CHAPTER 4

MODIFIED PRECODER AND DECODER DESIGN AND ANALYSIS FOR MIMO SYSTEM WITH OPTIMAL POWER ALLOCATION
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This Chapter provides the discussion on MSE design for the MIMO systems under the power constraints, i.e., to modify the precoder and decoder of the MIMO system which has been tried to formulate as a low complexity and effective processing technique to deal with the channel intersymbol interference and fading which provides better diversity and improved data rate.

4.1. INTRODUCTION

The aim of this Chapter design and analyze some scheme to reduce the loss of performance from the multi antenna wireless communication systems due to non-ideal antenna placement, insufficient antenna spacing and non-isotropic scattering environment. This has motivated to design a precoder or power loading schemes for multi-antenna wireless communication systems by exploiting the statistical information of the MIMO channels. The receiver has to be designed in such a manner so that the partial Channel state information may reach to the transmitter via MIMO channel like the correlation coefficients of the channel.

The mean square error linear precoding and decoding design has been tried to form for the downlink with taking [Schubert et al. (2005)] as the reference for the system. Earlier work has been done on the channel with perfect Channel state information conditions, whereas this Chapter deals with the channels with partial channel state information. The new algorithm has been tried based on the Karus-Kuhn-Tucker (KKT) conditions with the Cramer Rao bounds for the channel with partial CSI conditions. The KKT conditions are not able to achieve the global optimum solution for the partial CSI conditions; hence the proposed APASBCE scheme has been implemented with it for making it more robust and stable.
4.2. SYSTEM MODEL

Now moving towards the capacity analysis after enhancing the estimate perfection, which leads to get the optimized covariance matrix \( Q \) for which an adaptive precoder and decoder \((A, B)\) has been assumed at the transmitter and receiver end. \( A \) and \( B \) are \( M_T \times r_q \) and \( r_q \times M_R \) matrices with \( r_q = \text{rank}(\hat{H}) \). Let \( x \times 1 \) a zero mean i.i.d. unit variance data vector. Then \( x = AD \) and \( Q = E(xx^H) = AA^H \).

![Figure 4.1. Blind estimation of transmitted symbols using the MIMO systems with precoders and decoders](image)

### 4.3. MEAN SQUARE ERROR REDUCTION APPROACH

From Figure 4.1, we have, \( y = HAD + n = \hat{H}AD + EAD + n \) and then received vector after the decoder is given by \( R = BY \). Defining the MSE matrix \((A, B)\) with expectation w.r.t. \( x, n \) and \( E \), as -

\[
\text{MSE}(A, B) = E[(R - D)(R - D)^H]
\]

\[
= E\left\{B(\hat{H} + E)A - I_{r_q}\right\}DD^H + B(\hat{H} + E)A - I_{r_q}\}^H + \sigma_n^2BB^H
\]

\[
= B\hat{H}AA^H\hat{H}^H - B\hat{H}A - AA^H\hat{H}^H + I_{r_q} + B\left\{\sigma_n^2tr(\text{AA}^H)R + \sigma_n^2I_{M_s}\right\}B^H
\]

(4.3.1)

where, \( B\left\{\sigma_n^2tr(\text{AA}^H)R + \sigma_n^2I_{M_s}\right\}B^H \) is \( R_{n_r} \). To reduce the MSE, (3.4.10) can be written as -

\[
\min_{(A, B) \in \mathbb{R}^{M_T \times M_R}} \ln|\text{MSE}(A, B)|
\]

(4.3.2)
where, \( \ln \) is natural logarithm. Finding the optimized solution for (3.4.10) is related with the optimized solution of (4.3.2) with a global maximum existing for (3.4.10) and global minimum existing for (4.3.2).

With partial CSI, the minimum MSE design minimizes the trace of the MSE matrix whereas on the other hand the maximum mutual information minimizes its log determinant, which is the capacity lower bound achievable with a gaussian input distribution [Minhua and Blostein (2009)].

An optimized precoder and decoder pair may be obtained by solving and satisfying the Karush-Kuhn-Tucker (KKT) which will also satisfy the condition \( A'U,U'B' \) where, \( U \) is an arbitrary \( M \times M \) unitary matrix and \( A' & B' \) are the optimized pair set. According to KKT condition, the Lagrangian multiplier can be considered as -

\[
\zeta \left[ \left( A_j, B_j \right)_{j=1}^K, \mu^\prime \right] = \text{MSE} + \mu^\prime \left[ \sum_{j=1}^K \text{tr} \left( A_j A_j^H \right) - P_f \right]
\]

(4.3.3)

where, \( \mu^\prime \) is the Lagrange multiplier associated with the sum power constraint.

\[
\min_{A_j, r(AA^H) \leq r} \min \text{tr} \left[ \text{MSE} \{ A, B \} \right] \quad \text{(4.3.4)}
\]

The minimizing \( B \) for the inner unconstrained minimization is shown as -

\[
B = A^H \hat{H}^H \left[ \hat{H} A A^H \hat{H} A^H + \sigma_n^2 \text{tr} (Q_r A A^H) Q_r + \sigma_n^2 I_{M_x} \right]^{-1}
\]

(4.3.5)

which is the linear MMSE data estimator with given \( \hat{H} \) and \( A \) [Kay (1993) ; Poor (1994)]. Here, \( Q_r \) and \( Q_r \) are the covariance matrices for the transmitter and receiver. Substituting (4.3.5) in (4.3.4), the problem in (4.3.2) can be formulated as-

\[
\min_{A_j, r(AA^H) \leq r} \text{tr} \left[ \text{MSE} (A) \right] \quad \text{(4.3.6)}
\]

where,

\[
\text{MSE} (A) = \text{I} + A^H \hat{H}^H \left[ \sigma_n^2 \text{tr} (Q_r A A^H) Q_r + \sigma_n^2 I_{M_x} \right]^{-1} \hat{H} A - 1
\]

