CHAPTER-III

MATHEMATICAL MODEL

GOVERNING MAGNETIC FIELD

EFFECT ON BIO MAGNETIC FLUID

FLOW AND ORIENTATION OF RED

BLOOD CELLS

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MATHEMATICAL MODEL GOVERNING MAGNETIC FIELD EFFECT ON BIO MAGNETIC FLUID FLOW AND ORIENTATION OF RED BLOOD CELLS

3.1. INTRODUCTION

A bio-magnetic fluid is a fluid that exists in a living creature and its flow is influenced by the presence of a magnetic field. The most characteristic bio-magnetic fluid is the blood, which can be considered as a magnetic fluid because the red blood cells contain the hemoglobin molecule, a form of iron oxides, which is present at a uniquely high concentration in the mature red blood cells. It is found that the erythrocytes orient with their disk plane parallel to the magnetic field [26] and also that the blood possesses the property of diamagnetic material when oxygenated and paramagnetic when deoxygenated [47]. In order to examine the flow of a bio-magnetic fluid under the action of an applied magnetic field, Haik et.al. [21] developed a mathematical model for the Bio-magnetic Fluid Dynamics (BFD) in which the saturation or static magnetization is given by the Langevin magnetization equation. BFD differs from Magneto Hydro Dynamics (MHD) in that it deals with no electric current and the flow is affected by the magnetization of the fluid in the magnetic field. In MHD, which deals with conducting fluids, the mathematical model ignores the effect of polarization and magnetization.

During the last decades an extensive research work has been done on the fluid dynamics of biological fluids in the presence of magnetic field due to bioengineering and medical applications [22, 57, 49] The effect of magnetic field on fluids is worth investigating due to its innumerable applications in wide spectrum of fields. The study of interaction of the magnetic field or the electromagnetic field with fluids have been documented e.g. among
nuclear fusion, chemical engineering, medicine, high speed noiseless printing and transformer cooling.

One of the most exciting areas of technology to emerge in recent year is MEMS (micromechanical Systems), where engineers design and build systems with physical dimensions in micrometers, e.g. MEMS-based biosensors or macro scale heat exchangers. The transport of momentum and energy in miniaturized devices is diffusion limited because of their very low Reynolds numbers. Using ferro-fluids in these applications and manipulating the flow of ferro-fluids by external magnetic field can be a viable alternative to enhance convection in these devices.

Ferro-fluids are non-conducting fluids and the study of the effect of magnetization has yielded interesting information. In equilibrium situation the magnetization property is generally determined by the fluid temperature, density and magnetic field intensity and various equations, describing the dependence of static magnetization on these quantities. The simplest relation is the linear equation of state. It can be assumed that the magneto-thermo-mechanical coupling is not only described by a function of temperature, but by an expression involving also the magnetic field strength [37]. This assumption permits us not to consider the ferro-fluid far away from the sheet a Curie temperature in order to have no further magnetization. This feature is essential for physical applications because the Curie temperature is very high (e.g. 1043 Kelvin degrees for iron) and such a temperature would be meaningless for applications concerning most of ferro-fluids. So instead of having zero magnetization far away from the sheet, due to the increase of fluid temperature up to the Curie temperature this formulation allows us to consider whatever temperature is desired and the magnetization will be zero due to the absence of the magnetic field sufficiently far away from the sheet [73].
Moreover, ferro-fluids are mostly organic solvent carriers having ferromagnetic oxides, acting as solute. Ferro-fluids consist of colloidal suspensions of single domain magnetic particles. They have promising potential for heat transfer applications, since a ferro-fluid flow can be controlled by using an external magnetic field [18]. However, the relationship between an imposed magnetic field, the resulting ferro-fluid flow and the temperature distribution is not understood well enough. The literature regarding heat transfer with magnetic fluids is relatively sparse.

