be significant compared to the heat transfer by convection.

Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, astrophysical flows, solar power technology etc. Raptis and Perdikis (2003) analyzed the effects of thermal radiation on a moving vertical plate in the presence of mass diffusion. Sharma et al. (2008) investigated radiation effects on steady free convective flow along a moving vertical porous plate in the presence of magnetic field and heat source/sink. Recently Abdelkhalek (2008) investigated radiation and dissipation effects on unsteady MHD micropolar flow past an infinite vertical plate in porous medium with time dependent suction. More recently Singh and Kumar (2009e) studied radiation effects on the exact solution of free convective oscillatory flow through porous medium in a rotating vertical porous channel.

Effects of Hall Current

Hall, a graduate student at John Hopkins University discovered the "Hall effect" in 1879. Hall’s original experiments were limited to solid-metallic conductors. A thin, flat strip of width $b$ and thickness $d$ was traversed by a current $I$. Two fine wires were connected at equipotential points on opposite edges of the strip and in turn join to the terminals of a sensitive galvanometer. When the magnetic field $\vec{H}$, was introduced at right angles to the face of the strip, the galvanometer gave a steady deflection. The voltage indicated by
the galvanometer is known as the Hall voltage and is directly proportional to both current and magnetic field.

It is a well known fact that if the mean free path is much larger than the electron Larmor radius, electrons will be able to gyrate freely round the magnetic lines of force several times before suffering collisions. Consequently, the electrons and ions appear to be tied with the lines of force in a way and this reduces their mobility transverse to the magnetic field; the whole current will not flow along the electric field. This tendency of electric current to flow across an electric field in the presence of a magnetic field is called ‘Hall effect’. The Hall effect is more pronounced in the strong magnetic field or in the case of ionized gas (degree of ionization is small).

The effect of magnetic field on the flow of plasma at a constant pressure gradient is weakened as the Hall parameter increases. This is due to the decrease in the conductivity in the direction of the induced electric field with an increase in the Hall parameter. In many astrophysical and geophysical situations as well as in engineering problems Hall effects are important. In particular Hall effects are likely to be present in the case of ionosphere and outer layers of the solar atmosphere. The generalized Ohm’s law including Hall currents [Cowling (1957)] is of the form:

\[
\vec{J} + \frac{\omega_e \tau_e}{B_0} (\vec{J} \times \vec{B}) = \sigma \left(\vec{J} \times \vec{B} + \frac{1}{e n_e} \nabla p_e + \vec{E}\right)
\]  

(1.50)

where \(\omega_e\) is the electron frequency, \(\tau_e\) is the electron collision time, \(\sigma\) is the electrical conductivity, \(e\) is the electron charge, \(p_e\) is the electron pressure and \(n_e\) is the number density of electron.

Singh (1983) investigated Hall effects on an oscillatory MHD flow in the Stokes problem past an infinite vertical porous plate. Sato (1961) and Tani (1962) have considered the Hall effect in an incompressible viscous flow of an ionized gas with tensor conductivities in channels. They found that the inclusion of Hall currents gives rise to cross flow i.e. a flow at right angle to the primary flow in a channel in the presence of a transverse magnetic field. Attia (2006) investigated time varying hydromagnetic Couette flow with heat transfer of a dusty fluid in the presence of uniform suction and injection considering the Hall effect. Singh et al. (2007) studied hydromagnetic convective flow past impermeable plate in rotating system for dusty fluid with Hall current. Recently combined effects of Hall current and rotation on free convection MHD flow in a porous channel have been analyzed by Singh and Kumar (2009b).


**Soret Effects**

If two regions in a mixture are maintained at different temperatures so that there is a flux of heat, it has been found that a concentration gradient is set up. In a binary mixture, one kind of a molecule tends to travel toward the hot region and the other kind toward the cold region. This is called the “Soret effect”.

