CHAPTER 5
SPACE TIME BLOCK CODING
This Chapter provides discussion on MIMO systems with respect to channel capacity, system model, channel models including the focus on BER of STBC using adaptive semiblind channel estimation scheme with less number of training symbols and high code rate Space time block codes.

5.1. INTRODUCTION

Analysis of adaptive semiblind channel estimation scheme for MIMO antenna array systems with different code rate space time block coding (STBC) has been performed using the adaptive pilot assisted modulation scheme proposed earlier. Semi blind channel estimation method provides the best trade-off in terms of bandwidth overhead, computational complexity and latency. The result after using MIMO systems shows higher data rate and longer transmit range without any requirement of additional bandwidth or transmit power. This Chapter presents the detailed analysis of diversity coding techniques using MIMO antenna systems. Different STBC schemes have been explored and analyzed with the different code rate STBC techniques using MATLAB environment and the simulated results have been compared in the semiblind environment which shows the improvement even in highly correlated antenna arrays, and is found close to the condition when channel state information (CSI) is known to the channel.

5.2 DIFFERENT TYPES OF STBC CODES

5.2.1. STBC For Real Constellations

Considering $M_T \times M_R$ transmission matrix with variables $s_1, s_2, ..., s_{ns}$ satisfying [Tarokh et al. (1999a); Vucetic and Yuan (2005)] -

$$S_{M_T} \cdot S_{M_T}^T = c \left| S_1 \right|^2 + \left| S_2 \right|^2 + \ldots + \left| S_{ns} \right|^2 \cdot I_{M_T}$$

where, $c$ is a constant and $I_{M_T}$ is a $M_T \times M_T$ identity matrix, STBC can achieve a full diversity of order of 1. Square STBC matrix $S_{M_T}$ with real constellation exist if and only if the number of transmit antennas
are \( M_T = 2, 4, \) or \( 8 \). These codes offer full transmit diversity of \( M_T \) due to their full rate \( R = 1 \). The real transmission matrices for 2, 4 and 8 transmit antennas are given by -

\[
S_2 = \begin{bmatrix}
  s_1 & s_2 \\
  -s_2 & s_1 
\end{bmatrix}, \quad S_4 = \begin{bmatrix}
  s_1 & s_2 & s_3 & s_4 \\
  -s_2 & s_1 & -s_3 & s_4 \\
  -s_3 & s_4 & s_1 & -s_2 \\
  -s_4 & -s_3 & s_2 & s_1 
\end{bmatrix}
\]

\[
S_8 = \begin{bmatrix}
  s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\
  -s_2 & s_1 & s_4 & s_5 & -s_6 & -s_7 & -s_8 & s_7 \\
  -s_3 & -s_4 & s_2 & s_7 & s_8 & -s_5 & -s_6 & s_5 \\
  -s_4 & s_3 & -s_2 & s_1 & s_8 & -s_7 & -s_6 & s_6 \\
  -s_5 & -s_6 & -s_7 & s_1 & s_2 & s_3 & s_4 & s_3 \\
  -s_6 & s_5 & -s_8 & s_7 & -s_2 & s_1 & -s_3 & s_4 \\
  -s_7 & s_8 & s_5 & -s_6 & -s_3 & s_4 & s_1 & -s_4 \\
  -s_8 & -s_7 & s_6 & s_5 & -s_4 & -s_3 & s_2 & s_1 
\end{bmatrix}
\]

(5.2.2)

At the receiver end, the received expressions are based on Alamouti’s model with the simplicity of having only real symbols and therefore no conjugate symbol in the equations. Thus the received expressions \( r_{M_T, M_R} \) for any number of received antennas becomes -

\[
r_{1, M_R} = r_{M_R} t = h_{1, M_T} s_1 + h_{2, M_T} s_2 + n_{1, M_R}
\]

\[
r_{2, M_R} = r_{M_R} t - T = h_{1, M_T} s_2 + h_{2, M_T} s_1 + n_{2, M_R}
\]

\[
r_{r_{1, M_T}} = r_{M_T} t = h_{1, M_T} s_1 + h_{2, M_T} s_2 + h_{3, M_T} s_3 + h_{4, M_T} s_4 + n_{1, M_R}
\]

\[
r_{r_{2, M_T}} = r_{M_T} t - T = -h_{1, M_T} s_2 + h_{3, M_T} s_1 - h_{4, M_T} s_3 + h_{4, M_T} s_4 + n_{2, M_R}
\]

\[
r_{r_{3, M_T}} = r_{M_T} t + 2T = -h_{1, M_T} s_3 + h_{2, M_T} s_4 + h_{5, M_T} s_1 - h_{6, M_T} s_2 + n_{3, M_R}
\]

\[
r_{r_{4, M_T}} = r_{M_T} t + 3T = -h_{1, M_T} s_4 - h_{2, M_T} s_3 + h_{5, M_T} s_2 + h_{6, M_T} s_1 + n_{4, M_R}
\]

(5.2.4)

where, \( n_{1, M_R}, n_{2, M_R}, n_{3, M_R}, n_{4, M_R} \) are independent noise samples, \( h_{1, M_T} \) is the channel transfer function from the \( M_T \) transmit antenna and \( M_R \) denotes the receive antenna. Received signals are then combined for two transmit antennas as -

\[
\tilde{s}_1 = \sum_{M_T=1}^{M_T} r_{1, M_T} h_{1, M_T} + r_{2, M_T} h_{2, M_T}
\]

\[
\tilde{s}_2 = \sum_{M_T=1}^{M_T} r_{1, M_T} h_{2, M_T} - r_{2, M_T} h_{1, M_T}
\]

(5.2.5)
Similarly, the received signal for four and eight transmit antenna can also be derived. Alamouti STBC does not require CSI at the transmitter and can be used with two transmit antennas and 1 receive antenna with accomplishment of full diversity of 2. It reduces the effect of fading at receiver station at the cost of some additional antenna elements at the transmitter end. If having more antennas is not a problem, then this scheme is appropriate for getting full diversity of $2M_R$ with two transmit antennas.

5.2.2. STBC For Complex Constellation

For STBC with complex constellation, if $M_T \times M_R$ transmission matrix with variables $s_1, s_2, ..., s_n$ satisfies [Tarokh et al. (1999a) ; Vucetic and Yuan (2005)] -

$$S_{M_T} \cdot S_{M_R}^T = c |S_1|^2 + |S_2|^2 + ....... + |S_n|^2 I_{M_T}$$ (5.2.6)

where, $c$ is a constant and $I_{M_T}$ is a $M_T \times M_T$ identity matrix, STBC can achieve full diversity of the order of 1. The full rank diversity was introduced by Alamouti is considered as the simplest STBC with complex constellation and it is also the only $M_T \times M_R$ STBC code with complex constellation, which is the only STBC achieving full rate of 1 for a full diversity of 2. For the case of 3 transmit antennas, [Tarokh et al. (1998)] made block codes with 1/2 and 3/4 code rate and full diversity $3M_R$. The aim of using higher number of transmit antennas on generalized STBC is to achieve high rate with full diversity, minimum coding delay $n_s$ and minimum decoding complexity. Examples of half rate complex transmission matrices achieving full diversity for three and four transmit antennas are given as -

$$S^c_3 = \begin{bmatrix}
   s_1 & s_2 & s_3 \\
   s_1 & -s_4 & s_2 \\
   -s_4 & s_3 & s_2 \\
   s_1 & s_2 & s_3 \\
   -s_2 & s_1 & -s_4 \\
   -s_3 & s_4 & s_1 \\
   -s_4 & -s_3 & s_2 \\
   s_1 & s_2 & s_3 \\
   -s_2 & s_1 & -s_4 \\
   -s_3 & s_4 & s_1 \\
   -s_4 & -s_3 & s_2 \\
   s_1 & s_2 & s_3 \\
\end{bmatrix}, \quad S^c_4 = \begin{bmatrix}
   s_1 & s_2 & s_3 & s_4 \\
   -s_2 & s_1 & -s_4 & s_3 \\
   -s_3 & s_4 & s_1 & -s_2 \\
   -s_4 & -s_3 & s_2 & s_1 \\
   -s_1 & s_2 & s_3 & s_4 \\
   -s_2 & s_1 & -s_4 & s_3 \\
   -s_3 & s_4 & s_1 & -s_2 \\
   -s_4 & -s_3 & s_2 & s_1 \\
\end{bmatrix}$$ (5.2.7)