An overview of prior research on heat transfer in ferro-fluid flows e.g. thermo magnetic forced convection and boiling, condensation and multiphase flow are presented [18]. Many researchers are seeking new technologies to improve the operation of existing oil-cooled electromagnetic equipment. One approach suggested in literature is to replace the oil in such devices with oil-based ferro-fluids, which can take advantage of the pre-existing leakage magnetic fields to enhance heat transfer processes. In [69] present results of an initial study of the enhancement of heat transfer in ferro-fluids in magnetic fields which are steady but variable in space. Finite element simulations of heat transfer to a ferro-fluid in the presence of a magnetic field are presented for flow between flat plates and in a box. The natural convection of a magnetic fluid in a partitioned rectangular cavity was considered [84]. It was found that the convection state may be largely affected by improving heat transfer characteristic at higher Rayleigh number when a strong magnetic field was imposed. The influence of a uniform outer magnetic field on natural convection in square cavity was presented. It was discovered that the angle between the direction of temperature gradient and the magnetic field influences the convection structure and the intensity of heat flux. Numerical results of combined natural and magnetic convective heat transfer through a ferro-fluid in a cube enclosure were presented [64]. The purpose of this work was to validate the theory of magneto convection. The magneto convection is induced by the presence of
magnetic field gradient. The Curie law states that magnetization is inversely proportional to temperature. That is why the cooler ferro-fluid flows in the direction of the magnetic field gradient and displaces hotter ferro-fluid. This effect is similar to natural convection were cooler, denser material flows towards the source of gravitational force.

The effect of magnetic field on the viscosity of ferro convection in an anisotropic porous medium was studied [51]. It was found that the presence of anisotropic porous medium destabilizes the system, whereas the effect of magnetic field dependent viscosity stabilizes the system. In this paper the investigated fluid was assumed to be incompressible having variable viscosity. Experimentally it has been demonstrated in prior research that the magneto viscosity has exponential variation, with respect to magnetic field. As a first approximation for small field variation, linear variation of magneto viscosity has been used [51]. The effect of magnetic field dependent (MFD) viscosity (magneto viscosity) on ferro convection in a rotation sparsely distributed porous medium has been studied [74]. The effect of MFD viscosity on thermo solutal convection in ferromagnetic fluid has been considered for a ferromagnetic fluid layer heated and solute from below in the presence of a uniform vertical magnetic field [66]. Using the linearized stability theory and the normal mode analysis method, an exact solution was obtained for the case of two free boundaries.

One of the problems associated with drug administration is the inability to target a specific area of the body. Among the proposed techniques for delivering drugs to specific locations within the human body, magnetic drug targeting [75] surpasses due to its non-invasive character and its high targeting efficiency. A general phenomenological theory was developed and a model case was studied, which incorporates all the physical parameters of the problem. A hypothetical magnetic drug targeting system, utilizing high gradient magnetic separation principles, was studied theoretically using FEMLAB simulations [53]. This new approach uses a ferromagnetic wire placed at a bifurcation point inside a blood vessel and an
externally applied magnetic field, to magnetically guide magnetic drug carrier particles through the circulatory system and then to magnetically retain them at a target site.

The mathematical model for the Bio-magnetic Fluid Dynamics is based on the modified Stocks principles and on the assumption that besides the three thermodynamic variables P, ρ and T the bio-magnetic fluid behavior is also a function of magnetization M [21]. Under these assumptions, the governing equations for incompressible fluid flow are similar to those derived for Ferro Hydro Dynamics (FHD) [56].

3.2. ORIENTATION OF ERYTHROCYTES IN A MAGNETIC FIELD

Magnetic fields have long been assessed for their beneficial and adverse influence on the body [36, 76] and applied to various aspects of medical treatment [5]. However, only a few attempts have been made to scientifically determine their effects or elucidate the mode of action. On the other hand, the frequency of exposure to strong magnetic fields has increased with the rapid advances in science and technology, such as magnetic-resonance image diagnosis (MRI) and passenger transport systems based on the principle of magnetic levitation [67]. Therefore, it has become necessary to more systematically elucidate the influence of magnetic fields on the body. A number of excellent reports have in recent years been presented concerning their influence [71].