(4.3.7)

The feasible set of (4.3.6) is \( \{ A | r(AA^H) \leq P_f \} \), a Frobenius norm of radius \( \sqrt{P_f} \) [Boyd and Vandenberghe (2004)]. The objective function of (4.3.6) is continuous at all points of the feasible set. According to Weierstrass theorem [D.P.Bertekas (1999)], a global minimum for (4.3.6) has been evolved which also exists for (4.3.2). Based on (4.3.4), (4.3.5), (4.3.6), (4.3.7), the associated KKT condition can be found as -

\[
A_k \hat{H}^H_k = B_k \left[ \sum_{j=1}^K \hat{H} A_j A_j^H \hat{H}^H_j + \sigma_n^2 I_{M_x} + \sum_{j=1}^K \sigma_n^2 \text{tr} \left( A_j A_j^H \right) \sum J \right]
\]

(4.3.8)
\[ \hat{H}^k B_k^T = \left[ \hat{H}_K^T \left( \sum_{j=1}^{K} B_j^T B_j \right) \hat{H}_K + \mu I_{M_x} + \sigma_n^2 \sum_{j=1}^{K} \text{tr}(B_j B_j^T) I_{M_x} \right] A_k \] (4.3.9)

\[ \mu \geq 0, \quad \sum_{j=1}^{K} \text{tr}(A_j A_j^T) \leq P_T \quad \text{for } k = 1, \ldots, K \] (4.3.10)

\[ \mu \left( \sum_{j=1}^{K} \text{tr}(A_j A_j^T) P_T \right) = 0 \] (4.3.11)

Since, the objective and constraint functions of (4.3.2) are continuously differentiable w.r.t. \( A, B \) and the feasible point of (4.3.2) satisfy the regularity condition [D.P.Bertsekas (1999)], and the global minimum should satisfy the first order KKT conditions associated with (4.3.2). With these properties, it is concluded that the KKT conditions are necessary for local and global minimum both. The relation between Lagrange multiplier and receive decoder satisfying the KKT condition hold the following identity

\[ \mu = \left( \sigma_n^2 / P_T \right) \sum_{k=1}^{K} \text{tr}(B_k B_k^T) \] (4.3.12)

The algorithm based on the condition (4.3.8), (4.3.9), (4.3.10), (4.3.11), is shown in table 4.1.

**Table 4.1. Algorithm to calculate the optimum precoder.**

1) Initialize \( A_k, k = 1, \ldots, K \), satisfying the power constraint with equality and are non zero.
2) Update \( B_k \) using (4.3.8), for \( k = 1, \ldots, K \).
3) Update \( \mu \) using (4.3.12),
4) Update \( A_k \) using (4.3.9) for \( k = 1, \ldots, K \).
5) If the termination condition is met, stop; otherwise go back to step (2).

Based on the above method, (4.3.1) can be formulated to find the optimum structure of the \( A \) and \( B \) as -

\[ A' = \left[ \sigma_n^2 \alpha R + \mu I_{M_y} \right]^{1/2} V \Phi_A \] (4.3.13)

\[ B' = \Phi_B V \left[ \sigma_n^2 \alpha R + \mu I_{M_y} \right]^{1/2} \times \hat{H} H \left[ \sigma_n^2 \text{tr} \left( R A' A'^T \right) R + \sigma_n^2 I_{M_x} \right]^{1/2} \] (4.3.14)

where

\[ \Phi_A = \left[ I - A^{-1} \right]^{1/2} \] (4.3.15)

\[ \Phi_B = \left[ I - A^{-1} \right]^{1/2} A^{-1} \] (4.3.16)

And,
\[ \alpha = \text{tr} \left[ B' R \left( \sigma_n^2 \text{tr} \left( R A' A'' \right) R + \sigma_n^2 I_{M_r} \right)^{-1} \times \tilde{H} A' \right] \]  
\[ (4.3.17) \]
\[ \mu = \frac{\sigma_n^2}{P_f} \text{tr} \left[ B' \left( \sigma_n^2 \text{tr} \left( R A' A'' \right) R + \sigma_n^2 I_{M_r} \right)^{-1} \times \tilde{H} A' \right] \]  
\[ (4.3.18) \]

Matrix \( V \) and \( \Lambda \) are defined by the following eigenvalues decomposition,
\[ \left[ \sigma_n^2 \alpha R + \mu I_{M_r} \right]^{1/2} \tilde{H}^H \]
\[ \times \left[ \sigma_n^2 \text{tr} \left( R A A'' R + \sigma_n^2 I_{M_r} \right) \right]^{-1} \]
\[ \times \tilde{H} \left[ \sigma_n^2 \alpha R + \mu I_{M_r} \right] = \left[ V \quad \tilde{V} \right] \left( \begin{array}{cc} \Lambda & 0 \\ 0 & \tilde{\Lambda} \end{array} \right) \left[ V \quad \tilde{V} \right]^H \]  
\[ (4.3.19) \]

The entries of \( \tilde{\Lambda} \) are zero. \( V \) is the \( M_r \times r \) matrix composed of the eigenvectors corresponding to non-zero eigenvalues. \( \Lambda \) values are arranged in nonlinear order without the loss of generality. By putting \((4.3.13), (4.3.14) \) into \((4.3.17), (4.3.18) \), \( \mu \) and \( \alpha \) unknowns can be formulated in two equations which can be easily calculated.

The updated algorithm based on the previous algorithm, which calculated the optimum results with the property of reducing the value of objective function at each iteration, has been shown in Table 4.2. With increased number of iterations, the algorithm gives the \( \Lambda' \) and \( B' \) upto a unitary transform and a unique optimum covariance matrix can be obtained using \( Q' = A' A'' \).

