6

Conclusion

In the concluding chapter, we summarize our results which we have presented in the thesis and indicate possible future work arising out of the approach presented here.

6.1 Summary

In this thesis, we studied two one dimensional generalised versions of the Kitaev’s model. Since they reduce to a non-interacting Hamiltonians, it became possible for us to give analytic solutions.

In the Tetrahedral model, we found out numerically that the translationally invariant fluxes through the unit cells, namely $u^L_i = -1$ and $u^R_i = -1$ is the ground state sector of the model. We gave a generic expression for fermionic gap. The closed form solution for the zero modes of the Majorana fermions has been derived. Unpairing and manipulation of zero Majorana modes tuning $Z_2$ flux configurations has been shown. The regions in parameter space for homogenous chains has been shown where the zero modes occur. We further showed that there is a large parameter space for inhomogenous chains where the unpaired modes occur. Another important result we proved in the model using transfer marix method that every state of the system has a $2^{N/4}$ fold degeneracy, where $N$ is the number of sites.

The XY-Ising model was studied under periodic boundary condition and we solved the model exactly for $J_x = J_y$ for all flux configurations and proved that for $J_x = J_y$ the ground state lies in translationally invariant flux sector $u_i = -1$. We
gave an expression for fermionic gap. The closed form solution for the zero modes of the Majorana fermions was derived and unpairing and manipulation of zero Majorana modes was shown. The topological nature of the zero mode has been found. Further, it was shown that qubit made up of degenerate Majorana modes of the model are protected from decoherence by environmental perturbations.

We also studied ground state properties of the XYZ-Ising model for all $J_x, J_y$ and $J_z$ numerically which agree with analytical results in extreme limits. The zero temperature phase diagram of the system was plotted by numerically calculating the ground state in various sectors. It was found that the model undergoes a first order phase transition for $J_z < 0$ and for $J_z > 0$ the phase transition is described by a topological order parameter called winding number.

Finally, physical realisation of XY-Ising model with its commuting operator has been achieved.

6.2 Outlook

There are some loose ends in our work which we would like to tie up in future.

We have studied zero temperature phase diagram of the XYZ-Ising model. In future, we would like to study finite temperature behaviour of the model.

In our scheme, the flux configuration sector we want to access, we make it a ground state sector. At low temperature the system will be in ground state, therefore, the system would access the required flux configuration sector. But in order to be at low temperature system would have to interact with the environment. Still it has to be shown that how the system will go from one flux configuration sector to another flux configuration sector interacting with environment.

Our aim, above all, was to design a logic gate which would perform quantum computation on a qubit. In future, we would like to embark upon this venture.