Chapter-1

INTRODUCTION

This thesis is devoted to the study of some stochastic models in Inventories and Queues which are physically realizable, though complex. It contains a detailed analysis of the basic stochastic processes underlying these models. Many real-world phenomena require the analysis of system in stochastic rather than deterministic setting. Stochastic models are becoming increasingly important for understanding or making performance evaluation of complex systems in a broad spectrum of fields such as operations research, computer science, economics, telecommunication, engineering etc. Our aim is to have an improved understanding of the behaviour of such models, which may increase their applicability. Some variants of inventory systems with random lead times, non-identically distributed inter-arrival demand times, state dependent demands, perishable commodities, varying ordering levels etc. are considered. Also we study some finite and infinite capacity single server queueing systems with single/bulk services, vacation to the server; transient as well as steady state solutions of the systems are obtained in certain cases. Each chapter in the thesis is provided with self
This chapter gives a brief introduction to the subject matter and some related topics. It gives a concise survey of some important developments in the area of inventories and queues. Some basic notions in renewal theory and Markov renewal theory are supplemented. Finally an outline of the results obtained is given.

1.1 INVENTORY THEORY - AN OUTLINE

An inventory is an amount of material stored for the purpose of sale or production. Inventory management of physical goods or other products or elements is an integral part of logistic systems common to all sectors of the economy, such as business, industry, agriculture, defence etc. In an economy that is perfectly predictable, inventory may be needed to take advantage of the economic features of a particular technology, or to synchronize human tasks, or to regulate the production process to meet the changing trends in demand. When uncertainty is present, inventories are used as protection against risk of stockout.

The existence of inventory in a system generally implies the existence of an organized complex system involving inflow, accumulation, and outflow of some
commodities or goods or items or products. The regulation and control of inventory must proceed within the context of this organized system. Thus inventories, rather than being interpreted as idle resources, should be regarded as a very essential element, the study of which may provide insight to the aggregate operation of the system. The analysis of inventory system defines the degree of inter-relationship between inflow, accumulation, and outflow and identifies economic control method for operating such systems.

**Analysis of Inventory systems**

Inventory systems may be broadly classified as continuous review systems or periodic review systems. In continuous review systems, the system is monitored continuously over time. In periodic review systems, the system is monitored at discrete, equally spaced instants of time. An analysis of an inventory consists of the following steps: (1) determination of the properties of the system, (2) formulation of the inventory problem, (3) development of a model of the system, and (4) derivation of a solution of the system.

**Inventory policies – Decision variables**

An inventory policy is a set of decision rules
that dictate 'when' and 'how much' to order. Several policies may be used to control an inventory system; of these, the most important policy is the (s,S) policy. Under this particular policy, whenever the position inventory (sum of onhand inventory and outstanding orders) is equal to or less than a value s, a procurement is made to bring its level to S. Under a continuous review system, the (s,S) policy will usually imply the procurement of a fixed quantity Q = S - s of the commodity, while in periodic review systems the procurement quantity will vary. The (s,S) policy incorporates two decision variables s and S. The variable s is called the reorder level, which identifies when to order, while S - s identifies how much to order.

Objective function

In an inventory problem, the objective function may take several forms, and these usually involve the minimization of a cost function or the maximization of a profit function. The planning period or horizon period, which is the length of time over which the system is assumed to operate, may be either finite or infinite. For a finite horizon period, the total cost (profit) experienced over the entire horizon may be the criterion; alternatively the criterion may be average of the total cost (profit) per unit time. On the other hand, if the horizon is infinite,
the long run average total cost (profit) experienced over the infinite horizon, is selected as the criterion. In stochastic models expected values of costs are measured. The cost function, in general, consists of the additive contribution of the procurement cost, the holding cost, and the shortage cost.

The inventory models are usually characterized by the demand pattern and the policy for replenishing the stock in the store. The replenishments ordered may arrive after a time lag $L$, which may be fixed or a random variable. This time lag $L$ is called the 'lead time'. The time interval during which the inventory is empty is termed as a dry period.