When the influence of a magnetic field on the body is to be assessed, it is necessary to clarify whether the magnetic field is alternating or static. It must be clarified whether it is uniform or gradient in nature. It is also necessary to clarify the intensity of the magnetic field, duration of magnetic action, and reaction characteristic of the body to the magnetic field. This was somewhat obscure in many of the previous reports. The possibility cannot be ruled out that such obscurity has caused some confusion in the understanding of the effects of magnetic
fields on the body. In addition, it has posed the problem to setting stricter guidelines on the acceptable limits of exposure to magnetic fields [1, 19, 79].

When the literature was reviewed only for the orientation of high molecular body components in static magnetic fields, reports on the orientation of fibrinogen, [72, 83] retinal cells, [40] sickled cells, [45] etc were found. The orientation of fibrinogen and retinal cells is caused by the diamagnetic anisotropy retuned by the protein α-helix structure and lipid belayed in the biologic membranes. On the other hand, elongated stickled cells after deoxygenating are oriented with their longitudinal axes at right angles to such magnetic fields. This phenomenon is ascribable to Para magnetic anisotropy retained by the heme of hemoglobin that is polymerized in fiber by deoxygenating.

In the present work, the mathematical model, describing the bio-magnetic fluid flow, is presented and relations are given, expressing the dependence of the saturation magnetization $M_0$ on the temperature and the magnetic field intensity. A simplification of this mathematical model is used to obtain numerical solution of the differential equations describing the blood flow in a rectangular channel under the action of a magnetic field.

This chapter deals with the orientation of normal erythrocytes in a static magnetic field and heat transfer in bio-magnetic fluid is explained using various equations. It is hoped that these results will be useful in elucidating the influence of magnetic fields on the body and as basic data for setting guidelines on acceptable limits exposure to magnetic fields.

### 3.3. MATERIALS AND METHODS

#### 3.3.1. Materials

Regent – grade sodium citrate, sodium chloride, potassium chloride, glucose, sodium phosphate, sodium hydrosulfite, sodium nitrite, and gelatin were used.
3.3.2. Preparation of Erythrocytes

The fresh blood collected from healthy donors was mixed with a 1/10 volume of 3.1% sodium citrate. After 5 minutes of centrifugation at 3,000 rpm, the plasma and Buffy coat were removed. After washing with three portions of an isotonic phosphate–buffered saline (PBS) solution (90mmol/L NaCl, 5 mmol/L KCl, 5.6 mmol/L glucose, 50 mmol/L Na-phosphate, pH 7.4, saturated with air), oxygenated erythrocytes (containing diamagnetic oxyhemoglobin) were obtained. These were added with sodium hydrosulfite (25 mmol/L), kept anaerobic in nitrogen gas, and used as deoxygenated erythrocytes (containing paramagnetic deoxyhemoglobin). In addition, the washed oxygenated erythrocytes were allowed to react with sodium nitrite (20mmol/L) and washed five times. After adjusting the pH to 5.7 isotonic PBS solution, oxidized erythrocytes containing methemoglobin (high-spin state, paramagnetic) were obtained [82].

3.4. CHEMICAL STRUCTURE OF HEAMOGLOBIN:

In blood erythrocytes play vital role. In erythrocytes hemoglobin is the most important composition. The chemical structure and helical structure of Hemoglobin is given in Fig 3.1 & 3.2.
Fig. 3.1: Chemical structure of hemoglobin

Fig. 3.2: The helical structure of hemoglobin having $\alpha$ & $\beta$ chains
3.5. DETECTOR FOR ERYTHROCYTES ORIENTATION IN A STRONG MAGNETIC FIELD.

