APPENDIX 1:
STOCKONOMICS
MACROECONOMICS OF THE INDIAN STOCK MARKETS ON INDIA INC

INDIAN STOCK MARKET
INTRODUCTION

- Stock/Share Market mean the same. Both terms describe an exchange in which buyers & sellers of stock/share may trade in a market with high liquidity.

- SHARE: - is a unit issued by a company at time of raising fund from market. A certificate issued to a person who applies for it and is given at a pre-determined value by a company. It is the smallest unit of ownership that may be bought /sold on/off an exchange, called share trading.

- STOCK: - With respect to stock market is total number of shares a person has in one company or in many companies. Stocks can be traded publicly in the stock market.

- STOCK MARKET: - 1) An organized market, where securities of Government/Semi- Government/Corporate entities are bought/sold. 2) Established for the purpose of assisting /regulating/controlling business in buying/selling/dealing in securities. 3) In stock market only those securities listed in the stock exchange are transacted.4) Individuals alone can buy and sell securities.

Types of Stock market:-

<table>
<thead>
<tr>
<th>PRIMARY MARKET</th>
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<tr>
<td>SECONDARY MARKET</td>
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<tr>
<td>STOCK EXCHANGES</td>
</tr>
</tbody>
</table>

- The First Group Of Investors To Whom A New Issue Of A Security Or Share Is Sold.
- The Primary Market Consists Of Issuer And The First Buyers Of The Share Issue.
- The Primary Market At Times Can Be More Volatile Than Secondary Market As It Is Difficult To Determine The Underlying Value Of New Issues.

- A Market Where Investors Purchase Securities, Assets Or Shares From Other Investors Rather Than From Issuing Companies Themselves.
- National Exchanges Such As National Stock Exchange
- (NSE) & Bombay Stock Exchange (SENSEX) Are Secondary Markets.
**TYPES OF STOCKS:**

1. BLUE CHIP: Carries the highest value. Large, established firms, long record of profit, growth, Dividend pay-out, & good reputation.

2. Penny: Low priced, speculative stocks, very risky. Stocks issued by companies with an erratic history of revenue and earnings.

3. Income: Stocks that pay higher than average dividends over a sustained period. Dividends tend to be paid by established companies with stable earnings.

4. Value: Stocks that are currently selling at a low price. Companies with good earnings and growth potential but stock prices do not reflect this are called value company stocks. These stocks are only temporarily out of favour and may experience growth.

**MARKET CAPITALIZATION:**

1. A measure of size of a business enterprise.

2. Total $ Market Value of all of a company’s fully paid outstanding equity shares, calculated by multiplying a company’s shares outstanding by the current market price per share (M.P.P.S)

3. Based on the size of the market cap, companies are divided primarily into large cap, mid cap, & small cap stocks.
   - Large cap: -us $ 10 billion to us $ 100 billion
   - Mid cap : -us $ 1 billion to us$ 10 billion

**MARKET CONDITIONS:**

- Bull Market:
  - Financial Market Conditions In Which Prices Of Securities Or Portfolios Are Rising Or Expected To Rise.
  - Bull Markets Are Characterized By Optimism, Investor Confidence And Expectations That Strong Results Will

- Bear Market:
  - A Market Condition In Which Prices Of Securities Are Falling.
  - Investors Anticipate Losses In A Bear Market And Selling Continues, Pessimism Only Grows.
Small cap: -us $ 100 million to us$ 1 billion.

In India two reference benchmarks being popularly used are the BSE—SENSEX & NSE—S&P CNX NIFTY. In this study we will take up only NSE Nifty (fifty stocks). But a brief on BSE now follows:

**BSE CREDENTIALS: - (source: - World Federation of Exchanges) :-**

(WFE)

WFE regulates 52 stock exchanges across the globe promoting regulatory standards.

- Largest number of listed companies in the world. 5379 as at 31\textsuperscript{st} May 2014.
- 10\textsuperscript{th} largest exchange globally in terms of Market Capitalization.
- 4\textsuperscript{th} most liquid exchange globally for Index Options.
- 9\textsuperscript{th} largest in the world in terms of number of trades in equity shares.
- 3\textsuperscript{rd} largest in the world in terms of number of currency options contract traded.
- 4\textsuperscript{th} largest in the world in terms of number of currency futures contracts traded.
- Full bouquet of products including equity, equity derivatives, currency derivatives, Interest rate derivatives, Debt products, Security Lending & Borrowing platform, IPO, SME Platform, Mutual Funds, and ETF’s, & Offer for Sale (OFS).
- Partnership with S&P DOW JONES INDICES on Index products.
- Exchange Technology from DEUTSCHE BOERSE group ( Eurex platform)
- Cross listing of benchmark index— S&P BSE SENSEX in BRIC countries.

Over the past one year, the BSE SENSEX has witnessed the highest rise among all major stock market indices in the world. Between November 26, 2013 & November 26, 2014, the value of the Index increased by 39%. Among the 24 indices of the world’s largest stock exchanges that TOI analysed, 11 registered double digit growth and the South Korean KOSPI was the only major index that witnessed a decline in this period. If we look at the five year returns, then the SENSEX would be the 6\textsuperscript{th} best
in the world. Between November 25, 2009 & November 26, 2014, the NASDAQ Index rose by 120%, highest in the world¹.

<table>
<thead>
<tr>
<th>NAME OF INDEX</th>
<th>FROM</th>
<th>FROM</th>
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<th>NEGATIVE</th>
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<tbody>
<tr>
<td>5 YR RETURN</td>
<td>3 YR RETURN</td>
<td>1 YR RETURN</td>
<td></td>
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</tbody>
</table>

**FIGURES IN PERCENTAGE TERMS**

<table>
<thead>
<tr>
<th>NAME OF INDEX</th>
<th>FROM</th>
<th>FROM</th>
<th>FROM</th>
<th>NEGATIVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) BSE Sensex India</td>
<td>65</td>
<td>80.9</td>
<td>39</td>
<td>-</td>
</tr>
<tr>
<td>2) Hang Seng Hongkong</td>
<td>6.6</td>
<td>36.3</td>
<td>1.8</td>
<td>-</td>
</tr>
<tr>
<td>3) Ibovespa Brazil</td>
<td>-</td>
<td>0.4</td>
<td>7.1</td>
<td>18.9</td>
</tr>
<tr>
<td>4) Dow Jones USA</td>
<td>70.4</td>
<td>58.7</td>
<td>10.9</td>
<td>-</td>
</tr>
<tr>
<td>5) Mex Ipc Mexico</td>
<td>42.4</td>
<td>29.2</td>
<td>8.9</td>
<td>-</td>
</tr>
<tr>
<td>6) Nasdaq USA</td>
<td>120</td>
<td>96.1</td>
<td>19.2</td>
<td>-</td>
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<tr>
<td>7) Shenzhen Se China</td>
<td>-</td>
<td>9.4</td>
<td>19.3</td>
<td>20.8</td>
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<tr>
<td>8) Dax Germany</td>
<td>70.9</td>
<td>80.5</td>
<td>6.7</td>
<td>-</td>
</tr>
<tr>
<td>9) Micex Russia</td>
<td>17.2</td>
<td>53.1</td>
<td>3.6</td>
<td>-</td>
</tr>
<tr>
<td>10) Cac 40 France</td>
<td>14.8</td>
<td>53.1</td>
<td>2.2</td>
<td>-</td>
</tr>
<tr>
<td>11) Nikkei 225 Japan</td>
<td>84.1</td>
<td>113</td>
<td>12</td>
<td>-</td>
</tr>
</tbody>
</table>

1) SOURCE: - TIMES OF INDIA Dated: - 2nd December 2014—page 9—

**STATISTICS**

**A YEAR OF GROWTH (REFER TABLE—Returns of Financial Markets – cut off 26th November 2014) (BLOOMBERG SOURCE)**

“Sensex drops on Infosys, global cues: - Infosys founders reduced their stakes in the company, and also on account of global cues the benchmark SENSEX lost 339 points or 1.2% to close at 28119 points, the biggest drop since December 10. The Nifty was down 100 points or around 1.2% and ended at 8438 points. The market correction was led by stocks of IT (Information Technology), metals and financial services companies. The INFOSYS stock slipped more than 5% as almost 3.3 crore shares were sold in small blocks. The shares were sold in the range of Rs, 1988-2014. The scrip
fell 4.9% to close at Rs. 1969 on BSE. Barring the FMCG index, this closed up around 0.8%, all other indices ended in red. The market breadth remained negative with 1154 stocks advancing against 1792 stocks declining.

NIFTY continued to trade below the psychological level of 8500 points on selling pressure from stocks in IT, Banks, Realty & to some extent consumer durables. The I.I.P (Index of Industrial Production) and Inflation data that are due in the coming week should drive the market. If the data is strong and reflective of improving economic growth, then the market correction could be contained. Market observers expect Volatility and minor correction in the coming days, and the market should remain sideways in coming trading sessions opined experts”.


NSE

- One of the largest stock exchanges in India.
- Biggest in terms of daily trading and turnover
- Thousands of companies listed with NSE and divided into different categories primarily depending upon market cap.
- Primarily stocks listed are divided into three categories on the basis of Market Capitalization: - LARGE, MID, SMALL. Our study has for the population earmarked as S&P CNX NIFTY (fifty) stocks encompassing 12 industries /sectors.
- Large cap stocks: - 1) Biggest & most reputed companies among all the listed companies in the stock exchange. 2) Mostly companies that are in business for years and making significant growth in terms of profit and asset accumulation. 3) Large cap stocks are considered for inclusion in NIFTY that is the PRIME INDEX OF INDIA. (NSE).
- Mid cap: - 1) Moderate market cap. Market cap between USD 2 BILLION TO USD 10 BILLION. 2) These stocks have great investment proposition as they have all the signs of rising in the market and give you good returns on your investment.
- Small cap: - 1) Market cap between USD 200 MILLION TO USD 2 BILLION. 2) Relatively new companies 3) More risky as these companies take too long to rise in the market.4) Not wise to invest in these companies for
long term, but you can invest in these stocks and do some margin trading if you have definite and trustworthy tips.

- Besides these three prominent stock categories in NSE, there are other categories like Microcap & Penny stocks. While Microcap segment has companies less than USD 300 MILLION Capital. Penny stocks are low priced stocks. Besides these classifications, stocks at NSE are also categorized on the basis of sectors/industries.

- NSE or National fifty comprises of 50 stocks, while BSE (SENSEX) consists of 30 stocks.

- Key terminology:
  1) Shareholding pattern:- Shares of a company may be held by promoters, financial institutions including foreign institutions, banks and other companies besides being held by public at large.

- FREE FLOAT: - Number of shares held by non-promoters or generally referred to as public holdings. These include holdings by banks and financial institutions.

- How is an INDEX calculated?
  1. Weights assigned based on market cap or free float. Change in price is first multiplied by weight assigned and then multiplied by total number of shares or free float shares to arrive at impact on Index.
  2. NSE has switched over to FREE FLOAT based method w.e.f 26th June 2009. Sensex is calculated by free float method and Nifty was calculated prior to 26/6/2009 by Market cap method. Categories like Government holdings in the capacity of strategic investor, shares held by promotes through ADR/GDR, strategic stakes by corporate bodies, investments under FDI (Foreign Direct Investments), and cross holding among others would be excluded, while computing under free float method.

**WHY KNOW HOW AN INDEX IS CALCULATED?**

1. Sensex & Nifty are barometers of INDIAN Stock Market and generally used as a benchmark to evaluate performance of a portfolio. Mutual Funds in general and Index based mutual funds maintain their portfolios based on Index weights. We can therefore expect buying in companies that have gained weight or selling off shares of those companies held which are losing weight.
2. Structuring an Investment Portfolio is about allocating funds to different assets like shares, gold, risk-free assets, realty and the ilk. Structuring an equity portfolio means allocating funds to buy shares of different industries and companies within the Index so as to have a diversified portfolio.

3. Asset allocation based on economic trends and portfolio restructuring based on principles of diversification account for almost 95% of success in the investment game. Market timing, Industry selection, & Stock selection account for only a very small percentage.

DIFFERENCE BETWEEN NSE & BSE AND WHY SHOULD WE HAVE TWO EXCHANGES?

- Share/Stock Market is a massive ground for trading company stocks and derivatives, at a pre-determined or agreed price. These stocks are listed as securities in a stock exchange. A stock exchange is a kind of secure platform for stock brokers and traders for trading in stocks, bonds and other financial instruments. They also provide a platform for capital events such as payment of income & dividends. Securities traded on a stock exchange include shares of companies, unit trusts, derivatives to pooled investment products and bonds. Every stock exchange has a key index which keeps track of most important or most traded stocks in that exchange.

- Two most prominent stock exchanges in India are the National Stock Exchange (NSE) & Bombay Stock Exchange (BSE), though other stock exchanges too exist, BSE & NSE accounts for majority of Equity trading in India (Over USD 1.6 TRILLION).

- NSE is located at Maharashtra, India and it is regarded as 9th largest in the world in terms of market cap. To trade in a company’s share over the stock exchange it has to be listed there. There are over 1552 listings in NSE. However to 50 most traded companies (Market cap basis), shares are kept track of using NSE’s key index called NIFTY (National Stock Exchange fifty).

- Bombay Stock Exchange (BSE) is located in the renowned Dalai Street, Mumbai. It is the 8th largest stock exchange in the world and 4th largest in Asia, on the basis of market capitalization of companies listed there (USD
1.63 TRILLION as on December 2010), with maximum number of companies listed, and this exchange is supposed to be the oldest stock exchange in Asia as well. The key index for BSE is SENSEX (Sensitive Index). Alike Nifty, which tracks the progress of top 50 company shares, Sense tracks the progress of top 30 company share values. Sensex is regarded as the driver of Stock markets in India domestically.

- In 1850’s, for stock price discussions, (Gujjus & Parsis met) under banyan trees. Over time, the location of these meetings changed many times and number of brokers constantly increased. This nomadic movements ended in Dalal Street in 1874 and in 1875, became an official organization known as “The Native Share & Stock Brokers Association”. The BSE was first recognized stock exchange as per the Securities Contract Regulation Act (S.C.R.A.), by the Indian Government. BSE Sensex was created with the intention of regulating as well as measuring its performance. Contrast to this, NSE, however, does not have a long lineage. Some of the leading Financial Institutions promoted the BSE along with The Government of India, and was incorporated in the name of a tax paying company in November 1992. Only after a year BSE, came to be recognized as a full-fledged stock exchange.

- FREE FLOAT CAP METHOD has been adopted for calculating Index value in case of both BSE & NSE. The float and shares are used for trading other than the company’s outstanding shares. This method’s advantage is in its rationality. By not including restricted stocks, it eliminates their effect on Index calculations.

- While BSE is owned by the Government of India, a group of Financial Institutions like Banks, Insurance companies and others mutually own NSE. NSE’s ownership and management operate as two disparate entities.

- On the issue of why do we need both, Number of listings is higher in BSE, whereas trade volume is more in NSE. NSE is more liquid, as there are more number of buyers and sellers at a particular time for any listed stock in NSE than in BSE.

- NSE has maintained its slot as world’s largest bourse in terms of volume in equity segment for the first quarter of 2013, while two bourses from China have replaced NYSE & NASDAQ of US at 2nd & 3rd rank respectively.
• NSE logged in a total of 36.6 Crore trades in EQUITY SEGMENT DURING January – March 2013, making it the world’s largest exchange in terms of number of equity trades as per the latest data compiled by World Federation of Exchanges (W.F.E.). However, NSE, showed a month-on-month decline in terms of number of equity trades. From 13.8 crore in January 2013, number of trades dipped to 11.6 crore in February 2013 & fell further down to 11 Crore in March 2013.

• BSE another major Indian Bourse, which recorded a total of 7.7 Crore equity trades in January to March 2013, is placed at 7th spot. However like NSE, monthly equity trades on BSE fell to 2 crore in MARCH 2013, from 3 Crore in January 2013.

• While total number of listed companies is much larger in case of BSE, the exchange lags behind NSE, in terms of volume and value of trades.

• NSE is followed by China’s SHENZHEN SE & SHANGHAI SE, in the second and third positions respectively, with nearly thirty Crore equity trades each.
APPENDIX 2
Statistics & Applications to Finance.

Researcher, in his pursuit of estimating the population parameters [Mean (central tendency), standard deviation(σ) (Dispersion/ Scedasticity), Symmetric/Asymmetric, & kurtosis) of having described study of the population characteristics by studying the sample statistic (Mean (x), std deviation (s), coefficient of skewness to test Data set symmetric or not, kurtic nature of the curve, of the data distribution, under study, is assessed, purported to enhance, sound judgment and management principles benefiting all mankind.)

Brief of statistical metrics:-
I. Measures of central tendency:
Arithmetic Mean {Simple, Weighted, combined, moving }, Geometric mean{ Wealth ratio/Price Relative/Cumulative Wealth Index, time weighted Rate of Return}, Natural logarithms (lognormal) Returns and the Arithmetic average family.

II Measure of Dispersion
1. Traditional 5 number summary:
   Smallest value --------Q1--------Median Q2 -------- Q3----------Largest value
2. Variation
   a) Range(R) = H-L, H is highest & L is lowest value of given data distribution (discrete, continuous etc.)
   b) Inter Quartile Range : Q3 - Q1
   c) Variance: 
      \[ s^2 = \sum_{i=1}^{n} \left( \frac{x_i - \bar{x}}{n-1} \right)^2 \]
      As sample size increases, the difference between ÷ by N or N-1 becomes smaller and smaller
   d) Standard deviation (σ):- Traditional measure of risk, risk such as V.A.R. & other sophisticated tools not considered in this study.
e) Coefficient of variation \( C.V.\% = \frac{Std.Dev}{Mean} \times 100 \). Lower the C.V. \% better.

Eg. Rahul Dravid is consistent batsman compared to Sachin Tendulkar? Most critical measure for evaluation of performance metrics.

**III. SHAPE OF DISTRIBUTION:**

How data scatters itself in the plane/space, two types of distribution generally speaking

a. Discrete series /function/probability/mass/density/frequency. Distribution: stepwise, ladder distribution,

b. Discontinuous series etc.

Pertinent to our study, is **Continuous** distribution.

1. Continuous probability density function: for a curve, such that Area under curve, over interval, i.e. probability that X, falls into that interval can be found by summing the probabilities in that interval.

**Salient characteristic of this distribution**

1. Area under a continuous probability density \( f_n = 1 \).
2. \( P (a<= x<=b) \), that R.V. (X) value will fall into a particular interval from a to b is equal to the area under the density curve between the points (values) a & b.
3. Area under a continuous provability density \( f_0=1 \).
4. \( P (a<=x<=b) \), that R.V. (X) value will fall into a particular interval from a to b is equal to the area under the density curve between the points (Values) a&b.

**NOTE:** - **SHAPE OF DATA DISTRIBUTUION:**

1. Normal probability density function in which the curve is bell shaped, having a single peak. The mean if of distribution lies at the centre of curve and the curve is symmetrical around a vertical line erected at the mean. The tails of curve extended indefinitely parallel to horizontal axis.

Normal probability distribution function:

\[
\text{Rel.Freq: } \quad f(x) = \frac{e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}, \quad -\infty < x < +\infty
\]
\[ \pi = 3.1416, \quad e = \text{Euler’s Numbers}=2.71828 \]

- Also called “\text{GAUSIAN DISTRIBUTION}”: German Mathematician, Karl Friedrich Gauss”, early 19\textsuperscript{th} century, while studying daily changes in stock market index.

THE NORMAL LAW OF ERROR STANDS OUT IN THE EXPERIENCE OF MANKIND AS ONE OF THE BROADEST GENERALISATIONS OF NATURAL PHILOSOPHY. IT SERVES AS THE GUIDING INSTRUMENT RESEARCHERS IN THE PHYSICAL AND SOCIAL SCIENCES AND IN MEDICINE AGRICULTURE AND ENGINEERING. IT IS AN INDISPENSABLE TOOL FOR ANALYSIS AND INTERPRETATION OF BASIC DATA OBTAINED BY OBSERVATION AND EXPERIMENT.