The quantitative analysis of the inventory system started with the work of Harris (1915), who formulated and obtained the optimal solution to a simple inventory situation. Wilson (1934) rediscovered the same formula and was successful in popularizing its use. The formula is an expression for an optimal production lot size given as a square root function of a fixed cost, an investment or holding cost, and the demand. It is often referred to as the 'simple lot-size formula' or the 'economic order quantity (EOQ) formula', or the 'Wilson formula'. A stochastic inventory problem was analysed for the first
time by Masse (1946). After that several studies were made in this direction (See Arrow, Harris and Marschak (1951), Dvoretsky, Kiefer and Wolfowitz (1952). Dvoretsky et al. obtained the conditions under which optimum inventory levels can be found. The development of the theory upto 1952 have been summarized by Whitin (1953).

A valuable review of the problems in the probability theory of storage systems is given by Gani (1957). A systematic account of probabilistic treatment in the study of inventory systems using renewal theoretic arguments is given in Arrow, Karlin and Scarf (1958). Hadley and Whitin (1963) deals with the applications of such models to practical situations. Tijms (1972) gives a detailed analysis of the inventory systems under (s,S) policy. The cost analysis of different inventory systems is given in Naddor (1966). A practical treatment of the (s,S) lost sales model can be found in the recent books by Silver and Peterson (1985) and Tijms (1986).

A detailed review of the work carried out in (s,S) inventory systems upto 1966 can be found in Veinott (1966). We refer to the monograph by Ryshikov (1973) for inventory systems with random lead times. Sivazlian (1974) considers an (s,S) inventory model in which unit demands of items
occur with arbitrary interarrival times between demands, but with zero lead time. His results are extended by Srinivasan (1979) to the case in which lead times are i.i.d. random variables having a general distribution. Sahin (1979) considers an (s,S) inventory system in which demand quantities are nonnegative real valued random variables with constant lead time. Again in Sahin (1983) an (s,S) inventory system in which demand quantities (positive integer valued), lead times and interarrival times between consecutive demands are all independent and generally distributed sequences of i.i.d. random variables, is discussed. She obtained binomial moments for the inventory deficit. Thangaraj and Ramanarayanan (1983) consider an inventory system with random lead times and having two ordering levels. Kalpakam and Arivarignan (1985) have studied an (s,S) inventory system having one exhibiting item subject to random failure time and obtained the limiting distribution of position inventory by applying the techniques of semi-regenerative process. Ramanarayanan and Jacob (1987) also analyse an (s,S) inventory system with random lead times and bulk demands. An (s,S) inventory system with rest periods to the server has been analysed by Daniel and Ramanarayanan (1988).
The earliest work on the decay (perishability) problem is due to Ghare and Schrader (1963) who considered the generalization of the standard EOQ model without shortages. Their model was extended to more general types of deterioration by Covert and Philip (1973) and Shah (1977). Nahmias (1982) reviews various models and objective functions in the analysis of such inventory systems. Motivated by the study of blood bank models Kaspi and Perry (1983, 1984) and Perry (1985) have studied inventory systems for perishable commodities in which lifetime of the items stored are fixed as well as random variables. They utilized the analogy between these systems and a queueing system with impatient customers to study the process of the lost demand, the number of units in the system, etc.

A continuous review \((s,S)\) inventory system in random environment is analysed by Feldman (1978). Richards (1979) analyses an \((s,S)\) inventory system with compound Poisson demand. Algorithms for a continuous review \((s,S)\) inventory system in which the demand is according to a versatile Markovian point process is given by Ramaswami (1981). Approximation for the single-product production-inventory problem with compound Poisson demand and two possible production rates where the product is continuously added to inventory can be seen in De Kok, Tijms and Van aer Duyn Schouten (1984). Using Markov decision drift processes,
Hordijk and Van der Duyn Schouten (1986) examines the optimality of (s,S) policy in a continuous review inventory model with constant lead time when the demand process is a superposition of a compound Poisson process and a continuous deterministic process.

Stidham (1974) has introduced and studied a wide class of stochastic input-output systems. The system is fed by an exogenous stochastic input process. The quantity in the system builds over time as a result. At a certain (random) time instant all the quantity in the system is instantaneously removed (cleared) and the situation allowed to repeat itself. Such systems are called stochastic clearing systems. Its applications to bulk-service queues and (s,S) inventory systems are given by Stidham (1977, 1986). In a generalized stochastic clearing system, the system contents are restored to a level m ( > 0 ), rather than zero, at each clearing instant. With inventory defined as the negative of system contents, the generalized model covers (s,S) inventory systems with continuous or periodic review. In his paper Stidham (1986) discusses the optimality of the clearing parameters.