The matrix $S^c_3$ code transmit 4 symbols every 8 time intervals, and therefore has rate 1/2. For both schemes, flat fading channel are assumed to be constant over 8 symbol periods. Thus, following the real constellation derivations, the received signal for three transmit antenna can be expressed as -
\begin{align}
    &r_{1,M_t} = r_{M_t} t = h_{1,M_t} s_1 + h_{2,M_t} s_2 + h_{3,M_t} s_3 + n_{1,M_t} \\
    &r_{2,M_t} = r_{M_t} t + T = -h_{1,M_t} s_2 + h_{2,M_t} s_1 - h_{3,M_t} s_3 + n_{2,M_t} \\
    &r_{3,M_t} = r_{M_t} t + 2T = -h_{1,M_t} s_3 + h_{2,M_t} s_2 + h_{3,M_t} s_1 + n_{3,M_t} \\
    &r_{4,M_t} = r_{M_t} t + 3T = -h_{1,M_t} s_4 - h_{2,M_t} s_3 + h_{3,M_t} s_2 + n_{4,M_t} \\
    &r_{5,M_t} = r_{M_t} t + 4T = h_{1,M_t} s_1 + h_{2,M_t} s_2^* + h_{3,M_t} s_3^* + n_{5,M_t} \\
    &r_{6,M_t} = r_{M_t} t + 5T = -h_{1,M_t} s_3^* + h_{2,M_t} s_1^* - h_{3,M_t} s_4^* + n_{6,M_t} \\
    &r_{7,M_t} = r_{M_t} t + 6T = -h_{1,M_t} s_1^* + h_{2,M_t} s_2^* + h_{3,M_t} s_3^* + n_{7,M_t} \\
    &r_{8,M_t} = r_{M_t} t + 7T = -h_{1,M_t} s_4^* - h_{2,M_t} s_3^* + h_{3,M_t} s_2^* + n_{8,M_t} \\
\end{align}

\[(5.2.8)\]

Received signals are then combined to retrieve the original transmitted symbols using maximum likelihood detection to minimize the decision metric as -

\begin{align}
    \tilde{s}_1 &= \sum_{M_t=1}^{M_t} r_{1,M_t} h_{1,M_t}^* + r_{2,M_t} h_{2,M_t}^* + r_{3,M_t} h_{3,M_t}^* + r_{4,M_t} h_{4,M_t}^* + r_{5,M_t} h_{5,M_t}^* + r_{6,M_t} h_{6,M_t}^* + r_{7,M_t} h_{7,M_t}^* + r_{8,M_t} h_{8,M_t}^* \\
    \tilde{s}_2 &= \sum_{M_t=1}^{M_t} r_{1,M_t} h_{1,M_t}^* - r_{2,M_t} h_{2,M_t}^* + r_{3,M_t} h_{3,M_t}^* + r_{4,M_t} h_{4,M_t}^* - r_{5,M_t} h_{5,M_t}^* + r_{6,M_t} h_{6,M_t}^* + r_{7,M_t} h_{7,M_t}^* + r_{8,M_t} h_{8,M_t}^* \\
    \tilde{s}_3 &= \sum_{M_t=1}^{M_t} \left( r_{1,M_t} h_{1,M_t}^* - r_{3,M_t} h_{3,M_t}^* - r_{4,M_t} h_{4,M_t}^* + r_{6,M_t} h_{6,M_t}^* + r_{7,M_t} h_{7,M_t}^* - r_{8,M_t} h_{8,M_t} \right) \\
    \tilde{s}_4 &= \sum_{M_t=1}^{M_t} \left( -r_{2,M_t} h_{2,M_t}^* + r_{3,M_t} h_{3,M_t}^* - r_{4,M_t} h_{4,M_t}^* + h_{6,M_t} h_{3,M_t} + r_{7,M_t} h_{2,M_t} - r_{8,M_t} h_{6,M_t} \right) \\
\end{align}

\[(5.2.9)\]

Similarly, the received signal for four transmit antenna can also be formed. If three transmit antennas are considered and, three symbols are transmitted every four time intervals, and therefore has code rate 3/4.

Example of 3/4 code rate complex transmission matrix for three transmit antennas is given as -

\[
    S_c^3 = \begin{bmatrix}
        s_1 & s_2 & \frac{s_3}{\sqrt{2}} \\
        -s_2^* & s_3^* & \frac{s_3}{\sqrt{2}} \\
        \frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & \frac{(-s_1 - s_1^* + s_2 - s_2^*)}{2} \\
        \frac{s_3^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} & \frac{(s_2 + s_2^* + s_1 - s_1^*)}{2}
    \end{bmatrix}
\]

\[(5.2.10)\]

It is known that the complexity at the receiver end increases linearly with the number of transmit antennas and the receive antennas. Indeed, for X receiving antennas, the expression of matrix will have X times more terms than that it has now. Performance of STBC for complex constellation matrices of 1
bit/s/Hz, 2 bit/s/Hz and 4 bit/s/Hz for two transmit antennas and 1/2 bit/s/Hz, 1bit/s/Hz and 2 bit/s/Hz for three and four transmit antennas have already been analyzed and the spectral efficiency of each type of matrices according to the modulation used have been summarized in Table 5.1,

<table>
<thead>
<tr>
<th>Number of Tx</th>
<th>Modulation</th>
<th>BPSK (1 bit/symbol)</th>
<th>QPSK (2 bit/symbol)</th>
<th>16-QAM (4 bit/symbol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Tx</td>
<td>1 bit/s/Hz</td>
<td>2 bit/s/Hz</td>
<td>4 bit/s/Hz</td>
<td></td>
</tr>
<tr>
<td>3 Tx</td>
<td>½ bit/s/Hz</td>
<td>1 bit/s/Hz</td>
<td>2 bit/s/Hz</td>
<td></td>
</tr>
<tr>
<td>4 Tx</td>
<td>½ bit/s/Hz</td>
<td>1 bit/s/Hz</td>
<td>2 bit/s/Hz</td>
<td></td>
</tr>
</tbody>
</table>

### 5.2.3. Orthogonal Space Time Block Codes

As shown earlier, examples of 1/2 and 3/4 code rate complex transmission matrices for four transmit antennas have been proposed by [Tarokh et al. (1999a)] which gave full diversity of $4M_r$. With four transmit antennas and code rate of 1/2 and 3/4, complex transmission matrices have been given as -

$$S_4^c = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & -s_3 & s_4 \\ -s_3 & s_4 & s_1 & -s_2 \\ -s_4 & -s_3 & s_2 & s_1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = \begin{bmatrix} \frac{s_1}{\sqrt{2}} \\ \frac{s_2}{\sqrt{2}} \\ \frac{s_3}{\sqrt{2}} \\ \frac{s_4}{\sqrt{2}} \end{bmatrix}$$

(5.2.11)

### 5.2.4. Quasi-Orthogonal Space Time Block Codes

Full rate STBC using complex symbols in its transmission matrix are not possible to achieve as we have seen in previous section. Indeed, the particular case of Alamouti code presented can only achieve full rate with full diversity which follows the rules of orthogonal design for simple decoding. [Jafarkhani (2001)] proposed the new STBC technique called quasi-orthogonal STBC (QOSTBC), which achieved full rate at the cost of higher complexity decoding. Quasi-orthogonal designs are attractive because of its achievement for higher code-rate than orthogonal designs and lower decoding complexity than non-orthogonal designs. [Jafarkhani (2001)] suggested -
Now, it is defined $V_i, i=1,2,3,4$, as the $i$th column of $A$, it is easy to see that $(V_i, V_j) = (V_i, V_j)$ = $(V_i, V_j) = 0$, where, $(V_i, V_j) = \sum_{i=1}^{4}(V_i)_j^*$ is the inner product of vectors $V_i$ and $V_j$. Therefore, the subspace created by $V_i$ and $V_j$ is orthogonal to the subspace created by $V_k$ and $V_m$. This orthogonality allows the calculation of the maximum likelihood decision metric which minimizes the following sum -

$$ f_{14}(s_1, s_4) + f_{23}(s_2, s_3) $$

(5.2.13)

where, $f_{14}$ is independent of $s_2$ and $s_3$ and $f_{23}$ is independent of $s_1$ and $s_4$, and are decoded separately.

Indeed the maximum likelihood detection is to find the pair $(s_1, s_4)$ and $(s_2, s_3)$ that minimizes $f_{14}(s_1, s_4)$ over all possible values of $(s_1, s_4)$ and minimizes $f_{23}(s_2, s_3)$ over all the possible values of $(s_2, s_3)$ pairs.

It seems that the complexity of the decoder is increases compared to the STBC decoder presented earlier. However the complexity of QOSTBC does not grow linearly as for STBC but exponentially with the number of transmit and receive antennas. Similarly the QOSTBC code with different rate and higher number of transmit antennas have also been proposed.

### 5.3. SYSTEM MODEL FOR STBC IMPLEMENTATION

The same system model has been considered with an $M_T \times M_R$ quasi static Rayleigh flat fading MIMO channel, where, $M_T$ and $M_R$ denote the number of transmit and receive antennas. The system is described by $y(k) = Hx(k) + n(k)$, where, $x$ is $[x_1(k), x_2(k), \ldots x_{M_T}(k)]^T$ which is the transmitted symbol vector of $M_T$ transmit with the symbol energy given by $E[|x_m(k)|^2] = \sigma^2$ for $1 < m < M_T$ and covariance matrix $Q = E(XX^H)$, $y$ denotes the received vector $y(k) = [y_1(k), y_2(k), \ldots y_{M_R}(k)]^T$ and $n(k) = [n_1(k), n_2(k), \ldots n_{M_R}(k)]^T$ is the complex valued Gaussian white noise vector at the receiving end for MIMO channels with energy $E[n(k)n^H(k)] = 2\sigma^2 l_{M_R}$ distributed according to $\mathcal{N}_c(0, \sigma^2 l_{M_R})$ assumed to be zero mean, spatially and temporally white and independent of both channel and data fades.