I. CHARACTERISTIC OF A NORMAL DISTRIBUTION.

1. For each pair \((\mu, \sigma)\), the curve of normal probability density function is bell-shaped and symmetric.
2. Normal curve is symmetric around a vertical line erected at the mean \((\mu)\), with respect to area under it, i.e. 50\% of area of curve lies on both sides of mean and reflect mirror image of shape of curve on both sides of mean \((\mu)\).
   \[ \therefore P(x \leq \mu) = P(x \geq \mu) = 0.5 \]
3. Mean = Median = mode occurs, when highest value of probability density function occurs when value of a random variable \(x = \mu\).
4. The two tails extend to infinity in go to direction and theoretically never touches the horizontal axis.
5. Total area under the curve = 1 or 100\% 
6. \(\mu\) may be negative, zero or positive and it is the central location of normally distributed data set \((\sigma)\), the spread longer \(\sigma\) wider and flatter is normal curve.
II. STANDARD NORMAL DISTRIBUTION

* Use of Z the standardized normal variate

\[ Z = \frac{X - \mu}{\sigma} \]

A normal probability distribution with \( \mu=0 \) and \( \sigma=1 \) is the standard normal probability distribution.

\( X \sim N \) (Mean, various / standard deviation) i.e. \( X \sim N(\mu,\sigma) \) and we define, std normal variate \( Z \) such that

\[ Z = \frac{X - \mu}{\sigma} \quad \text{i.e.} \quad Z \sim \text{stdN}(0,1) \]

III. Lognormal Distribution :- (LN) : Refer Appendix for understanding basic mathematics, covered under the ambit of “Power of E”. Continuous compounding.

\[ A = Pe^{rn}, \quad S_t = S_0e^r \]

- One needs to also acquaint with Taylor series Expansion where in, first term yields bond duration and second term its convexity.

- Exponential function finds its utility for option pricing using Taylor series for finding Greeks of the B.S. Model [Black – Scholes]i.e. Delta, Gamma, Vega etc. are used in hedging, portfolio insurance, financial engineering and management.

- \( \ln(e^x) = e^{\ln x} = x \)

- ASSET PRICES (Eg. Stock prices tend to follow a log normal distribution. This distribution allows only \( x_i \geq 0 \).

- Log-normal is like the normal distribution but with thinner (fatter) tails. Hence log-normal distribution is skewed.

- \( \ln S_t = S_0e^{r}, \) \( r \) the rate of return being an input to the exponent function and \( S_0 \) being constant, now taking \( \ln \) we have normal distribution left, also as you know, the rate of return follows normal distribution and also taking natural logarithm of exponential function, leaves you with input \( r \).
<table>
<thead>
<tr>
<th>Moment</th>
<th>Description</th>
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<tbody>
<tr>
<td>1st</td>
<td>Moment: Mean</td>
</tr>
<tr>
<td>2nd</td>
<td>Moment: Variance</td>
</tr>
<tr>
<td>3rd</td>
<td>Moment: Skewness</td>
</tr>
<tr>
<td>4th</td>
<td>Moment: Kurtosis</td>
</tr>
</tbody>
</table>

Skewness is a measure of lack of symmetry in the data, and is based on a statistic, that is a function of curved differences around Arithmetic mean/Lognormal mean Returns.

Kurtosis is a measure of relative concentration of values, in the center of distribution as compared with tails, and is based on differences around the arithmetic mean, raised to fourth power.

<table>
<thead>
<tr>
<th>Right (Positive) Skewed Distribution</th>
<th>Left (Negatively) Skewed Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median $X_{\text{largest}}$ is greater than median $- X_{\text{smallest}}$</td>
<td>$X_{\text{smallest}} &gt; X_{\text{median}}$</td>
</tr>
<tr>
<td>Also $Q_3 &gt; X_{\text{longest}} &gt; Q_1 &gt; X_{\text{smallest}}$</td>
<td>$X_{\text{smallest}} &gt; Q_3 &gt; Q_1 &gt; X_{\text{largest}}$</td>
</tr>
<tr>
<td>75% of data distribution of observation between $X_{\text{smallest}}$ to $Q_3$</td>
<td>Few small observations distort mean towards left tail. Heavy clustering of observations at night (high) side.</td>
</tr>
<tr>
<td>25% long right whisker at upper end of scale [BOX – AND – WHISKER PLOT] graph of 5 number summary</td>
<td>Long left whisker contains distribution of only smallest 25% of observations.</td>
</tr>
</tbody>
</table>

| 75% of observation between $Q_1$ to $X_{\text{largest}}$ |

19TH Century British Statesman Benjamin Disraeli: “3 kinds of lies: LIES, DAMNEDLIES & STATISTICS”, How data vary around the mean (Avg/ Exponential lognormal mean returns).

<table>
<thead>
<tr>
<th>Empirical Rule</th>
<th>The Bienayme-Chebyshev Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>In a bell shaped distribution approx 68% if observation are contained within a distance of $\pm 1$ std deviation around mean</td>
<td>For heavily skewed data this is used</td>
</tr>
</tbody>
</table>
Approx 95% within ±2s.d & 99.7% within ±3s.d

1/20 observation will be beyond 2s.d from mean ∴ observation not contained in interval μ±2σ could be considered OUTLIERS 3/100 will be beyond 3 stddevn from mean and hence observation in interval of μ±3σ are almost always considered outliers.

Given any data set, regardless of shape, the % of observation that are contained within distances around mean, must be at least

\[ \left( 1 - \frac{1}{k^2} \right) \times 100\% \]

For e.g. If k = 2, at least 75% of all observation in any data set are contained within a distance of ±2 std deviation around mean At least \( \frac{15}{16} \) (88.89%) are contained within ±4 std deviation around mean

<table>
<thead>
<tr>
<th>TAB 1: Interval</th>
<th>THE BIENAYME-CHEBYSHEV RULE</th>
<th>EMPIRICAL RULE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(μ-σ,μ+σ)</td>
<td>At least 0%</td>
<td>Approx 68%</td>
</tr>
<tr>
<td>(μ-2σ,μ+2σ)</td>
<td>At least 75%</td>
<td>Approx 95%</td>
</tr>
<tr>
<td>(μ-3σ,μ+3σ)</td>
<td>At least 88.89%</td>
<td>Approx 99.7%</td>
</tr>
</tbody>
</table>

⇒ Lognormal distributions are infinitely invisible and a data set which arises from log normal distribution has a symmetric Lorenz curve.

⇒ Economic Income of 97-99% of population is distributed log normally, wireless communication exhibits gauss law distribution and coefficient of friction, wear and tear also exhibits lognormal distribution [compounding, multiplicative growth rate, real wealth ratio etc.] as performance indicator, to be better based on lognormal distribution analysis.

⇒ \( X = \log_e[1+k] \) is normally distributed as -∞<x<∞ & in continuous time it’s called G.B.M.(Geometric Brownian Motion) & stock price distribution is lognormal, which forms basis of BLACK-SCHOLES MODEL.
Remember, Lognormal, plays a crucial role in human & ecological risk assessment, hence every asset needs to exploit their basic properties.

⇒ Log normal returns & Geometric mean returns give fairly good estimate of Weighted Rate of returns, which simple Arithmetic mean fails to deliver.

⇒ It does not accept negative values

⇒ Also using Symmetry of returns \( \ln \left( \frac{P_t}{P_{t-1}} \right) = - \ln \left( \frac{P_{t-1}}{P_t} \right) \)

When returns are continuously compounded when daily/monthly price ranges are small then returns calculated using simple arithmetic mean and continuous compounding gives identical values. Such is the power of ‘e’. NATURAL LOGARITHMS [LN]: used for base e logarithms e is EULERS and \( e \approx 2.7182 \).

If \( X \sim N(\mu, \sigma) \)…………….A Taking log to the base e [ LN ] on both sides of ………….A. We get, \( \ln[x] \sim N(\mu, \sigma) \)…………….B Where X is a log normal random variable and \( \ln[x] \) is a normal variable.

⇒ \( \ln[X] = \mu \) is median of \( N(\mu, \sigma) \) ∴ \( X \sim \exp [N(\mu, \sigma)] \)

⇒ \( (1 + \frac{1}{n})^n = ? \).

Let \( n \in N \), using Binomial Theorem, for +ve index.

\[
(1 + \frac{1}{n})^n = 1 + n \cdot \frac{1}{n} + \frac{n(n-1)}{2!} \cdot \frac{1}{n^2} + \ldots \ldots \\
\Rightarrow \lim_{n \to \infty} (1 + \frac{1}{n})^n = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots \Rightarrow \text{Exponential series } = e.
\]

\[
\therefore e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots
\]

which implies \( e > 2 \)

Similarly \( \frac{1}{4!} + \frac{1}{5!} + \ldots + \frac{1}{n!} = \frac{1}{4\cdot3\cdot2} + \frac{1}{5\cdot4\cdot3\cdot2} + \ldots + \frac{1}{n!} < \frac{1}{2\cdot2\cdot2} + \frac{1}{2\cdot2\cdot2} + \ldots \)

\[
\Rightarrow \frac{1}{4!} + \frac{1}{5!} + \ldots + \infty \leq \lim_{n \to \infty} \left[ \frac{1}{2^3} + \frac{1}{2^4} + \ldots + \frac{1}{2^{n+1}} \right]
\]

\[
\leq \frac{1/8}{1 - \frac{1}{2}}
\]

\[
\leq \frac{1}{4}
\]

\[
\therefore e \leq 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} = \frac{211}{12}
\]
∴ e < 3
∴ 2 < e < 3 \Rightarrow e \approx 2.71828(\text{constant})

N.B. For n ≥ 2, we have

\[(1 + \frac{1}{n})^{nx} = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots + \infty\]

\[1 + \frac{x}{n}^{(nx1)} + \frac{1}{n^2} + \ldots + \infty\]

\[\therefore 1 + x + \frac{x(x-1)/n}{2!} + \frac{x(x-1)(x-2)}{n^3} + \ldots + \infty\]

\[\lim_{n \to \infty} (1 + \frac{1}{n})^{nx} = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e^x \ldots \text{Exponential series.}\]

1. \(P_t = P_0 e^{Rt}\) where \(R_t = \ln (1+k)\) & \(k = (e^{Rt} - 1) \times 100\%\)

Taking natural log on both sides,

2. G.M. Returns = Exp[ Avg log normal returns].

3. Gordon’s Constant growth model used:

\[1 + ke = \frac{D_1 + P_1}{P_0} = \frac{P_t}{P_0} \Rightarrow Ke\% = \{\frac{P_t}{P_0} - 1\} \times 100\]

Taking natural log on both sides,

\[\ln(1 + Ke) = \ln \left(\frac{P_t}{P_0}\right)\]

\[\therefore P_t = P_0 e^{Rt}\text{ where }e = \text{continuous compounding } R_t = \ln [1 + Ke]\]

Fundamental Pillar Of Finance

D.D.M : Dividend Discount Model( Gordon's Constant Growth Model)\(^1\)

CASE : I - Share Valuation with constant dividends:

The value of share with constant dividends is:

\[Po = \frac{D}{1 + ks} + \frac{D}{(1 + ks)^2} + \ldots + \frac{D}{(1 + ks)^n} n \to \infty (I)\]

Multiplying both sides by (1+Ks) gives:

\[Po = D + \frac{D}{1 + ks} + \ldots + \frac{D}{(1 + ks)^{n-1}} n \to \infty (II)\]

(II) – (I) Yields

\[Poks = D \left[1 - \frac{1}{(1+ks)}\right] n \to \infty +\]

\(^1\) Fundamental of Financial Management Prasanna Chandra Tata McGraw Hill pg:552-553
as $n \to \infty$, $\frac{1}{1+ks} \to 0$

so equation results in \[ P_0 = \frac{D}{ks} \]

CASE II: - Share Valuation with constant growth in dividends

\[ P_0 = \frac{D_1}{1+ks} + \frac{D_1(1+g)}{(1+ks)^2} + \ldots + \frac{D_1(1+g)^n}{(1+ks)^n+1} \quad n \to \infty \]

Multiply both sides $\frac{1+g}{1+ks}$ gives:

\[ P_0 \left( \frac{1+g}{1+ks} \right) = \frac{D_1(1+g)}{(1+ks)^2} \left[ 1 + \frac{D_1}{1+ks} \left( \frac{1+g}{1+ks} \right)^2 \right] \quad n \to \infty \]

Further simplifying:

\[ \frac{P_0(ks-g)}{1+ks} = D_1 \left[ \frac{1}{1+ks} - \frac{(1+g)^{n+1}}{(1+ks)^n+2} \right] \quad n \to \infty \]

as $n \to \infty$, $\frac{(1+g)^{n+1}}{(1+Ks)^n+2} \to \infty$ because $g < k$

hence

\[ \frac{P_0(ks-g)}{1+ks} = \frac{D_1}{1+ks} \]

this means

\[ P_0 = \frac{D_1}{ks-g} \]

Where, $P_0$ = Price per share at beginning of years, 0

$Y_0$ = Earnings, per share at end of year 0.

1-$b$ = Fraction of earnings from distribute by way of dividend.

$b$ = Fraction ploughed back into business.

$K$ = Rate of return required by shareholders.

$r$ = Rate of return earned on investments made by from.

$br$ = Growth rate of earnings and dividends.

Using dividend capitalism approach, the price per share is

\[ P_0 = \frac{D}{1+K_S} + \frac{D}{(1+K_S)^2} + \frac{D}{(1+K_S)^3} + \ldots + \frac{D}{(1+K_S)^n} \]
\[
E_0 (1-b) + E_1 (1-b) + E_2 (1-b) + \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ ld
\[
\begin{array}{ccc}
1 & E_0 + E_0 b k_e & E_0 (1 + b k_e) b = E_0 (1 + b k_e) \\
2 & E_0 (1 + b k_e) + E_0 (1 + b k_e) b k_e & E_0 (1 + b k_e) b^2 = E_0 (1 + b k_e) 2(1 - b) \\
\vdots & E_0 (1 + b k_e)^n & E_0 (1 + b k_e) b^n = E_0 (1 + b k_e)^2 (1 - b) \\
\end{array}
\]

\[K_e\text{ in the given equation is cost of Equity capital.}\]

\[P = \sum_{i=1}^{\infty} \frac{E}{(1 + K_e)^i} \text{ [D.D.M.]}\]

We can deduce that \[K_e = \frac{E_1}{P}\]

**Cost of Capital & Capital budgeting decisions:**

Besides simple Arithmetic mean, time weighted Rate of Return, we also have money weighted Rate of Return, which is that rate, which equates \[\sum \text{PV of cash inflows} = \sum \text{PV of cash outflows},\] as you experience in a firm’s capital budgeting exercise, instrumental in the firms cash operating from input, which reflects on price of share & growth.

It is the same rate of return which is the I.R.R. [Risk adjusted rate of return/ Discount Rate ] used in evaluating investment decisions. On CAPEX [Capital expenditure / budget] decisions which also paves way for growth and development of the market.

In our discussion on risk free rate, when it comes to C.M.L. (C.A.P.M.), we also make a cursory mention of this I.R.R.

**REFERENCES**

APPENDIX 3

Stochastic Calculus

1) EFFICIENT MARKET
An Efficient Market is defined as a market where there are large number of Rational, Profit-Maximizes, actively competing, with each trying to predict future market value of individual securities and where important current information is almost freely available to all participants. In an efficient market, competition amongst many intelligent participants, leads to a situation where, at any point in time, actual prices of individual securities already reflect the effects of information based on events that have already occurred, and on events, which as of now, the market expects to take place in future. In an Efficient Market at any point in time, actual price of a security will be a good estimate of its intrinsic value.

In Financial Markets, dynamics of Stock Prices are reflected by uncertain movements of their value over time. One possible reason for Random Behavior of Asset Price is the E.M.H. (Efficient Market Hypothesis)—which basically states two things:

a) The past history of stock price is fully reflected in present prices.
b) Markets respond immediately to any new information about the stock.

These two assumptions implies that stock price changes are a “Markov Process”, which means that Expected Future Value of a Stock depends only on its current price. Predictions remain uncertain and may be only expressed in terms of Probability Distributions. In this context, modeling stock price is concerned with modeling the arrival of new information, which affects the price. Therefore two important learning’s are: 1) Probability Distribution, and 2) Information. These play a major role in modeling future stock prices. In other words, the future price of a stock can be predicted within a certain level of exactitude, if one can anticipate new information about stock.

2) MARKOV PROPERTY
States that the Expected Value of a Random Variable St, conditional on all past events only depends on the immediate previous value of the Random Variable St-1. This property can be expressed as:

\[E[St/I t-1] = E [ St/St-1] , \text{ where I t-1 is the Information available up to t-1.}\]
3) **ITO’S LEMMA**

KIYOSHI ITO (September 7, 1915 --- November 2008):- Was a Japanese Mathematician whose work is now called ITO Calculus. The basic concept of this calculus is the ITO Integral and among the most important results is the ITO’s LEMMA. The ITO Calculus facilitates mathematical understanding of random events. His theory is widely applied in various fields and is perhaps best known for its use in Financial Mathematics.

Most important result about manipulation of Random Variable, that is required to ITO’s processes. It is as a function of the Random Variable, as what Taylor’s theorem is to function of deterministic variable.

Consider a function of two variables \((x, t)\), Taylor’s expansion of this function up to the second order is expressed as:

\[
\begin{align*}
\text{d} G &= \frac{\partial G}{\partial X} \text{d} X + \frac{\partial G}{\partial t} \text{d} t + \frac{1}{2} \left( \left( \frac{\partial^2 G}{\partial X^2} \right) (\text{d} X)^2 + \left( \frac{\partial^2 G}{\partial t^2} \right) (\text{d} t)^2 \right),
\end{align*}
\]

where

\[
\begin{align*}
\text{d} X &= a \text{ d} t + b \text{ d} B, \\
\text{d} G &= \frac{\partial G}{\partial X} (a \text{ d} t + b \text{ d} B) + \frac{\partial G}{\partial t} \text{ d} t + \frac{1}{2} \left( \left( \frac{\partial^2 G}{\partial X^2} \right) (b^2 \text{ d} t) \right) \\
\text{d} G &= (\frac{\partial G}{\partial X}(a) + \frac{\partial G}{\partial t}) \text{ d} t + \frac{1}{2} \left( \left( \frac{\partial^2 G}{\partial X^2} \right) (b^2) \right) \text{ d} t + (\frac{\partial G}{\partial X}(b) \text{ d} B)
\end{align*}
\]

This is the ITO’s LEMMA, THAT CAN BE APPLIED TO ANY STOCHASTIC VARIABLE THAT FOLLOWS AN ITO PROCESS.

4) **PRICE ADJUSTMENT RULE**

If there is a Market Surplus or Glut (Excess Supply), prices fall, ending the glut. Whilst a Shortage (Excess Demand), causes prices to rise. However instead of price adjustment—or, more likely, simultaneously with price adjustment—quantities may adjust: A Market Surplus leads to a cut back in quantity supplied, while a shortage causes a cut-back in the quantity demanded. The ‘Short Side’ of the Market dominates, with the quantity demanded constraining supply in the first case and limited quantity supplied constraining demand in the second.

5) **BROWNIAN MOTION**:- Robert. Brown a Botanist first realized the Brownian motion (also called WIENER PROCESS). NORBERT.WIENER (1894-1964) at MIT; in 1920’s. He explained Brownian motion in terms of Modern Probability Theory. Further Robert. Brown when trying to describe motion exhibited by particles immersed in a gas or liquid discovered this type of motion, hence named
after Brown. The particles were essentially being bombarded by molecules present within the matter causing displacement or movement.

**FISCHER BLACK (1938-1995)**: in 1975. Due to his early demise, Black did not share the Noble Prize awarded to Scholes & Merton in 1997.

**MYRON SCHOLES**: (Born in 1941) at a press conference at STANFORD, in October 1997, after the announcement that he and Merton had been awarded the NOBEL Prize for Economics.

**ROBERT C. MERTON**: Son of Sociologist Robert. K. Merton. Born in 1944, he was the first to understand fully the game-theoretic nature of BLACK-SCHOLES PRICING.