In the case of random lead times, the concept of vacations to the server during dry period is introduced in inventory system by Daniel and Ramanarayanan (1987, '88).
Several other models with vacations to the server, finite backlog of demands, bulk demands, varying ordering levels etc. can be found in Jacob (1987).

1.2 QUEUEING THEORY – AN OUTLINE

The development of queueing theory started with the publication of Erlang's paper (1909) on the M/D/1 queueing system. For this system, which has constant service times and a Poisson arrival process, Erlang explained the concept of statistical equilibrium. This paper touched the essential points of queueing theory, and for a long time research in queueing theory concentrated on questions, first time discussed by Erlang.

Until 1940, the majority of the contribution to queueing theory was made by people active in the field of telephone traffic problems. After the Second World War, the field of operations research rapidly developed and queueing applications were also found in production planning, inventory control and maintenance problems. In this period, much theoretically oriented research on queueing problems were done.

In the fifties and sixties, the theory reached a very high mathematical level (see Cohen (1969) and Takacs(1962)).
Advanced mathematical techniques like transform methods, Wiener-Hopf decomposition and function theoretic tools were developed and refined. This research resulted in a number of elegant mathematical solutions.

In particular, noting the inadequacy of the equilibrium theory in many queue situations, Pollaczek in 1934 began investigations of the behaviour of the system in a finite interval. Since then, there appeared considerable work in the analytical behavioural study of queueing systems. The trend towards the analytical study of the basic stochastic processes of the system has continued, and queueing theory has proved to be a fertile field for researchers who wanted to do fundamental research on stochastic processes involving mathematical models.

For the time dependent analysis of the system, more sophisticated mathematical procedures are necessary. For instance, for an M/M/1 queue, under statistical equilibrium, the balance of state equations is simple and the limiting distribution of the queue size is obtained by recursive arguments and induction. But for the time dependent solution, the use of transforms is necessary. The time dependent solution was first given by Bailey (1954b) and Ledermann and Reuter (1956). While Bailey used the
method of generating functions and the differential equations satisfied by them, Ledermann and Reuter used spectral theory for their solution. Champernowne's (1956) combinatorial method and Conolly's (1958) difference equation techniques are also aimed at the transient solution for the system size in an $M/M/1$ queue system. Parthasarathy (1987) suggests a simple and direct approach for the same.

To analyse the case of $M/G/1$ queues, Palm (1943) and Kendall (1953) have used the method of regeneration points and imbedded Markov chain which continue to have a tremendous influence on queueing theory. Kendall's exposition created a new technique for analysing certain queueing models which are not Markovian. His approach made the analysis of the transient behaviour of queueing systems much more accessible. The method of supplementary variables investigated by Kendall (1951) and Cox (1955) is extensively discussed in the book by Gnedenko and Kovalenko (1968).

The study of bulk queues is considered to be originated with the pioneering work of Bailey (1954a). It may be said to have begun with Erlang's investigations of $M/E_k/1$ queue, for its solution contains implicitly the solution of the model $M^k/M/1$. Bailey studied the stationary behaviour of a single server queue having simple Poisson
input, intermittently available server and service in
batches of fixed maximum size. The results of this study
are given in terms of probability generating functions,
the evaluation of which requires determining the zeroes
of a polynomial. This study was followed by a series of
papers involving the treatment of queueing processes with
group arrivals or batch service. Gaver (1959) seems to be
the first to take up specifically queues involving group
arrivals followed by Jaiswal (1960, 1961, 1962), Bhat(1964)
and others. For more details on bulk queues, one may refer
detailed treatment of queueing systems and for further
references, one may refer any one of the standard books
on the subject like Saaty (1961), Takacs (1962), Cohen (1969),
Prabhu (1965, 1980), Gnedenko and Kovalenko (1968),
Cooper (1972), Gross and Harris (1974), Kleinrock (1975)

Queueing systems in which the service process is
subject to interruptions resulting from unscheduled break­
downs of servers, scheduled off periods, arrival of customers
with pre-emptive or non-preemptive priorities or the server
working on primary and secondary customers arise naturally.
The impact of these service interruptions on the performance
of a queueing system will depend on the specific interaction
between the interruption process and service process.
Queueing models with interruptions and their connection to priority models were first studied by White and Christie (1958), who considered the case with exponential service, on-time and off-time distributions. Their results were extended by Gaver (1962), Keilson (1962), Avi-Itzhak and Naor (1962) and Thiruvengadam (1963) to models with general service time and off-time distributions but exponential on-times. When the on-periods are not exponential, the problem becomes very difficult and one such model is studied by Federgruen and Green (1986).