The channel model considered here denoted by $H = R_R^{1/2}H_w R_T^{1/2}$ [Da-Shan et al. (2000)] with $R_r$ & $R_T$ representing the normalized transmit and receive correlation matrices with identity matrix. The entries of $H_w$ are independent and identically distributed (i.i.d.) $\mathcal{N}_c(0,1)$. 

\[ S_{40} = \begin{bmatrix} s_1, s_2, s_3, s_4 \\ -s_2, s_4, -s_4^*, s_3 \\ s_2^*, s_1, s_4, s_2 \\ -s_4^*, -s_3, -s_3^*, s_1 \\ s_1^*, -s_1, s_2^*, -s_4 \end{bmatrix} \] (5.2.12)
Figure 5.1. System block model using the Alamouti’s STBC method.

Assuming $H$ remains constant during the transmission of a codeword, and taking independent values from one codeword to another. The realization of $H$ is assumed to be partially known at the receiver, but not at the transmitter. A system block diagram using Alamouti’s method is shown in Figure 5.1. The transmitting symbols are encoded according to orthogonal STBC scheme. A pilot sequence is inserted in the transmission of every $X$ symbol to easily estimate the channel. This pilot sequence will be reduced with the implementation of the proposed adaptive semi-blind estimation scheme in [Kumar and Saxena (2012a)]. A different pilot scheme has been used for each channel and these orthogonal pilot sequences enable the receiver to decouple pilot sequences from the combined signals for each channel at a receive antenna. The transmitted sequence from each antenna faces the flat fading channel with Additive White Gaussian Noise (AWGN). From the received sequences, the pilot symbols are extracted and passed through the channel estimation process. The channel estimation process further passes these detected estimated symbols to the decoder. Then STBC signals are detected, using the estimated channel information as if they were actual path gains. The transmitted symbols have been considered having empty slots left in its codeword matrix for maintaining the orthogonality between the symbols of the vector.

Assuming the block transmission scheme with block length $T$, the $n^{th}$ received data block can be expressed as:

$$y(k) = D(k, \omega_0) x(k) + n(k)$$  \hspace{1cm} (5.3.1)

where,
here, $D(k, \omega_b)$ is defined as -
\[
D(k, \omega_b) = \text{diag} \left\{ e^{i \omega_b (k-1)T + j} \right\},
\]
where, the $(.)^T$ denotes the transpose in the second exponential term. If a slow fading environment is considered, the time becomes much longer than the data block length $T$. The matrix $X(k) = X\{s(k)\}$ can be treated as a mapping transform the $k^{th}$ block to $T \times K$ complex matrix of transmit signals, where, $S(k) = \{s^{(1)} \ s^{(2)} \ldots \ s^{(L)}\}^T$ is the $k^{th}$ symbol vector alphabet set of length $L$, i.e., set of all possible symbol vector. The entries of $S(k)$ are assumed to be randomly drawn from a constant modulus constellation, that is $|S_L(k)| = 1$. Using this assumption, the requirement for estimating the norm of the channel matrix can be raised.

The $T \times K$ matrix $XS(k)$ is called an OSTBC if all elements of this matrix are linear functions of the $K$ complex variables $s_1(k), s_2(k), \ldots, s_L(k)$, and their complex conjugates, and if for any arbitrary $s(k)$, it satisfies the following property -
\[
X(S(k))X^H(S(k)) = \|S(k)\|^2 I_k
\]
where, $I_k$ is the $K \times K$ identity matrix, $\|\|$ is the Euclidean norm, and $(\cdot)^H$ denotes the Hermitian transposition. The calculation of the basis function of OSTBC can be denoted as -
\[
X(S(k)) = \sum_{k=1}^{K} \left[ C_k \text{Re}\{S_k(n)\} + C_{k+K} \text{Im}\{S_k(n)\} \right]
\]
where,
\[
C_k = \begin{cases} 
X(e_k), & \text{for } 1 \leq k \leq K \\
X(je_{k-K}), & \text{for } K+1 \leq k \leq 2K
\end{cases}
\]
where, $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ denote the real and imaginary parts, respectively and $k \times 1$ vector $e_k$ is defined as the $k^{th}$ column of the identity matrix $I_k$. It is known that OSTBC are completely defined by its basis.
matrices $\{C_k\}_{k=1}^{2K}$. If the channel frequency offset is not available then the (5.3.1) can be rewritten in vector form without going for the calculation of the basis function of the OSTBC and the $2M_T T \times 2K$ real valued matrix can be denoted as, $X(H) = \begin{bmatrix} C_1 H & C_2 H & \ldots & C_{2K} H \end{bmatrix}$, which captures both the effects of the OSTBC and the channel. The $X(H)$ matrix follows the decoupling property, i.e., its columns have identical norms and are orthogonal to each other,

$$X(H)X^T(H) = \|H\|_F I_{2K} \quad (5.3.3)$$

where, $\|\cdot\|_F$ denotes the Frobenius norm of a matrix. $X(k,S(k))$ follows the basis matrices and referred to as a time varying OSTBC.

### 5.4. DESIGN CONDITION AND DECODING METHOD

It has been found that $XS(k)$ denote an OSTBC for $M_T$ transmit antennas which transmit $k$ information symbols $x_1(k), x_2(k), \ldots, x_{M_T}(k)$ with having empty slots left in its codeword matrix for orthogonality, we then found $k + \lambda$ information symbols transmitting high code rate with full diversity $X_{n,k+\lambda}$ from $XS(k)$ as -

$$X_{n,k+\lambda} = XS(k) + PW_{\lambda} \quad (5.4.1)$$

where, $W_{\lambda}$ is the codeword matrix with $\lambda$ additional information symbols to be transmitted from empty slots of $XS(k)$, and $P$ is the optimization matrix wise entries are complex design parameters to be determined by the rank and determinant criteria, and both $XS(k)$ and $PW_{\lambda}$ are non-overlapping entries.

Owing to the non-orthogonal structure of the information symbols as unknown deterministic parameters, it is required to apply ML estimation approach to jointly estimate the symbols and pilots both. To obtain the ML estimates of all these parameters, the log-likelihood function need to be maximized. Hence the parameter estimates can be found by solving the following optimization problem -

$$\max \max_h \log f(y(1), y(2), \ldots, y(n) | S, h) \quad (5.4.2)$$

where, $f(y(1), y(2), \ldots, y(n) | S, h)$ is the likelihood function computed for $N$ snapshots $y(n)_{n=1}^N$, and $S$ is the set of all possible values of the transmitted symbols received. It is not easy to solve (5.4.2) because its computational cost grows exponentially in $N$. To simplify the optimization problem in (5.4.2), we have to maximize the expectations and minimize the error in the estimates for which,

$$\hat{X} = \arg \min_{x_1, x_2, \ldots, x_{n+\lambda}} \|Y - X_{n,k+\lambda} H\|^2 \quad (5.4.3)$$
Now the elimination of terms coming from additional transmitted symbols from empty slots of $XS(k)$ will be tried by computing intermediate signals from the received signals for all possible values of the additional symbols $x_{k+1}, x_{k+2}, \ldots, x_{k+\delta}$ in $W_x$ as -

$$Z = Y - PW_xH$$

(5.4.4)

and the optimization problem in (5.4.2) can be rewritten as -

$$\max_{h, S} \log f\{y(1), y(2), \ldots, y(n) | S, h\}$$

(5.4.5)

Since, the noise vector $n(k)$ are zero mean i.i.d. Gaussian with covariance matrix,

$$Q\{n(k)\} = \frac{\sigma^2}{2} I_{2M_r}$$

the likelihood function for any $y(n)$ can be expressed as -

$$f\{y(n) | H\} = \left(\frac{1}{(\pi\sigma^2)^{\frac{M_r T}{2}}}\right) e^{-\frac{1}{2} E\{X(n, h)H(h)\}}$$

(5.4.6)

here, $E\{\}$ denotes the statistical expectation. Taking into account that all $\{y(n)\}_{n=1}^{N}$ are independent random vectors, the obtained value is -

$$f\{y(1), y(2), \ldots, y(n) | H(n), h\} = \prod_{n=1}^{N} f\{y(n) | H(n), h\}$$

(5.4.7)

Using (5.4.6) and (5.4.7), the problem in (5.4.5) can be formulated as shown in (5.4.3) and can also be written as -

$$\min_{h, S} \sum_{n=1}^{N} \left\|y(n) - X(n, h)H(n)\right\|^2$$

(5.4.8)

It is known in (5.4.8), that the $n^{th}$ term of the sum is minimized with,

$$H(n) = \frac{1}{\|h\|^2} X^T(n, h)y(n)$$

(5.4.9)

where, (5.4.9) follows from the fact that $X(n, h)$ satisfies the decoupling property that has been discussed earlier in (5.3.3). Using this relation, the objective function in (5.4.8) can be concentrated w.r.t. $\{H(n)\}_{n=1}^{N}$ and after such concentration, the latter optimization problem can be shown as -

$$\min_{h, S} \sum_{n=1}^{N} \left\|y(n) - \frac{X(n, h)X^T(n, h)y(n)}{\|h\|^2}\right\|^2$$