The Wiener Process has no drift but you would see one if you looked at a particular part of picture. Drift is easily obscured by volatility. As can be observed in case of Bajaj Auto, refer above, there seems to be upward drift, but as apparent drift is always produced by volatility, the evidence of drift is not strong from a theoretical perspective. Also drift cannot be estimated very well from the data, but volatility can be estimated from data, BLACK SCHOLES PRICING, ignores drift. It only uses volatility, which can be estimated from past data, which is enough to determine an ARBITRAGE PROCESS.

**Definition of Brownian Motion**: The Brownian Motion is a fundamental process that serves as a part of many different processes. It refers to either:

1) The physical phenomenon that small particles immersed in a fluid, move randomly, or

2) The Mathematical Models used to describe these random movements.

It is scaling limit of Random Walk in one dimension as the time-step goes to zero that is, the number of steps becomes large. The family portraits shows, that scaled random walk tends to a Brownian Motion when the number of steps at random in fig 3 increases.

**Properties of Brownian Motion**: 

1) Continuity: Bt has a continuous path and B (t = 0) = zero.

2) Normality: The increment of the Brownian process in the time interval of length t between two moments S & (S+t), is B S+t - Bs. This increment is Normally distributed, with Mean zero and Variance equal to time increment t.
3) Markov Property :- The conditional distribution $B(t)$ given information up to time $S < t$ depends only on the $B(S)$.

**Generalized Random Walk:-**

The generalized Random Walk also called “BROWNIAN MOTION WITH DRIFT”, is a Stochastic process $B_t$. For given constants $\mu$ and $\sigma$, the process has following form:

$$B_t = \mu t + \sigma W_t$$

Where $t$ represents time and $W_t$ is a random walk process as described before under the section “Random Walk”. $W_t$ can be written as:

$$W_t = \varepsilon \sqrt{t}$$

Where $\varepsilon$ is a random number drawn from a standard normal distribution. The process described by 5.3 above is Normally distributed with Mean $\mu \cdot t$ and Variance $\sigma^2 \cdot t$.

If we define mean of Brownian motion process as $E(B_t)$ and its variance as $V(X)$, we have:

$$E[B_t] = E[\mu t + \sigma W_t]$$

$$= E[\mu t] + E[\sigma W_t]$$

$$= \mu t + \sigma E[W_t]$$

This is because the function mean is linear. Using the fact that mean of Random Walk is zero, i.e.

$$E[W_t] = 0$$

we finally have:

$$E[B_t] = \mu t$$

For the Variance of the process $B_t$, we have, according to function variance we have:

$$\text{Variance } (B_t) = \text{Var}(\mu t + \sigma W_t)$$

$$= E[(\mu t + \sigma W_t)^2] - [E(\mu t + \sigma W_t)]^2$$

Using again the linearity property of mean function, first term in the right side of this equality is equal to:
\[ E \left[ (\mu t)^2 \right] + E \left[ (\sigma W_t)^2 \right] + 2 E \left[ \mu \sigma W_t \right] \]

\[ = (\mu t)^2 + \sigma^2 t \cdot E \left[ (W_t)^2 \right] + 2 \mu t \sigma E \left[ W_t \right] \]

The second term of equality is equal to mean of the process \( E[B_t] \), because of \( E[W_t] = 0 \), we have

\[ E \left[ (\mu t + \sigma W_t)^2 \right] = (\mu t)^2 + \sigma^2 E[W_t^2] + 0 \]

According to definition of Variance we have:

\[ E \left[ W_t^2 \right] = \text{Var} (W_t) + \left[ E (W_t) \right]^2 = t + 0 = t \text{, therefore:} \]

\[ E \left[ (\mu t + \sigma W_t)^2 \right] = (\mu t)^2 + \sigma^2 E \left[ W_t^2 \right] + 0 \]

\[ = (\mu t)^2 + t \]

Finally, \( \text{Var} (B_t) = (\mu t)^2 + t - (\mu t)^2 = t \)

The above figure shows the Generalized Brownian process given by [5.3], with its components. It consists of a random walk process (below) with a drift (line). The result is an increasing process if the constant \( \mu \) is positive or a decreasing one if the constant \( \mu \) is negative.

If we now compare the closing price process plot in Fig. with the generalized Brownian Motion in Fig

We can conclude that the two processes show the same behavior in time. This statement is the first Step towards the mathematical modeling of the stock’s closing price.

6) STOCK PRICE MODELING

Notice the similarities between Brownian Motion & the stock price process. However there is doubt about Brownian motion as a global model for stock behavior. This is because of property (2) of Brownian motion, which says that the process is Normally Distributed with mean equal to zero and hence negative with probability \( \frac{1}{2} \), whereas stock price normally grows at some rate. One way to solve this problem is to model the stock price as a sum of a positive, deterministic function of time and a Brownian Motion.
In the real world of Financial Markets, Investors & Financial Analysts are generally more interested in profit or loss of stock over a time period that is, increase or decrease in price, than the price itself. Therefore modeling a behavior of stock price can be made through its relative change in time. We use Stochastic Differential Equations (S.D.E.) to model stock returns, and its solution used to find a mathematical model of stock price.

**STOCK RETURN**

Akin to the particle being bombarded in Brownian Motion, Stock prices deviate from a steady state, as a result of being jolted by trades that is ask and bid in Financial Markets. Consider price $S_t$ at time $t$ and an expected rate of return $\mu$, then return or relative change in its price during the next period of time $dt$ can be decomposed in two parts:

Part 1 :- A predictable, deterministic, anticipated, non-stochastic part that is expected return from the stock held during a time period $dt$. This return $= \mu S_t dt$.

Part 2:- A stochastic and unexpected part, which reflects the random changes in stock price during time interval $dt$, as response to external effects such as unexpected news on stock.

A reasonable assumption is to take this contribution, as proportional to stock for some constant $\sigma$ and a random walk process $dB_t$, this unexpected part of return $= \sigma S_t dB_t$.

This definition of daily return leads to the stochastic differential equation followed by stock price.

$$d S_t = \mu S_t dt + \sigma S_t dB_t \quad \text{Aliter}$$

$$\frac{d S_t}{S_t} = \mu dt + \sigma dB_t$$

The stochastic differential equation 6.1 is Brownian Motion with “DRIFT “ Followed by stock price $S_t$. Equation 6.2 is Instantaneous rate of return on $S_t$. The return on $S_t$ in period of time $dt$ follows an ITO’S process, which can be written for every time interval $dt$ between two consecutive instants as follows:

$$\frac{d S_t}{S_t} = d(\ln S_t) = \ln(S_t) - \ln(S_{t-1}) = \ln(\frac{S_t}{S_{t-1}})$$

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therefore, \( \ln(St / St-1) = \mu dt + \sigma dB \)

**SOLUTION OF STOCHASTIC DIFFERENTIAL EQUATION:**

Solution \( St \) of formula can be found by applying the now famous ITO’s formula.

For any function \( G(X, t) \) of two variables \( X \& t \) where \( X \) satisfies following S.D.E.

\[
d X = a dt + b dB_t \quad \text{where} \quad a \text{ and } b \text{ are some constants,} \quad \text{and} \quad dB_t \text{ is a Brownian motion.}
\]

The general form of ITO’s formula is:

\[
d G = \left( \frac{\partial G}{\partial X} \right) a + \left( \frac{\partial G}{\partial t} \right) dt + \frac{1}{2} \left( \frac{\partial^2 G}{\partial X^2} \right) b^2 dt + \left( \frac{\partial G}{\partial X} \right) dB_t
\]

Let’s consider function: \(-G = \ln(St)\) where \( St \) satisfies formula and applying it we get:

\[
\frac{\partial G}{\partial S} = \frac{1}{St}, \quad \frac{\partial^2 G}{\partial S^2} = -\frac{1}{St^2}, \quad \text{and} \quad \partial G/\partial S = 0
\]

Inserting these into the ITO’s formula yields as follows:

\[
d (\ln St) = \left( \mu St^* \frac{1}{St} + \frac{1}{2} \sigma^2 St^2 \times (-\frac{1}{St^2}) \right) dt + \sigma St^* (\frac{1}{St}) dB_t
\]

\[
= \ln(St) - \ln(St-1) = \ln\left(\frac{St}{St-1}\right) = (\mu - \frac{1}{2} \sigma^2) dt + \sigma \epsilon \sqrt{dt}
\]

It follows that \( d\ln(St) \) has a Brownian Motion with DRIFT. It has a Normal Distribution with mean = \( (\mu - \frac{1}{2} \sigma^2) dt \) and variance \( \sigma^2 * dt \).

Solution of \( St \) can then be easily written as follows:

\[
\ln(St) = \ln St - 1 + (\mu - \frac{1}{2} \sigma^2) dt + \sigma \epsilon \sqrt{dt}
\]

Hence: \(-St = St-1 * \exp(\mu - \frac{1}{2} \sigma^2) dt + \sigma \epsilon \sqrt{dt}\)

The solution is Geometric Brownian Motion Model of future stock price. It can be extended to any period of time \( t \). In this case future stock price \( St \) can be derived from initial value \( S0 \), just by applying on period of time \( t \) when time \( dt = t \).

**PARAMETRIC ESTIMATION**
DRIFT & VOLATILITY of Stock, supposed to be known, and if not available needs to be estimated from Historical Data, which is the starting step undertaken in this study. For example refer Bajaj auto file, considering the daily lognormal returns of the Bajaj auto stock prices for the five year period under study, we obtained the mean daily return of 0.000954392 (0.095%) and daily volatility average 0.027302911 (2.73%). Our study delves into continuous-time series, hence the daily mean return obtained for the study period with respect to Bajaj Auto is penalized for the risk or volatility, therefore to get the annualized returns for the stock, hitherto referred as drift or annualized or adjusted mean return we proceed as under:

Step 1:- 0.095% = μ – (1/2 σ²). Hence we get the daily μ as equal to (0.095% + 0.5*σ²) = 0.13271165%.

Step 2:- Annualized Return = 0.13271165%*252 days in a year (assumed 252 trading days here throughout this study) = 33.44% (refer also the next section for this on volatility)

Step 3:- Annualized Historical volatility = daily mean standard deviation * sqrt(252 trading days in a year) = 0.027302911* sqrt(252) = 0.433420275 (43.34%)

VOLATILITY

Is a constant characteristic of a stock expressed as an annualized percentage. It gives an idea about stability of Stock Price. Relatively high volatility means that the stock price varies continuously within relatively large intervals, measured by the standard deviation of Price Returns (Fairly established standard practice). Practical way of estimating empirically, the Stock Volatility is to observe historical data at fixed intervals of time, say daily closing prices.

Denote Si as stock closing price at the end of ith trading period and τ is the length of time interval between two consecutive trading periods expressed in years, where τ = ti – ti – 1 for positive integer i. If u I is logarithm (natural) of daily return on stock price over short time interval τ that is, ui = ln(Si/Si-1) for i= 1,2,3,.................n

Then the unbiased estimator u bar of logarithmic returns is given by:

ubar = 1/n (Σui), And the estimator of standard deviation of ui’s is given as:-
\( v = \text{square root}\left[ \frac{1}{n-1} \sum (u_i - \bar{u})^2 \right] \text{ or} \)

\( \sqrt{\frac{1}{n-1} \sum u_i^2 - \frac{1}{n*(n-1)} \sum u_i^2} \) where \( \bar{u} \) is unbiased estimator of logarithmic returns \( u_i \).

We have seen that the estimator of standard deviation of daily returns on stock = \( \sigma \times \sqrt{\tau} \). It follows that \( \sigma \) can simply be estimated as \( \sigma = v / \sqrt{t} \)

With a standard error equal to \( \sigma / (\sqrt{2n}) \).

**DRIFT:** Using result given in above, and an unbiased estimator of daily return given above, we get the expected annual rate of return or drift \( \mu \) as follows:

\( \bar{u} = (\mu - \frac{1}{2} \sigma^2) \tau \), thus

\( \mu = (\bar{u} / \tau) + \left( \frac{1}{2} \sigma^2 \right) \)

**When is method of scaling Volatility appropriate?**

1) Returns are Independently & Identically distributed.

2) Prices are log normally distributed (that is their behavior is generated by Geometric Brownian Motion)

3) Volatility is calculated from log stock prices.

However, daily (or high frequency) stock prices are generally not independently and identically distributed, as well as having more noise than less frequency data. In this case Diebold et al (1998) showed that scaling volatility to a longer time horizon can amplify fluctuations in the data, scaled volatilities are often overestimated.

**VOLATILITY CLUSTERING:** Volatility is the conditional variance of Asset Return, a measure used to quantify the riskiness of an asset. Empirical studies reveal that volatility tends to increase with the square root of time, as time increases and the increase is continuous with it being rare to observe Volatility Jumps. Volatility is usually stationary, meaning it varies within a fixed range and is hence finite. There are a number of different types of Volatility (title of this study), such as Historical Volatility, which is calculated from past Asset Returns of asset or Implied Volatility, which is obtained inversely or indirectly (Black Scholes model for Pricing Options
In Financial Derivatives), from a Model that has been accepted. It is worth noting that Volatility does not actually imply a direction of dispersion.

Volatility Clustering is a phenomenon of spells of high Amplitude that alternates with spells of low amplitude. That is to say, if at time t, the volatility is high then, at next consecutive time t+1, the asset return will also tend to have high volatility. This characteristic contradicts Independent & Identically distributed assumption that is traditionally assumed to be a stylized fact of log returns. Empirical studies of Returns have shown that extreme values do not normally appear unpredictably but rather they happen after the occurrence of larger than normal movements in return value. Periods of high volatility in returns will commonly follow a “CRASH v/s BUBBLE” pattern swinging from higher than average positive values to lower than average negative values and back again. During periods of low volatility, also called “DOLDRUMS”, returns stay much closer to mean value with little deviation

**HISTORY OF HISTORICAL VOLATILITY SO FAR (SINCE 1971)**

**CHRONOLOGY:**

1) 1971:- Fixed Exchange Rate System (Bretton Woods Agreement) broke down leading to flexible currency exchange rates (this was the genesis of volatility in the currency markets).

2) 1973:- First oil crisis, created high volatility in Interest rates, and was followed by high Inflation – which not only marked the start of volatility in interest rates but also volatility in GDP.

3) 1987:- US STOCK MARKETS collapse in October, was harbinger of extreme volatility in the EQUITY MARKETS, this event was also a landmark event due to two reasons:–
   a) Portfolio Insurance Models as applied to equities became a major subject for analysis both for practitioners & academicians.
   b) For the first time Option Traders realized that the BLACK-SCHOLES MODEL of constant volatilities was unstable and the phenomenon of VOLATILITY SMILE was discovered.

4) 1989:- Japanese Stock Market (NIKKEI 225), crashed towards end of this year and within three years the Stock Index had lost more than 50% of its value, which led to a long and dark period in Japanese Capital Markets and real economy which lasted
for almost 11 years and its after effects caused high volatility in both the Equity & Bond markets.

5) 1994:- Bond markets in US, crashed which was eventually followed by 6 consecutive hikes in short term interest rates, and once again it was proved beyond doubt that, volatility is not only the preserve of equity markets but is also rampant in bond markets.

6) 1997:- The Asian Financial crisis in July 1997, brought back nightmares of Foreign Exchange volatilities which eventually wiped out about 75% of the dollar capitalization of equities in Malaysia, Korea, Indonesia, & Thailand.

7) 1998:- Russian debt default ignited, another round of financial crisis in the global financial markets and finally ended up with the bankruptcy of Long term Capital Management, the behemoth hedge fund.

8) 2000:- Finally after a long bull run the sunset on NASDAQ In early 2000 and the values of Internet, Technology & New Economy Stocks melted away in decline that had no precedent which lasted for almost four years, this was when the equity volatilities reached an all time peak (as measured by VIX—SO CALLED FEAR GAUGE)

9) 2008 Crisis and when will next crisis be?

**STOCK PRICE DISTRIBUTION**

Having so far covered the mathematical form of Geometric Brownian Motion, to better understand the model, we need to derive other necessary features like the Probability Density Function of Mean and Confidence intervals.

**LN DENSITY FUNCTION**

A Random variable Q has a ln distribution if V = ln(Q), had a Normal Distribution. Suppose that V is Q (m,s) that is, it is Normally Distributed with mean m and standard deviation s. The probability density function for V is given by:

\[ f(V) = \frac{1}{\sqrt{2\pi}s} \exp \left[ -\frac{1}{2}(V-m/s)^2 \right] \]

The lognormal probability density function of Random Variable can be obtained, realizing that for equal probability under Normal & Log normal probability density functions, incremental areas should also be equal, hence \( f(V)dV = h(Q)dQ \), hence dividing both sides by dQ we get:
\[ h(Q) = f(V) \frac{dV}{dQ} \]

Using the first order derivative for \( V = \ln(Q) \) i.e., \( dV = dQ/Q \). By putting \( dV \) in \( h(Q) \) we get:

Probability density function of lognormal distribution of variable \( Q \) as:

\[ h(Q) = \left( \frac{1}{\sqrt{2\pi s^2}} \right) \exp \left[ -\frac{1}{2} (\ln(Q) - m^2)^2 \right] \]

Mean Lognormal Distribution:

Using probability density function, mean of variable \( Q \), is given by the following integral

\[ E[Q] = \int Q h(Q) dQ \text{, by putting } Q = \exp(V) \text{ & } dQ = \exp(V) dV \text{ IN THE ABOVE INTEGRAL WE HAVE:} \]

\[ E[Q] = \int (Q/sQ \sqrt{2\pi}) \exp(-1/2(\ln(Q) - m^2)^2) dQ \]

\[ = \int \left( \frac{1}{s\sqrt{2\pi}} \right) \exp(V) \exp[-1/2(v-m/s)^2]dV \]

\[ = \int \left( \frac{1}{s\sqrt{2\pi}} \right) \exp[ - (v-m^2)^2/2s^2] \exp(ms^2+s^4/2s^2) dV. \]

\[ = \exp(m+(s^2/2)) \int \left( \frac{1}{s\sqrt{2\pi}} \right) \exp[ - (v-m-s^2)^2/2s^2] dV \]

The integral in this expression is integral of a Normal Density function with Mean \((m+s^2)\) & standard deviation \(s\) and is hence equal to 1. It follows that the expectation of lognormal variable \( Q \) is:

\[ E[Q] = \exp( m + (s^2/2) \]

**STOCK EXPECTED VALUE**

According to results of previous section, the variable \( Y = \ln St \) is Normally Distributed with Mean \( \ln(S0)+(\mu - \frac{1}{2} \sigma^2) \) * \( t \) and variance \( \sigma^2 \) * \( t \).

The Expected value \( E[St] \) of stock price at future time \( t \) is given by:

\[ E[St] = \exp[ \ln(S0) + (\mu - \frac{1}{2} \sigma^2) \ast t + \sigma^2 \ast t ] \]

\[ = \exp[(\ln S0) \ast \exp((\mu +\sigma^2/2) \ast t)] \]

\[ = S0 \ast \exp((\mu +\sigma^2/2) \ast t) \]
CONFIDENCE INTERVALS

The variable \( \ln (S_t) \) follows a Normal Distribution with Mean \( \ln (S_0) + (\mu - 1/2\sigma^2) \cdot t \) and Variance \( \sigma^2 \cdot t \). A confidence interval of 95% level of the variable \( \ln (S_t) \) is given by:

\[
[\ln(S_0) + (\mu - 1/2\sigma^2) \cdot t] - 1.96 \cdot \sigma \sqrt{t} \leq \ln (S_t) \leq [\ln(S_0) + (\mu - 1/2\sigma^2) \cdot t] + 1.96 \cdot \sigma \sqrt{t}
\]

Hence by taking exponentials in 8.5 equation we get the 95% confidence interval of lognormally distributed variable \( S_t \), which takes the following form.

\[
\exp(\ln(S_0) + (\mu - 1/2\sigma^2) \cdot t) - 1.96 \cdot \sigma \sqrt{t} \leq S_t \leq \exp(\ln(S_0) + (\mu - 1/2\sigma^2) \cdot t) + 1.96 \cdot \sigma \sqrt{t}.
\]

DRIFT AND NOISE

- Stock prices have been studied for over a century in order to detect technical patterns with very little success. But Index returns have been found to be a Random Walk with an upward drift.
- On a daily basis, or weekly basis, the upward drift or “INDEX DRIFT” as it is also called is indeed imperceptible.
- Mathematically, Index is considered as a “SUB-MARTINGALE”. A Martingale is a random series in which each number is equal to the previous number plus a random factor. For a Martingale, most accurate prediction for the next number is the previous number, because random factor is unpredictable and is only a source of error.
- A sub-Martingale is a class of Martingales in which the base number is steadily increasing over time, but still has a random component.
- Over longer periods, Index drift overwhelms noise:– The combination of noise and Index drift helps explain why both “Buy & Hold” and trading strategies work on Index. Buy and Hold focuses on drift and trading profits from noise. Both Drift & Noise have the potential to push the price upward.