A detailed analysis of single server queueing system with server failures is given in Gnedenko and Kovalenko (1968).

Another variation of the interruption model is the vacation model. In this the queueing system incurs a start-up delay whenever an idle period ends or server takes vacation periods. The vacation model includes server working on primary and secondary customers also.

Motivated by the study of cyclic queues, Miller (1964) analysed a system in which the server goes for a vacation (rest period) of random duration whenever it becomes idle. He also considered a system in which server behaves normally but the first customer arriving to an empty system has a special service time. These types of systems and similar ones were also examined by Welsch (1964), Avi-Itzhak,
Maxwell and Miller (1965), Cooper (1970), Pakes (1973),
Lemoine (1975), Levy and Yechiali (1975), Heyman (1977),
Van der Duyn Schouten (1978), Shantikumar (1980, 1982),
Scholl and Kleinrock (1983) etc.

All the models having rest periods, set-up time, 
starter, interruptions etc. can be jointly called as 
vacation models. While the queue with interruption has 
preemptive priority for vacation, other types of vacations 
have least priority among all work with vacation taken 
when the system is empty. Variations of vacation models 
are available with single and multiple vacations and 
exhaustive and non-exhaustive service disciplines.

A queueing system in which the server taking exactly 
one vacation at the end of each busy period, is called a 
single vacation system. When the system becomes empty, 
server starts a vacation and he keeps on taking vacations 
until, on return from a vacation, at least one customer is 
present. This is called a multiple vacation system. We 
say that a vacation model has the property of exhaustive 
service in case each time the server becomes available, 
he works in a continuous manner until the system becomes 
empty. Systems with a vacation period beginning after 
every service completion, or after any vacation period
if the queue is empty is known as the single service discipline. According to Bernoulli schedule discipline the server begins a vacation period if the queue is empty. If at a service completion the queue is not empty, then service is resumed with fixed probability $p$ and with probability $1-p$ a vacation commences. Single service disciplines and exhaustive service disciplines are special cases of the Bernoulli schedule discipline. Another variant of the vacation model is that the server goes for vacation after serving a random number of customers.

Vacation systems with exhaustive service discipline are analysed by several authors. See for example, Levy and Yechiali (1975), Hayman (1977), Courtois (1980), Shantikumar (1980), Scholl and Kleinrock (1983), Lee(1984), Fuhrmann (1984), Doshi (1985), Servi (1986 a), Levy and Kleinrock (1986), Keilson and Servi (1986 b) etc. Systems without exhaustive service discipline are considered by Ali and Neuts (1984), Neuts and Ramalhoto (1984), Fuhrmann and Cooper (1985), Keilson and Servi (1986 b,c) and Servi (1986 a). The case of Bernoulli schedule discipline is introduced by Keilson and Servi (1986 a) and further studied by Servi (1986 b).
The main results in the vacation system is the delay analysis by decomposition. The stochastic decomposition property of $M/G/1$ queueing system with vacation says that the (stationary) number of customers present in the system at a random point in time is distributed as the sum of two or more independent random variables, one of which is the (stationary) number of customers present in the corresponding standard $M/G/1$ queue (i.e. the server is always available) at a random point in time. For more details on queueing systems with vacations one may refer to Doshi (1986).

All the above models assume the existence of stationary distribution and study the queue length and waiting time distributions. The time dependent behaviour as well as steady state behaviour of $M/G/1$ and $G/M/1$ queueing systems are extensively studied by Bhat (1968) in which bulk arrival and bulk service queues are considered and the behaviour of the waiting time process is obtained. Some aspects of the dynamic behaviour of $M/G/1$ queues with vacations is studied by Keilson and Servi (1986 c). An attempt to find the transient solution of $M/G^{a,b}/1$ queue with vacation using matrix convolution has been made in Jacob and Madhusoodanan (1987). But they have remarked that the solution in that form is not numerically tractable.
1.3. RENEWAL THEORY

Renewal processes are the simplest, regenerative stochastic processes. Let \( \{ X_n, n=1,2,\ldots \} \) be a sequence of non-negative independent identically distributed random variables with common distribution function \( F(.) \) and assume that \( \Pr \{ X_n=0 \} < 1 \). Since \( X_n \) is non-negative, \( E(X_n) \) exists.