(5.4.10)
This function can further be solved in a simple manner and found with the existence of traces of matrix as -

\[
\sum_{n=1}^{N} \left\| y(n) - \frac{X(n,h)X^T(n,h)y(n)}{\|h\|} \right\|^2
\]

\[
= \sum_{n=1}^{N} \frac{2}{\|h\|} X(n,h)X^T(n,h)y(n)y^T(n)
\]

\[
+ \frac{1}{\|h\|} \left( \frac{X(n,h)X^T(n,h)y(n)y^T(n)X(n,h)X^T(n,h)y(n) + \|y(n)\|^2}{\|h\|} \right)
\]

\[
= \frac{1}{\|h\|} \sum_{n=1}^{N} X(n,h)X^T(n,h)y(n)y^T(n) + \text{constant}
\]

\[
= \frac{1}{\|h\|} \sum_{n=1}^{N} \text{tr} \left( X(n,h)X^T(n,h)y(n)y^T(n) \right) + \text{constant} \quad (5.4.11)
\]

Now with the little replacements in the expression for convenience, relation can be denoted as -

\[
\text{vec}^T \left\{ X(n,h) \right\} \left( I_{2K} \otimes y(n)y^T(n) \right) \text{vec} \left\{ X(n,h) \right\}
\]

\[
= h^T \Phi^T(n) \left( I_{2K} \otimes y(n)y^T(n) \right) \Phi(n) \quad (5.4.12)
\]

where, \( \Phi(n) \) is \( 4KM_f T \times 2M_f N \) matrix whose kth column can be defined as, \( \left[ \Phi(n) \right]_k = \text{vec} \left\{ X(n), e_i \right\} \),

where, \( e_i \) is the kth column of the Identity matrix \( I_{2MN} \) and \( \otimes \) is the Kronecker matrix product. Now putting (5.4.12) in (5.4.11), the concentrated optimization problem can be denoted as -

\[
\max_k \frac{h_k^T \Psi(\phi_k) h_k}{\|h\|^2}
\]

(5.4.13)

where,

\[
\Psi(\phi) = \sum_{n=1}^{N} \Phi^T(n) \Phi(n) \left( I_{2K} \otimes y(n)y^T(n) \right) \quad (5.4.14)
\]

(5.4.14) is \( 2M_f N \times 2M_f N \) real matrix which depends on the received data vectors \( \{ y(n) \}_{n=1}^{N} \) and the carrier frequency offset \( \phi \). Further, this can be solved and the carrier frequency offset can be derived using (5.4.13) -

\[
\hat{\phi} = \arg \max_{\phi} \lambda_{\text{max}} \left\{ \Psi(\phi) \right\}
\]

(5.4.15)

where, \( \Psi \) denotes the largest eigenvalues of matrix. And further for the estimates of channel, we receive -

\[
\hat{h} = \mathbb{P} \left\{ \Psi(\hat{\phi}) \right\}
\]

(5.4.16)
where, $P(\cdot)$ denotes the normalized principal eigenvector of a matrix with the assumption of no multiplicity in the largest eigenvalues of $\Psi(\hat{\omega})$. Now for those specific OSTBCs that results in $\Psi(\hat{\omega})$ with multiple largest eigenvalues, $h$ belongs to the subspace spanned by the corresponding multiple principal eigenvectors of $\Psi(\hat{\omega})$, and as a result, the blind technique is not applicable using this method of detection. Hence, the semiblind technique proposed in [Kumar and Saxena (2012a)] will be utilized which uses the small number of training symbols both in time and frequency axis adaptively and decoded at the receiver end according to the requirement of the channel. Using this method, it searches for all the possible combinations of $x_{k+1}, x_{k+2}, \ldots, x_{k+\hat{d}}$, we use the decoding procedure of $X_s(t)$ to obtain conditional estimates to get the weight vectors in (15) of [Kumar and Saxena (2012a)]. An adaptive method of increasing pilot symbols in the empty slots have been proposed in the same and then the robust estimation method have been found for getting the correct combination of $x_{k+1}, x_{k+2}, \ldots, x_{k+\hat{d}}$. This method uses a small number of training symbols after some iteration. Finally we minimize the decision metric in (5.4.3) for $x_1, x_2, \ldots, x_k, x_{k+1}, x_{k+2}, \ldots, x_{k+\hat{d}}$ over all possible values of $x_{k+1}, x_{k+2}, \ldots, x_{k+\hat{d}}$. It means instead of searching all possible values of $x_{k+1}, x_{k+2}, \ldots, x_{k+\hat{d}}$ and suffering from $M^{\hat{d}+1}$ metric computations, only searching with a decoding complexity of $M^\hat{d}$, and obtaining estimates of $x_1, x_2, \ldots, x_k$, which needs the additional decoding complexity of $kM_T^\hat{d}$ per each step of $M^\hat{d}$ calculations. This method of estimating and detecting is somewhat similar to ML detection technique and therefore we obtain the total decoding complexity of $kM_T^\hat{d} = kM_T^{\hat{d}+1}$.

Now it is known that $h$ belongs to the subspace spanned by $\{u_i\}_{i=1}^a$, we have -

$$h = \sum_{i=1}^a \alpha_i u_i = U\alpha$$

(5.4.17)

where, $U = [u_1, u_2, \ldots, u_a]$ and $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_a]^T$. The proposed semiblind channel estimation scheme has been utilized to obtain the estimate of $U$ in a blind way and meanwhile estimating the vector $\alpha$ using the training symbols as low as possible. It is known that the number of entries in $\alpha$ is much less than that in $h$, and this semiblind estimator will require very less training data than the direct training based channel estimator obtaining all entries of $h$ in a non-blind way. For this purpose, it is required to estimate the value of $\alpha$ and taking short time average of the detected estimates and then further process it to give the branch metric which then further, will processed for giving the selected estimates with minimum branch metric which gives the minimum surviving states with minimum value from the $\hat{H}$ and eventually the possible block of transmitted sequence.
The ML estimate for the STBC system of the vector $\mathbf{a}$ can be written as:

$$\hat{\mathbf{a}} = \sqrt{(h_i - \hat{h})(h_i - \hat{h})^T}$$  \hspace{1cm} (5.4.18)

this, further will give:

$$\hat{\mathbf{a}} = (\mathbf{X} \mathbf{X}^T)^{-1} \mathbf{X}^T \mathbf{r}$$  \hspace{1cm} (5.4.19)

where, $r = \mathbf{X} \mathbf{a} + \mathbf{n}$. This estimate can be used to obtain the coefficients $\{\alpha_i\}_{i=1}^n$ from these few training symbols to resolve the ambiguity in the channel vector estimate. To ensure that the ML estimate in (5.4.19) is unique and it is required that $2M_rT \geq n$ and for known non-identifiable OSTBCs, $n=4$ holds true and therefore as $T \geq 2$, the condition $2M_rT \geq n$ is satisfied for any number of receive antennas for which it is required to have code rate of STBC should be higher than 1 which will be further designed in next section.

### 5.5. HIGH CODE RATE DESIGN METHOD

In order to achieve energy efficient STBC codes with high code rates, It is required to construct the $M'X[i]^2$, a rotated version of the complex lattice with source information $\mathbf{X}[i]^2$, where, $M'$ is a complex unitary matrix, so that there is no shaping loss in the signal constellation emitted by the transmitting antenna as shown in [Belfiore et al. (2005)].

For any given $M_r$ and $G$ column groups of the matrix and $T$ be the block length, then $T = M_r + 2(G - 1)$. Assuming that $M$ is even, the higher code rate STBC will be designed as:

$$\mathbf{G}_{M_r,T,G} = A'_{M_r,T,G} + jB'_{M_r,T,G}$$  \hspace{1cm} (5.5.1)

where, the real and imaginary matrices $A'_{M_r,T,G}$ and $B'_{M_r,T,G}$ of size $T \times M_r$ are given by:

$$A'_{M_r,T,G} = \begin{bmatrix} s^1_R & s^2_R \\ -s^2_R & s^1_R \end{bmatrix}$$  \hspace{1cm} (5.5.2)

$$B'_{M_r,T,G} = \begin{bmatrix} s^1_I & s^2_I \\ -s^2_I & s^1_I \end{bmatrix}$$  \hspace{1cm} (5.5.3)

where, $s_R^i$ and $s_I^i$ are real and imaginary parts of $S'$, where, $(i = 1,2)$ that is given by:
where, $x' = (i-1)G \frac{M_T}{2}$ and the $G^{th}$ diagonal layer from left to right written as $\frac{M_T}{2} \times 1$ vector $X_{i}^{v} (i=1,2; G=1,2,...G)$, given by -

$$X_{i}^{v} = \begin{bmatrix}
    s_{x'+1} & 0 & \cdots & 0 \\
    s_{x'+M_T} & s_{x'+2} & \ddots & \vdots \\
    \vdots & \ddots & \ddots & \ddots \\
    0 & s_{x'+(G-1)M_T} & \cdots & s_{x'+M_T}
\end{bmatrix}$$

The symbol rate of the STBC code $\Theta_{M_T, T, G}$ is given by -

$$\text{Code rate} = \frac{L}{\frac{M_T}{2} \times G - 2}$$

Which is the same as that of STBC decoding proposed in [Wei et al. (2010) ; Wei et al. (2012)]. For a large value of G, the code rate can be up to $\frac{M_T}{2}$ and similarly for large elements on transmitting side, i.e., $M_T$, the code rate can be up to $G$. For the design of STBC with odd antenna elements, it is supposed to design an STBC for $M_T + 1$ transmit antennas with the last antenna to be shut down, i.e., when the $M_T$ is odd, the STBC is obtained by selection of first $M_T$ columns of the STBC designed for $M_T + 1$ antennas.