MOMENTUM, MEAN-REVERSION AND MARKET CYCLES
Mean reversion is the tendency for a price or factor or Economic Indicator to revert to a mean value over time. For example the US G.D.P. rates tended to be mean reverting over a period of few years. Average annual rate for Us over past 28 years is 3%, and after years in which it is lower, it tends to rise and vice-versa.

Momentum is the opposite of Mean-Reversion, and is the tendency for a variable to keep moving in the same direction. Both Mean Reversion & Momentum are not mutually exclusive, so long as they exist in different time frames.

For over 80 years of available stock market data, neither Mean reversion ,Nor Momentum has been conclusively evidenced in major broad stock market indices, by any leading academic research.

7) **GEOMETRIC BROWNIAN MOTION – MODEL (G.B.M.)**

The assumptions on which G.B.M. model is most widely used in Stock Price Modeling, meet the Financial Market Laws & Rules, imposed by Market Efficiency Hypothesis. These rules and laws suppose that only present information about a stock being efficient is employed to determine the future price of this stock. According to G.B.M, the returns on a certain stock in successive and equal periods of time are independent and identically distributed random variable which is normally distributed, thus they form a Markov Process.

Theoretically, G.B.M, serves to be a good way of modeling future stock prices. Albeit the shortcoming of using this model for pricing behavior over shorter periods of time, but it is more accurate when used for modeling stock prices over longer periods of time. This is due to the fact that the expected rate of return and volatility of a stock are assumed to be constant. To assess model accuracy, it is hence recommended to model these parameters as a Stochastic function of time and not as constants. Thus G.B.M is a Stochastic Markov Process.

8) **STOCK PRICE BEHAVIOR**

**STOCK DEFINITION: -** In Finance, a Stock represents a share in the ownership of an Incorporated company. Stocks are evidences of ownership, or Equity. Investors buy stocks in the hope that it will yield Income by way of Dividends and Capital Appreciation, or growth in Value of such Capital Investments from time to time. Shares of widely held companies are traded on Stock Market. Stockholding is
popular because stocks represent ownership of capital that can be easily transferred by means of Organized Trading in Stock Markets.

**STOCK PRICE PROCESS**

Refer Fig Below :- which displays how the price of a Financial Stock varies with time.(Example Zee Entertainment)

The Horizontal (x axis), represents time in years, and the Vertical (y axis) is the daily closing price of a stock during one year.

![Stock Price Chart]

The price behavior shows the same behavior as a Stochastic Process called “Brownian Motion”. Thus some properties of stock price process can be derived from those of Brownian motion process.

**RANDOM WALK (DRUNKARDS WALK):** Is the first step in understanding Brownian Motion. A Random Walk is a formalization of the intuitive idea of taking successive steps. The simplest random walk is a path constructed according to following rules:-

For an Integer n, n>0, we define Random Walk Process at time t, \{ W_n(t), t > 0 \} as follows:-

1) Initial value of process is \(-W_0 (0) = 0\).
2) The layer spacing between two successive jumps is equal to 1/n.
3) The Up & Down jumps are equal, each having same size of $1/\sqrt{n}$, occurring with equal probability.

In other words, if we consider a sequence of Independent Binomial Variable $X_i$, taking values +1 or -1, with equal probability that is $1/2$, then value of Random Walk at the $i^{th}$ step is defined as follows:

$$W_n(i/n) = (W_n(i-1/n) + (X_i/\sqrt{n}) \text{ for all } I >= 1$$

The above figure shows the first two steps of $W_n$. At time 1, process has only two values with same probability. It moves up to value $\sqrt{1/n}$ with 0.5 probability and moves down to value $-1/\sqrt{n}$ with same 0.5 probability. In this case the Mean & Variance of process are:

$$E[W_n(1)] = 0.5 \times 1/\sqrt{n} + 0.5 \times (-1/\sqrt{n}) = 0.$$  

$$\text{Variance} \ [W_n(1)] = E \left[ W_n(1) - E(W_n(1)) \right]^2 = 0.5 \times (1/\sqrt{n} - 0)^2 + 0.5 \times (-1/\sqrt{n} - 0)^2 = 1/n.$$  

Using same procedure to obtain the value of Random Walk at time 2, which equals $2/n$. Random Walk at time 2 is obtained as follows:

$$W_n(2/n) = (W_n(1/n) + (X_i/\sqrt{n})$$

As shown in figure above, $W_n(2/n)$ can have three different states:

The state $2*(1/\sqrt{n})$ with probability 0.25 if $W_n(1/n) = \sqrt{n}$ and $X_i = +1$  

The state 0 with probability $2*0.25 = 0.5$ if from state $\sqrt{n}$ the random walk goes down that is, $X_i = -1$, or jumps up from state $-1/\sqrt{n}$ that is $X_i = +1$.

In the state $-2/\sqrt{n}$ process goes down from state $-1/\sqrt{n}$ with probability 0.25, and here $X_i = -1$.

The Mean & Variance of Random Walk process after second step can be calculated in same way as before. Thus we have:

$$E[W_n(2/n)] = (2/\sqrt{n})(0.25) + 0*0.5 + (-2/\sqrt{n})(0.25) = 0 \text{ and hence the variance:}$$
Variance \[ W_n \left( \frac{2}{n} \right) \] = \( E \left[ W_n \left( \frac{2}{n} \right) - E \left( W_n \left( \frac{2}{n} \right) \right) \right]^2 \)

\[ = 0.25 \left( -2 \sqrt{n} \right)^2 + 0.25 \left( 0 \right)^2 + 0.25 \left( 2 \sqrt{n} \right)^2 = \frac{2}{n}. \]

As is illustrated using various plots in this study, the process starts from value zero and randomly makes a series of three successive “up” jumps followed by 5 successive down jumps. At the end of random walk seems to show a different behavior like an up and down process. But what does the random walk look like if \( n \) gets larger and larger that is the intervals of time becomes smaller and smaller?

Looking at the above familial portraits, appear to be setting down towards something as the number of steps \( n \) increases. The moves of Size \( \frac{1}{\sqrt{n}} \) seems to force some kind of convergence. According to probability theory the process \( W_n(t) \) has a Normal Distribution with Mean zero & Variance equal to \( t \). Of course this needs proof.

**PROOF:**-Let \( X_1, X_2, \ldots X_n \), be a series of \( n \) independent and identically distributed Binomial Random Variables taking values -1 and +1 with probability equal to 0.5.

Starting from \( W_n(0) = \text{zero} \), we have:

\[ W_n(\frac{1}{n}) = W_n(0) + \frac{X_1}{\sqrt{n}} = \frac{X_1}{\sqrt{n}} \]

\[ W_n(\frac{2}{n}) = W_n(\frac{1}{n}) + \frac{X_2}{\sqrt{n}} \text{ which equals } = \frac{X_1}{\sqrt{n}} + \frac{X_2}{\sqrt{n}} = \frac{1}{\sqrt{n}}(X_1 + X_2) \]

\[ \cdots \]

\[ W_n(i-1/n) = W_n(i-2/n) + \frac{X_i}{\sqrt{n}} = \frac{1}{\sqrt{n}}(X_1 + X_2 + \cdots + X_i) \]

\[ W_n(i/n) = W_n(i-1/n) + \frac{X_i}{\sqrt{n}} = \frac{1}{\sqrt{n}}(X_1 + X_2 + X_3 + \cdots + X_i) \text{ for all } i \geq 1 \]

And so using recursive formulae, we get value of Random Walk at time after \( n \) steps starting from initial value at time zero \( (W_n(0)) \) that is:-

\[ W_n(t) = \frac{1}{\sqrt{n}} \sum X_i \]
We multiply Numerator & Denominator, by $\sqrt{t}$, we have:

$$W_n(t) = \sqrt{t} \left( \sum Xi / \sqrt{nt} \right)$$

According to the Central Limit Theorem (C.L.T.), Equation tends to standardized Normal Distribution with Mean zero and Variance 1 i.e. $N(0,1)$. Hence random walk process $W_n(t)$ tends to $N(0,t)$

Stochastic Calculus is very important in Mathematical Modeling of Financial processes, due to the underlying random nature of Financial Markets. Most academic articles in Finance have a ‘pure’ mathematical theme.

**THE MARKOV PROPERTY**

The Expected Value of Random Variable $S$ (Stock price), conditional upon all of the past events only depends on the previous value $S(t-1)$. This is the Markov Property.

We say that the Random Walk, has no memory (forgetfulness property or Memory less), beyond where it is now. Note that it does not have to be the case that the expected value of random variable $S$, is same as previous value.

This can be generalized to say that, given information about $S_j$, for some values of $1 \leq j < 1$, then the only information that is of use to us in estimating $S_i$ is the value of $S_j$ for largest $j$ for which we have information. All the financial models have Markov property. This is of fundamental importance in modeling in Finance. Also we will show examples where system has a small amount of memory, meaning that one or two other pieces of information are important. And couple of examples where all of Random Walk Path contains relevant information.

**THE MARTINGALE PROPERTY**

If in a coin tossing experiment say, you know how much money you have won say after the fifth toss. Your expected winnings after sixth toss, and indeed any number of tosses, if we keep playing, are just the amount you already hold. That is the conditional expectation of your winnings at any time in the future is just the amount you already hold.

$$E[S_i / S_j, j<i] = S_j.$$  

**QUADRATIC VARIATION**
We now define the quadratic variation of random walk. This is defined by: \( \Sigma (S_j - S_{j-1})^2 \). Because you either win or lose an amount $1 after each toss, \( |S_j - S_{j-1}| = 1 \). Thus quadratic variation is always i. \( \Sigma (S_j - S_{j-1})^2 = i \), we use the coin tossing experiment, that will lead us to a continuous time random walk.

**BROWNIAN MOTION**

Let us restrict time allowed for 6 tosses to a period \( t \), so each toss will take a time \( t/6 \), second the size of the bet will not be $1, but \( \sqrt{t/6} \), this new experiment clearly still possesses, both the Markov & Martingale properties, and its quadratic variation measures over the whole experiment is \( \Sigma (S_j - S_{j-1})^2 = 6 \ast (\sqrt{t/6})^2 = t \). The experiment is set such that the quadratic variation is just the time taken for the experiment. We can speed up the game by having \( n \) tosses in the allowed time \( t \), with an amount \( \sqrt{t/n} \), riding on each throw. Again the Markov & Martingale properties are retained and the quadratic variation is still \( \Sigma (S_j - S_{j-1})^2 = n \ast (\sqrt{t/n})^2 = t \). Now making \( n \) larger and larger, speeding up the game, decreasing the time between tosses, with a smaller amount for each bet. But we choose new scaling very carefully, the time-step is decreasing like \( 1/n \), but bet size only decreases by \( 1/\sqrt{n} \).

As we go to limit \( n = \infty \), the resulting random walk stays finite. It has an expectation, conditional on a starting value zero, of \( E[ S(t)] = 0 \), and a variance \( E[ S(t)^2] = t \). \( S(t) \) is used to denote the amount you have won or the value of random variable after a time interval \( t \). The limiting process for this random walk as the time steps go to zero is called Brownian Motion, and will denote it by \( X(t) \).

The important properties of Brownian motion are as follows:

1) **Finiteness:**- Any other scaling of the bet size or ‘increments’ with time step would have resulted in either a random walk going to \( \infty \) in a finite time, or a limit in which there was no motion at all. It is important that the increment scales with square root of time step.

2) **Continuity:**- Paths are continuous, there are no discontinuities. Brownian Motion is the continuous –time limit of our discrete time random walk.

3) **Markov:**- The conditional distribution of \( X(t) \) given information up until \( r < t \) depends only on \( X(r) \).

4) **Martingale:**- Given information up until \( r < t \) the conditional expectation of \( X(t) \) is \( X(r) \).
5) Quadratic Variation:- If we divide the time 0 to t, in a partition with n+1 partition points ti = it/n, then \( \sum (X(tj) - X(tj-1))^2 \) tends to t (technically almost surely).

6) Normality:- Over finite time increments ti-1 to ti, \( X(Ti) - X(ti-1) \) is Normally Distributed with mean zero and variance ti - ti-1.

Having built up the idea and properties of Brownian Motion from a series of experiments, we can discard the experiment to leave the Brownian Motion defined by its properties. These properties will be very important for our Financial Models.

**STOCHASTIC INTEGRATION:-**

We define a stochastic integral by:

\[
W(t) = \int f(\tau) dX(\tau) = \text{Lim} \sum f(tj-1)(X(tj) - X(tj-1))
\]

with \( t j = jt/n \). The function f(t) which is being integrated is evaluated in the summation at left hand point tj-1. It will be crucially important that each function evaluation does not know about the random increment that multiplies it, that is the integration is non-anticipatory. In financial terms, we will see that we take some action such as choosing a portfolio and only then does the stock price moves. This choice of integration is natural in finance, ensuring that we use no information about the future in our current actions.

**STOCHASTIC DIFFERENTIAL EQUATIONS (S.D.E.)**

Stochastic Integrals are important for any theory of stochastic calculus since they can be meaningfully defined (and this definition leads to some important properties). It is very common to use a shorthand notation for expressions such as:

\[
W(t) = \int f(\tau)dX(\tau)
\]

That short hand comes from differentiating and is \( dW = f(t) dX \)

Here \( dX \) IS INCREMENT IN x that is a normal random variable with mean zero and standard deviation \( \sqrt{dt} \). Equations are meant to be equivalent. One of the reasons for this shorthand is that equation looks a lot like an ordinary differential equation. We do not do further step of dividing by dt to make it look exactly like an ordinary differential equation because then we would have the difficult task of defining \( dX/dt \). Pursuing this idea further, imagine what might be meant by:

\[
dW = g(t)dt + f(t)dX.
\]

This is simply short hand for:

\[
W(t) = \int g(r)dr + \int f(r)dX(r)
\]
Equations like this are called stochastic differential equations. Their precise meaning comes however from the technically more accurate equivalent stochastic integral

**THE MEAN SQUARE LIMIT**

This term is useful in the precise definition of stochastic integration.

Examine the quantity:\[ E[ \sum (X(t_j) - X(t_{j-1}))^2 - t^2] \]

Where \( t_j = jt/n \). This can be expanded as:

\[
E \left[ \sum (X(t_j) - X(t_{j-1}))^4 + 2 \sum \sum (X(t_j) - X(t_{j-1}))^2 (X(t_j) - X(t_{j-1}))^2 - 2t \sum (X(t_j) - X(t_{j-1}))^2 + t^2. \right.
\]

Since \( X(t_j) - X(t_{j-1}) \) is Normally distributed with mean zero and variance \( t/n \), we have:

\[
E \left[ (X(t_j) - X(t_{j-1}))^2 \right] = t/n \quad \text{and} \quad E \left[ (X(t_j) - X(t_{j-1}))^4 \right] = 3t^2 / n^2
\]

Thus becomes:

\[
n * \frac{3t^2}{n^2} + n(n-1) \frac{t^2}{n^2} - 2t n \frac{t}{n} + t^2 = 0 \frac{1}{n}.
\]

As \( n \) tends to \( \infty \) this tends to zero. We therefore say that \( \sum (X(T_j) - X(t_{j-1}))^2 = t \), in the mean square limit. This is often written for obvious reasons as

\[
\int (dX)^2 = t. \quad \text{Hence equality referred to subsequently is equality in mean square sense.}
\]

**FUNCTIONS OF STOCHASTIC VARIABLES AND ITO’s LEMMA**

If we consider a function:- \( F(X) = X^2 \), if it's true, then \( dF = 2X dX \) ? NO. The ordinary rules of calculus do not generally hold in a stochastic experiment? Then what are the rules of calculus? Thus we set about to derive the most important rule of stochastic calculus. ITO’S LEMMA. The derivation is Heuristic rather than being rigorous, bit its transparent, we play with various time scales, first time scale is very very small. We will denote this by \( dt/n = h \).

This time scale is so small that the function \( F(X(t+h)) \) can be approximated by a TAYLOR SERIES.
\[ F(X(t+h) - F(X(t)) = (X(t+h)-X(t))d F/d X (X (t))+1/2 (X(t+h)-X(t))^2 d^2 F/d X^2 (X (t)) + \cdots \]

From this it follows and using relevant approximations the above expression approximates to simply,

\[ F(X(t+nh)) - F(X(t)) = F(X(t+dt))-F(X(t)) \]

The second is just the definition of \( \int (d F / d X) * d X \), and the last is \( 1/2 d^2 F / d X^2 (X (t)) \)dt in mean square sense. Thus we have:

\[ F( X(t+dt)) - F( X(t)) = \int (d F / d X )( X (r) )d X (r) + 1/2 \int d^2 F / d X^2 (X (r)) ) d r \]

We can now extend this to longer time intervals from zero up to \( t \), over which \( F \) does vary substantially to get:

\[ F (X(t)) = F( X(0) ) + \int d F / d X ( X (r) ) d X (r) + 1/2 \int d^2 F / d X^2 (X (r)) ) d r \]

This is the integral version of ITO’s LEMMA, which is usually written as:

\[ d F = d F /d X + 1/2 d^2 F / d X^2 )dt \] (ITO’S LEMMA)
APPENDIX 4:
NumXL

Part: - I: NumXL

INTRODUCTION: - NUMXL is a suite of time series add-ins for Microsoft Excel. Once loaded, NumXL integrates scores of time series functions, along with a rich set of user interfaces and tools to assist in data analysis.

Module: - Description

1. Data Preparation
2. Descriptive Statistics
3. Time series Smoothing
4. Correlogram Analysis
5. Modeling
6. Calibration
7. Residual Diagnosis
8. Forecast.

Module 1:- Data preparation:-

a) Data layout in Excel b) Data Sampling c) Stationarity d) Homogeneity e) Outliers (Influencers) f) Concentration of Values.

a) Data layout in Excel: - Display the dates and closing stock prices in adjacent columns in the same spreadsheet. Although the date component is not needed for modeling, it gives us a general idea about the chronological order of the values. We would choose for our study throughout ascending order, which means the first value corresponds to the earliest observation. Num XL assumes an ascending order by default.

b) Data Sampling: - We begin by examining the sampling assumptions. A Time Series will generally contain observations that are equally spaced over time, where the value of each observation is available (that is there are no missing values). Please note that the sample period is the trading day (not the calendar day). We need to visually inspect the data to see that it meets the sampling assumptions, defined by Econometric & Time series theories:

(Source: - NumXL free download)
1) Is the underlying process stable? (Homogeneity).
2) Does the Variance & Auto-Covariance remain the same throughout the sample span? (Stationarity).
3) Do we have observations with unusual values? (Outliers/Influencers).
4) Are the values of observations well spread out?

c) Stationarity: - For this, we are mainly concerned with the stability of Variances&Covariances throughout the sample. This assumption is pivotal for time series theories, so how do we check for it? Paradoxically, we begin by testing for non-stationary conditions, primarily: 1) The presence of unit root (RANDOM WALK) &/or, 2) The presence of deterministic trend. If we cannot find them we conclude that the data is stationary. For this we plot using original data for a deterministic trend or random walk (possibly with a drift).

d) Homogeneity: - We need to ascertain whether the underlying process underwent any structural changes during the span of the sample data under observation. Structural changes are those changes caused by events that permanently alter the statistical properties of the stochastic process. A structural change can be triggered by new changes in policies, passing new laws, or any major development (exogenous) during the span of the sample. To examine the homogeneity or lack thereof, we shall look at the data plot along with WMA (Weighted Moving Average) & EWMA (Exponentially Weighted Moving Average), and try to identify any permanent changes in the mean, variance or any signs of trend or Random Walk. We shall use the 20 day (Window—h) equally weighted moving average along with the original data, to deduce any evidence of a sudden permanent change in the underlying process mean.

e) Outliers: - An outlier is an observation, which is numerically distant from rest of data that is one that appears to deviate markedly from other members of the sample in which it occurs. The mere presence of outliers in our data may change the mean level in the uncontaminated time series, or it might suggest that the underlying distribution has fat tails. A quick way to do this is to use the robust measure of IQR (Inter Quartile Range which is Q3-Q1.