Let \( S_0 = 0, S_n = X_1 + X_2 + \ldots + X_n \) for \( n \geq 1 \), and let \( F_n(x) = \Pr \{ S_n \leq x \} \) be the distribution function of \( S_n \).

Since \( X_i \)'s are i.i.d., \( F_n(x) = F^n(x) \).

Define the random variable

\[
N(t) = \sup \{ n \mid S_n \leq t \}
\]

Then the process \( \{ N(t), t \geq 0 \} \) is called a renewal process.

If for some \( n \), \( S_n = t \), then the \( n^{th} \) renewal is said to occur at time \( t \); \( S_n \) gives the time of the \( n^{th} \) renewal and is called the \( n^{th} \) renewal epoch. The random variable \( N(t) \) gives the number of renewals in the interval \( (0,t] \).

The function \( M(t) = E[N(t)] \) is called the renewal function of the process. It is easy to see that

\[
N(t) \geq n \iff S_n \leq t
\]
Thus the distribution of $N(t)$ is given by

$$\Pr\{N(t) = n\} = F^n(t) - F^{n+1}(t)$$

where $F^n(t)$ denotes the $n$-fold convolution of $F(t)$ with itself ($F^0(t) \equiv 1$)

and the expected number of renewals is given by

$$M(t) = \sum_{n=1}^{\infty} F^n(t)$$

Its derivative

$$m(t) = M'(t) = \sum_{n=1}^{\infty} f^n(t)$$

is the renewal density function, assuming the density function $f(t)$ exists. $m(t)$ is the expected number of renewals per unit time. Let us give another interpretation of renewal density, which is very important in practical applications, in the following way:

$$m(t)dt = M(t+dt) - M(t)$$

$$= \sum_{n=1}^{\infty} [F^n(t+dt) - F^n(t)]$$

$$= \sum_{n=1}^{\infty} \Pr\{t < S_n \leq t+dt\}$$
We have $\Pr\{\text{more than one renewal point in } (t, t+dt)\} \rightarrow o(dt)$ as $dt \rightarrow 0$. Therefore

$$\lim_{dt \rightarrow 0} m(t)dt = \Pr\{S_1 \text{ or } S_2 \text{ or } S_3 \text{ or } \ldots \text{ lies in } (t, t+dt)\}$$

i.e., $m(t)$ is the probability of a renewal in $(t, t+dt)$.

Now, suppose that the first interoccurrence time $X_1$ has a distribution $G(\cdot)$ which is different from the common distribution $F(\cdot)$ of the remaining interoccurrence times $X_2, X_3, \ldots$.

As before define

$$S_0 = 0, \quad S_n = \sum_{i=1}^{n} X_i$$

and

$$N_D(t) = \sup \{ n \mid S_n \leq t \}$$

The stochastic process $\{N_D(t), t \geq 0\}$ is called a Delayed or Modified renewal process.

Here we have

$$\Pr\{N_D(t) = n\} = G * F^{*(n-1)}(t) - G * F^n(t)$$
so that the modified renewal function is

\[ M_D(t) = E[N_D(t)] = \sum_{n=0}^{\infty} G^*F^n(t) \]

The modified renewal density function is given by

\[ m_D(t) = M_D'(t) = \sum_{n=0}^{\infty} g*f^n(t) \]

provided that the density functions \( g(x) = G'(x) \) and \( f(x) = F'(x) \) exist.

Now, consider a stochastic process \( \{ X(t), t \geq 0 \} \) with state space \( \{ 0, 1, 2, \ldots \} \), having the property that there exist time points at which the process (probabilistically) restarts itself. That is, suppose that with probability one, there exists a time \( T_1 \), such that the continuation of the process beyond \( T_1 \) is a probabilistic replica of the whole process starting at 0. Note that this property implies the existence of further times \( T_2, T_3, \ldots \), having the same property as \( T_1 \). Such a stochastic processes is known as a regenerative process.