This optimum covariance matrix reduces to the capacity results as obtained in [Sampath et al. (2001); Telatar (1999)] if the estimated variance has been considered as zero. With the partial CSIR knowledge, the optimum transmitters for maximized \((3.4.10) \) and minimized \((4.3.2) \) share the same structure differing only in power allocation. Finally, it is seen that the optimum solution for \((4.3.2) \) as \( \Lambda' \) and \( B' \) gives the optimum solution for \((3.4.10) \) using \( Q' = A' A'' \) which further shows the global maximum for \((3.4.10) \) and global minimum in \((4.3.2) \).

**Table 4.2. Updated Algorithm to calculate the optimum precoder with the property of reducing the objective function.**

1) Initialize \( \Lambda \)
2) Update \( R_{n'} \) using \((3.4.2) \) with \( Q = \Lambda \Lambda'' \)
3) Update \( B \) using \( B = \Lambda'' \tilde{H}^H (\tilde{H} A A'' \tilde{H}^H + R_{n'})^{-1} \)
4) Update \( \mu \) and \( \alpha \).
5) Update \( A \) using \( A = \left[ \mu I + \alpha \sigma_n^2 R_{M_r} \right]^{-1} \tilde{H}^H B'' \) such that \( \text{tr} \left( A A'' \right) = P_f \)
6) If \( \| A - A_{j-1} \|_4 \leq \epsilon \), stop; otherwise go back to step 2.
Table 4.3. Number of Transmit antennas and its corresponding range of block length for system with receive antennas.

<table>
<thead>
<tr>
<th></th>
<th>Optimal $N_T$</th>
<th>4</th>
<th>5</th>
<th>$\geq 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Power allocation with 20dB</td>
<td>Range of $T$</td>
<td>8 to 15</td>
<td>16 to 30</td>
<td>$\geq 31$</td>
</tr>
<tr>
<td>Optimal Power allocation with 20dB</td>
<td>Range of $T$</td>
<td>8 to 16</td>
<td>17 to 38</td>
<td>$\geq 39$</td>
</tr>
<tr>
<td>Equal Power allocation with 30dB</td>
<td>Range of $T$</td>
<td>8 to 22</td>
<td>23 to 46</td>
<td>$\geq 47$</td>
</tr>
<tr>
<td>Optimal Power allocation with 30dB</td>
<td>Range of $T$</td>
<td>8 to 26</td>
<td>27 to 60</td>
<td>$\geq 61$</td>
</tr>
</tbody>
</table>

Table 4.3 shows the optimal number of transmit antennas obtained using (2.4.57), (4.3.12), (4.3.17), (4.3.18), (4.3.19) and its corresponding range of block length for fixed SNR values. It is observable that $N_T$ can exceed $N_r$ with block range increase at high SNR values.

4.4. AUGMENTATION OF MIMO-STBC SYSTEM

After the precoder design and making it robust with MSE reduction, it is required to modify the precoder and decoder of the MIMO systems according to the requirement of STBC codes which are going to be used in next Chapter. It is known that the space time encoder at the transmitter takes a set of modulated symbols as in (3.4.11) and maps them onto a $TC_{N_L}$ code word matrix of unitary space time modulated constellation matrices set $S = \{ S_i S_j \} = I_{M_c}, \quad l = 1,2,\ldots,q$, where, $q$ denotes the constellation size. Generally the space time modulated constellations follow the property like in orthogonal designs [Tarokh et al. (1999a)] using -

$$\left( S_i - S_j \right) \left( S_i - S_j \right)^\dagger = \beta_{ij} I_{N_T}$$  \hspace{1cm} (4.4.1)

where, $\beta_{ij}$ is the scalar and $S_i, S_j \in S$.

Now for the STBC, when partial CSI has been assumed i.e the transmitter receives the partial information, the transmitted code length of two consecutively received symbols $y(k)$ and $y(k-1)$ detected at the receiver, is detected simply by applying the ML detection rule as -
\[ \hat{S}_q = \arg \max_{S_q \in \mathcal{S}} \|y(k) - y(k-1)\hat{h}S_q\|^2 \]
\[ = \arg \max_{S_q \in \mathcal{S}} \text{Re}\left\{ y(k-1)\hat{h}S_q y^*(k) \right\} \tag{4.4.2} \]

assuming that the scattering channel matrix \( \mathbf{H}_s \) will remain constant during the reception of two consecutive received signal blocks \( y(k-1) \) and \( y(k) \), which can be expressed in vector form as

\[ y(k-1) = \sqrt{E} h \kappa_y(k-1) + n(k-1) \]
\[ y(k) = \sqrt{E} h \kappa_y(k) + n(k) \]
\[ = y(k-1)S_q + w(k) \tag{4.4.3} \]

where,
\[ y(k) = \left( \text{vec}\left\{ \mathbf{Y}(k) \right\} \right)^T, \quad \kappa_y(k) = \mathbf{I}_{N_s} \otimes \left( \mathbf{A} \mathbf{X}(k) \right), \quad h = \left( \text{vec}\left\{ \mathbf{H}_s \right\} \right)^T, \quad n(k) = \left( \text{vec}\left\{ \mathbf{N}(k) \right\} \right)^T, \]
\[ S_q = \mathbf{I}_{N_s} \otimes S_q \quad \text{and} \quad w(k) = n(k) - n(k-1)S_q. \]

Based on the above expression, the receiver will select \( S_q \) when \( S_j \) was actually sent as the \( k^{th} \) information matrix if,
\[ \|y(k) - y(k-1)S_j\|^2 \leq \|y(k) - y(k-1)S_q\|^2 \tag{4.4.4} \]
\[ y(k-1)\left( \mathbf{I}_{N_s} \otimes \left( (S_j - S_q)(S_j - S_q)^\dagger \right) \right)y^*(k-1) \leq 2\text{Re}\left\{ w(k)(S_q - S_j)^\dagger y^*(k-1) \right\} \]