Lower limit: - Q1 – 1.5*IQR Upper limit: - Q3 + 1.5*IQR
f) Concentration of values: - To detect the issues associated with the concentration of values, we ask the following questions.

- Are the volatility/ Variance changing in relation to observation levels?
- Are the data values capped or floor leveled?
- Does the distribution show a skew in either direction?

If we find any of the above, we need to employ, data transformation techniques on the input data stream, such as the Box-Cox Transformation. Statisticians George Box & David Cox developed a procedure to identify an appropriate exponent lambda, which is used to transform the data into a normal shape. The lambda value indicates the power to which all data should be raised in order to do this, the Box-Cox Transformation searches from $\lambda = -5$ to $\lambda = +5$ until the optimal value for lambda is obtained. We have already incorporated this in the data by choosing the value of lambda as equal to zero, implying by doing so we are getting our data values close to Normality. The log (Box-Cox with zero lambdas) transformation improves the distribution of values, especially at the right tail.

Goal: - We wish to have the values of the observations to be distributed close to a Normal Distribution.

**ARCH Model**

ARCH models based on the variance of the error term at time it depends on the realized values of the squared error terms in previous time periods. The model is specified as:

$$y_t = u_t$$

$$u_t \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2$$

This model is referred to as ARCH ($q$), where $q$ refers to the order of the lagged squared returns included in the model. If we use ARCH (1) model it becomes

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2$$

(3)

Since $h_t$ is a conditional variance, its value must always be strictly positive; a negative variance at any point in time would be meaningless. To have positive conditional...
variance estimates, all of the coefficients in the conditional variance are usually required to be non-negative.

\( \alpha_0 > 0, \quad \alpha_1 \geq 0 \)

**GARCH Model**

Bollerslev (1986) and Taylor (1986) developed the GARCH \((p, q)\) model. The model allows the conditional variance of variable to be dependent upon previous lags; first lag of the squared residual from the mean equation and present news about the volatility from the previous period, which is as follows:

\[
h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}
\]

(4)

In the literature most used and simple model is the GARCH \((1, 1)\) process, for which the conditional variance can be written as follows:

\[
h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1}
\]

(5)

Under the hypothesis of covariance stationarity, the unconditional variance \(h\) can be found by taking the unconditional expectation of equation 5.

We find that

\[
h = \alpha_0 + \alpha_1 h + \beta_1 h
\]

(6)

Solving the equation 5 we have

\[
h = \frac{\alpha_0}{1 - \alpha_1 - \beta_1}
\]

(7)

For this unconditional variance to exist, it must be the case that \(\alpha_1 + \beta_1 < 1\) and for it to be positive, we require that \(\alpha_0 > 0\).
**Exponential GARCH**

Exponential GARCH (EGARCH) proposed by Nelson (1991) who has form of leverage effects in its equation. In the EGARCH the following form gives model the specification for the conditional covariance:

$$
\log(h_t) = \alpha_0 + \sum_{j=1}^{q} \beta_j \log(h_{t-j}) + \sum_{i=1}^{p} \alpha_i \frac{u_{t-i}}{\sqrt{h_{t-i}}} + \sum_{k=1}^{r} \gamma_k \frac{u_{t-k}}{\sqrt{h_{t-k}}}
$$

(9)

Two advantages stated in Brooks (2008) for the pure GARCH specification; by using $\log(h_t)$ even if the parameters are negative, will be positive and asymmetries are allowed for under the EGARCH formulation.

In the equation $\gamma_k$ represent leverage effects which accounts for the asymmetry of the model. While the basic GARCH model requires the restrictions the EGARCH model allows unrestricted estimation of the variance (Thomas and Mitchell2005:16).

If $\gamma_k < 0$ it indicates leverage effect exist and if $\gamma_k \neq 0$ impact is asymmetric. The meaning of leverage effect bad news increase volatility.

Applying process of GARCH models to return series, it is often found that GARCH residuals still tend to be heavy tailed. To accommodate this, rather than to use normal distribution the Student’s t and GED distribution used to employ ARCH/GARCH type models (Mittnik et al. 2002:98).

The news impact is asymmetric on the other words bad news increase volatility. In the E-GARCH model negative and significant leverage effect parameter indicating the existence of the leverage effect in returns. If GED parameter ($r$) equals two it means normal distribution if it is less than two it means leptokurtic distribution. In all models $r$ is less than two and statistically significant which indicate that RPX is leptokurtic. This result is consistent with the skewness values shown in Table 1.

After all model are estimated ARCH effect is tested and except GJR-GARCH model in normal distribution is rejected at 10% level, the null hypothesis that there is no ARCH effect cannot be rejected for all models (see: Appendix).
The in-sample evidence provides the history performance of models. We estimate estimated variance for all models for full sample period using static forecast. Then we compare forecasting performance of models which are used in this paper. We consider four statistics as the forecasting error criterions which are employed in Wang and Wu (2012). Four measures are used to evaluate the forecast accuracy for full sample namely, Mean Square Error (MSE) and Mean Absolute Error (MAE):

\[
MSE_1 = n^{-1} \sum_{t=1}^{n} (\sigma_t^2 - \hat{\sigma}_t^2)^2
\]

\[
MSE_2 = n^{-1} \sum_{t=1}^{n} (\sigma_t - \hat{\sigma}_t)^2
\]

\[
MAE_1 = n^{-1} \sum_{t=1}^{n} |\sigma_t^2 - \hat{\sigma}_t^2|
\]

\[
MAE_2 = n^{-1} \sum_{t=1}^{n} |\sigma_t - \hat{\sigma}_t|
\]

Where, \( n \) is the number of forecasts, \( \sigma_t^2 \) is the actual volatility and \( \hat{\sigma}_t^2 \) is the volatility forecast at day \( t \).

**Conclusion**

This paper examined three GARCH models namely GARCH, GJR-GARCH and EGARCH model with three different distributions which are Normal, Student-t and Generalized Error distributions for comparing their forecasting power for volatility of the return of Czech stock market, PX index.

All models are employed and their coefficients are interpreted. The results show that significant ARCH and GARCH effects are present in the data indicates that return volatility can be characterized by significant persistence and asymmetric effects consistent with papers Rockinger and Urga (2012), Haroutounian and Price (2010), Vošvrda And Žikeš (2004) which has GARCH type models to Czech stock market.

Finally we compare the in-sample-forecasting performance of all nine models. The evidence shows that the EGARCH model has a best-forecast performance under most of the loss criterions.
### Legend: - Description of Various Parameters in these Models

<table>
<thead>
<tr>
<th>SNO.</th>
<th>SYMBOL</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>$\mu$</td>
<td>LONG-RUN MEAN (mu) (all models)</td>
</tr>
<tr>
<td>2)</td>
<td>$\varnothing 1$</td>
<td>First Coefficient of AR Component (ARMA MODEL)</td>
</tr>
<tr>
<td>3)</td>
<td>$\varnothing 1$</td>
<td>First Coefficient of AR Component (ARMA MODEL)</td>
</tr>
<tr>
<td>4)</td>
<td>$\sigma$</td>
<td>Standard deviation of Residuals/Innovation (ARMA MODEL)</td>
</tr>
<tr>
<td>5)</td>
<td>LLF</td>
<td>Uses the log-likelihood method to measure goodness of fit (all models)</td>
</tr>
<tr>
<td>6)</td>
<td>AIC</td>
<td>Uses the Akaike Information criterion method to measure goodness of fit.</td>
</tr>
<tr>
<td>7)</td>
<td>CHECK</td>
<td>examines the values of model’s parameters for stability, stationarity, invertibility and causality. (Value 1 mostly ideal for all models)</td>
</tr>
<tr>
<td>8)</td>
<td>KURTOSIS</td>
<td>Excess kurtosis (aka fat tails)</td>
</tr>
<tr>
<td>9)</td>
<td>NOISE?</td>
<td>Determines if observations are not significantly Auto-correlated.</td>
</tr>
<tr>
<td>10)</td>
<td>NORMAL?</td>
<td>Determines if observations are Normally Distributed.</td>
</tr>
<tr>
<td>11)</td>
<td>ARCH?</td>
<td>Determines if observations exhibit a significant ARCH Effect.</td>
</tr>
<tr>
<td>12)</td>
<td>TARGET</td>
<td>Targets the desired sample statistic value.</td>
</tr>
<tr>
<td>13)</td>
<td>SIG?</td>
<td>Determines if the calculated statistics are significantly different from the Target value.</td>
</tr>
<tr>
<td>14)</td>
<td>$\alpha_0$</td>
<td>Constant in the conditional volatility equation. (GARCH MODELS)</td>
</tr>
<tr>
<td>15)</td>
<td>$\alpha_1$</td>
<td>First coefficient of ARCH component (GARCH MODELS)</td>
</tr>
<tr>
<td>16)</td>
<td>$\beta_1$</td>
<td>First coefficient of GARCH component (GARCH MODELS)</td>
</tr>
<tr>
<td>17)</td>
<td>$\nu$</td>
<td>Degrees of freedom of the Innovations/Shocks distribution. (t distribution)</td>
</tr>
<tr>
<td>18)</td>
<td>$\lambda$</td>
<td>Risk-Premium lambda (GARCH M MODEL)</td>
</tr>
<tr>
<td>19)</td>
<td>$\gamma_1$</td>
<td>First Leverage coefficient (EGARCH MODEL)</td>
</tr>
</tbody>
</table>
GARCH Analysis:-

Enables us to model and forecast the conditional variance or volatility, instead of conditional mean of a variable. The analysis of conditional variance is useful on account of a) pricing of options b) improving the estimation of forecast intervals.

Here the conditional variance in time t depends on past errors, and variances. They are designed to model time varying volatility, in particular. Volatility Clustering, a feature often displayed by financial market series. The variance at time t is expected to be higher when past errors and variances were higher in past and vice versa.

The phenomenon of time varying volatility is well known and generated a vast body of econometric literature following the seminal contributions by Engle (1982), Bollerslev (1986) & Taylor (1986), introducing the (generalized) Auto regressive Conditional heteroscedastic (GARCH) process and stochastic volatility models respectively.

Descriptive Statistics: - Example Hindustan Unilever (2009-2014)

<table>
<thead>
<tr>
<th>Descriptive Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AVERAGE:</td>
<td>0.08%</td>
</tr>
<tr>
<td>STD DEV:</td>
<td>1.64%</td>
</tr>
<tr>
<td>SKEW:</td>
<td>1.05</td>
</tr>
<tr>
<td>EXCESS-KURTOSIS:</td>
<td>9.20</td>
</tr>
<tr>
<td>MEDIAN:</td>
<td>0.02%</td>
</tr>
<tr>
<td>MIN:</td>
<td>-7.99%</td>
</tr>
<tr>
<td>MAX:</td>
<td>16.03%</td>
</tr>
<tr>
<td>Q 1:</td>
<td>-0.84%</td>
</tr>
<tr>
<td>Q 3:</td>
<td>0.99%</td>
</tr>
</tbody>
</table>
Summary

- Mean (Average) is close to zero.
- Density (mass) distribution is significantly positively skewed.
- Density distribution has fat tails.
- Half of the observations lie between minus 0.84% to + 0.99%.
- Median is lesser than average, distribution is positively skewed (right skew), and that the distribution has right fat tails.
- Quartiles (Q1, Q3), inscribes 50% of values in the sample. Interquartile range (IQR) is used to characterize data, when there may be extremities that may skew the data. IQR is a relatively robust statistic (also sometimes called “Resistant”) compared to range (Max value – Min value), and the standard deviation.

Significance Test

<table>
<thead>
<tr>
<th>Significance Test</th>
<th>5.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target P-Value</td>
<td>SIG?</td>
</tr>
<tr>
<td>--------------------</td>
<td>-------</td>
</tr>
<tr>
<td>0.000</td>
<td>5.25%</td>
</tr>
<tr>
<td>0.000</td>
<td>0.00%</td>
</tr>
<tr>
<td>0.000</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

Table 2: - Significance values given are for mean, skewness and kurtosis respectively.

Null hypothesis for mean: - Sample estimate is close to zero. Since the p value as reported in table 2 above is 5.25 % which is greater than our level of significance (5 %), which is the probability of committing a Type-1 error, or rejecting the null hypothesis when indeed it's true. Whenever the p-value is lesser than 5 % we decide to reject the null hypothesis. Here we accept the null hypothesis that the sample mean is zero (close to zero, and hence infer statistically that the mean daily return on Hindustan lever stock for the study period (2009-2014) which is 0.08% is indeed close to zero, which implies that differences if any can be attributable to sampling fluctuation and thereby accept our null hypothesis of saying that the mean log daily returns on Hindustan lever ltd stock is
zero. But the same cannot be said for skewness and kurtosis p values as the null hypothesis states H0: – Skewness is zero and kurtosis is zero. Observe that the p value of significance obtained for both is zero percent which being individually less than 5% we reject the Null hypothesis thereby inferring that the sample log returns on Hindustan Lever stock is asymmetric and has fat tails at the right side of the data distribution.

### White Noise/Normal Distribution/Arch Effect

<table>
<thead>
<tr>
<th>Test</th>
<th>p-value</th>
<th>SIG?</th>
</tr>
</thead>
<tbody>
<tr>
<td>White-noise</td>
<td>63.50%</td>
<td>TRUE</td>
</tr>
<tr>
<td>Normal Distributed?</td>
<td>0.00%</td>
<td>FALSE</td>
</tr>
<tr>
<td>ARCH Effect?</td>
<td>0.00%</td>
<td>TRUE</td>
</tr>
</tbody>
</table>

Table: - 3:-Let’s address the following questions.

**QUESTION 1:-** Does The Log returns Time series of Hindustan Lever stock for the selected period of study five years (1/4/2009 to 31/3/2010) exhibit WHITE NOISE? (Aka no serial correlation) or (no auto correlation)?

To answer this question, we shall use the descriptive statistics wizard and check the white noise test (LJUNG-BOX) TEST function. Observe that the p value of significance is very high at 63.5% and significance displays true remark, meaning thereby that Null Hypothesis is accepted and the answer to our question is yes, this financial time series of Hindustan Lever ltd stock’s log returns does not exhibit significant serial correlation, as mentioned earlier if the computed p value happens to be greater than 5% level of significance, we accept the null hypothesis.

**QUESTION 2:-** On the question of whether the log returns are normally distributed—we infer that the p value which is 0% is less than 5% we reject the null hypothesis which states that the log normal distribution on Hindustan Lever stock is normally distributed, is false meaning thereby that there is statistical evidence to the effect stating that the data on daily log returns of this stock for the period under study is not normally distributed.

Coupled with the fact that the descriptive section results displayed shows that the daily log returns time series distribution possesses fat tails (excess kurtosis, value = 9.20) which may occur if the squared returns are correlated (aka ARCH EFFECT).
QUESTION:-3:- Does the daily log returns on Hindustan Lever stock exhibit an ARCH Effect? That is, is the squared daily log returns correlated or more like a white noise distribution?

Examining the ARCH effect test results, we conclude that squared returns are serially correlated which implies that it's possible that we encounter a conditional heteroscedasticity in log returns. Observe that the p value is 0%, which is less than 5 %, and hence we reject the null hypothesis or say that it is true, which means we reject the null hypothesis that the squared returns are not serially (aka auto correlated).

Correlogram Analysis:-

Figure shows: - Hindustan Lever auto correlation (ACF) and partial auto correlation functions (PACF):-
While examining the correlogram analysis of squared log returns refer plot as well as tabled values:

Firstly correlogram analysis is a key tool used to explore the interdependency of observation values, and it can also serve the purpose of identifying an opportune model and thereby estimate the order of its components. In our example we found that daily log returns are not correlated but their squared values are indeed correlated. As a result it's opportune to employ ARCH/GARCH MODEL for modeling Hindustan lever stock returns to trace the actual or close to actual volatility.

**Time Series Modeling**

NuM XL supports numerous time series models: - ARMA (Acronym for auto regressive moving average model), ARIMA, AIRLINE, AND GARCH MODEL OR ITS FAMILY. We have restricted our study by using (p,q) the orders of the ARMA ,GARCH,GARCH-M & EGARCH Models to order (1,1) only most popular and commonly used in financial Time Series Modeling of Stock/Index Returns.

<table>
<thead>
<tr>
<th>Lag</th>
<th>ACF</th>
<th>UL</th>
<th>LL</th>
<th>PACF</th>
<th>UL</th>
<th>LL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.55%</td>
<td>5.55%</td>
<td>-5.55%</td>
<td>0.55%</td>
<td>5.55%</td>
<td>-5.55%</td>
</tr>
<tr>
<td>2</td>
<td>-2.71%</td>
<td>5.56%</td>
<td>-5.56%</td>
<td>-2.71%</td>
<td>5.56%</td>
<td>-5.56%</td>
</tr>
<tr>
<td>3</td>
<td>-2.45%</td>
<td>5.56%</td>
<td>-5.56%</td>
<td>-2.40%</td>
<td>5.56%</td>
<td>-5.56%</td>
</tr>
<tr>
<td>4</td>
<td>3.10%</td>
<td>5.57%</td>
<td>-5.57%</td>
<td>3.06%</td>
<td>5.56%</td>
<td>-5.56%</td>
</tr>
<tr>
<td>5</td>
<td>-3.56%</td>
<td>5.57%</td>
<td>-5.57%</td>
<td>-3.72%</td>
<td>5.56%</td>
<td>-5.56%</td>
</tr>
<tr>
<td>6</td>
<td>-2.88%</td>
<td>5.58%</td>
<td>-5.58%</td>
<td>-2.73%</td>
<td>5.57%</td>
<td>-5.57%</td>
</tr>
<tr>
<td>7</td>
<td>2.17%</td>
<td>5.59%</td>
<td>-5.59%</td>
<td>2.21%</td>
<td>5.57%</td>
<td>-5.57%</td>
</tr>
<tr>
<td>8</td>
<td>-0.82%</td>
<td>5.59%</td>
<td>-5.59%</td>
<td>-1.28%</td>
<td>5.57%</td>
<td>-5.57%</td>
</tr>
<tr>
<td>9</td>
<td>-6.41%</td>
<td>5.60%</td>
<td>-5.60%</td>
<td>-6.22%</td>
<td>5.57%</td>
<td>-5.57%</td>
</tr>
<tr>
<td>10</td>
<td>-2.41%</td>
<td>5.60%</td>
<td>-5.60%</td>
<td>-2.19%</td>
<td>5.57%</td>
<td>-5.57%</td>
</tr>
</tbody>
</table>
Let us begin by considering a GARCH (1, 1) Model:

\[ y_t = \mu + \alpha_t \]
\[ \alpha_t = \sigma_t \epsilon_t \]
\[ \epsilon_t \sim \phi(0, 1) \]
\[ \sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_t^2 + \beta_1 \sigma_{t-1}^2 \]

Using NuM XL toolbar, locate and click on GARCH icon. The GARCH wizard dialog box pops up. In input data field specify cells range for the sample data. Next enter the values of ARCH & GARCH component orders as one (1). For innovations distribution we shall use the default as Gaussian distribution, although we have the famous t distribution as well as the GENERAL ERROR DISTRIBUTION for the GARCH FAMILY, WHICH IS WORKED OUT IN DETAILS.

Next we instruct the GARCH wizard to generate and augment the “GOODNESS OF FIT” calculations and along with it the “RESIDUALS DIAGNOSIS” sections in model output table. By default selected cell is used for output range value. Following table will be generated in your worksheet.