Using a superconducting magnet, a uniform static magnetic field (8 T in maximum) was allowed to occur in a space measuring 60 diameter)*80 mm. The cylindrical sample portion measuring 50 (diameter)* 60 mm contained a spectroscopic cell holder for the samples, a temperature- controlling water circulator, etc. It could be smoothly introduced into the magnetic field and removed from it along the guide way. The main parts of the optical analyzer and constant-temperature water bath were installed apart from the magnet. He-Ne laser rays for measuring the intensity of transmittance (T %) were introduced into and removed from the sample portion using an optical fiber. In addition, the same equipment was installed outside the magnet and used to obtain the control values. During the experiment, the temperature within the cells was monitored using a temperature sensor and maintained at 24.0 ± 0.5 c in both sample and control cells (Fig 3.3).
3.6. THE GOVERNING EQUATIONS

3.6.1. Magneto Static and Quasi-Static Fields

Under certain circumstances, it can be helpful to formulate the problems of electromagnetic analysis in terms of the electric scalar potential \( V \) and the magnetic vector potential \( A \). They are given by the equalities [31]

\[
B = \nabla \times A, \quad E = -\nabla V - \frac{\partial A}{\partial t} \tag{3.6.1}
\]

The defining equation for the magnetic vector potential is a direct consequence of the magnetic Gauss’ law. The electric potential results from Faraday’s law. Using the definitions of the potentials and the constitutive relation \( B = \mu_0(H + M) \), Ampere’s law can be rewritten as

\[
\sigma \frac{\partial A}{\partial t} + \nabla \times \left( \mu_0^{-1} \nabla \times A - M \right) - \sigma v \times (\nabla \times A) + \sigma \nabla V = J^e. \tag{3.6.2}
\]

The equation of continuity, which is obtained by taking the divergence of the above equation, gives us the equation

\[
-\nabla \left( \sigma \frac{\partial A}{\partial t} - \sigma v \times (\nabla \times A) + \sigma \nabla V - J^e \right) = 0 \tag{3.6.3}
\]

These two equations give us a system of equations for the two potentials \( A \) and \( V \). In the static case we have the equations

\[
-\nabla \left( -\sigma v \times (\nabla \times A) + \sigma \nabla V - J^e \right) = 0, \tag{3.6.4}
\]
\[ \nabla \times \left( \mu_0^{-1} \nabla \times A - M \right) - \sigma \nabla \times (\nabla \times A) + \sigma \nabla V = J^e. \quad (3.6.5) \]

The term \( \sigma \nabla \times (\nabla \times A) \) represents the current generated motion with a constant velocity in a static magnetic field, \( J^B = \sigma \nabla \times B^e \). Similarly the term \( \sigma \nabla V \) represents a current generated by a static electric field, \( J^E = \sigma E^e \).

If \( v = 0 \) the equations decouple and can be solved independently. The formulation is the single equation

\[ \nabla \times \left( \mu_0^{-1} \nabla \times A - M \right) = \overline{J}^e. \quad (3.6.6) \]

The conductivity cannot be zero anywhere when the electric potential is part of the problem, as the dependent variables would then vanish from the first equation.

Simplifying to a two – dimensional problem with perpendicular currents that are 0 it should be noted that this implies that the magnetic vector potential has a nonzero component only perpendicularly to the plane

\[ A = (0,0,A_z). \quad (3.6.7) \]

Ampere’s law can be rewritten as

\[ \nabla \times \left( \mu_0^{-1} \nabla \times A_z - M \right) = 0. \quad (3.6.8) \]

Along a system boundary reasonably far away from the magnet we can apply a magnetic insulation boundary condition \( A_z = 0 \). solving equation (3.6.8) together with the constitutive relation we can get
\[ B = \left( \frac{\partial A_z}{\partial y}, -\frac{\partial A_z}{\partial x}, 0 \right), \quad H = \frac{1}{\mu_0} B - M. \]  \hspace{1cm} (3.6.9) 