For given \( y(k-1) \), the term on the left hand side of (4.4.4) is a constant and the term on the right hand side is a Gaussian random variable. Let,
\[ u = 2\text{Re}\left\{ w(k)\Delta_{i,j}y^*(k-1) \right\}, \quad \text{where,} \quad \Delta_{i,j} = S_j - S_i, \]
the conditional mean on the received signal \( y(k-1) \) can be formulated as
\[ m_{u(k-1)} = E\left\{ 2\text{Re}\left\{ w(k)\Delta_{i,j}y^*(k-1) \right\} \right\} \]
\[ = 2\text{Re}\left\{ E\left\{ w(k) | y(k-1) \right\} \Delta_{i,j}y^*(k-1) \right\} \tag{4.4.5} \]

now, substituting \( w(k) = n(k) - n(k-1)S_q \) and observing \( E\{n(k) | y(k-1)\} = 0 \), (4.4.5) can be simplified to
\[ m_{u(k-1)} = -2\text{Re}\left\{ m_{n(k-1)y(k-1)}S_q\Delta_{i,j}y^*(k-1) \right\} \]
\[ = -2\text{Re}\left\{ m_{n(k-1)y(k-1)} \left( \mathbf{I} - s_s s_s^\dagger \right)y^*(k-1) \right\} \tag{4.4.6} \]

where, \( m_{n(k-1)y(k-1)} = E\{n(k-1) | y(k-1)\} \). Now using the minimum mean square error estimator results,
\[ m_{n(k-1)y(k-1)} = E\{n(k-1)\} + \left( y(k-1) - E\{y(k-1)\} \right) \times \Sigma_{n(k-1),y(k-1)}^{-1} \Sigma_{y(k-1),n(k-1)} \]
\[ \times \Sigma_{n(k-1),y(k-1)}^{-1} \Sigma_{y(k-1),n(k-1)} \]

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where,
\[
\Sigma_{y(k-1),y(k-1)} = E\{y^T(k-1)y(k-1)\} = E_x\kappa_x^T(k-1)r_h\kappa_x^T(k-1) + \sigma_n^2 I_{N_rN_x} \tag{4.4.7}
\]
and,
\[
\Sigma_{y(k-1),n(k-1)} = E\{y^T(k-1)n(k-1)\} = \sigma_n^2 I_{N_rN_x} \tag{4.4.8}
\]
since, \(E\{n(k-1)\} = 0\) and \(E\{y(k-1)\} = 0\), the mean becomes -
\[
\bar{m}_{\mu(y(k-1))} = \sigma_n^2 y(k-1) \times (E_x\kappa_x^T(k-1)r_h\kappa_x^T(k-1) + \sigma_n^2 I_{N_rN_x})^{-1} \tag{4.4.9}
\]
now, substituting (4.4.9) into (4.4.6) will give the conditional mean \(\bar{m}_{\mu(y(k-1))}\) as -
\[
\bar{m}_{\mu(y(k-1))} = E\{\mu \mid y(k-1)\} = 2 \text{Re}\left\{\bar{m}_{\mu(y(k-1))} \left(I_{N_rN_x} - S_\mu S_\mu^T\right) y^T(k-1)\right\}
\]
where, \(\bar{m}_{\mu(y(k-1))} = \sigma_n^2 y(k-1)(\kappa_x^T(k-1)r_h\kappa_x^T(k-1) + \sigma_n^2 I_{N_rN_x})^{-1}\) and the conditional variance has been formulated similarly which has been shown as -
\[
\sigma_n^2 \left(I_{N_rN_x} - S_\mu S_\mu^T\right) y^T(k-1) \Delta_y y^T(k-1)
\]
where, \(\Delta_y = \sigma_n^2 \left(I_{N_rN_x} - S_\mu S_\mu^T\right) - \sigma_n^2 (E_x\kappa_x^T(k-1)r_h\kappa_x^T(k-1) + \sigma_n^2 I_{N_rN_x})^{-1}\)

Based on (4.4.4), the probability of error \(P_e\), on the received signal \(y(k-1)\) can be observed as -
\[
P_e(S_i \mid S_j) = \int_{\mathbb{C}} \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(u - \bar{m})^2}{2\sigma^2}\right) du \tag{4.4.10}
\]
Mean \(\bar{m}_{\mu(y(k-1))}\) becomes non zero and variance \(\sigma_n^2 \left(I_{N_rN_x} - S_\mu S_\mu^T\right) y^T(k-1)\) becomes complicated to solve which leads the \(P_e\) to calculate very difficult. Although, at high SNR levels, the average probability of error can be assumed to be upper bounded as [Lamahewa et al. (2007)] -
\[
P_e(S_i \mid S_j) \leq \frac{1}{2} \frac{1}{\det \left( I + \frac{1}{8}(\mathbf{Z}_h + I_{N_rN_x}) \right)} \tag{4.4.11}
\]
where, \(\bar{\gamma} = E_s / \sigma_n^2\), is the average SNR at each receive antenna and \(Z_{\text{th}} = \kappa_e (k-1) r_{\text{th}} \kappa_z (k-1)\). The space time modulated constellations will follow the property as shown in (4.4.1). If the code length of the STBC is assume equivalent to \(N_t\), each code word matrix \(S_i\) in \(S\) will satisfy the unitary property \(S_i S_i^T = I\) and \(S_i^T S_i = I\) for \(i = 1, 2, \ldots\). Similarly, \(X(k)\) will also satisfy the unitary property \(X(k) X(k)^T = I\) and \(X(k)^T X(k) = I\) for \(k = 1, 2, \ldots\). Applying (4.4.1) in (4.4.11) and using the unitary property of \(X(k)\) and the determinant identity \(\det (A A^T) = \det (A^T A)\), the correlation matrix can be denoted as -

\[
\mathbf{r}_t = (U_R \otimes U_T) (\Lambda_R \otimes \Lambda_T) (U_R \otimes U_T)^T
\]