### 5.6. IMPLEMENTATION OF HIGH CODE RATE DESIGNS

#### 5.6.1. For Three Antenna Elements

In this section, a new STBC code with code rate of 1.5 and 2 has been achieved for three transmit antennas with the use of the design procedure shown in the previous section. According to (5.4.1), using the design method as shown in previous section, for the transmitting six symbols using three antennas, i.e., code rate 2 can be found using the value of $G = 2$. 
With the optimization matrix,

\[
A'_{3,6,4} = \begin{bmatrix}
    s_1 & s_2 & s_3 \\
    -s_2^* & s_1 & -s_6^* \\
    s_7 & s_8 & s_3 \\
    -s_8^* & s_7 & -s_4^*
\end{bmatrix}, \quad B'_{3,6,4} = \begin{bmatrix}
    s_3 & s_4 & s_7 \\
    -s_4^* & s_3 & -s_8^* \\
    -s_6 & s_6 & s_1 \\
    -s_6^* & s_5 & -s_2^*
\end{bmatrix}
\]

(5.6.1)

With the optimization matrix,

\[
G = \begin{bmatrix}
    1 & 1 & e^{j\theta} \\
    1 & 1 & e^{j\theta} \\
    e^{j\theta} & e^{j\theta} & 1 \\
    e^{j\theta} & e^{j\theta} & 1
\end{bmatrix}
\]

where, \(A'_{3,6,4}\) and \(B'_{3,6,4}\) denotes the Real and Imaginary matrices for the 6 symbols per 3 transmit antennae with code rate 2. This matrix has been derived from the optimization matrix of \(G = e^{j\theta}I_4\), where, \(I_4\) is the 4x4 identity matrix. After continuous simulation search, maximum coding rate of \(\Theta_{3,6,4}\) has been found by sacrifice of some constellation angle \(\theta\) for the optimum value of 15.98°, which gave minimum determinant value of 0.15 for the QPSK modulation technique.

Now secondly, the real and imaginary matrices for code rate 1.5 which transmits six information symbols per three time intervals is obtained as -

\[
A'_{3,6,3} = \begin{bmatrix}
    s_1 & s_2 & s_3 \\
    -s_2^* & s_1 & -s_6^* \\
    0 & 0 & s_3 \\
    0 & 0 & s_4^*
\end{bmatrix}, \quad B'_{3,6,3} = \begin{bmatrix}
    s_3 & s_4 & 0 \\
    -s_4^* & s_3 & -s_6^* \\
    -s_6 & s_6 & s_1 \\
    -s_6^* & s_5 & -s_2^*
\end{bmatrix}
\]

(5.6.2)

With the optimization matrix

\[
G = \begin{bmatrix}
    1 & 1 & e^{j\theta} \\
    1 & 1 & e^{j\theta} \\
    e^{j\theta} & e^{j\theta} & 1 \\
    e^{j\theta} & e^{j\theta} & 1
\end{bmatrix}
\]

Maximum coding rate of \(\Theta_{3,6,3}\) has been found by making constellation angle \(\theta\) for the optimum value of 44.96°, which gave minimum determinant value of 0.32 for the QPSK modulation technique. Hence in this, it can be easily seen that the complexity has been reduced upto \(4M^2_7\) for \(M_7 = 4\) and \(\lambda = 2\).

### 5.6.2. For Four Antenna Elements

In this section, another STBC code with higher code rate of 1.3, 1.5 and 2 has been achieved for four transmit antennas with the use of the design procedure shown in the earlier section. According to
(5.4.1), using the design method as shown in previous section, for the transmitting six symbols using four antennas, i.e., code rate 1.3 can be found using the value of $G = 2$ as -

$$A'_{4, 6, 2} = \begin{bmatrix} s_1 & 0 & s_5 & 0 \\
- s_5 & 0 & s_1 & 0 \\
-s_7 & -s_6 & s_3 & s_2 \\
0 & -s_8 & 0 & s_4 \end{bmatrix}, \quad B'_{4, 6, 2} = \begin{bmatrix} s_1 & 0 & s_5 & 0 \\
0 & s_4 & 0 & s_8 \\
0 & s_4 & 0 & s_8 \\
s_7 & s_6 & -s_3 & -s_2 \end{bmatrix}$$ (5.6.3)

Maximum coding rate of $\Theta_{4,6,2}$ has been found by making constellation angle $\theta$ for the optimum value of 1.04°, which gave minimum determinant value with the QAM modulation technique.

With the QPSK modulation technique, for code rate 2, we found the following matrix -

$$A'_{4, 8, 4} = \begin{bmatrix} s_1 & s_2 & s_5 & s_6 \\
-s_2^* & s_1^* & -s_6^* & -s_5^* \\
s_7 & s_8 & s_3 & s_4 \\
-s_8^* & s_7^* & -s_4^* & s_3^* \end{bmatrix}, \quad B'_{4, 8, 4} = \begin{bmatrix} s_1 & s_2 & s_5 & s_6 \\
-s_2^* & s_1^* & -s_6^* & -s_5^* \\
-s_4 & s_7 & s_8 & s_3 \\
-s_8^* & s_7^* & -s_4^* & s_3^* \end{bmatrix}$$ (5.6.4)

With optimization matrix,

$$G = \begin{bmatrix} e^{i\theta} & e^{i\theta} \\
e^{i\theta} & e^{i\theta} \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \end{bmatrix}$$

Maximum coding rate was found for the code rate of 2, with the angle $\theta = 13.3^\circ$ of QPSK signal constellation with symbols on the two axes rotation to achieve full diversity.

Similarly, for the code rate 1.5 with the four antenna, using QPSK modulation rotation at $\theta = 90^\circ$, can be obtained by making $s_8 = 0$ and which resulted as -

$$A'_{4, 8, 3} = \begin{bmatrix} s_1 & s_2 & s_5 & s_6 \\
-s_2^* & s_1^* & -s_6^* & -s_5^* \\
0 & 0 & s_3 & s_4 \\
0 & 0 & -s_4^* & s_3^* \end{bmatrix}, \quad B'_{4, 8, 3} = \begin{bmatrix} s_3 & s_4 & 0 & 0 \\
-s_4^* & s_3^* & 0 & 0 \\
s_5 & s_6 & s_1 & s_2 \\
-s_6^* & s_5^* & -s_2^* & s_1^* \end{bmatrix}$$ (5.6.5)

With the optimization matrix

$$G = \begin{bmatrix} 1 & 1 & e^{i\theta} & e^{i\theta} \\
1 & 1 & e^{i\theta} & e^{i\theta} \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \end{bmatrix}$$
5.7. DECODING AND ESTIMATION METHOD

A decoding method for four antenna element is being described here, in which the receiver calculates the intercepted received signals from the channel using (5.4.4) for all the combinations of $s_{k+1}, s_{k+2}, \ldots, s_{k+l}$ to obtain the ML estimates of $s_1, s_2, \ldots, s_k$. Therefore, for the given values of $s_5, s_6, s_7$ and $s_8$ which can only be obtained with the help of reduced form of (5.4.4),

$$Z = \Theta_{4,8,4}H + N$$

(5.7.1)

It is required to minimize the decision metric obtained with the help of (5.4.3) for all possible values and obtained conditional ML estimates of $s_1, s_2, \ldots, s_k$ which need additional decoding complexity of $kM_T$ per each step of $M_T^k$ calculations. Therefore, we get a total decoding complexity of $MM_T^k = kM_T^{k+1}$. The receiver follows the decoding procedure of $\Theta_{4,8,4}$, and it is observed that $Z$ which is component of $Z$ is calculated from $y_i$ which is component of $Y$, $i$ and $j$ are the column and row of the corresponding matrix. The receiver combines the received intercepted signals to obtain -

$$\hat{y}_1 = \sum_{i=1}^{M_T} h_{i,1}^* z_{i,1} + h_{i,1}^* z_{i,1}^* ; \quad \hat{y}_2 = \sum_{i=1}^{M_T} h_{i,2}^* z_{i,2} + h_{i,2}^* z_{i,2}^* ; \quad \hat{y}_3 = \sum_{i=1}^{M_T} h_{i,3}^* z_{i,3} + h_{i,3}^* z_{i,3}^* ; \quad \hat{y}_4 = \sum_{i=1}^{M_T} h_{i,4}^* z_{i,4} + h_{i,4}^* z_{i,4}^*$$