### 3.6.2. The Magnetic Field Intensity

In this chapter the considered flow is influenced by magnetic dipole. It is assumed that the magnetic dipole is located at distance \(|b|\) below the sheet at point \((a, b)\). The magnetic dipole gives rise to a magnetic field, sufficiently strong to saturate the fluid. In the magneto static case where there are no currents present, Maxwell-Ampère’s law reduces to \( \nabla \times H = 0 \). When this holds, it is also possible to define a magnetic scalar potential by the relation \( H = -\nabla V_m \) and its scalar potential for the magnetic dipole is given by

\[ V_m(X) = V_m(x_1, x_2) = \frac{\gamma}{2\pi} \frac{x_1 - a}{(x_1 - a)^2 + (x_2 - b)^2}. \]  \hspace{1cm} (3.6.10) 

Where the \( \gamma \) is the magnetic field strength at the source (of the wire) and \((a, b)\) is the position were the source is located.

### 3.6.3. Heat Transfer and Fluid Flow

The governing equations of the fluid flow under the action of the applied magnetic field and gravity field are: the mass conservation equation, the fluid momentum equation and the energy equation for temperature in the frame of Boussinesque approximation.

The mass conservation equation for an incompressible fluid is

\[ \nabla V = 0. \]  \hspace{1cm} (3.6.11) 

The momentum equation for magneto convective flow is modified from typical natural convection equation by addition of a magnetic term
\[ \rho_0 \left( \frac{\partial V}{\partial t} + V \cdot \nabla V \right) = -\nabla p + \nabla S + \alpha \rho_0 \varepsilon (T - T_0) k + (M \cdot \nabla) B \] (3.6.12)

Where \( \rho_0 \) is the density, \( V \) is the velocity vector, \( p \) is the pressure, \( T \) is the temperature of the fluid, \( S \) is the extra stress tensor, \( k \) is unit vector of gravity force and \( \alpha \) is the thermal expansion coefficient of the fluid.

The energy equation for an incompressible fluid which obeys the modified Fourier’s law is

\[ \rho_0 c \left( \frac{\partial T}{\partial t} + V \cdot \nabla T \right) = k \nabla^2 T + \eta \phi - \mu_0 T \frac{\partial M}{\partial T} ((V \cdot \nabla) H) \] (3.6.13)

Where \( k \) is the thermal conductivity, \( \eta \) is the viscosity and \( \eta \phi \) is the viscous dissipation

\[ \phi = \left\{ 2 \left( \frac{\partial v_1}{\partial x} \right)^2 + \left( \frac{\partial v_2}{\partial y} \right)^2 \right\} + \left( \frac{\partial v_1}{\partial y} \right)^2. \] (3.6.14)

The last term in the energy equation represents the thermal power per unit volume due to the magneto caloric effects.

**3.6.4. The Kelvin Body Force for Magneto Convective Flow**

The last term in the momentum equation represents the Kelvin body force per unit volume

\[ f = (M \cdot \nabla) B, \] (3.6.15)
This is the force that a magnetic fluid experiences in a spatially non-uniform magnetic field. We have established the relationship between the magnetization vector and magnetic field vector

$$M = x_m H$$  \hspace{1cm} (3.6.16)

Using the constitutive relation (relation between magnetic flux density and magnetic field vector) we can write the magnetic induction vector in the form

$$B = \mu_0 (1 + x_m) H$$  \hspace{1cm} (3.6.17)

The variation of the total magnetic susceptibility $x_m$ is treated solely as being dependent on temperature

$$x_m = x_m(T) = \frac{x_0}{1 + \alpha(T - T_0)}$$  \hspace{1cm} (3.6.18)

Finally, the Kelvin body force can be represented by

$$f = \frac{1}{2} \mu_0 x_m (1 + x_m) \nabla (H \cdot H) + \mu_0 x_m H (H \cdot \nabla) x_m$$  \hspace{1cm} (3.6.19)