**Illustration GARCH (1, 1)**

<table>
<thead>
<tr>
<th>GARCH(1,1)</th>
<th>Goodness-of-fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Param</td>
<td>Value</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.00</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>0.00</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.09</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.09</td>
</tr>
</tbody>
</table>

**Residuals (standardized) Analysis**

<table>
<thead>
<tr>
<th>AVG</th>
<th>STDEV</th>
<th>SKEW</th>
<th>KURTOSIS</th>
<th>Noise?</th>
<th>Normal?</th>
<th>ARCH?</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.96</td>
<td>0.67</td>
<td>3.65</td>
<td>TRUE</td>
<td>FALSE</td>
<td>TRUE</td>
</tr>
<tr>
<td>Target</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SIG?</td>
<td>FALSE</td>
<td>TRUE</td>
<td>TRUE</td>
<td>TRUE</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table one above shows the model’s parameters values which are set by a quick guess (initial guestimates) and hence are not optimal values at all, even the model to model
values differ significantly both in terms of optimality check as well as the parameters and its resulting outputs.

The model ought to be calibrated before we can gauge its fit or consider the same for forecasting purposes. In the top right table is displayed the goodness of fit measure readings for Hindustan Lever Daily log stock returns. The wizard created for us, the LLF (LOG LIKELIHOOD FUNCTION VALUES AND, AIC(AKAIKE INFORMATION CRITERION) formulas on the corresponding cells. At the bottom of this same table we obtained, RESIDUALS (STANDARDIZED) ANALYSIS, INTERPRETED AS UNDER:-

Average: FALSE: GOOD/DESIRABLE
Standard Deviation: - TRUE: - BAD/UNDESIRABLE
Skewness: - TRUE: - BAD / UNDESIRABLE
Kurtosis: - TRUE: - BAD/UNDESIRABLE
NOISE?: - TRUE: GOOD/DESIRABLE
Normal distribution?:- FALSE:- BAD/UNDESIRABLE
ARCH EFFECT ?:- TRUE:- BAD/UNDESIRABLE.

In the residual diagnosis table, the wizard created a series of statistical test (formulae), for standardized residuals that is \( \{\varepsilon_t\} \), to help us verify the GARCH assumption:-

\[ \varepsilon_t \sim i.i.d. \sim \mathcal{N}(0,1) \]

The generated formulas reference the model’s parametric cells and input data cells range, so when you calibrate (or modify) the values of model’s parameter latest and best (optimal values), which means from majority of undesirable features obtained using GARCH (1,1) MODEL, we have a suspect model, and hence we propose to ask the following questions to this model.

QUESTION 1:- What are the optimal values for GARCH (1, 1), given input data?
QUESTION 2:- Given the calibrated model, how well does the model fit the input data?
QUESTION 3:- Do the Residuals address the assumptions underlying the model?
QUESTION 4:- Are there similar models to consider (eg EGARCH, GARCH-M)? How do we rank them and decide which of them to use?

Next step in our analysis is the Model Calibration phase.
**Model Calibration:**

Here we need to find optimal values for the model’s parameters - a process referred to as “Calibration”. Once calibrated we can examine the residuals for the model’s assumptions and compare this model with other models. NuMXL supports numerous time series models, and fortunately, the calibration process using Num XL is the same for all models. In a nutshell, a calibration is an “Optimization Action Problem”, where we search for a set of parameter values that Maximizes the value of a Utility Function (log likelihood function—LLF) /and minimizes the AIC (Akaike Information Criterion), while complying with one constraint, the stability of the model.

What do we mean by model stability? For an ARMA Model, the underlying process has a finite unconditional (long-run) Mean & Variance (that is the roots of the characteristic equation are outside the unit circle). For GARCH &GARCH variant models, in addition to the constraint, the variance model must guarantee positive values for the conditional variance. Fortunately, NumXL, lumps together model specific constraints with a function: - ARMA_CHECK, GARCH_CHECK ETC) which returns one (1) for a stable model, otherwise reports a zero value.

We use the solver for this optimization process.

**Solver Parameters**

Set Objective: - $L$

To: - Max.

By changing variable cells: - $J$: $J$ $6$

Subject to the constraints: - $N$3 >= 0.99999

Non negativity of unconstrained variables

Solving method: - GRG Non-linear

Click on SOLVER button

After 14 odd iterations solver finds an optimum solution

Keep solver solution.—ok.

Once we accept the solver solution, new set of parameters values are copied to your work sheet.
The optimal results using GARCH (1, 1) are as follows:-

<table>
<thead>
<tr>
<th>GARCH(1,1)</th>
<th>Goodness-of-fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Param</td>
<td>Value</td>
</tr>
<tr>
<td>(\mu)</td>
<td>0.00</td>
</tr>
<tr>
<td>(\alpha_0)</td>
<td>0.00</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>0.38</td>
</tr>
<tr>
<td>(\beta_1)</td>
<td>0.15</td>
</tr>
</tbody>
</table>

### Residuals (standardized) Analysis

<table>
<thead>
<tr>
<th>AVG</th>
<th>STDEV</th>
<th>SKEW</th>
<th>KURTOSIS</th>
<th>Noise?</th>
<th>Normal?</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.00</td>
<td>0.51</td>
<td>2.79</td>
<td>TRUE</td>
<td>FALSE</td>
</tr>
<tr>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>TRUE</td>
<td>TRUE</td>
</tr>
<tr>
<td>FALSE</td>
<td>FALSE</td>
<td>TRUE</td>
<td>TRUE</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that LLF Changed from 3408.53, to 3443.81, yet the model is valid and stable as check value is maintained at 1.

The calibrated parametric values improved the overall fit of the model with input data, but is this the right model? We need to examine the assumptions of GARCH and whether they are met or not needs to be seen next.

Looking at the residual diagnosis table, we observe the following points: - barring the STD deviation result which has turned from true to false post optimization, rest all others maintain their status quo, and remember it’s the standard deviation (volatility) which is our subject topic under study.

In a nutshell, a time series model draws some patterns for evolution of values over time and assumes the error term (residuals/noise/shock), to be independent and following a particular probability distribution, could be Gaussian Innovation, or Student’s t (William Gosset), or generalized error distribution.

The objective of time series analysis is “Forecasting”, so by ensuring that our model properly fits the data and meets all assumptions, we can have faith in the projected forecast.

The standardized residual diagnosis includes the following hypothesis testing
1) Population Mean (H0: \( \mu = 0 \))
2) Population Standard Deviation (H0: \( \sigma = 1 \))
3) Population Skewness (H0: \( S = 0 \))
4) Population Excess Kurtosis (H0: \( K = 0 \))
5) Whiten Noise or Serial Correlation (H0: \( \rho_1 = \rho_2 = \rho_3 = \cdots = \rho_K = 0 \))
6) Normality Test
7) ARCH Effect Test.

   The first four tests examine the distribution center, dispersion, symmetry and fat end tails. The normality test compliments these tests by assuming a specific distribution a GAUSSIAN, where \( \varepsilon_t \sim \mathcal{N}(0,1) \)

   The white noise and ARCH EFFECT tests address a different concern namely, independence of residual observations: \( \varepsilon_t \sim i.i.d. \) since the independence test is quite a complex topic; we simplify it by examining the linear and quadratic order dependency.

Analysis of Residual diagnosis table:-

   a) The population mean (average) test shows that the sample average is zero (same as target). As a result the residuals distribution has a mean of zero.

   b) The population standard deviation (STDEV) test shows that the sample data standard deviation is 1 (same as target).

   c) Population skewness test shows that sample skewness is slightly different from zero, which tells us that the residual distribution is close to being symmetric, but not symmetric.

   d) The population excess kurtosis, indicates sample kurtosis to be significantly different from that of a normal distribution (that is excess kurtosis is zero). The residual distribution tails are not normal.

   e) The normality test shows the the standardized residuals are not sampled from a normal population.

   f) Now let us examine the interdependence concern among the values of residuals. The first order dependence (linear) or serial (auto) correlation using WHITE
NOISE TEST (LJUNG BOX), the test shows no sign of significant serial correlation.

g) Let us examine the second order dependence (quadratic), or ARCH EFFECT. The arch effect shows insignificant serial correlation in the squared residuals or the absence of an ARCH EFFECT.

As a result, the standardized residuals are independent and identically Non GAUSSIAN Distributed. Thus the GARCH Model assumption is not fully met, hence we check using EGARCH AND Use the innovation t distribution and the findings are reported as under:-

<table>
<thead>
<tr>
<th>EGARCH(1,1) &amp; t-dist(v)</th>
<th>Optimal</th>
<th>Goodness-of-fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Param</td>
<td>Value</td>
<td>LLF</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.00</td>
<td>3491.17</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>-3.02</td>
<td></td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>5.53</td>
<td></td>
</tr>
</tbody>
</table>

| Residuals (standardized) Analysis |
|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| AVG   | STDEV | SKEW  | KURTOSIS | Noise? | Normal? | ARCH? |
| 0.03  | 1.00  | 0.50  | 3.43     | TRUE  | FALSE   | FALSE |
| Target | 0.00  | 1.00  | 0.00     | 3.91   | TRUE    | TRUE  |
| SIG? | FALSE | FALSE | TRUE    | TRUE   |         |       |

Observe from this optimal table we have got the maximum value for the log likelihood function as 3491.17, bettering the previous value of 3443.81, but the other conclusions same as enlisted for seven points as above for the hypothesis testing mode. We have also tried and tested other models in a few cases using the MODELS LIKE :- ARMA (2,2), ARMA (3,3), GARCH (2,2), INTEGRATED ARMA (ARIMA (1,1)), Sarima And A Few More But We Choose To Narrow Done In Our Study To Employ Simple Arma And
Garch, And Egarch All (1,1) Models, Albeit Using The Innovations:- 1) Gaussian Or Normal Distribution 2) Studentized ( t Distribution 3) Generalized Error Function Distribution For Comparison Purposes.

PART II: -Time Series Forecasting
Num XL uses the following syntax commands:
Syntax
GARCH_FORECI (X, Order, Mean, Alphas, Betas, Innovation, V, T, Alpha-Level, Upper)
X: - is the univariate time series data (a one dimensional array of cells e.g. rows or columns).
Order is the time order of the data series (that is whether the first data point corresponds to the earliest or latest date (earliest date = 1 (default, and latest date =0)
Mean: - is the GARCH MODEL MEAN (mu)
Alphas: - are the parameters of the ARCH (p).
Betas: - are the parameters of the GARCH (q) component model (starting with the lowest lag)
Innovation: - is the probability distribution function of the innovations/residuals/shocks/noise. 1= Gaussian, 2= t Distribution, 3= Generalized Error Distribution Function. If missing, a Gaussian distribution is assumed.
θ (nu) is the shape parameter (or degrees of freedom) of innovations/residuals probability distribution function.
T is the forecast time horizon (expressed in terms of steps beyond end of the time series)
Alpha Level: - is the statistical significance level. If missing a default of 5% is assumed.
Upper: - If true, returns the upper confidence interval limit, otherwise returns lower limit.
REFERENCE
1.NumXL An Add in Feature of MS Excel, freely downloadable.


APPENDIX 5

Regression Fundamentals

Econometric Series (Financial Time series)

Gauss-Markov theorem assumptions (OLS estimation technique) (Ordinary Least Squares)

The Population Regression Function (P.R.F.) is expressed as:

\[ Y_i = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots + \beta_k X_{ki} + \epsilon_i \tag{A} \]

- Model is linear in parameters, and is correctly specified.
- Independent Variables (Xi’s) are Non-Stochastic.
- The Expectation of error term is zero (\( E[\epsilon_i] = 0 \))
- The Variance of error terms\( \sigma_i^2 \) is constant (Homoscedasticity).
- The error terms are uncorrelated – \( E[\epsilon_i, \epsilon_j] = 0 \), FOR \( \exists i \neq j \).
- There is no linear relationship between the Independent Variables in a Multiple Linear Regression model.

If the above assumptions hold true in a data set, we may infer that the OLS Estimators are B.L.U.E. (Best, Linear, Unbiased (Asymptotic), Estimator and are considered Efficient.

- Error terms are independent of explanatory variables. \( \text{Cov}(\epsilon_i, X_k) = 0 \)
- Explanatory Variables are measured without error, or the error term absorbs all the possible measurement errors.
- Explanatory Variables are not perfectly linearly correlated. There is no "Multicorrelation", otherwise the parameter of the relation becomes indeterminate and the method of least squares breaks down. (When\( r_{x_l, x_j} \neq 1 \)). On the other extreme if \( r_{x_l, x_j} \neq 0 \), the variables are called orthogonal and there are problems of multicollinearity while estimating parameters of relationships.
- Macro variables are correctly aggregated.
- Estimated relationships are identifiable.
- Relationships are correctly estimated.
A.S. Goldberger says “As Sample size increases, issues of statistical significance becomes much less important, but issues of economic (practical) significance becomes critical, as with very large samples almost any Null Hypothesis will be rejected”.

**Problems In Regression**

1) Heteroscedasticity
2) Multicollinearity
3) Autocorrelation.

1) **Heteroscedasticity**
   
   - Assumed $\sigma^2 = k^2X^2$, where $k$ (a constant) is estimated from the model, this problem is more likely to occur in cross-sectional data than time series data.
   
   - **CONSEQUENCES OF HETEROSCEDASTICITY**
     1) We cannot apply the formula as, $\text{VAR (}\beta_0 \text{ CAP, VAR (}\beta_1 \text{ CAP)}$, the estimators, to conduct the tests of significance.
     2) If $\sigma_i$ is heteroscedastic, then $\sigma^2_i$ is not a constant finite number. It changes with X. In the presence of heteroscedasticity, these OLS estimators do not have “Minimum Variance Property” Therefore estimators are inefficient in small samples & asymptotically inefficient in large samples. However the parametric estimates are still statistically unbiased, as expected values of $(\beta_0 \text{ CAP, } \beta_1 \text{ CAP)}$, do not involve $\sigma^2_i$ or do not require the assumption of constant $\sigma^2_i$.

2) **Multicollinearity**
   
   - If $r_{x_i,x_j} \neq 1$, the application of OLS depends on a crucial condition, that the explanatory variables are not perfectly linearly correlated. If this is violated that is if there exists a perfect linear correlation among some or all of the explanatory variables then the variables are multicollinear.
   
   - Ragnar Frisch “The broader definition of multicollinearity implies that it is not a condition that exists or does not exist, it is a phenomena inherent in most economic relationships due to the sheer nature of economic phenomena”.
   
   - Reasons for presence of multicollinearity
1) In various phases of business cycle, all economic variables tend to move in the same direction. Thus growth and trends in time series are serious causes of multicollinearity.

2) Some econometric models use lagged values of certain variables as independent explanatory variables, for example, consumption is explained with current as well as past income or investment is taken as a function of distributed lags of past level of economic activity.

- **Consequences of Multicollinearity**

1) Even if multicollinearity is high, OLS estimators are still BLUE.

2) Multicollinearity is essentially a sample regression phenomenon that is even if X’s are not linearly related in the population; they may be related in a particular sample from which we estimate the PRF.

3) L.R.KLEIN: “Multicollinearity is not necessarily a problem unless it is high, relative to the overall degree of multiple correlation among all variables simultaneously.

4) Increasing standard errors appear when we include inter correlated variables as explanatory variables in the function.

5) Due to large standard error, confidence intervals for corresponding population parameters tend to be larger and probability of accepting a false hypothesis that is “Type II Error” increases.

6) Presence of multicollinearity may give rise to danger of misspecification as we may reject a variable whose standard error appears high, although it may be an important determinant of the variation of dependent variable.

7) As estimated standard error increases with multicollinearity, the computed t values become smaller and lead to acceptance of Null hypothesis i.e.: \(-\beta’s = 0\).

8) There is high overall R²& insignificant t.

3) **Autocorrelation**

- Correlation between members of series of observations ordered in time or space as in a time series or cross sectional data.
• An important assumption of OLS is successive values of a random variable $\sigma_i$ are
temporarily independent: - $\text{COV}(\text{ui},\text{uj}) = 0$, for all $i \neq j$. Violation of this
assumption leads to stochastic variables being auto correlated or serially correlated.

• Sources of Autocorrelation

1) Inertia: - Given economic data in time series form, it exhibits business cycle as
states of nature of an economy, such as boom & recession, have momentum built
into them. Therefore in regression analysis of time series data, successive
observations are likely to be interdependent.

2) Excluded Variables: - The omitted explanatory variables, if auto correlated, exert
their influence on error or residual term, whose values then is auto correlated.
This is called quasi autocorrelation in the random disturbance error term.

3) Misspecified Form: - When the model is misspecified mathematically, different
from the true form, the errors or residual terms may be correlated.

4) Presence Of Lagged Variables: - Can lead to autoregression, as one explanatory
variable is lagged.

5) Data Manipulated: - Some interpolation and smoothing is done to time series
data, which affects the error term and successive error terms getting correlated.

• Consequences of Autocorrelation

1) When the stochastic disturbance (noise/error/residual) term shows serial
correlation value, as well as the standard error of the estimated parameter is
affected. However OLS estimates are statistically unbiased as long as the error
terms and explanatory variables are uncorrelated.

2) When the residuals are auto correlated the variances of OLS estimates are likely
to be large.

3) Variance of random term, may be seriously under estimated in the presence of
auto correlated error terms, which is most serious in case of positive correlation
of error terms and of positively auto correlated X’s, in successive time periods.

4) When the residuals are auto correlated, predictions based on OLS estimates will
be inefficient, as variance of such an estimate is larger than the variance of
estimate based on any other econometric method.
Residual Tests for Regression of In-Sample Realized Volatility

Future realized volatility of In-Sample period are estimated by the classical linear regression model which as stated before has following five major assumptions:

1) \( E[\varepsilon_t] = 0 \)
2) \( \text{VAR}(\varepsilon_t) = \sigma^2_{\infty} \)
3) \( \text{COV}(\varepsilon_i, \varepsilon_j) = 0 \)
4) \( \text{COV}(\varepsilon_t, X_t) = 0 \)
5) \( \varepsilon_t \sim N(0, \sigma^2) \)

These assumptions make OLS Technique have substantive desirable properties and the HO that the test concerning the parametric estimates can be conducted validly.

- Violations of any of these assumptions can lead to some problems, such as both the parameter estimates and the standard errors associated with them are wrong, and the distributions assumed for the test are not appropriate.
- In order to confirm that the volatility forecasts are efficient and our inferences based on the coefficient estimates of regression for in-sample realized volatility are correct, the residual diagnostic tests were conducted.

1) Due to coefficient \( \alpha \) are highly significant for all regressions, thus the first assumption is not violated, that is if a constant is included in the regression model, the first assumption that the mean of the residuals are equal to zero will never be violated.

2) In terms of \( xt \) of the fourth assumption as above, it denotes that the independent variable of the regression equation. If the independent variable is stochastic and uncorrelated with residual, then the estimates of the OLS are consistent and unbiased.

REFERENCE

2) My own notes
Appendix 6

Japanese Candlestick Plots (Basics)

LEGEND:
- Green color candle stick represent Bullish sentiments, while the red color candle stick represent bearish trend.

- These candlesticks are created exactly from the intraday prices: open, high, low, and close stock prices.
• The real benefit of using candlestick plots accrues due to better visual representation of the movement of stock prices, which facilitates quicker identification of short-term sentiments of the market.

• Candle sentiment (Psychology of the single candle)- answers three primary question:-
  1) Who is in control? 2) The Bulls? 3) The Bears?

• Green color candle sticks, implies buyers are willing to pay higher prices, size of the green candle, larger it is more bullish than small green candle sized bodies. Similarly the red candles indicate that the sellers are in control, and larger the size of the red body, more bearish sentiments are evident as compared to smaller red sized candles.

• Larger the tails on the lower end of red candle sticks, and larger the tails on the upper end of the green candle sticks implies significant rejection of lower prices/ significant rejection of higher prices respectively, thereby implying that in the first case of red candles of this bullet point, trend implied is bullish, and in the same vein for green colour candles bearish trend is implied.

• DOJI:- Equal tails above or below any candle suggests bearish rejection of higher prices and bullish rejection of lower prices, denotes sideways trend.

• We can observe three stages in the candle pattern plots:
  1) Trend  2) Pattern 3) Confirmation.

• We can decipher several patterns such as: -a) Bearish reversal pattern b) Bullish reversal pattern c) Sideways trend d) Bullish Engulfing pattern etc.