Eckert and Drake (1972) have pointed out that in a convective fluid when the flow of mass is caused by a temperature difference one cannot neglect the thermal diffusion effect (commonly known as Soret effect) due to its practical application in engineering and science. Usually this effect has a negligible influence on mass transfer, but it is useful in the separation of certain mixtures.

For instance thermal diffusion effect has been utilized for isotope separation and in mixtures between gases with very light molecular weight ($H_2, He$) and medium molecular weight ($N_2, air$) and it was found to be of a magnitude that cannot be neglected. Jha and Singh (1990) investigated the free convection flow of a viscous fluid past an infinite vertical plate moving impulsively in its own plane taking into account the Soret effect.

The equation of mass transfer when Soret effects are included is of the form:

$$\frac{DC^*}{Dt^*} = D\nabla^2 C^* + D_t \nabla^2 T^*, \quad (1.51)$$

where $D$ is molecular diffusivity and $D_t$ is thermal diffusivity.

Soret effects on free convection and mass transfer flow in the Stokes problem for an infinite vertical plate has been investigated by Jha and Singh (1990). Soret effects due to natural convection between heated inclined plates with magnetic fields have been analyzed by Raju *et al.* (2008). Singh and Kumar (2009f) studied Soret and Hall current effects on heat and mass transfer in MHD flow of a viscous fluid through porous medium with variable suction.

**Rotating Flows**

Rotating flows are an important part of classical fluid mechanics well known to give rise to interesting structures and instabilities. Since we live on a rotating earth, rotating flows are extremely important in geophysics- in the oceans or the atmosphere. Planetary rotation greatly influences flows of fluids in atmospheres and in oceans through the
Coriolis acceleration. In a rapidly rotating fluid, particles move across the pressure gradient (along isobars) due to the Coriolis force. In an engineering context, rotating flows are abundant, e.g. in hydraulic turbomachinery. Experimentally rotating containers offer possibilities for studying vortex motion cleanly and obtaining insight into phenomena such as tornados or bathtub vortices and their instabilities in the form of surface waves.

The equations of motion of an incompressible fluid in rotating system are of the form:

\[
\frac{D\vec{v}}{Dt^*} = -\frac{1}{\rho} \nabla p^* + \nu \nabla^2 \vec{v} - 2\vec{\Omega} \times \vec{v}.
\]  

(1.52)

In recent years a number of studies have appeared in the literature on rotating flows viz. Vidyanidhu and Nigam (1967). Injection/suction effects have also been studied extensively for horizontal porous plate in rotating frame of references by Mazumder et al. (1976) and Soundalgekar and Pop (1973) for different physical situation. Sattar and Alam (1995) studied Soret effects as well as transpiration effects on MHD free convection and mass transfer flow past an impulsively started vertical porous plate in a rotating fluid. Recently Singh and Kumar (2009a) analyzed an exact solution of an oscillatory MHD flow through a porous medium bounded by rotating porous channel in the presence of Hall current.

**Heat Source/Sink**

A heat sink is an environment or object that absorbs and dissipates heat from another object using thermal contact (either direct or radiant). Heat sinks are used in a wide range of applications, wherever efficient heat dissipation is required; major examples include refrigeration, heat engines, cooling electronic devices and lasers.

Heat sinks function by efficient transferring thermal energy (heat) from an object at a relatively high temperature to a second object at a lower temperature with a much greater heat capacity. This rapid transfer of thermal energy quickly brings the first object into thermal equilibrium with the second, lowering the temperature of the first object, fulfilling the heat sink's role as a cooling device. Efficient function of heat sink relies on rapid transfer of thermal energy from the first object to the heat sink, and the heat sink to the second object.

Heat sink performance (including free convection, forced convection, liquid cooled, and any other combination thereof) is a function of material, geometry, and overall surface heat transfer coefficient. Generally, forced convection heat sink thermal
performance is improved by increasing the thermal conductivity of the heat sink materials, increasing the surface area (usually by adding extended surfaces, such as fins or foam metal) or by increasing the overall area heat transfer coefficient (usually by increasing fluid velocity, such as adding fans, pumps etc.).