Using equation (3.6.19) we can write Eq.(3.6.12) and (3.6.13) in the form, respectively

$$\rho_0 \left( \frac{\partial v}{\partial t} + V \cdot V \right) = -v \nabla p + \nabla S + \alpha \rho_0 g (T - T_0) \kappa + \frac{1}{2}$$

$$\frac{1}{2} \mu_0 x_m (1 + x_m) \nabla (H \cdot H) + \mu_0 x_m H (H \cdot \nabla) x_m$$  \hspace{1cm} (3.6.20)

And
\[
\rho_o C \left( \frac{\partial T}{\partial t} + V \cdot \nabla T \right) = k \nabla^2 T + \eta \phi - \mu_0 T \frac{\partial \left( x_m H \right)}{\partial T} \cdot \left( V \cdot V \right) H. \tag{3.6.21}
\]

### 3.6.5. The Brinkman Equations for Porous Media

Fluid and flow problems in porous media have attracted the attention of industrialists, engineers and scientists from varying disciplines, such as chemical, environmental, and mechanical engineering, geothermal physics and food science. There has been increasing interest in heat and fluid flows through porous media.

The Brinkman equations describe flow in porous media where momentum transport by shear stresses in the fluid is of importance. The model extends Darcy’s law to include a term that accounts for the viscous transport, in the momentum balance, and introduces velocities in the spatial directions as dependent variables in combination with the continuity equation

\[
\rho_o \frac{\partial V_B}{\partial t} = -\nabla p_B + \nabla \cdot S_B + \frac{\eta}{k_p} V_B + F \tag{3.6.22}
\]

Where \( \eta \) is the viscosity, \( k_p \) is the permeability of the porous structure (unit: \( m^2 \)).

The Brinkman equations applications are of great use when modeling combinations of porous media and free flow. The coupling of free media flow with porous media flow is common in the field of chemical engineering. This type of problems arises in filtration and separation and in chemical reaction engineering, for example in the modeling of porous catalysts in monolithic reactors.

Flow in the free channel is described by the Navier-Stokes equations and the mass conservation equation described in previous sections. In the porous domain, flow is described by the Brinkman equations according
\[
\rho_0 \frac{\partial V_B}{\partial t} = -\nabla p_B + \nabla . S_B + \frac{\eta}{k_p} V_B \quad (3.6.23)
\]

And

\[
\nabla . V_B = 0 \quad (3.6.24)
\]

3.7. THE DIMENSIONLESS EQUATIONS

For simplicity the preferred work choice is to work in non-dimensional frame of reference. Now some dimensionless variables will be introduced in order to make the system much easier to study. Moreover some of the dimensionless ratios can be replaced with well-known parameters: the Prandtl number \( \text{Pr} \), the Rayleigh number \( \text{Ra} \), the Eckert number \( \text{Ec} \), the Reynolds number \( \text{Re} \), the Darcy number \( \text{Da} \) and the magnetic number \( \text{Mn} \), respectively:

\[
\text{Pr} = \frac{\eta_0}{\rho_0 K}, \quad \text{Ra} = \frac{\alpha \rho_0 g h^3 \delta T}{\eta_0 K},
\]

\[
\text{Ec} = \frac{V_r^2}{c \delta T} = \frac{K^2}{c \delta T h^2}, \quad \text{Re} = \frac{h \rho_0 V_r}{\eta_0} = \frac{\rho_0 K}{\eta_0},
\]

\[
\text{Da} = \frac{h^2}{k_p}, \quad \text{Mn} = \frac{\mu_0 H_r^2}{\rho_0 V_r} = \frac{\mu_0 H_r^2 h^2}{\rho_0 K^2}. \quad (3.7.1)
\]