• A pictorial representation of candlesticks plots as displayed below, have been attempted for all fifty stocks and NSE Nifty index, which displays candlestick plots on the top panel, and volume of stock traded in the bottom panel.
Candlestick Charting Pattern
A) Computation of Degrees, given Price

Case 1: O is at the centre of the grid. Angles in Degrees = MOD ( 180* \( P^\frac{1}{2} \) - 225), 360.

Case 2: 1 is at the centre of the grid. Angles in Degrees = MOD(( 180* (P - 1)^\frac{1}{2} - 225),360).

Case 2: For example, NSE Market Index was quoted in news at 8797, dated 3rd February 2015, page 20, “SENSEX at 1-wk low, ahead of RBI Policy”, we want to find the angle given this index value.

B) Computation of the Ring number given the stock price/Index value?

Ring Number = Round ((( SQRT(Price) - 0.22/2),0)

Illustrative example on NSE Market (Nifty), our population, quoted at 8797.

Step 1: Find the ring number given this value index at 8797?

Ring number = [sqrt (8797) - 0.22] ÷ 2 = 46.78616189

Step 2: Round up this to get answer as Ring Number 47. We say that 8797 lies on the ring which is numbered 47 or cycle number 47, whether clockwise or anti clockwise, does not matter.

Step 3: Find the 315° number to 8797?

\[ = (\text{Ring number} \times 2 + 1)^2 \]

\[ = (47 \times 2 + 1)^2 \]

\[ = 9025, \text{ which is the ending of 47th ring, on which the square of 95 an odd number ends. Hence 9025 is at 315° from the given index 8797.} \]

Step 4:- To find the zero degree angle on this ring?

\[ = [\text{Ring number} \times 2 + 1]^2 - [7 \times \text{Ring number}] \]

\[ = 9025 - 329 \]

\[ = 8697 \]
Thus the zero degree angle from which cycle/ring number 47 begins is 8697.

Step 5:- What is the angle between 8697 and 8797?

Angle = Sum ( Price - Zero angle )/ Ring number\(\div45\)

\[
= \frac{(8797-8697)}{45}
\]

= 95.75°

Hence 8797 is at an angle of approx 96 degrees.

Step 6:- What is the Number which is at 45 degrees to the number 8797.

= 95.75 \+ 45

= 140.7°

Step 7: = ( Ring Number * 2 +1)² - ( 7 * Ring number) + ( \(\frac{Ring Number}{45}\) * Angle)

= 8843.

It can be said that 45 Degree being the most important angle, there will be a trend reversal obtained when the index touches 8843 level.
C) The Gann Square of Nine-- A square root calculator.

1) Gann Wheel & Squares, one of the cornerstones of Gann: Part of Gann's methodologies, had to do with the idea of "Squaring Price & Time". When trying to graph using a Gann tool, the charts should be set to a 1x1 ratio or your results will not be true.

2) Results from your Square of Nine are telling you the next most likely Resistance Price levels up and/or down from price measured. This would be your basic calculator.

3) Another concept is "Squaring the Circle", the box containing the square of nine should also have a circle around it, so that each corner of the box, touches the inside of the circle, this allows us to divide the square up in degrees of 360.

4) Blue cross (Cardinal Cross),+ sign, and the yellow diagonals or the ordinal cross (X shape), numbers lying on these two crosses, are considered very important numbers, when looking for next move up or down, and it divides the square into 45 degree increments. 13 is 45 degrees from 11.
5) Focussing on the Price, is only one-half of what he square of nine represents, the degrees here are also associated with time. A complete 360 degrees rotation is equal to adding 2 to the root of a number, then a 1 degree will correspond to a movement which would be 2/360 which is 0.00056 added or subtracted from the root number.

Illustration:- Same case of Nifty 8797 as on 3rd February 2015.

Let us assume a start value of 8797

a) root of 8797 = 93.79 ~ 94.

b) increments of 45 degrees, we add 0.25 (to root)

<table>
<thead>
<tr>
<th>Degree</th>
<th>Working</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(94)^2</td>
<td>8836</td>
</tr>
<tr>
<td>45</td>
<td>(94.25)^2</td>
<td>8883.0625</td>
</tr>
<tr>
<td>90</td>
<td>(94.50)^2</td>
<td>8930.25</td>
</tr>
<tr>
<td>135</td>
<td>(94.75)^2</td>
<td>8977.5625</td>
</tr>
<tr>
<td>180</td>
<td>(95)^2</td>
<td>9025</td>
</tr>
<tr>
<td>225</td>
<td>(95.25)^2</td>
<td>9072.5625</td>
</tr>
<tr>
<td>270</td>
<td>(95.50)^2</td>
<td>9120.25</td>
</tr>
<tr>
<td>315</td>
<td>(95.75)^2</td>
<td>9168.0625</td>
</tr>
<tr>
<td>360</td>
<td>(96)^2</td>
<td>9216</td>
</tr>
</tbody>
</table>
These would be the significant levels found on the crosses every 45 degrees. If we wish to work the levels below our starting value, we would subtract 0.25 increments instead of adding it. This is one of Gann's Basics. What is missing is time element, to match up the prices with. When both time and price are squared the technique becomes even more better at Forecasting likely outcomes in the stock prices and Index levels.

c) Since start point is 8797, a target of 8883 would then be the trade you would be looking for.

d) Air in sacred geometry is represented by an Octahedron( Made up of 8 triangles)

e) W.D. Gann Time cycles: W. D. Gann's forecasting methods involved looking back at the past with time cycles of various lengths from annual seasonal trends of 1 year to 5,7,10,20,30,45,60,84,90 and 100 year cycles. Behind these cycles are planetary periods of various lengths, and harmonics of their cycles, and sinusoidal cycles through the 360 degrees of the Zodiac-- wheels within wheels. Each planet has an energy, vibration, number and scale. Price always seeks the gravity centre.
The Earth is a magnet, which is surrounded by a vast sea of Electromagnetic energy. Most of this energy comes from the Sun. Humans also produce electromagnetic energy through their nervous systems. Water covers, most of the earth and our bodies contain 85% water. The Moon affects the water on Earth (tides), and eclipses have a drastic effect on the Earth's magnetic field. Then is it illogical to conclude that these cosmic events, could also have an effect on human behaviour, and Investor Psychology? Where does the word "Lunatic" come from? John. Kepler, known as one of the founders of Astronomy stated: "The Planets forming angles (Aspects), upon the earth, by their luminous beams of strength tostirup the virtue of sublunary things, have compelled my unwilling belief".

Patrick Mikula:- book on "Definitive guide to Forecasting using W.D.Gann's square of nine

"Spring Equinox on March 21 is aligned on 0° - 360°.

Summer Solstice on June 21 is aligned to 90°.

Autumnal Equinox on September 22 is positioned on 180°.

Winter Solstice on December 21 is placed on 270°.

From Spring Equinox to Summer Solstice there are 92 days, and from Summer Solstice to Autumnal Equinox there are 93 days, and 90 days each from Autumnal

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1 Source:- Hans Hannula, Ph.D. "Using Gann with Astrophysics to validate turning points"
Equinox to Winter Solstice & from Winter Solstice to Spring Equinox, except in a leap year.

When Gann used 24 hours of a day, he assigned 6am to $0^\circ - 360^\circ$ mark. The Earth rotates 1 degree every 4 Minutes, so 24 hours are divided evenly into $360^\circ$, 4 minutes increments around the circle. 1 year equals 365.24 days.

$0^\circ$ March 21, 6 am, Spring (Vernal) Equinox, East, Aries.

$1^\circ$ March 22, 6.04 am

$9^\circ$ March 30, 6.36 am

g) Fibonacci numbers$^2$: 44, 67.5, 266, 360 degrees. Gann taught us that at the beginning of each annual Earth cycle commencing at the Spring Equinox (Vernal Equinox), Gann indicated that by counting days from March 21st, that many of Nature's, invisible secrets would be revealed. By no means was the Stock Markets divorced from, this natural calendar rhythm.

h) Eleventh Commandment

Please grasp this universal law." Life deals out no Random Events". "More you scratch at the truth, the more it will itch".

Take a closer look at the number 365.24, multiply by 7, and we get 2556.68 days, which is equal to 7 years of course, and notorious seven year itch.

Mind boggling 2556.68 days equals to exactly 365.24 weeks. This new information totally vindicates the perfect time cycle, as being 365.24 and not 360.

i) SUN-MOON Confluence:

The Sun & Moon are Conjunct at New Moon, and unless, overshadowed behaviour aspects, the Stock Markets can be expected to, move upward at that time. The Sun & Moon are in square aspects($90^\circ$), at 1st Quarter and last Quarter and a Bear Market is the normal outcome. When the angle between the Sun & Moon is $180^\circ$, they are said to be in opposition, at the Full Moon, which usually brings in its wake a somewhat

---

$^2$ Source:- Granville Cooley:- "The Gann Side of Fibonacci Number"
Bullish Market, but is not as positive as the New Moon. Sun & Moon Trine occurs at 120°, which comes half-way between opposition and conjunction, which are indicators of Bull Markets.

j) Gann divided the square into eight equal parts to produce eight triangles. The strongest (Support/Resistance), point is at the centre where all angles cross. Four angles cross at these point. For example if the top of a stock is say 28, this square of 28x28, would represent squaring the price by time, for the simple reason that if we have 28 points up in Price, and we move over 28 spaces in time, we Square the Price with Time. Hence when the stock has moved over 28 days, 28 weeks, or 28 months, it will be squaring its price range of 28. As Gann rightly says, there are three types of angles: The vertical, The Horizontal, and the Diagonal, which we use to measure Price & Time. We use the Square of Odd & Even Numbers to get not only, the proof of Market Movements, but the cause.

References

1) Source:- Hans Hannula, Ph.D- "Using Gann with Astrophysics to validate turning points"

2) Source:- Granville Cooley:- "The Gann Side of Fibonacci Number"
APPENDIX 8

Portfolio Management
Exhibit 1

A. Summary Of Various Investment Options (Investment Universe)

<table>
<thead>
<tr>
<th>AVENUES/</th>
<th>CURRENT</th>
<th>RISK</th>
<th>LIQUIDITY/</th>
<th>TAX</th>
<th>CONVENIENCE</th>
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<td>OPTIONS/</td>
<td>YIELD</td>
<td>gain/loss</td>
<td>MARKETABILITY</td>
<td>SHIELD</td>
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<tr>
<td>VEHICLES</td>
<td>(Income + (clg_opg)</td>
<td>pur.price</td>
<td>topg price</td>
<td></td>
<td></td>
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<td>A) Equity Shares</td>
<td>low</td>
<td>high</td>
<td>high</td>
<td>fairly high</td>
<td>high</td>
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<td>B) Non Convertible Debentures-N.C.D.)</td>
<td>high</td>
<td>negligible</td>
<td>low</td>
<td>average</td>
<td>nil</td>
</tr>
<tr>
<td>C) Equity Schemes</td>
<td>moderate</td>
<td>low</td>
<td>high</td>
<td>high</td>
<td>high</td>
</tr>
<tr>
<td>D) Debt Schemes</td>
<td>low</td>
<td>high</td>
<td>high</td>
<td>no tax on sum</td>
<td>very high</td>
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<tr>
<td>E) Bank Deposits</td>
<td>moderate</td>
<td>nil</td>
<td>negligible</td>
<td>high</td>
<td>nil</td>
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<tr>
<td>F) Public Provident Fund(PPF)</td>
<td>nil</td>
<td>moderate</td>
<td>nil</td>
<td>average</td>
<td>sec 80 c</td>
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<tr>
<td>G) Life Insurance Policy (LIC)</td>
<td>nil</td>
<td>moderate</td>
<td>nil</td>
<td>average</td>
<td>sec 80 c</td>
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<td>H) Residential House</td>
<td>moderate</td>
<td>moderate</td>
<td>negligible</td>
<td>low</td>
<td>high</td>
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<tr>
<td>I) Gold &amp; Silver</td>
<td>nil</td>
<td>moderate</td>
<td>average</td>
<td>average</td>
<td>nil</td>
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B. 

<table>
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<tr>
<th>INVESTOR</th>
<th>SPECULATOR</th>
<th>GAMBLER</th>
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<tr>
<td>1) Investor planning horizon</td>
<td>1 year or more</td>
<td>few days to month</td>
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<td>2) Risk disposition</td>
<td>risk averse</td>
<td>risk lover</td>
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<td>3) Return expectation</td>
<td>Modest</td>
<td>very high</td>
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<td>4) Decision Making</td>
<td>Fundamentals</td>
<td>Technical analysis</td>
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<tr>
<td>5) Leverage</td>
<td>own funds,</td>
<td>borrows to supplement own</td>
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</table>

C. Investors decision making approaches

1) Fundamental
2) Psychological
3) Academic
4) Eclectic.

D. Common Errors in Investment Management

1) Inadequate comprehension of Risk and Return.
2) Investment policy vague.

---

1 Investment Analysis and Portfolio Management second edition, Prasanna Chandra, Tata McGraw Hill, Pg 9
3) Naïve extrapolation of past performance.
4) Cursory Decision Making.
5) Simultaneous Switching.
6) Misplaced love for cheap stocks.
7) Over or Under Diversification.
8) Buying familiar company shares.
9) Wrong attitude towards losses and gains.
10) Speculative tendencies

Portfolio construction, management and performance evaluation metrics, assuming each portfolio can be taken as a mutual fund.

- The tighter, or more peaked, the probability distribution, the more likely it is that the actual outcome will be close to the expected value, and consequently the less likely it is that the actual return will end up far below the expected return. Thus tighter the probability distribution, lower the risk assigned to a stock.
- In a Market dominated by risk-averse Investors, Riskier securities must have higher expected returns, as estimated by the Marginal Investor, than less risky securities. If this situation does not exist, buying and selling in the Market, will force it to occur. Diversification, does nothing to reduce risk, if the Portfolio consists of perfectly positively correlated stocks. As a rule, risk of a Portfolio, will decline, as the number of Stocks in the Portfolio increases. It is impossible to form completely riskless stock portfolios. Almost half of riskiness, in an average individual stock can be eliminated, if the stock is held in a reasonably well diversified portfolio, which is one containing 20 odd stocks, in a number of different sectors/industries. The part of a stock's risk, that can be eliminated is called Diversifiable risk, while the part that cannot be diversified away or eliminated is called Market Risk. Diversifiable risk, is caused by such random events as law suits, strikes, successful and unsuccessful marketing programs, winning or losing a major contract and other events that are unique to a firm. As these events are random, their effects on a portfolio, can be eliminated by diversification--- bad events in one firm will be offset by good events in another. Market risk, stems from factors, that systematically affect most firms: War, Inflation, Recession, High Interest Rates. Since most stocks are negatively affected by these factors, market risk cannot be eliminated by diversification.
- C.A.P.M. :: " The relevant risk of an individual stock, is its contribution to the risk of a well diversified portfolio (which is much smaller than stand alone risk). Why
accept risk, that can easily be eliminated? The risk that remains after diversification, is market risk, or risk that is inherent in the market, and it can be measured by the degree to which a given stock tends to move up or down, with the Market.

**Basic Assumptions of C.A.P.M**

1) All Investors focus on a single holding period, and they seek, to maximize, the Expected Utility of their Terminal Wealth by choosing, among alternative portfolios on the basis of each portfolio, expected return, and standard deviation.

2) All Investors, can borrow or lend, an unlimited amount at a given risk-free rate, and there are no restrictions on short sales, of any asset.

3) All Investors have identical estimates of expected return, variances, and covariance among all assets, that is, Investors have homogenous expectations.

4) All assets are perfectly divisible, and perfectly liquid (Marketable at the going price).

5) There are no transaction costs.

6) No taxes

7) All Investors are price takers, that is, all investors assume that their own buying or selling activity, will not affect stock prices.

8) Quantities of all assets are given and fixed.

*Beta:* - The benchmark for a well diversified stock portfolio, is the market portfolio, which is a portfolio, consisting of all stocks, therefore, the relevant risk of an individual stock, which is called its beta coefficient, is defined under the CAPM, as the amount of risk, that the stock contributes to the market portfolio. $\beta_i = \frac{\sigma_i}{\sigma_m}$

*SML:* - Graphical representation of the CAPM. Represents on the vertical (y axis), return on the individual security or portfolio and on the horizontal (x axis), is the beta of the security/portfolio

Slope of the S.M.L = $\frac{R_p - R_f}{\beta_p} = \frac{R_m - R_f}{\beta_m}$

Therefore : $R_p - R_f = \beta_p ^* [ R_m - R_f ]$

* C.A.L :* - Is the locus of portfolios constructed by adjusting the proportion of wealth invested in risky assets. CAL & Utility Indifference curves, depicts how investors choose different portfolios.

$U = R_p - 0.005 A \sigma_p^2$ (For a risk neutral investor $A=0$)................ (A)
1) Each Individual investor must find Investment on the CAL, that maximizes Utility (Objective).

2) Portfolio Return: \( R_p = R_f + y ( R_s - R_f ) \)

3) Portfolio Risk: \( \sigma_p = y \sigma_s \)

Putting 2 & 3 in A, above we get: \( U = R_f + y(R_s - R_f) - 0.005*A (y^2 \sigma_s^2) \) ------ (4)

\[
\frac{\partial U}{\partial y} = 0, \quad \text{we get} \quad \frac{y}{0.01A\sigma_s^2} = \frac{R_s - R_f}{R_f - R_f} = 0
\]

*CML:* 1) Generates a line on which efficient frontiers can lie. 2) Preferred Investment strategies, plot along a line, representing alternative combo of risk and return obtainable by combining the market portfolio with borrowing or lending called CML. 3) Slope of CML is considered as reward per unit of risk borne. 4) In essence, CML, provides a place where time and risk can be traded and their prices determined by forces of Demand & Supply. Internal rate of return, can be thought of as price of time, and slope of CML, as price of risk. 5) Equilibrium in the Capital Market, can be characterized by \( R_f ( y \text{ intercept, reward for waiting) & slope of CML (Reward per unit of risk borne).} \)

Slope of the CML: \[
\frac{R_m - R_f}{\sigma_m} = \frac{R_p - R_f}{\sigma_p}
\]

Equation of CML: \[
R_p - R_f = \sigma_p [ \frac{R_m - R_f}{\sigma_m} ]
\]

*CML is a special case of SML Relationship.

* Diversification is the difference between Returns corresponding to beta implied by total risk of Portfolio and return corresponding to its actual beta.

* Why does risk matter if it does not hurt the Investor?. Given that an Investment made money, what difference does it make how the money was made.

* As time passes, the value of the portfolio composition changes, and value of the composition beta also changes, that is the beta of a self-contained stock, or a portfolio often declines, hindsight is an inappropriate perspective of Investment Decision Making.

- If Markets are efficient there is no such thing as undervalued and overvalued assets.
- Beta must be estimated, and this estimate is shrouded with substantial uncertainty.
Approaches to Investment Decision Making

- **Fundamental:** 1) Conduct Fundamental Analysis to establish certain value anchors. 2) Earn superior returns by buying undervalued securities (Securities whose Intrinsic Value is greater than the current quoted latest market price of the asset, and selling overvalued securities, which has a lower intrinsic worth as compared to the quotation of the stock price now.

- **Technical:** Assesses state of Market Psychology, mood of Investors, and relative strengths of supply and demand forces. Burton. G. Malkiel :- "Castles-in-the-Air."

- **Psychological:** J. M. Keynes:- "A conventional Valuation, which is established as an outcome of the Mass Psychology, of a large number of ignorant individuals, is liable to change violently as a result of a sudden fluctuation of opinion due to factors which do not really make much of a difference to the prospective yield.".

- **Academic:** 1) Stock Markets quickly react quite reasonably efficient and rationally to flow of information. Hence stock prices reflect Intrinsic values fairly well. 2) Stock Price Behavior corresponds to a Random Walk that is, successive price changes are independent that is, we cannot use past prices to predict future price behavior. 3) There seems to be a positive relation between risk and return in the capital markets., $E[R] \propto$ Systematic risk/Market Risk/Non-Diversifiable Risk.