Now the optimization problem can be considered by taking the logarithm of probability of error in (4.4.12) subject to \(tr(\mathbf{AA}^T) = N_t\) as -

\[
\min \log \det \left( I_{N_t N_t} + \frac{\beta_{i, j}}{8 + \beta_{i, j}} r_{\text{th}} \left( I_{N_z} \otimes \mathbf{A} \mathbf{A}^T \right) \right)
\]

Probability of the error is the main element in performance of a communication system. The precoder will be designed by considering \(\beta = \min_{i, j} \{ \beta_{i, j} \}\). The optimization problem is similar to the [Yi et al. (2004)] who derives the optimum precoder in closed form by considering MISO channels. Now considering \(U_R \mathbf{B}_{U_R}^T\) as the Eigen Value Decomposition (EVD) equivalent to \(U_R \mathbf{A} U_R^T\) and similarly for the transmitter, EVD \(U_T \mathbf{A} U_T^T = U_T \mathbf{A} U_T^T\) and using the kronecker product identity \((\mathbf{A} \otimes \mathbf{C})(\mathbf{B} \otimes \mathbf{D}) = \mathbf{A} \mathbf{B} \otimes \mathbf{C} \mathbf{D}\), the correlation matrix can be denoted as -

\[
\mathbf{r}_t = (U_R \otimes U_T) (\mathbf{A} \otimes \mathbf{A}^T) (U_R \otimes U_T)^T
\]
\[
\min - \log \det \left( I_{N_rN_s} + \frac{\beta^T}{8 + \beta} r_n \left( I_{N_s} \otimes (AA^\dagger) \right) \right) \tag{4.4.15}
\]

where, \( \beta = \min_{i,r} \left\{ \beta_{i,j} \right\} \) over all possible codewords. Now substitute (4.4.14) in (4.4.15) for formulating \( r_n \) by considering,

\[
Q_{lc} = \frac{\beta^T}{8 + \beta} U_T^\dagger AA^\dagger U_T
\]

then, the objective function (4.4.15) becomes -

\[
\min - \log \left[ I_{N_rN_s} + \left( \Lambda_R \otimes \Lambda_T \right) \left( I_{N_s} \otimes Q_{lc} \right) \right]
\]

subject to -

\[
Q_{lc} \geq 0, tr \{Q_{lc}\} = \frac{\beta^T N_r}{8 + \beta} \tag{4.4.16}
\]

By applying Hadamard’s inequality on \( I_{N_rN_s} + \left( \Lambda_R \otimes \Lambda_T \right) \left( I_{N_s} \otimes Q_{lc} \right) \) gives that this determinant is maximized when \( \left( \Lambda_R \otimes \Lambda_T \right) \left( I_{N_s} \otimes Q_{lc} \right) \) is diagonal. Therefore, \( Q_{lc} \) must be diagonal as \( \Lambda_R \) and \( \Lambda_T \) both diagonal. Since, \( \left( \Lambda_R \otimes \Lambda_T \right) \left( I_{N_s} \otimes Q_{lc} \right) \) is a positive semi-definite diagonal matrix with non-negative entries on its diagonal, \( I_{N_rN_s} + \left( \Lambda_R \otimes \Lambda_T \right) \left( I_{N_s} \otimes Q_{lc} \right) \) forms a positive definite matrix which leads to show that the objective function of optimization problem is convex. Therefore, the optimization problem (4.4.16) above is a convex minimization problem because the objective function and inequality constraints are convex and equality constraints are having finite response.

Now the optimum \( Q_{lc} \) is diagonal and diagonal entries of \( Q_{lc} \) are found by solving the optimization problem,

\[
\min - \sum_{j=1}^{N_r} \sum_{i=1}^{N_T} \log \left( 1 + (\Lambda_T)_{i,j} Q_{lc} \left( \Lambda_R \right)_{j,j} \right)
\]

subject to, \( Q_{lc} \geq 0 \)

\[
1^T Q_p = \frac{\beta^T N_r}{8 + \beta} \tag{4.4.17}
\]

where, \( Q_p = [Q_1, Q_2, ..., Q_{N_r}]^T \) and 1 denoting the vector of all ones.

The precoder \( A \) is obtained by forming,

\[
A = \sqrt{\frac{8 + \beta}{\beta^T}} U_T Q_{lc}^{1/2} U_n^T
\]
where, $Q_{c} = Q_{1}, Q_{2}, ..., Q_{N_{T}}$ and $U_{n}$ is any unitary matrix.

Now considering the Lagrange multipliers $\xi \in \xi$ for the inequality constraints $-Q_{p} \leq 0$ and $\xi_{e} \in \xi$ for the equality constraint $1^{T}Q_{p} = \beta \bar{T}N_{T}/(8 + \beta)$, the Karush-Kuhn-Tucker condition shows, $Q_{p} \geq 0, \xi_{i} \geq 0, 1^{T}Q_{p} = \beta \bar{T}N_{T}/(8 + \beta), \xi_{e}Q_{c,i,i} = 0$, with $i = 1, 2, ..., N_{T}$,

$$-\sum_{j=1}^{N_{T}} (\Lambda_{R})_{j,i}(\Lambda_{T})_{i,j} - \xi_{i} + \xi_{e} = 0, \quad i = 1, 2, ..., N_{T} \tag{4.4.18}$$