Since now primes will not be written (old variables symbols will be used) but it is important to remember that they are still there. The dimensionless form of Navier-Stockes (3.6.20) and thermal diffusion (3.6.21) equations are as follows:

\[
\frac{\partial V}{\partial t} + V . \nabla V = -\nabla p + \text{RaPr} \left( T - \frac{T_0}{\delta T} \right) k + \text{Pr} \nabla . S + Mn f \quad (3.7.2)
\]

and
\[
\frac{\partial T}{\partial t} + V \nabla T = \nabla^2 T + \text{Pr} \text{Ec} \eta \phi + M_n \text{Ec} T \frac{\partial (x_m H)}{\partial T} ((V \nabla) H)
\]  
(3.7.3)

Where

\[
f = \frac{1}{2} x_m (1 + x_m) \nabla H^2 + x_m H ((H \nabla) x_m)
\]  
(3.7.4)

And

\[
x_m = x_m (T(X)) = \frac{x_0}{1 + (\alpha \delta T) \left( T(X) - T_0 \right) \delta T}
\]  
(3.7.5)

Dimensionless Brinkman equations are as follows

\[
\frac{\partial V_B}{\partial t} = -\nabla p_B + \text{Pr} \nabla S_B + \text{Pr} Da V_B.
\]  
(3.7.6)

In the presence of magnetic field Kelvin body force is added

\[
\frac{\partial V_B}{\partial t} = -\nabla p_B + \text{Pr} \nabla S_B + \text{Pr} Da V_B + M_n f.
\]  
(3.7.7)

3.8. MAGNETIZATION EQUATIONS

3.8.1. Saturation Magnetization Equation
In equilibrium situation the magnetization property is generally determined by the fluid temperature, density and various equations, describing the dependence of $M_0$. The simplest relation is the linear equation of state

$$M_0 = K(T_c - T)$$  \hspace{1cm} (3.8.1)

Where $K$ is a constant called pyromagnetic coefficient and $T_c$ is the Curie temperature. Above the Curie temperature the biofluid does not subjected to magnetization.

Another equation for magnetization, below the Curie temperature is given by

$$M_0 = M_1 \left( \frac{T_c - T}{T_1} \right) ^\beta$$  \hspace{1cm} (3.8.2)

Where $\beta$ is the critical exponent for the spontaneous or saturation magnetization. For Iron $B = 0.368$, $M_1 = 54$ Oe and $T_1 = 1.45$ K.

A linear equation involving the magnetic intensity $H$ and Temperature $T$ is given as

$$M_0 = KH(T_c - T)$$  \hspace{1cm} (3.8.3)

Finally Higashi et. all (26) found that the magnetization process of red blood cells behaves like the following function, known as Langevin function,

$$M_0 = mN \left[ \coth \left( \frac{\mu_0 mH}{kT} \right) - \frac{kT}{\mu_0 mH} \right]$$  \hspace{1cm} (3.8.4)

Where $m$ is the particle magnetization, $N$ is the number of particles per unit volume and $k$ is the Boltzmann’s constant.
In all the above cases the magnetization $M_0$, is dependent on the temperature $T$ of the fluid. In non-isothermal case, it is also consider in the mathematical model, describing the problem under consideration, the energy equation containing the temperature $T$ of the fluid.

This equation can be written as

$$\rho C_p \frac{DT}{Dt} + \mu_0 T \frac{\partial M_0}{\partial T} \nabla \cdot (\nabla H) = k \nabla^2 T + \eta \phi$$

(3.8.5)

Where $k$ is the coefficient of thermal conductivity of the fluid, $C_p$ the specific heat and $\phi$ the dissipation function.

### 3.9. HEAT TRANSFER IN FERRO-FLUID IN CHANNEL

Considered flow takes place in channel between two parallel flat plates. The length of the channel is $L$ and distance between plates is $h$.