- **Eclectic:** Uses all approaches. 1) Fundamental (Basic Standards/Benchmarks) 2) Technical:- Mood of Investors 3) Market is neither well ordered, as Academic Nor as Speculative as Psychological. It is characterized by some imperfections, it seems to react reasonably efficiently and rationally to flow of information.

- **SENSEX:** A Simple Arithmetic Average of Price Relatives of 30 sensitive shares. Factors attributable to Sensex movements are :-a) Industrial growth, in real terms. b) Secular inflation rate c) Shift in the average P/E: More mature the market more meaningful the risk-return parity.

- **Risk:** Business risk-- default risk, Inflation risk-- reduction in purchasing power, Interest rate risk- increase in interest rate lower is the price, and market risk:- investor's psychology.
- **Burton. Malkiel**: 'Risk and Risk alone, determines the degree to which Returns will be above or below average and thus decides the valuation of any stock relative to the market.

<table>
<thead>
<tr>
<th>TECHNICAL ANALYSIS</th>
<th>UNDervalued</th>
<th>Overvalued</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak</td>
<td>Wait</td>
<td>Sell</td>
</tr>
<tr>
<td>Strong</td>
<td>Buy</td>
<td>Wait</td>
</tr>
</tbody>
</table>

1) Buy only stocks that are expected to have above-average earnings growth for five or more years. 2) Never pay more for a stock than its firm foundation value. 3) Look for stocks, whose stories of anticipated growth are the kind on which investors can build castles in the air.

- **Francis Galton** (Cobb Douglas Production function) - "In probability theory, a log-normal distribution is a cumulative probability distribution function of a random variable (takes only positive real values), whose log is normally distributed. Therefore if Random Variable is log-normally distributed then \( X = \log Y \) has a normal distribution. Similarly if \( X \) has a normal distribution, \( Y = \exp (X) \) has log-normal distribution. Multiplicative product of many positive independent random variables, then that variable is modelled as log-normal. \( \ln \ N (\mu, \sigma^2) \).

- Security analysts are not paid to be historians, they are paid to be prophets.

- Use of Leverage, can dramatically shift an Investor's return.

- Logarithms reduce the effect of extraordinary deviations from Normality. Logs reduce the impact of extreme values that distorts true distribution. A safe $ is worth more than a risky $.

- Greater the dispersion in number series, greater gap between the Arithmetic Mean & Geometric Mean, use of Geometric Mean, reduces the likelihood of nonsense answers with Financial Rates of Returns. Mostly in finance we are interested in rate of return that equates a Present Value with a series of future expected values.

- Standard Error (S.E.), is Standard Deviation of a 'would be' distribution of Mean values computed from many different samples of exactly the same size from the same population.

- 68% of the distribution lie within 1 S.E of Mean, 95% within 2 S.E, and 99% within 3 S.E from the mean.
• **Asset Pricing Models**


2) **Exhibit 2: Portfolio Theory**

Portfolio theory suggests, that if all investors have the same (homogenous) expectations about returns and their (co)variances and they wish to trade-off risk against return, then they will all hold Market Portfolio. 

**Fair Value and Efficient Frontier.** The Fair Value (V), of a stock is the Discounted Present Value (D.P.V), of future expected dividends, as future dividends are uncertain (Fig:1), the discount rates (k), should reflect the riskiness of these dividend payments.

\[
V = E \left[ \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \cdots + \frac{D_t}{(1+k)^t} \right] \quad \text{(1)}
\]

where \(D_t\) are dividend payments (\(t = 1, 2, 3, \cdots\) years), \(k\) is risk adjusted discount rate for year (\(t = 1, 2, 3, \cdots\) years).

\(E\) is the “Expected Value”, with expectations formed at time \(t=0\). (Refer Appendix 1), and as per this formula, it does not require investors to consider holding the stock forever.

**Efficient Market Hypothesis (E.M.H.)**—(The current price is correct price and reflects expected future earnings from project):

- **Rational Expectations (RE):** Investors use all available relevant information, to forecast asset returns (or prices) and they do not make systematic forecast errors (i.e. their forecast errors are random, with a mean of zero).

- **Informational Efficiency (I.E.):** Equilibrium excess returns (i.e. returns adjusted for risk and transaction costs) are independent of currently available information.

- **Abnormal Profits:** (A.P.): It is impossible to consistently make abnormal profits (i.e. profits adjusted for risk and transaction costs) using a ‘stock picking’ strategy.
If $P < V$ (Then ‘buy’ the stock, and If $P > V$ (Then short (sell) the stock.) Invest in stocks if the N.P.V (Net Present Value) is positive.

Calculating the ‘Fair Value’

Case: 1: Using the ‘Yield Curve’
Case: 2: Using the CAPM(Capital Asset Pricing Model.)
Case: 3: Using the ‘SML (Security Market Line)
Case: 4: Modigliani-Miller Theorem.

- Individuals do not have different comparative advantages in the acquisition of information.
- These rational traders instantaneously ‘move’ market prices to equal ‘Fair Value’, as found by the PV (Present Value) of future dividends.
- If current and past information is immediately incorporated into current prices, then new information or ‘news’ should cause changes in prices.’ News’ by definition, is unforecastable, then price changes (or returns) should be unforecastable, which means no information at time $t$, or earlier should help improve the forecast of returns (to reduce the forecast error made by individual). E.M.H., provides a ‘benchmark’, in Finance literature, against which, we can judge the actual behavior of asset returns (and prices).

Exhibit: 3 Naive Diversification
Exhibit 4: **SHARPE’S SINGLE INDEX MODEL (MARKET MODEL)**

\[ R_i = \alpha_i + \beta_i * R_m + \epsilon_i. \]

Where:
- \( R_i \) is the return on security I, \( R_m \) is the return on the market index.
- \( \alpha_i \) is the constant return (Y intercept)
- \( \beta_i \) is the measure of sensitivity of the security I’s return to the return on the market index.
- \( \epsilon_i \) is the error term/Stochastic disturbance term/Noise term/Residual Variance?

Assumptions:

1) The error term has an expected value of zero and has finite variance.
2) The error term is not correlated with the return on the market portfolio
   Covariance (Rᵢ, Rₘ) = zero
3) Securities are related only through their common response to the return on the market index.
   Which implies that the error term of security I and security j are uncorrelated or
   Covariance (Rᵢ, Rⱼ) = zero
4) As constant term (aᵢ) and βᵢ*Rₘ are uncorrelated, the variance of the stock returns can be expressed as
   Variance (Rᵢ) = Variance (aᵢ + βᵢ*Rₘ + eᵢ)
   i. = Variance (βᵢ*Rₘ) + Variance (eᵢ)
   ii. = βᵢ²σₘ² + σ²(eᵢ)
   iii. = Systematic Risk + Idiosyncratic Volatility.

Based on Sharpe’s Single Index Model, we can arrive at the inputs required for applying the Markowitz Model.
1. E (Rᵢ) = aᵢ + βᵢ * E (Rₘ)
2. Variance (Rᵢ) = βᵢ² [Variance (Rₘ)] + Variance (eᵢ)
3. Covariance (Rᵢ, Rⱼ) = βᵢ*βⱼ*Variance (Rₘ)

Exhibit 5: EUGENE. FAMA’s DECOMPOSITION OF EXCESS RETURNS.
Fama’s Decomposition of Excess Returns is a performance attribution of Excess Returns i.e. Rp (Return of Portfolio) less Rf (risk free returns). This method can be used when an investor specifies a particular target level of risk (i.e., beta) is given then we can further decompose the systematic risk into investor’s risk and manager’s risk.
1. **RP due to manager’s Risk premium taking ability**: If the manager actually takes a different level of risk than the target level (i.e., the actual beta was different than the target beta) then part of the risk premium was due to the extra risk that the manager’s took.

\[
RP_{\text{manager’s risk}} = (\beta_p - \beta_T)(R_m - R_f)
\]

2. **RP due to Investors Target**: It is the Risk Premium that portfolio earns due to the target beta of the investor (risk taken by the investor).

\[
RP_{\text{investorRisk}} = \beta_T(R_m - R_f)
\]

3. **RP due to Diversification**: It is RP that the portfolio earns due difference between the return that should have been earned according to the CML and the return that should have been earned according to the SML. If the portfolio is perfectly diversified, this will be equal to 0. It shows the return due to unsystematic risk in the portfolio.

\[
RP_{\text{Diversification}} = \left[ RFR + \frac{\sigma_p}{\sigma_m}(R_M - RFR) \right] - \left[ RFR + \beta_p(R_M - RFR) \right] = (R_M - RFR) \left( \frac{\sigma_p}{\sigma_m} - \beta_p \right)
\]

4. **RP due to Net selectivity**: It is RP that the portfolio earns due ability of managers to select stocks. \(RP_{\text{Net selectivity}} = R_P - \left[ RFR + \frac{\sigma_p}{\sigma_m}(R_M - RFR) \right] \)

**Graph 7 E : Utility Indifference Curves**
Graph 7F: Security Market Line

Graph 7G: Time Series and Cross Sectional

Time series graph of HUL

Cross Sectional

Stock Prices

TIME

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Graph 7H: Normal Distribution

Graph 7I: Log Normal Distribution

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Dear Respondents,

This Questionnaire is aimed to describe the traditional (Economic) & behavioral factors that influences individual decision making process in the Indian Stock Market. Data collected through this process will be kept confidential as it is purely an Academic Research.

* Required

Name *

1. Age group (In Years) *

- 18 - 25
- 26 - 35
- 36 - 45
- 46 - 55
- 56 – 65
- 65 & Above

2. Gender *

- Male
- Female

3. Qualification *

- Under graduate
- Graduate
- Post graduate
- Doctorate
- Other: 

4. Occupation *

- Business
- Housewife
- Retired
- Student
- Professional
- Employee
- Other: 

5. Annual Income (In Indian Rupees) *

- NIL
- Less than 2 Lacs
- 2 – 5 Lacs
- 5 – 10 Lacs
- 10 – 15 Lacs
- 15 – 20 Lacs
- 20 – 30 Lacs
- 30 – 50 Lacs
- 50 Lacs – 1 Crore
- Above 1 Crore

6. What is your investment objective? *

- Income
- Growth
- Stability
- Capital preservation
- Increased Terminal Wealth
- Other: 

7. What is/are your preference(s) & Investment Constraints? *

- Liquidity
- Investment Horizon
- Taxes and Regulatory
- Unique Circumstances
8. Generally, what proportion of your Income / Wealth is invested? *

- Below 25%
- 25 – 50%
- 50 – 75%
- 75 – 100%
- More than 100%

9. What are your preferences in terms of various Investment Avenues available? *

- Equity Shares
- Money Market Instruments
- Life Insurance Policies
- Precious Objects
- Non-Marketable Financial Assets
- Bonds
- Mutual Funds Schemes
- Real Estate
- Financial Derivatives
- Commodities Market

10. What criteria (if any) do you adopt for evaluation of an Investment Avenue? *

- Rate of Return
- Risk
- Marketability
- Tax Shield / Shelter
- Convenience
- Liquidity
- Safety / Security
- None

11. What is your source of investment advice? *

- News/ TV channels
- Newspapers
- Advisers / Certified Financial Planners
12. If you are an Investor, what induces you to stay invested in a particular stock? *
- Dividend Paying Stock
- Organic Growth
- Inorganic Growth
- Future Prospects
- Principles & Policies

13. From how many years are you investing in the Stock Market? (In Years) *
- Less than 1
- 1 – 3
- 3 – 5
- 5 – 10
- More Than 10

14. What are your preferences when it comes to Sectoral Investments? *
- Banking
- Pharmaceuticals
- FMCG
- Telecom
- Oil / Gas / Power
- Diversified
- I.T.
- Venture Funds
- Other: 

15. What are the annual average returns delivered by your fund in the last 3 years? *
- 5-10%
- 10-25%
- 25-50%
- 50-100%
- NIL or Negative Returns
16. During the next five years, you expect your annual revenue to? *

- Decline significantly
- Decline slightly
- Remain about the same
- Increase slightly
- Increase significantly

17. If you reinvest the amount earned from your portfolio, how much percentage of it would you plan to reinvest? *

- Less than 10%
- 10-25%
- 25-50%
- Greater than 50%
- 100%

18. What proportion of your Investment during a given period, you generally wish to liquidate? *

- Upto 75% within 12 Months
- Upto 25% within 12 Months
- Between 50 – 75% in the next 2 – 3 Years
- Between 25 – 50% in the next 2 – 3 Years
- Won’t be selling for atleast the next 5 Years

19. Once withdrawal of Money from the Investment begins, how long do you expect the withdrawals to continue? *

- Less than 1 Year
- 1-3 Years
- 3-6 Years
- 6-10 Years
- More than 10 Years

20. What is your biggest concern in terms of your Investments getting affected? *

- Political Situation & Government Policies
- Fall in Market Index
- Rise in GDP / National Income
5/16/14

PH.D – Questionnaire for Collection & Analysis of Primary Data

☐ Inflation
☐ International Effects / F.D.I / F.I.I
☐ No Concern

21. Your general preference (If any), in the kind of stocks you would Trade / Invest in? *
☐ Blue Chip
☐ Large Cap
☐ Mid Cap
☐ Small Cap
☐ Mixed / Diversified
☐ Venture Funds
☐ International Investments

22. What is your take on Investment risks V/S Investment objectives? *
☐ To get back the Amount Deposited / Invested
☐ To get Investment Returns above savings & time deposit rates (upto 10%)
☐ To achieve Low to Moderate Capital Appreciation on Investments (10-15%)
☐ To achieve Moderate to High Capital Appreciation on Investments (15-25%)
☐ To achieve Good Capital Appreciation on Investments (25-40%)
☐ To achieve Super Normal Capital Appreciation on Investments (40% and above)

23. What kind of Risk Appetite do you perceive to possess? *
☐ No Risk
☐ Low Risk
☐ Medium Risk
☐ High Risk
☐ Very High Risk

24. What is the source of funds employed to invest / trade in the Stock Market? *
☐ Savings
☐ Loan
☐ Pledge / Hypothecation of Asset
☐ Inheritance
☐ Goodwill
25. What is the frequency of Trading / Investing in Investments in the Stock Market? *

- Daily
- Weekly
- Monthly
- Seasonal
- Cyclical
- Occasionally
- Rarely
- Never

26. What do you think determines the choice of Asset Mix in your Portfolio? *

- Proportion of Stocks & Units / Shares of equity oriented mutual funds.
- Proportion of Bonds (Fixed Income)
- Appropriate stock – bond mix
- Risk Tolerance
- Investment Horizon

27. What portfolio strategy (if any) you adopt, once the asset mix decision is made? *

- Active Portfolio
- Passive Portfolio
- Constant Proportion Portfolio Insurance
- Constant Mix
- Naive
- No Strategy

28. What do you think is your commonly used approach to Investing? *

- Fundamental
- Technical
- Eclectic
- Academic
- Practical
- Behavioural
- None of these

29. What do you think are the qualities required for successful Investing? *

https://docs.google.com/forms/d/13csI7U-1oCuWzJXErTlb2Sk7gI0e7g0mrDr8XmLwjc/viewform
30. What approaches, according to you, can be adopted by Investors to succeed in the Investment Game? *
- Naïve
- Physically Difficult
- Intellectually Difficult
- Psychologically Difficult
- Mix of Traditional & Behavioural Approaches

31. Your reactions when there is a drastic reduction in the value of your Investments? *
- Buy more of Such Investments
- Wait for the Market to Correct / Recover
- Panic Button
- Worry Compounded
- Liquidate Immediately

32. Your personal experience on realized returns on your Investments vis-à-vis Horizon (Time Frame)? *
- Long-Term Investments in Stocks always pays
- Short-Term Investments Pays
- Intuition
- Burnt my fingers in the Stock Market
- No Gain / No Losses Overall

33. If you are a Global Investor, which type of risk according to you exists? *
- Political
- Currency
- Custodial
34. Share Prices according to you are influenced by? *
   - Fundamental Factors
   - Technical Factors
   - Psychological Factors
   - All of these
   - None of these

35. What Screening Criteria, do you adopt (if any) for considering Investing in shares in the Secondary Market? *
   - Size of the Company more than Rs. 100 Crores & Equity Base of over Rs. 20 Crores
   - Competitive Position
   - Industry Prospects
   - Price - Earning Ratio
   - Dividend & Bonus Records
   - Reputation of Management

36. Have you ever felt that in Investing, human beings display irrationality? *
   - Yes
   - No
   - Can't Say

37. Selecting stocks, according to you is somewhat Confusing, Difficult & Frustrating? *
   - Strongly Agree
   - Agree
   - Neutral
   - Disagree
   - Strongly Disagree

38. Stock Prices are much more volatile than Dividends? *
   - Strongly Agree
   - Agree
   - Neutral

- Sustainable Competitive Advantage
- Low Price Stock Availability
- Hungry Management Dominance
- Huge External Opportunities for a company
- No Government Regulation on Pricing

40. What is the Investment philosophy of your firm? *

- Value Investing
- Fundamentals Investing
- Growth Investing
- Socially Responsible Investing
- Technical Investing

41. In your opinion, if an Investor’s focus is only on growth stocks, his investment philosophy should revolve around? *

- Sound Benchmarking for Selecting Growth Stocks
- Invest in Growth Stocks without much concern for the price
- Hold Growth Stocks as long as they remain Growth Stocks
- Identify Growth situations which have not been fully discounted by the market

42. How do you track the Status & Progress of your Investments? *

- Manual Reports
- Spreadsheets
- Quickbooks & Accounting Packages
- CRM or Database tools
- Other: [ ]

43. Do you make use of any statistical tools while calculating expected returns on your portfolio? *

- Yes
- No
44. Does your portfolio have any shares traded in the NASDAQ or DOW JONES? *
   [ ] Yes
   [ ] No
   [ ] Don't Know

45. Do you think that you have adequate Insurance Coverage, in case you face huge losses in your Investments? *
   [ ] Yes
   [ ] No
   [ ] Don't Know

46. Are you Bullish or Bearish on the Indian Stock Market? *
   [ ] Bullish
   [ ] Bearish
   [ ] Depends

47. What best describes your Investment Experience? *
   [ ] Beginner (No Investment Experience)
   [ ] Moderate (Comfortable with Fixed Income Securities)
   [ ] Knowledgeable (Involved in Equity Trading)
   [ ] Experienced (Exposed to Sophisticated Tools & Instruments)

48. Just a few weeks after you have invested in a stock, its price declines by 20%, what would you do, if the fundamentals of the stock have not changed? *
   [ ] Sell
   [ ] Buy
   [ ] Do Nothing

49. Refer to the Previous Question, If that stock is part of a portfolio designed to meet investment goals, what would you do, if it were 5 Years away? *
   [ ] Sell
   [ ] Buy
   [ ] Do Nothing
50. Refer to the Previous Question, What would you do, If the goal were 15 Years away? *

☐ Sell
☐ Buy
☐ Do Nothing

51. Refer to the Previous Question, what would you do, If the goal were 30 Years away? *

☐ Sell
☐ Buy
☐ Do Nothing

52. You bought a stock as a part of your Retirement Portfolio. Its price rises by 20% after 4 weeks, what would you do if the fundamentals of the stock have not changed? *

☐ Sell
☐ Buy
☐ Do Nothing

53. You are investing for Retirement which is say atleast 15 Years away, What would you do? *

☐ Invest in Money Market Mutual Fund or a Guaranteed Investment Contract
☐ Invest in a Balanced Mutual Fund that has a Stock : Bond mix of 1:1
☐ Invest in an Aggressive Growth Mutual Fund

54. As a prize winner, you have been given some choice, which one would you choose? *

☐ Rs. 5 Lacs in Cash
☐ A 50% Chance to get Rs. 12.5 Lacs
☐ A 20% Chance to get Rs. 37.5 Lacs

55. A Good Investment Opportunity has come by your way. To participate in it, you have to borrow money. Would you take a Loan? *

☐ No
☐ Yes
☐ Perhaps

56. If you are working in a company, which is planning to go public after 3 years, is offering stock to the employees. Until it goes public you cannot sell your shares, your Investment however has a potential of multiplying 10 times when the company goes public how much money would you invest? *

☐ Nothing
☐ 3 Months’ Salary
☐ 6 Months’ Salary

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