$\xi_{i}$ can be eliminated since it can be treated as a slack variable, giving the new KKT conditions:

$$Q_{p} \geq 0, 1^{T}Q_{p} = \frac{\beta \bar{T}N_{T}}{(8 + \beta)},$$

$$Q_{p}\left[\xi_{e} - \sum_{j=1}^{N_{T}} (\Lambda_{R})_{j,i}(\Lambda_{T})_{i,j}Q_{c,i,j}\right] = 0, \quad i = 1, 2, ..., N_{T} \tag{4.4.19}$$

$$\xi_{e} \geq \sum_{j=1}^{N_{T}} (\Lambda_{R})_{j,i}(\Lambda_{T})_{i,j}Q_{c,i,j}, \quad i = 1, 2, ..., N_{T} \tag{4.4.20}$$

when, $N_{R} = 1$, the optimal solution to (4.4.17) is given by the “water-filling” solution as shown in [Telatar (1999)]. The optimal $Q_{p}$ for this case can be driven for different antenna configurations. For $N_{R} > 1$, the problem in finding the optimal $Q_{p}$ for given $(\Lambda_{T})_{i,j}$ and $(\Lambda_{R})_{j,i}$ is the case that, there are multiple terms that involve $Q_{p}$ on (4.4.19). Hence the optimization problem in (4.4.17) can be viewed as a generalized water-filling problem.

As shown above, the optimal $Q_{c}$ is diagonal with $Q_{c} = diag\{Q_{1}, Q_{2}, ..., Q_{N_{T}}\}$ and optimal spatial precoder $A$ is obtained by forming -

$$A = \sqrt{\frac{8 + \beta}{\beta \bar{T}}}U_{T}Q_{c}^{1/2}U_{n}^{*} \tag{4.4.21}$$

where, $U_{n}$ is a unitary matrix.
4.5. OPTIMUM PRECODER FOR DIFFERENT ANTENNA CONFIGURATIONS USING STBC

4.5.1. For MISO Channel

The optimization problem is similar to the “water-filling” problem case in the information theory [Telatar (1999)] which has the optimal solution -

\[ Q_i = \begin{cases} 
\frac{1}{w'} - \frac{1}{(A_T)_{i,j}}, & w' < (A_T)_{i,j} \\
0, & \text{otherwise,} 
\end{cases} \]

(4.5.1)

where, the water level \(1/w'\) is chosen to satisfy -

\[
\sum_{i=1}^{N_T} \max \left(0, \frac{1}{w'} - \frac{1}{(A_T)_{i,j}} \right) = \frac{\beta N_T}{8 + \beta}
\]

4.5.2. For MIMO Channel

For MIMO antenna with two receivers, the optimum \(Q_i\) will be formulated as -

\[ Q_i = \begin{cases} 
A + \sqrt{K'}, & w' < (A_T)_{i,j} (M_{R1} + M_{R2}) \\
0, & \text{otherwise,} 
\end{cases} \]

(4.5.2)

where \(w'\) is chose to satisfy -

\[
\sum_{i=1}^{N_T} \max (0, A' + \sqrt{K'}) = \frac{\beta N_T}{8 + \beta}
\]

\[
A' = \frac{2M_{R1}M_{R2}(A_T)^2_{i,j} - w'(A_T)_{i,j} (M_{R1} + M_{R2})}{2w'M_{R1}M_{R2}(A_T)^2_{i,j}}
\]

and

\[
K' = \frac{w'^2 (A_T)^2_{i,j} (M_{R1} - M_{R2})^2 + 4M_{R1}^2M_{R2}^2(A_T)^4_{i,j}}{2w'M_{R1}M_{R2}(A_T)^2_{i,j}}
\]

Likewise the precoders can be formulated for \(N_T\) and \(N_R\) numbers of antenna in the MIMO system in a generalized form. Further, the simulation results have been shown in the results section with the comparison from the existing results available in the literature.
4.6. CAPACITY ANALYSIS AND RESULT DISCUSSION

The broad capacity analysis has been performed by deriving the order statics techniques, reducing the MSE, maximizing the mutual information by implementing the proposed algorithm followed by the improved results in enhancement of overall capacity and reduction in pilot symbols for different combination of MIMO antennas. Figure 4.2. shows capacity of the different MIMO antenna combinations and their capacity increase with the increase in transmitting and receiving antenna elements for the condition when CSI is known to transmitter with the spatial heterogeneous behavior.

![Figure 4.2(a). Capacity analysis for different antenna configurations with known CSI (with waterfilling)](image)
Figure 4.2(b). PDF distribution for different antenna configurations with known CSI (with waterfilling)

Figure 4.3. Comparison of capacity analysis for 2x2 and 4x4 MIMO antenna with known CSI and unknown CSI
Figure 4.4. Comparison of capacity analysis with unknown CSI for 2x2 and 4x4 MIMO antenna for existing and proposed estimation scheme.

It is clearly visible from the existing analysis and simulated results that the capacity increases with the increase in transmitter and receiver antenna elements. It is known that with CSI, zero forcing beamforming completely removes the interference between the transmitting antennas in,

$$\hat{C} = \log_2 \left( 1 + \rho \| \mathbf{H}_k \|_2^2 \right)$$  
(4.6.1)

Now this capacity reduces to two different forms depending upon the number of antennas [Sohn et al. (2010)] -

$$C = \mathbb{E} \left[ \sum_{k \in \mathcal{K}} \log_2 \left( 1 + \rho_k \| \mathbf{H}_k \|_2^2 \right) \right]$$  
(4.6.2)

$$C = \sum_{j=1}^{M} \mathbb{E} \left[ \log_2 \left( \max_{k \in \mathcal{K}_j} \rho_k \| \mathbf{H}_k \|_2^2 \right) \right]$$  
(4.6.3)

Relation (4.6.2) is found when channel gain is reduced since zero forcing beamforming reduces to negligible with proper selection of semi orthogonal parameters, i.e., \( \lim_{\epsilon \to 0} \| \mathbf{H}_k^H \mathbf{\epsilon} \|_2^2 = 1 \) and (4.6.3) is found
with \( \max_{k \in \mathbb{C}} \left\{ \rho_k \| H_k \|^2 \right\} \gg 1 \) with large number of antennas. Figure 4.2 and Figure 4.3 shows that the capacity increases with the increase in number of antennas \( K \). It has been observed that the analysis has been found near to practical propagation scenario on comparing the analytical and simulated results and the difference between then has come out from the loose bound of extreme order statistics for small number of antennas and non-trivial semi orthogonal parameter \( \varepsilon \).