These received estimates can now be utilized for the estimation of ML estimates for $s_i$, where $i=1,2,\ldots,4$ manipulated for estimating higher code rate estimates -

$$s_i^{ML} = \arg \min_{s_i} \left[ (\beta' |s_{1,i} - \alpha' s_{2,i}|^2) + (\alpha' |s_{3,i} - \beta' s_{4,i}|^2) \right]$$

$$s_2^{ML} = \arg \min_{s_i} \left[ (\beta' |s_{2,i} - \alpha' s_{3,i}|^2) + (\alpha' |s_{4,i} - \beta' s_{3,i}|^2) \right]$$

$$s_3^{ML} = \arg \min_{s_i} \left[ (\beta' |s_{3,i} - \alpha' s_{1,i}|^2) + (\alpha' |s_{2,i} - \beta' s_{1,i}|^2) \right]$$

$$s_4^{ML} = \arg \min_{s_i} \left[ (\beta' |s_{4,i} - \alpha' s_{3,i}|^2) + (\alpha' |s_{1,i} - \beta' s_{3,i}|^2) \right]$$

(5.7.2)

where, $A'$ and $B'$ denote for the Real and Imaginary part of STBC codes $\alpha' = \sum_{i=1}^{M_T} |h_{i,1}|^2 + |h_{i,2}|^2$, $\beta' = \sum_{i=1}^{M_T} |h_{i,3}|^2 + |h_{i,4}|^2$ and $\hat{s}_1 = \text{Re}(\hat{y}_1) + j \text{Im}(\hat{y}_1)$, $\hat{s}_2 = \text{Re}(\hat{y}_2) + j \text{Im}(\hat{y}_2)$, $\hat{s}_3 = \text{Re}(\hat{y}_3) + j \text{Im}(\hat{y}_3)$, $\hat{s}_4 = \text{Re}(\hat{y}_4) + j \text{Im}(\hat{y}_4)$. 

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It is observed that a total decoding complexity of $4M_T^2$ rather than $M_T^8$ by minimizing (5.4.3) for all the possible values as discussed earlier has been achieved.

Now for the implementation of the estimated symbols with the semiblind channel estimation, it is required to deploy the scheme proposed in [Kumar and Saxena (2012b)], although it can also be implemented with [Kumar and Saxena (2012a)], but the procedure to estimate has been refined in the second method by modifying precoder and decoder at the transmitter and receiver side implementing the same estimation technique used in [Kumar and Saxena (2012a)]. It can be seen in (26) of [Kumar and Saxena (2012b)] that the weight vectors are not sufficient to estimate the symbols correctly in the semiblind environment with partial CSI conditions and an adaptive estimation method was tried as shown in [Kumar and Saxena (2012a)] for getting optimal linear minimum mean square error (MMSE) estimate for the channel path gain at the $m^{th}$ symbols period where, the weighting coefficients $h(i,q)$ explicitly depend on the symbol position $q$. For each $q$, $h(i,q)$ can be obtained by solving the adaptive method as discussed in [Kumar and Saxena (2012a)] which provides the unknown estimate sequence $\alpha$ to avoid sacrifice of tracking ability of channel. These estimated symbols can be used to obtain the coefficients $\{\alpha_i\}_{i=1}^{N_a}$ from these few training symbols to resolve the ambiguity in the channel vector estimate. The minimum path metric with its short time average of long detected sequence $\mu_k$ was detected which has then utilized to calculate the minimum branch metric $\mu_k$ for all possible estimated vectors for tracking surviving state with minimum value of channel coefficients $\hat{H}$. Then these metrics are utilized to update the weight vectors in (26) of [Kumar and Saxena (2012b)] of $m^{th}$ spatial equalizer at each step with increase in processing steps $k$. [Kumar and Saxena (2012b)] has discussed about the capacity analysis of the proposed semiblind channel estimation scheme with modified precoder and decoder.

In this thesis, Bit Error Rate performance analysis has also been taken care. A comparative chart has been shown in Table 5.2 for showing the used number of antennas for different schemes and their symbol transmission rate with different coding rates.
Table 5.2. Comparison of different STBCs for different transmitting antenna and code rates

<table>
<thead>
<tr>
<th>Design Approach</th>
<th>Number of Transmitting Antenna</th>
<th>Number of Symbols per block</th>
<th>Code rate</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial Multiplexing</td>
<td>M_T</td>
<td>M_T</td>
<td>M_T</td>
<td>Less</td>
</tr>
<tr>
<td>Alamouti</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>Less</td>
</tr>
<tr>
<td>OSTBC</td>
<td>3,4</td>
<td>3,4</td>
<td>1/2</td>
<td>Medium</td>
</tr>
<tr>
<td>OSTBC</td>
<td>3,4</td>
<td>3,3</td>
<td>3/4</td>
<td>High</td>
</tr>
<tr>
<td>OSTBC</td>
<td>4</td>
<td>6,8</td>
<td>1.3</td>
<td>Very High</td>
</tr>
<tr>
<td>OSTBC</td>
<td>3,4</td>
<td>4</td>
<td>2</td>
<td>Very High</td>
</tr>
<tr>
<td>QOSTBC</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>High</td>
</tr>
</tbody>
</table>

5.8. CODING PERFORMANCE

Performance analysis and improvement observed in the MIMO systems using different antenna configurations utilizing STBC using different modulation scheme with the implementation of adaptive pilot assisted semiblind channel estimation scheme for the partial CSI condition proposed earlier in [Kumar and Saxena (2012a)] has been shown in this section. It is known that the performance for the different STBC coding schemes degrades when more bits per symbol are transmitted, but we have simulated upto 9bps/Hz with higher code rate STBCs which has shown relatively good results as compared with the existing saying. For the general simulation case for known channel models, it is obtained that, the best performance is obtained by using higher number of transmitting and receiving antenna elements. However for any modulation case with low SNR values, three transmitting antenna STBC system with code rate 1/2 gives better results than the four antenna element system STBC with code rate 3/4 even though the gain for the said is higher. When the simulation was tried with more number of antenna elements at the transmitting side with code rate 1/2, they gave better results than the 3/4 code rate type STBC systems. The possible reason for this is that the higher rate of four transmitting antenna element system causes lower channel gain per symbol and therefore BER for particular SNR. If we consider equal data rate and simulate the 16-QAM scheme and 64-QAM modulation scheme for the code rate, i.e., 1/2 and 3/4, for three and four antenna element system, it is easily observable that the S^C_1 and S^C_2 with code rate 3/4 using 16-QAM (4 Bits/symbol) gave the 3 Bits/Sec/Hz data rate, whereas S^C_1 and S^C_2
with code rate 1/2 using 64-QAM (6 Bits/symbols) gave the same 3 Bits/Sec/Hz data rate. Hence we decided to show the comparative analysis of QPSK and 16-QAM modulation scheme for different antenna configurations with maintaining the correlation coefficient of 0.5.

In this section, we have evaluated the BER performance, and the received constellation comparisons for different modulation schemes using STBC with different code rate, for their constellation angle for appropriate modulation for achieving the exact code rate and diversity. Also the capacity comparison is shown for the improvement seen with different antenna configurations with different channels.

Throughout the simulations, the noise variance has been considered between the 3dB to 20dB level for different scenarios. Alamouti’s STBC has been simulated as discussed in (5.2.7) using QPSK and 16-QAM modulation technique for 3x3 transmitting and receiving antennas configuration with code rate 1/2 and full diversity which has been shown in Figure 5.2 which shows that the received signal using APASBCE started to identify and separate the received signal after 12dB in case of 16-QAM and after 17.4 for QPSK modulation schemes. It is seen that the semiblind result gave better result than the existing results available in the literature as the number of iterations reaches upto the level when the symbols are easily identifiable at the receiver end.

![Figure 5.2](image-url)

**Figure 5.2.** Comparative results of Alamouti’s model using QPSK and 16-QAM modulation technique for 3x3 trans-receivers antennas with APASBCE based simulation using the STBC found in (5.2.7) with code rate 1/2 and diversity 1.
Further, in Figure 5.3 and Figure 5.4, OSTBC has been compared for 3x3 and 4x4 antenna system with APASBCE scheme using QPSK and 16-QAM modulation technique for code rate 1/2 as discussed in (5.2.7) using $S^C_3$ and $S^C_4$ for 3x3 and 4x4 antenna configuration, whereas for code rate 3/4, the simulation has been performed for 3x3 antenna configuration using $S^C_3$ as discussed in (5.2.10) and for 4x4 antenna configuration using $S^C_4$ as discussed in (5.2.11) which shows that code rate 1/2 is performing better than code rate 3/4 for both 3x3 and 4x4 antenna configurations, as discussed earlier in this section but Figure 5.4 shows significant improvement in the BER between the simulated semiblind results as compared with the Figure 5.3. This happens because of the increase in number of antenna elements, as the number of elements increases, the symbol rate increases and further by utilizing the proper code rate STBC, the BER may be enhanced up to some extent which has been shown in the Figure 5.4. Figure 5.3 shows the improved results for 16-QAM modulation with 3x3 antenna configuration for code rate 1/2 as compared with the code rate 3/4 for the same, by 3.2 dB at the level of $10^{-4}$ BER level, whereas for QPSK modulation scheme, improvement is 8dB which is much better as compared than the 16-QAM modulation scheme. Similarly in Figure 5.4, QPSK again performs better as compared to 16-QAM modulation scheme, where, the improvement of 1.6dB and 1.9dB is obtained using code rate of 1/2 and 3/4 respectively.