From the above it follows that \( \{ T_1, T_2, \ldots \} \) forms a renewal process; and we shall say that a cycle is completed every time a renewal occurs. For details on renewal theory, one may refer to Cox (1962), Feller (1965), Ross (1970) or Cinlar (1975b) among others.
1.4 MARKOV RENEWAL PROCESSES

A Markov renewal process $\{(X_n, T_n); n \geq 0\}$ has two constituents; $\{X_n : n \geq 0\}$ is a homogeneous Markov chain whilst $(T_{n+1} - T_n)$ is the sojourn time in $X_n$ (throughout, $T_0 = 0$). Hence we can think of $X_n$ as the state entered at $T_n$ and left at $T_{n+1}$. Given $\{X_n : n \geq 0\}$, the $\{T_{n+1} - T_n : n \geq 0\}$ are independent and the distribution of $(T_{n+1} - T_n)$ depends on $\{X_n : n \geq 0\}$ through $X_n$ and $X_{n+1}$ only. We assume that the sojourn times are always strictly positive. When the initial state is $i$, that is $X_0 = i$, the time of returns to state $i$ form an ordinary renewal process; whilst the visits to $j \neq i$ form a delayed renewal process (the delay being the time that elapses until the first visit to $j$). Thus as Cinlar (1969) puts it 'the theory of Markov renewal processes generalizes those of renewal processes and Markov chains and is a blend of the two'.

The semi-Markov matrix $Q$ has its $(i,j)^{th}$ entry

$$Q(i,j,t) = \Pr \{X_{n+1} = j, T_{n+1} - T_n \leq t \mid X_n = i\}$$

so that $\sum_j Q(i,j,t)$ is the distribution function of the sojourn time in $i$ and $P(i,j) = Q(i,j,\infty)$ is the transition matrix of the Markov chain $\{X_n : n \geq 0\}$. The Markov
renewal function is \( R(i,j,t) = \sum_{n=0}^{\infty} Q^n(i,j,t) \), where

\[
Q^{n+1}(i,j,t) = \sum_{k=0}^{t} Q(i,k,du) Q^n(k,j,t-u) \quad \text{for } n \geq 0
\]

and \( Q^0(i,j,t) = I(i,j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \)

There are processes which are, in general, non-Markovian and yet possess the strong Markov property at certain selected random times. Then, imbedded at such instants, one finds a Markov renewal process.

Let \( Y = \{Y(t), t \geq 0\} \) be a stochastic process defined on a probability space \((\Omega, \mathcal{F}, P)\) with a topological state space \(E\), and suppose that the function \( t \mapsto Y(t, \omega) \) is right-continuous and has left-hand limits for almost all \( \omega \in \Omega \).

A random variable \( T: \Omega \rightarrow [0, \infty] \) is called a stopping time for \( Y \) provided that for any \( t < \infty \), the occurrence or non-occurrence of the event \( \{T \leq t\} \) can be determined once the history \( \{Y(u); u \leq t\} \) before \( t \) is known.

The process \( \{Y(t), t \geq 0\} \) is said to be semi-regenerative if there exists a Markov renewal process \( (X,T) = \{(X_n,T_n), n \geq 0\} \) with finite state space such that

(a) for each \( n \geq 0 \), \( T_n \) is a stopping time for \( Y \);

(b) for each \( n \geq 0 \), \( X_n \) is determined by \( \{Y(u); u \leq T_n\} \).
(c) for each $n \geq 0$, $m \geq 1$, $0 \leq t_1 < t_2 < \ldots < t_m$, and function $f$ defined on $E^m$ and positive,

$$E_i[f(Y(T_n+t_1), \ldots, Y(T_n+t_m))/ Y(u); u \leq T_n] = E_j[f(Y(t_1), \ldots, Y(t_m)] \text{ on } \{X_n = j\}$$

where $E_i$ and $E_j$ refer to expectations given the initial state for the Markov chain $X$.

The theory of Markov renewal processes provides a useful framework for the analysis of many complex stochastic systems. For a summary of basic results and applications of Markov renewal theory one may refer to two excellent survey papers by Cinlar (1969, 1975a).