With partial CSI, \( H \neq \hat{H} \), the interference remain in the reception of the signals at the receiver end due to which denominator in (4.6.1) exists. Using [Jindal (2006); Taesang et al. (2007)], the capacity reduces to:

\[
C = E \left[ \sum_{k \in \mathbb{C}} \log_{2} \left( 1 + \frac{\rho_k \| H_k \|^2 \| \hat{H}_k \|^2 \hat{H}_k}{1 + \rho_k \| H_k \|^2 \left( \sin^2 \theta_k \right) \sum_{j \in \mathbb{C}} \beta_j} \right) \right] \quad (4.6.4)
\]

where, \( \beta_j \) denotes a distributed random variable with parameter \((1, M+N-1)\). Fig.5(b) shows the probability distribution function (PDF) of \( \sin^2 \theta_k \) which has been found using cumulative distribution function (CDF) of \( \sin^2 \theta_k \) and further been derived as:

\[
f(\sin^2 \theta(x)) = \begin{cases} (M + N - 1)2^{(M+N-1)}x^{M-2} & , \quad 0 \leq x \leq \xi \\ 0 & , \quad x \geq \xi \end{cases}
\]

where, \( \xi \) is a parameter.

Figure 4.3 and Figure 4.4 shows the capacity for the unknown CSI or partial CSI condition for 2x2 and 4x4 antenna configurations. It has been observed that with lower SNR conditions, significant improvement in capacity has been seen with the increase in number of transmitting and receiving elements. Whereas in case when the SNR is high, capacity of MIMO system with spatial channel becomes interference limited and this interference limited capacity increase with the quantization factor \( B \). Figure 4.3 shows that the capacity of the MIMO system with known CSI is better than the capacities with partial CSI or unknown CSI which has been limited due to lower bound conditions. Figure 4.4 shows the comparison of capacity with partial CSI and the new capacity found using adaptive method with partial CSI, which shows improvements at the lower SNR conditions as compared with the capacity with the partial CSI. At higher SNR conditions, this estimation scheme has also the interference limited bound with them but still a good result has been found using this estimation scheme as compared with the existing partial CSI condition, which has been found using the following bounds [Vu and Paulraj (2005)] -
\[
\lim_{B \to M^*} \frac{C}{M^* B / M^* - 1} = 1
\]
i.e.,
\[
C > \sum_{k=3} \left[ \mathbf{E} \left\{ \log_2 \left( |\tilde{\mathbf{H}}_k \mu_k|^2 \right) \right\} - \mathbf{E} \left\{ \log_2 \left( \sin^2 \theta_k \right) \sum_{j \neq k} \beta_j \right\} \right]
\] (4.6.5)
\[
C > \sum_{k=3} \left[ \mathbf{E} \left\{ \log_2 \left( |\tilde{\mathbf{H}}_k \mu_k|^2 \right) \right\} - \log_2 \left\{ \mathbf{E} \left( \sin^2 \theta_k \right) \mathbf{E} \left( \sum_{j \neq k} \beta_j \right) \right\} \right]
\] (4.6.6)
\[
C > M \left\{ \log_2 (1 - \xi) + \frac{\xi}{M \log_2 (M - 1)} + \frac{B}{B} \right\}, \quad \xi = 2^{\frac{-B}{M^* N - 1}}
\] (4.6.7)

Relation (4.6.5) has been found with the fact of growth of \( \rho_k \| \mathbf{H}_k \|^2 \), (4.6.6) comes from the Jensen's inequality and (4.6.7) has been formulated for the proposed estimation method for capacity with partial CSI knowledge.

Figure 4.5. Received 16-QAM constellation for the channel with known CSI.
Figure 4.6. Received 16-QAM constellation for the channel with partial CSI.

Figure 4.7. Received 16-QAM constellation for the channel with partial CSI using proposed estimation scheme and correlation coefficient of 0.5.
Figure 4.8. Received 16-QAM constellation for the channel with partial CSI using proposed estimation scheme for SNR 35dB and correlation coefficient 0.1.

Figure 4.5 and 4.6, shows the comparison of the received constellation for 16-QAM modulation scheme for both channels, i.e., with known CSI knowledge and partial CSI knowledge, in which it can be seen that the symbols in the partial CSI case, are more diverged but still detectable, whereas in Figure 4.7 the 16-QAM constellation has been received at the receiver end for the channel with partial CSI knowledge implemented with the proposed estimation, which shows less divergence of the symbols at the receiver end when the system is simulated with correlation coefficient of 0.5. Similarly, if the higher SNR case is considered, the received constellation is shown in Figure 4.8 which shows the divergence of the received symbols in good quantity. These noise or the incorrect estimated symbols can be rectified by increasing the number of iterations in the channel estimation process.
Figure 4.9. Comparison of Ergodic capacity with proposed partial CSI, known CSI with waterfilling and known capacity with equal power with the correlation coefficient of 0.5 with 2x2 antenna configuration.