Figure 5.3. OSTBC comparison for 3x3 antenna systems with the APASBCE based simulation using QPSK and 16-QAM modulation techniques for code rate 1/2 and 3/4 using $S^C_3$. 
Figure 5.4. OSTBC comparison for 4x4 antenna systems with the APASBCE based simulation using QPSK and 16-QAM modulation techniques for code rate 1/2 and 3/4 using $S_4^C$.

Figure 5.5. OSTBC comparison for QPSK modulation for 4x4 antenna systems with the APASBCE based simulation of the same using the STBC with code rate 1.3 and diversity 1.
Next, Figure 5.5 shows the OSTBC comparison for QPSK with existing results in literature with the newly found results using APASBCE based STBC modulation for 4x4 antenna configuration with code rate 1.3 and full diversity found in (5.6.3). This shows that the semiblind result starts to perform better after 18.8dB with the existing results and kept maintaining this improvement up to higher level of SNR.

Figure 5.6 shows the comparison of QOSTBC and OSTBC comparison for 4x4 antenna systems with the APASBCE based simulation using QPSK and 16-QAM modulation techniques for code rate 1 and 1.3 using $S^C_4$.

Figure 5.6, shows the comparison of QOSTBC and OSTBC comparison for 4x4 antenna systems with the APASBCE based simulation using QPSK and 16-QAM modulation techniques for code rate 1 and 1.3 using $S^C_4$ in (5.2.12) and $A'$ and $B'$ in (5.6.3). Again it is obtained that the APASBCE based 16-QAM and QPSK modulation is performing better than the normal modulation using code rate 1.3 with 8bps/Hz datarate. For 16-QAM, we found the received signal improves after 14.2dB and for QPSK, it is 11.4dB before reaching the level of BER of $10^{-2}$ that shows the performance of the APASBCE scheme based modulation. Similarly OSTBC comparison for 4x4 antenna system with APASBCE based simulation using QPSK with the existing results in the literature for code rate 1.3 with 8 bps/Hz and 9 bps/Hz using $A'$ and $B'$ in (5.6.3) has been shown in Figure 5.7. As discussed earlier that higher rate of transmitting antenna element system causes lower channel gain per symbol and therefore BER for particular SNR value, it is seen in this Figure that low rate i.e 8 Bps/Hz, QPSK modulation is performing better than the
higher rate, i.e., 9 Bps/Hz modulation and further again, the APASBCE method gave the better results at 16.7 dB for 9 Bps/Hz rate and 11.8 dB for 8 Bps/Hz rate before reaching the level of $10^{-2}$ BER as shown.

**Figure 5.7.** OSTBC comparison for 4x4 antenna systems with the APASBCE based simulation using QPSK and the existing results for code rate 1/3 with 8 bps/Hz and 9 bps/Hz.

**Figure 5.8.** OSTBC comparison of 4x4 antenna systems with the existing result and APASBCE based simulation results using QPSK and 16-QAM modulation technique for code rate 2 with 4 bps/Hz.
Figure 5.9. OSTBC comparison of 4x4 antenna systems with the existing result and APASBCE based simulation results using QPSK and 16-QAM modulation technique for code rate 1.5 with 3 bps/Hz.

Similarly, Figure 5.8 and 5.9 has been shown for the comparison of OSTBC using APASBCE based QPSK and 16-QAM modulation with the existing results available in the literature for code rate 2 and 1.5 with 4 bps/Hz and 3 bps/Hz utilizing $A$ and $B^*$ in (5.6.4) and (5.6.5) respectively. In Figure 5.8, the APASBCE based QPSK and 16-QAM modulation start to perform better than the existing QPSK and 16-QAM modulation for 4x4 antenna configuration with code rate 2 at SNR level of 16.2dB and 20.8 dB respectively and similarly in figure 5.9, it achieves 11.8dB and 18.9dB before reaching the level of $10^{-3}$ BER which is quite good as compared to the existing one.

It is observed in Figure 5.8 for QPSK modulation, that the system is able to maintain 1.2dB gain at the level of 25dB of SNR, for the STBC with code rate 2, whereas in case of 1.5 code rate STBC simulations, as depicted in Figure 5.9 was able to maintain less amount of gain nearly of 0.8dB but at the SNR level of 20dB. On contrary, for 16-QAM modulation as evident in Figure 5.8 and 5.9, the effect of maintaining gain is not of same quantum and maintains the gain of 2.8dB and 2dB for APASBCE based STBC with code rate 2 and 1.5 respectively at the higher level of SNR, i.e., 25dB and 30dB.

The decrease in maintaining less SNR gain has occurred because of the loss of training symbols at the receiver end for which the algorithm again started to track the symbols and after finding the sufficient amount of training symbols, it kept to maintain the gain in SNR levels again. The comparative result
analysis has been shown in Table 5.3 for different STBCs with different transmitting antenna configurations with their respective code rates.

Table 5.3. Comparison result analysis of different STBCs for different transmitting antenna with their code rates.

<table>
<thead>
<tr>
<th>Modulation used</th>
<th>Antenna Configuration</th>
<th>Code rate</th>
<th>STBC used</th>
<th>APASBCE based Semiblind CE improvement</th>
<th>Reference Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>QPSK</td>
<td>3x3</td>
<td>1/2</td>
<td>$S^C_1$ in (5.2.7)</td>
<td>At 12 dB</td>
<td>Alamouti’s Code comparison as shown in Figure 5.2.</td>
</tr>
<tr>
<td>16-QAM</td>
<td>3x3</td>
<td>1/2</td>
<td>$S^C_1$ in (5.2.7) and $S^C_3$ in (5.2.10)</td>
<td>After 3.2 dB SNR level with BER $10^{-7}$ as compared with code rate 3/4</td>
<td>OSTBC comparison as Shown in Figure 5.3.</td>
</tr>
<tr>
<td>QPSK</td>
<td>3x3</td>
<td>1/2</td>
<td>$S^C_4$ in (5.2.7) and $S^C_4$ in (5.2.11)</td>
<td>After 1.6 dB SNR level with BER $10^{-7}$ as compared with code rate 3/4</td>
<td>OSTBC comparison as Shown in Figure 5.4.</td>
</tr>
<tr>
<td>16-QAM</td>
<td>4x4</td>
<td>1/2</td>
<td>$A^C' \text{ and } B^C'$ in (5.6.3)</td>
<td>At 18.8 dB</td>
<td>OSTBC comparison as Shown in Figure 5.5.</td>
</tr>
<tr>
<td>QPSK</td>
<td>4x4</td>
<td>1.3</td>
<td>$S^C_4$ in (5.2.12)</td>
<td>At 14.2 dB SNR before reaching the BER level of $10^{-2}$</td>
<td>QOSTBC and OSTBC comparison as shown in Figure 5.6.</td>
</tr>
<tr>
<td>16-QAM</td>
<td>4x4</td>
<td>1.3</td>
<td>$A^C' \text{ and } B^C'$ in (5.6.3)</td>
<td>At 11.8 dB SNR before reaching the BER level of $10^{-2}$</td>
<td>OSTBC comparison as Shown in Figure 5.7.</td>
</tr>
<tr>
<td>QPSK</td>
<td>4x4</td>
<td>1.3 (8Bps/Hz)</td>
<td>$A^C' \text{ and } B^C'$ in (5.6.3)</td>
<td>At 16.7 dB SNR before reaching the BER level of $10^{-2}$</td>
<td>OSTBC comparison as Shown in Figure 5.8.</td>
</tr>
<tr>
<td>QPSK</td>
<td>4x4</td>
<td>1.3 (9Bps/Hz)</td>
<td>$A^C' \text{ and } B^C'$ in (5.6.4)</td>
<td>At 16.2 dB</td>
<td>OSTBC comparison as Shown in Figure 5.9.</td>
</tr>
<tr>
<td>16-QAM</td>
<td>4x4</td>
<td>2</td>
<td>$A^C' \text{ and } B^C'$ in (5.6.4)</td>
<td>At 20.8 dB</td>
<td>OSTBC comparison as Shown in Figure 5.8.</td>
</tr>
<tr>
<td>QPSK</td>
<td>4x4</td>
<td>2</td>
<td>$A^C' \text{ and } B^C'$ in (5.6.5)</td>
<td>At 11.8 dB SNR before reaching the BER level of $10^{-3}$</td>
<td>OSTBC comparison as Shown in Figure 5.9.</td>
</tr>
<tr>
<td>16-QAM</td>
<td>4x4</td>
<td>1.5</td>
<td>$A^C' \text{ and } B^C'$ in (5.6.5)</td>
<td>At 18.9 dB SNR before reaching the BER level of $10^{-3}$</td>
<td>OSTBC comparison as Shown in Figure 5.9.</td>
</tr>
</tbody>
</table>

It is seen that the semiblind result gave better result than the existing results available in the literature as the number of iterations reaches upto the level, when the symbols are easily identifiable at the receiver end. The received constellation for different antenna configurations, with different modulation schemes utilizing the APASBCE technique has been shown in the further Figures. These received constellations shows that how much rotation is required for achieving the particular value of code rate required for these STBC techniques to modulate through the channel using APASBCE scheme and to receive the symbols perfectely at the receiver.
Figure 5.10 and 5.11 shows the received constellations for Alamouti’s 3x3 antenna system with code rate 1 which was found ISI free at SNR level of 23dB for both 16-QAM and QPSK APASBCE modulation scheme with the constellation rotation angle of 36.89° and 8.92° respectively. Similarly, Figure 5.12 and Figure 5.13 shows the received constellation for OSTBC antenna system for 3x3 and 4x4 configurations with code rate 1/2, found ISI free at SNR level of 35dB and 25dB with constellation rotation angle of 1.5° and 18.48° respectively, using QPSK APASBCE based modulation.