The corresponding boundary conditions for dimensionless variables are assumed:

- For the upper wall ($0 \leq x \leq L, y=1$): the upper wall temperature is kept at constant temperature $T_u / \delta T$. The velocity is 0 (no slip condition).
- For the lower wall ($0 \leq x \leq L, y=0$): the lower wall temperature is kept at constant temperature $T_i / \delta T$. The velocity is 0 (no slip condition).
- For inlet (the left wall) ($x=0, 0 \leq y \leq 1$): the temperature is varying linearly from $T_i / \delta T$ to $T_u / \delta T$ and is given by equation
\[
T_m = \frac{T_u - T_i}{\partial T} \, y + \frac{T_i}{\partial T} \text{ where } \delta T = |T_u - T_i|. \text{ There is a parabolic laminar flow profile given by equation } u_{in} = -4 \frac{u_0}{u_r} y(y - 1) \text{ for } y \in (0, 1) \text{ at the inlet end.}
\]

- For outlet (the right wall) \((x = L, 0 \leq y \leq 1)\): the convective flux is assumed for temperature, \(n(-k\nabla T) = 0\). Pressure outlet is also assumed, \((-pI + S)n = -p_0n\), where \(p_0\) is the dimensionless atmospheric pressure.

The following initial conditions for dimensionless variables are assumed: the fluid is motionless, the pressure is zero and the temperature is varying linearly from lower to upper wall.

The time dependent flow is considered for dimensionless time \(t \in (0, 0.5)\), the problem is solved with COMSOL code using direct UMFPACK linear system solver. Relative and absolute tolerances used in calculations are 0.05 and 0.005, respectively. The following values of temperatures are assumed \(T_i = T_0, T_u = T_0 + \delta T\) where \(T_0 = 300K\) and \(\delta T = 30K\).

### 3.10. THE QUANTITIES FOR FERROFLUID FLOWS

It can be observed that the maximum value of the magnitude of the velocity field of the flow in the channel under the magnetic dipole increases due to the value of the magnetic number.

The flow was relatively uninfluenced by the magnetic field until its strength was large enough for the Kelvin body force to overcome the viscous force. It can be observed that the cooler ferro-fluid flows in the direction of the magnetic field gradient and displaced hotter
ferro-fluid. This effect is similar to natural convection where cooler, more dense material flows towards the source of gravitational force. Ferro-fluids have promising potential for the heat transfer applications because a ferro-fluid flow can be controlled by using an external magnetic field.

The corresponding boundary conditions for dimensionless variables in channel flow are assumed:

- For free-porous structure interface: $V_b = V$. These conditions imply that the components of the velocity vector are continuous over the interface between the free channel and the porous domain.
- For the upper domain walls; the temperature is kept at constant temperature $T_u / \delta T$. The velocity is 0 (no slip condition)
- For the lower domain walls: the temperature is kept at constant temperature $T_l / \delta T$. The velocity is 0 (no slip condition)

The following initial conditions for dimensionless variables are assumed: the fluid is motionless, the pressure is zero and the temperature is $T_1 / \delta T$.

The time dependent flow is considered for dimensionless time $t \in (0, 0.1)$, the problem is solved with COSMOL code using direct UMFPACK linear system solver. Relative and absolute tolerance used in calculations is 0.05 and 0.005, respectively.

The following values of temperatures are assumed $T_1 = T_0$, $T_u = T_0 + \delta T$ where $T_0 = 300K$ and $\delta T = 30K$

The heat transfer in bio-magnetic fluid flowing in channel with porous walls is considered in four different flows with different magnetic susceptibility, inlet velocity or
permeability of the porous structure. The most interesting example of flow we can observe in the last considered flow. In this case the magneto convection is observed. We observe vortex created near the centre of magnetic dipole. Each vortex is moving from left to right where the magnetic field intensity is getting smaller.

3.11. CONCLUSION

In this section successfully prepared model for governing magnetic effects on red blood cells and present numerical simulation results of heat transfer in bio-magnetic fluid. The flow takes places in channel and in channel with porous walls. The two-dimensional time dependent flows are assumed viscous, incompressible and laminar. Above the channel magnetic dipole is located. The fluid is assumed to be electrically nonconducting. It is assumed also that there is no electric field effect. This magneto – thermo-mechanical problems is governed by dimensionless equations.