Figure 4.10. Comparison of Ergodic capacity with proposed partial CSI, known CSI with waterfilling and known capacity with equal power with the correlation coefficient of 0.5 with 4x4 antenna configuration.
Figure 4.9 and 4.10 shows the comparison of the capacity of the channel with partial CSI with proposed method, known CSI (waterfilled) and known CSI channel with equal power for two different MIMO antenna configurations of 2x2 and 4x4. The partial CSI capacity optimal power allocation improves from the known CSI capacity at 1.5dB and maintains this up to 16.5dB where, both partial CSI capacity and known CSI known capacity with waterfilling starts to decrease as with the equal power capacity of the known CSI capacity. At 20dB, the partial CSI approximates with the known CSI which then again improves after 22dB. This shows that, at higher SNR, the capacity for both partial CSI and known CSI approximates to the lower of the equal power capacity with known CSI. Figure 4.10 has been shown for 4x4 MIMO antenna for all the three said channel capacities comparison as said above. A number of variations have been found at the higher SNR levels for this 4x4 configurations. At lower SNR levels, the partial CSI channel with proposed scheme has the advantages over the known CSI and the capacity with equal power. With the increase in SNR at 18dB, channel capacity with known CSI start to decrease with equal power channel capacity, whereas the partial CSI capacity is still higher than both of the capacities. With some more increase in SNR at 18.8dB, capacity with partial CSI starts decreasing with equal power but remains in between the remaining two capacities. At 22.9dB, the capacity with partial CSI starts to decrease and both capacities, i.e., with partial CSI and with known CSI come less than the capacity with equal power. Further, at 24.6dB, the capacity with known CSI again increases as compared with the capacity with equal power, whereas capacity with partial CSI still remains less with both of them. At least, the capacity with partial CSI again improves and moves ahead of capacity with equal power at 26.5dB, but at this stage, too, capacity with known CSI is higher than the said two capacities. These large variations in the capacities are due to the channel mean and the variance, i.e., higher the variance, more variations will result which is due to the difference in the power allocation.
Figure 4.11. Analysis of the eigenvalues of the transmit covariance matrix for 2x2 MIMO antenna system.

Figure 4.12. Analysis of the eigenvalues of the transmit covariance matrix for 4x4 MIMO antenna system.
Figure 4.11 and 4.12 shows the mutual information with the eigenvalues of Q for two different antenna configurations, i.e., 2x2 and 4x4 with three different scheme comparisons, i.e., mutual information with known CSI (waterfilling), partial CSI and known CSI with equal power are shown. A zero mean variance one has been considered for the channel and the results shows that the optimal power for partial CSI follows the path for the known CSI channel in the case of 2x2 MIMO antenna systems. The mutual information (power allocation) at higher SNR for the partial CSI known channel approaches equi-power channel with known CSI as the channel with known CSI has approached.

At some instants, the mutual information deviates to follow the known CSI which occurs due to the occurrence of Jensen’s inequality covariance at those instants. With higher SNR and higher antenna elements, the optimized mutual information may converges to non equi-power channel with known CSI. But after implementing the adaptive estimation based scheme for 4x4 MIMO antenna systems, the channel with partial CSI also approaches equi-power channel as the channel with known CSI does, i.e., it follows the same path.

Figure 4.13. Comparative analysis of precoder based SER for 2x2 MIMO antenna configurations with correlation coefficient of 0.5.
Figure 4.14. Comparative analysis of precoder based BER for 2x2 MIMO antenna configurations with correlation coefficient of 0.5.

Figure 4.15. Comparative analysis of precoder based capacity for 2x2 MIMO antenna configurations with different correlation coefficients.
The performance comparison of the Symbol Error Rate has been shown in Figure 4.13 for which the comparison shows that the modified precoder based results proposed in this Chapter are giving better results than the scheme proposed in [Zhiguo et al. (2006)] which has shown better results than the SER shown in this [Zhiguo and Ward (2005)]. It is easily observable that there is a 2.4dB SNR enhancement at the level of $10^{-6}$ dB of SER as proposed by [Zhiguo et al. (2006)] which has been derived for the semiblind channel with two training symbols but less than the perfectly known channel due to the partial information received.

Figure 4.14 shows the comparison of the improvement in BER achieved by the proposed method in this Chapter with the BER of [Zijian and Leus (2006)] and [Lamahewa et al. (2007)]. The results shows that the proposed scheme is giving results better than the [Lamahewa et al. (2007)] and reaching very near to the BER achieved by the [Zijian and Leus (2006)] using jakes precoder model just 0.2 dB less SNR at the level of $10^{-6}$ dB of BER.

Figure 4.15 is showing the comparative study of the capacity achieved in this Chapter with the existing result observed in [Seo (2012)], who have shown his results for the correlation coefficient of 0.1 and 0.9 and the results found using the proposed scheme in this Chapter has shown much better results at the correlation coefficient of 0.5 only.

4.7. SUMMARY

A new adaptive algorithm which has been tried and implemented with the MIMO capacity lower bounds to maximize the capacity with the lower pilot symbols requirement in the partial CSI knowledge conditions for the spatial channels whose results have been shown and compared in previous section. Also the increase in mutual information and reduction in Mean square error has been tried with determination of optimum covariance matrix in precoder form for the transmit correlation matrix. The adaptive estimation based optimized precoded MIMO system shows the advantages in capacity enhancement as compared with the existing partial CSI conditions and equal power allocation scheme. Different estimation methods and comparison for transmitted training symbols have been investigated and their improved results based on adaptive estimation method have been shown with the consideration of different correlation coefficient values for the transmitting antennas. At last, based on the eigenvectors knowledge of transmit covariance, optimized capacity achieving eigenvalues of the transmit covariance matrix has been shown and found that the gap between the power of eigenvectors for higher antenna
configurations has been reduced and it started to follow the equi-power channel mutual information. The changes made in this Chapter have been shown by the yellow blocks made in Figure 4.16.

**Figure. 4.16.** Changes made in MIMO systems on the basis of proposed algorithms in this Chapter.