Further, Figure 5.14 and Figure 5.15 shows the received constellation for the OSTBC MIMO systems with code rate 3/4 and found the discriminating ISI free symbols at SNR level of 40dB for 3x3 antenna system and 30dB for 4x4 antenna system.

Similarly, Figure 5.16 and Figure 5.17 has been shown for the received constellation for the OSTBC MIMO systems with code rate 2 utilizing QPSK APASBCE based modulation scheme and found the discriminating ISI free symbols at SNR level of 40dB and 30dB respectively, for 3x3 and 4x4 antenna configurations.

These SNR levels have been taken in consideration since at these levels, the received signal were clearly able to be detected by the receiver using the APASBCE. As depicted in Figure 5.14 and 5.15, constellation rotation angle of 1.8° and 3.2° is required for identifying the received symbols using APASBCE scheme with QPSK modulation with code rate 3/4 at SNR 40dB and 30dB for 3x3 and 4x4 antenna respectively.

Similarly, Figure 5.16 and 5.17 shows the constellation rotation angle of 65.49° and 62.1° for the ISI free symbols at the above said respective SNR levels but with code rate 2.

**Figure 5.10.** Received constellation for Alamouti’s 3x3 antenna system with code rate 1 and distinguish level at SNR 23dB for 16-QAM modulation with angle 36.89°.
Figure 5.11. Received constellation for Alamouti’s 3x3 antenna system with code rate 1 and distinguish level at SNR 23dB for QPSK modulation with angle 8.92°.

Figure 5.12. Received constellation for OSTBC MIMO antenna systems with code rate 1/2 and distinguish level at SNR 35dB for 3x3 antenna system using QPSK modulation technique with angle 1.5°.
Figure 5.13. Received constellation for OSTBC MIMO antenna systems with code rate 1/2 and distinguish level at SNR 25dB for 4x4 antenna system using QPSK modulation technique with angle 18.48°.

Figure 5.14. Received constellation for OSTBC MIMO antenna systems with code rate 3/4 and distinguish level at SNR 40dB for 3x3 antenna using QPSK modulation technique with angle 1.8°.
Figure 5.15. Received constellation for OSTBC MIMO antenna systems with code rate $\frac{3}{4}$ and distinguishing level at SNR 30dB for 4x4 antenna system using QPSK modulation technique with angle 3.2°.

Figure 5.16. Received constellation for OSTBC MIMO antenna systems with code rate 2 and distinguishing level at SNR 40dB for 3x3 antenna system using QPSK modulation technique with angle 65.49°.
It is also observable that the received signal improves with the increase in the number of antenna elements at the receiving end. We have used QPSK and 16-QAM modulation schemes using the gray constellation mapping for the comparative study of the existing results available in literature with the proposed APASBCE [Kumar and Saxena (2012a)] scheme results for these mentioned modulation schemes. The capacity analysis and the improvement has already been discussed in [Kumar and Saxena (2012b)] for the APASBCE based scheme using the existing MIMO systems available in the literature. Comparative study of the capacity improvement has been shown in Figure 5.18 and Figure 5.19 using (48) (49) and (52) in [Kumar and Saxena (2012b)], where, the analysis has been done using 3x1, 3x3, 4x1 and 4x4 antenna systems for different channel numbers and obtained the improvement with the increase in the number of partial CSI channels. The complete result analysis for the received constellation for different antenna configurations, with different modulation schemes utilizing the APASBCE technique has been shown in the Figures shown above and their complete result analysis in form of chart has been shown in Table.5.4.
Table 5.4. Comparison of received constellation for different transmitting antenna with their code rates and required spatial constellation rotation angle.

<table>
<thead>
<tr>
<th>Modulation Type</th>
<th>Antenna Configuration</th>
<th>Code Rate</th>
<th>Discriminating SNR Level</th>
<th>Required Angle</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>16-QAM</td>
<td>3x3</td>
<td>1</td>
<td>23 dB</td>
<td>36.89°</td>
<td>Figure 5.10</td>
</tr>
<tr>
<td>QPSK</td>
<td>3x3</td>
<td>1</td>
<td>23 dB</td>
<td>8.92°</td>
<td>Figure 5.11</td>
</tr>
<tr>
<td>QPSK</td>
<td>3x3</td>
<td>1/2</td>
<td>35 dB</td>
<td>1.5°</td>
<td>Figure 5.12</td>
</tr>
<tr>
<td>QPSK</td>
<td>4x4</td>
<td>1/2</td>
<td>25 dB</td>
<td>18.48°</td>
<td>Figure 5.13</td>
</tr>
<tr>
<td>QPSK</td>
<td>3x3</td>
<td>3/4</td>
<td>40 dB</td>
<td>1.8°</td>
<td>Figure 5.14</td>
</tr>
<tr>
<td>QPSK</td>
<td>4x4</td>
<td>3/4</td>
<td>30 dB</td>
<td>3.2°</td>
<td>Figure 5.15</td>
</tr>
<tr>
<td>QPSK</td>
<td>3x3</td>
<td>2</td>
<td>40 dB</td>
<td>65.49°</td>
<td>Figure 5.16</td>
</tr>
<tr>
<td>QPSK</td>
<td>4x4</td>
<td>2</td>
<td>30 dB</td>
<td>14.2°</td>
<td>Figure 5.17</td>
</tr>
</tbody>
</table>

Further, the analysis of capacity which has already been discussed in Chapter 4 is again implemented with the higher code rate STBC systems which shows that the improved result are found for the partial channel state information which has been compared with the existing results for the known channel state information with 3x3, 3x1, 4x4, and 4x1 antenna configurations. Figure 5.18 and 5.19 shows the improved results of APASBCE based capacity which shows that, for 3x1 antenna systems with 8 channels, the proposed system started to enhance the capacity at SNR level of 19.3 dB. Similarly for 3x3 antenna systems with 4 channels and 8 channels gave the improved enhancement of capacity at the SNR level of 17.9 dB and 17.2 dB respectively. It also shows that the capacity improvement is related with the increase in the number of the channels. Again for 4x1 antenna systems with 8 channels, the improvement started at the level of 16.4 dB, whereas for 4x4 antenna systems with 4 channels, it shows the improvement after 19.2 dB. For 4x4 antenna systems, the improvement in capacity has been found initially at the level of SNR 12.8 dB but then decreases and again improved after the level of 15.7 dB which then maintained its improvement after 20.6 dB. This variation was caused because of the fading effect and channel estimation adaptively using APASBCE method for stabilizing the channel state information.
Figure 5.18. Capacity comparison results for 3x3 and 3x1 antenna configurations for both (a) known channel state information conditions and (b) with partial channel state information after application of APASBCE scheme.

Figure 5.19. Capacity comparison results for 4x4 and 4x1 antenna configurations for both (a) known channel state information conditions and (b) with partial channel state information after application of APASBCE scheme.
Finally, it can be easily seen that the results found in this Chapter are at par or better than the results developed by the [Basar and Aygolu (2009); Choqueuse et al. (2011); Loiola et al. (2011); Long et al. (2011); Yi-Sheng (2010)] in terms of constellation angle, BER and Capacity as shown in [Kumar and Saxena (2013)] which has been developed on the basis of the methods and models of the STBCs proposed in [Grau et al. (2008); Jafarkhani (2005); Jafarkhani (2001); Tarokh et al. (1999a), (1999b)].

5.9. SUMMARY

Conclusively, this Chapter shows the comparison of the existing 16-QAM and QPSK modulation schemes for different code rates with the result of STBC with code rate higher than 1 using different STBC techniques with the new improved results found using the proposed APASBCE scheme as discussed earlier, which shows the better BER result and improved capacity with less number of used training symbols and increasing the stability of the system by utilizing the minimum required number of training symbols. Also the constellation rotation for required angle has also been discussed for different higher code rate values for both 16-QAM and QPSK modulation schemes. These high code rate STBCs have been obtained and analyzed with improved results and the quantitative improvement has been discussed in this section.

Figure 5.20. Changes made in the MIMO system according to the implementations of the proposed scheme and modifications in STBC system.