Further, heat source/sink effect has its great applicability to ceramic tiles production problem, the study of heat transfer in the presence of heat source/sink has acquired new dimensions.

Ostrach (1958) studied unstable convection in vertical channel with heating from below including effects of heat source and frictional heating. Sharma et al. (2008) analyzed radiation effect on steady free convective flow along a uniformly moving vertical porous plate in presence of heat source/sink and transverse magnetic field. Recently Singh and Kumar (2009d) investigated heat and mass transfer in MHD flow of a viscous fluid through porous medium with variable suction and heat source.

**The Effect of Chemical Reaction**

In convective heat and mass transfer processes, diffusion rates can be altered tremendously by chemical reaction. The effect of chemical reaction depends on whether the reaction is heterogeneous or homogeneous. Muthucumaraswamy and Ganesan (2001) pointed out that chemically reacting flows are classified as heterogeneous or homogeneous depending on whether they occur at an interface or as a single phase volume. In particular, a reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration. In nature, the presence of pure water or air is not possible. Some foreign mass may be present either naturally or mixed with air or water. The presence of a foreign mass causes some kind of chemical reaction. The study of such type of chemical reaction processes is useful for improving a number of chemical technologies, such as food processing, polymer production and manufacturing of ceramics or glassware.

The mass conservation or concentration equation in the presence of chemical reaction has the following form in three dimensions:

\[
\frac{\partial C^*}{\partial t^*} + u^* \frac{\partial C^*}{\partial x^*} + v^* \frac{\partial C^*}{\partial y^*} + w^* \frac{\partial C^*}{\partial z^*} = D \left( \frac{\partial^2 C^*}{\partial x^{*2}} + \frac{\partial^2 C^*}{\partial y^{*2}} + \frac{\partial^2 C^*}{\partial z^{*2}} \right) + m^*,
\]

where \(m^*\) is a chemical reaction term and is finite. In some reactions the species of interest is a product of reaction \(m^* > 0\), while in others the species is being
consumed \( (m'' < 0) \). In the case of homogeneous reactions, the volumetric mass rate of production of a species can be expressed as

\[ m'' = K^n'' C^n'' , \quad (1.54) \]

where \( n \) is the order of reaction and \( K^n'' \) is the rate constant. In the case of a first-order reaction \( n = 1 \). In homogeneous reactions, the production or consumption of the species of interest takes place in the fluid, that is, wherever the species exists. In heterogeneous reactions, on the other hand, the reaction takes place on the surface of a catalyst; the rate of species production in this case may be expressed as

\[ m'' = K'_n C_0'' , \quad (1.55) \]

where \( C_0'' \) is the concentration at the surface, \( n \) is the order of the reaction, and \( K'_n \) is the reaction rate.

The concentration equation in the presence of homogeneous reaction is of the form:

\[ \frac{\partial C''}{\partial t'} + u \cdot \frac{\partial C''}{\partial x'} + v \cdot \frac{\partial C''}{\partial y'} + w \cdot \frac{\partial C'}{\partial z'} = D \left( \frac{\partial^2 C''}{\partial x'^2} + \frac{\partial^2 C''}{\partial y'^2} + \frac{\partial^2 C''}{\partial z'^2} \right) - K'_n C_0'' , \quad (1.56) \]

here negative sign indicates that the chemical reaction consumes the species of interest.

If a chemical reaction can either generate or consume a species, it will certainly influence the distribution of that species in the flow of the mixture. The best way to illustrate the effect of chemical reaction on mass convection is to focus on a very basic problem for which we already know the solution for the case when chemical reaction is absent. We will then solve the problem allowing for the presence of a chemical reaction, and comparing the two solutions, we will develop a feeling for the effect of chemical reaction.