1.5 AN OVERVIEW OF THE MAIN CONTRIBUTIONS OF THIS THESIS

The main concern of this thesis is the study of some complex stochastic models in Inventories and Queues. By studying the underlying stochastic processes of the models considered, transient state probabilities of the systems are obtained. Steady state results are attempted wherever possible. The associated optimization problems are also discussed for some models.
Renewal theory and Markov renewal theory provide elegant and powerful tools for analysing the underlying stochastic processes of the models considered in this thesis. By identifying the process as a regenerative or semi-regenerative one the transient as well as the steady state solutions are obtained.

Chapter 2 deals with a continuous review \((s,S)\) inventory system with independent non-identically distributed interarrival demand times and random lead times. Explicit expressions are obtained for the distribution of on-hand inventory. An optimization problem associated with this model and also the one associated with the model with zero lead time are discussed. Some numerical examples are considered and the optimal decision variables are obtained.

In chapter 3 we consider two models of \((s,S)\) inventory policy in which the quantity demanded by an arriving customer depends on the availability such that it does not exceed what is available on hand. The interarrival times between demands constitute a family of i.i.d random variables. Model–I assumes zero lead time. Using renewal theoretic arguments, the system state probability distribution at arbitrary time and also the limiting probability distribution are obtained. Optimal decision rule is also indicated.
In Model-II we study the situation with random lead time and in this case the inventory level probability distribution at arbitrary time is derived by applying the techniques of semi-regenerative process. The computation of limiting distribution is also indicated.

Chapter 4 is devoted to a continuous review (s,S) perishable inventory system having exponential life time distribution for the commodities in stock. The demand epochs form a renewal process and the probability distribution of demand magnitude depends on the time elapsed since the previous demand. Lead time is assumed to be zero. For this model the transient and limiting distributions of inventory level are derived by applying the techniques of semi-regenerative process. Some particular cases are also discussed.

In chapter 5 an (s,S) inventory policy with varying ordering levels and random lead times is studied. The quantity ordered is to bring the level back to S and the ordering level is determined based on the number of demands during the previous lead time subject to a maximum level c. Time-dependent system size probabilities are obtained. The correlation between the number of demands during a lead time and the next inventory dry period is obtained. Some illustrations are also given.
The last four chapters are concerned with queueing models. A queueing process of the type $E^k/G^a,b/l$ with server vacation is considered in chapter 6. The system is assumed to be of finite capacity. On completing the service of a batch if the server finds less than 'a' units (customers) waiting, he goes on vacation of random duration having a general distribution. If on return from vacation the number of units waiting is again less than 'a', the server extends his vacation for a random length of time independent of and having the same distribution as the previous one. This goes on until on return from vacation there are at least 'a' units in the system (multiple vacation). The transient system state probability distribution at arbitrary time point is obtained by identifying the regeneration points and using matrix convolutions. Virtual waiting time distribution is also obtained.

Chapter 7 deals with a service system with single and batch services. Customers arriving according to a homogeneous Poisson process enter the service station via a waiting room. At each time when all the customers in the service station are served out, the server scans the waiting room and if he finds less than or equal to a fixed number 'c' of customers he takes them to the service station and serves them one at a time according to FCFS (First Come First Served) rule. If he finds more than 'c' customers the
server serves them all together. Single/batch service times have general distributions. Here we consider three models. In the first model the server starts serving as soon as an arrival to an empty system takes place. In the second model when the system becomes empty the server goes on vacation of a random duration. Multiple vacation policy is assumed here. Using Markov renewal theoretic arguments the steady state and transient solution of the system state probabilities and virtual waiting time distributions for the two models are obtained. In Model-III a variant of the standard M/G/1 queue with single and batch services is considered. Here we assume that customers arrive at the service station according to a Poisson process with parameter $\mu$. At the end of each service, if the server finds more than $c$ customers waiting he serves them all together in a batch and if there are less than or equal to $c$ customers, he serves them one at a time according to FCFS rule. Limiting probabilities of the number of customers in the system is obtained explicitly by applying the techniques of semi-regenerative process.

In chapter 8 a single server queueing system with a finite waiting room is considered. The interarrival times of customers and service times have phase type distributions. An arriving customer finding the system full is lost.
Algorithmically tractable matrix formulas are obtained for the computation of stationary queue length distribution.

The last chapter deals with a finite capacity $M/G/1$ queueing system with server vacation schedules dependent on the number of customers it has served since the completion of the last vacation. Using Markov renewal theory the transient system state probabilities are derived. The virtual waiting time distribution of a customer in the queue is also obtained.